# Modeling and Design Optimization for Quadcopter Control System Using L1 Adaptive Control

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***Abstract*— Quadcopters have generated considerable interest in both the control community due to their complex dynamics and a lot of potentials in outdoor applications because of their advantages over regular aerial vehicles. This paper presents the design and new control method of a quadcopter using L1 adaptive control design process in which control parameters are systematically determined based on intuitively desired performance and robustness metrics set by the designer.**

***Keywords—quadcopter; UAV; design; modeling; automatic control system; L1 adaptive control***

## I. Introduction

Unmanned aerial vehicles (UAVs) have become increasingly prominent in a variety of aerospace applications. The need to operate these vehicles in potentially constrained environments and make them robust to actuator failures and plant variations has brought about a renewed interest in adaptive control techniques [6]. Model Reference Adaptive Control (MRAC) has been widely used, but can be particularly susceptible to time delays. A filtered version of MRAC, termed L1 adaptive control, was developed to address these issues and offer a more realistic adaptive solution [1].

The main advantage of L1 adaptive control over other adaptive control algorithms such as MRAC is that L1 cleanly separates performance and robustness [2]. The inclusion of a low-pass filter not only guarantees a bandwidth-limited control signal, but also allows for an arbitrarily high adaptation rate limited only by available computational resources. This parameterizes the adaptive control problem into two very realistic constraints: actuator bandwidth and available computation. In this paper we consider the output feedback version of L1 described in [3]. This single-input single-output (SISO) formulation has several advantages. Foremost, the internal system states need not be modeled or measured. All that is required is a SISO input-output model that can encompass the entire closed-loop system and be acquired using simple system identification techniques. Thus the adaptive controller can be wrapped around an already-stable closed-loop system [4], adding performance and robustness in the face of plant variations. It is also easy to predict the time-delay margin using standard linear systems analysis, and this margin has been confirmed experimentally. Finally, output-feedback L1 is relatively easy to implement in practice as will be seen in the experimental sections [5].

## II. Modeling of Quadcopter Dynamic

### A. Reference Systems of Quadcopter

A quadcopter is an under actuated aircraft with fixed pitch angle four rotors as shown in Figure 1. Modeling a vehicle such as a quadcopter is not an easy task because of its complex structure. The aim is to develop a model of the vehicle as realistically as possible.

A typical quadcopter have four rotors with fixed angles and they make quadcopter has four input forces, which are basically the thrust provided by each propellers as shown in Figure 1. There are two possible configurations for most of quadcopter designs “+” and “×”. An X-configuration quadcopter is considered to be more stable compared to + configuration, which is a more acrobatic configuration. Propellers 1 and 3 rotates counter clockwise (CW) , 2 and 4 rotates counter-counter clockwise (CCW). So that, the quadcopter can maintain forward (backward) motion by increasing (decreasing) speed of front (rear) rotors speed while decreasing (increasing) rear (front) rotor speed simultaneously, which means changing the pitch angle. This process is required to compensate the action/reaction effect (Third Newton’s Law). Propellers 1 and 3 have opposite pitch with respect to 2 and 4, so all thrusts have the same direction [7].

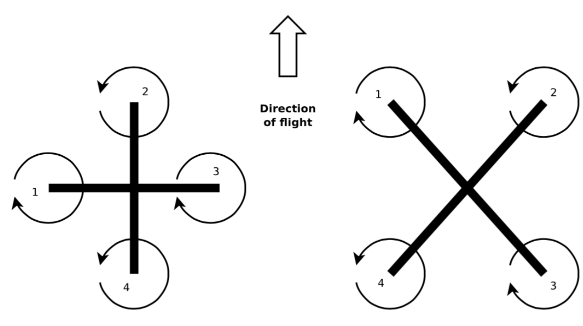


Fig. 1. Two main types of quadrocopter configuration.

There are two reference systems that have to be defined as a reference which are Inertial reference system (Earth frame-

XE, YE, ZE) and quadrotor reference system (Body frame-XB, YB, ZB). The reference system frames are shown in Figure 2. The dynamics of quadcopter can be describe in many different ways such as quaternion, Euler angle and direction matrix. However, in designing attitude stabilization control reference in axis angle is needed, so the designed controller can achieve a stable flight. In attitude stabilization control, all angle references in each axis must be approximately zero especially when take-off, landing or hover. It ensures that, the quadcopter body always is in horizontal state, when external forces are applied on it [8]. The quadcopter orientation can be defined by three Euler angles which are roll angle (Φ), pitch angle (θ) and yaw angle (φ).

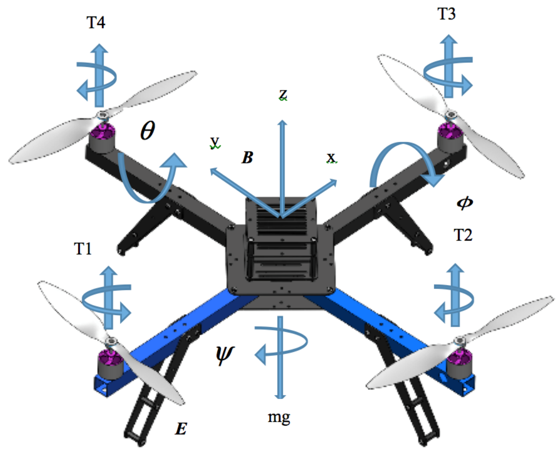


Fig. 2. Forces, moments and reference systems of a quadcopter.

Where,

$\omega_1, \omega_2, \omega_3, \omega_4$ : rotation speeds (angular velocity) of the propellers

$T_1, T_2, T_3, T_4$ : forces generated by the propellers

$F_i \propto \omega_i^2$ : on the basis of propeller shape, air density, etc.

*m*: mass of the quadcopter

*mg*: weight of the quadcopter

$\phi, \theta, \psi$ : roll, pitch and yaw angels

The position of the quadcopter is defined in the inertial frame *x,y,z*- axes with ξ. The attitude, i.e. the angular position, is defined in the inertial frame with three Euler angles η. Pitch angle $\theta$ determines the rotation of the quadcopter around the y-axis. Roll angle $\varphi$ determines the rotation around the x-axis and yaw angle $\psi$ around the z-axis. Vector $q$ contains the linear and angular position vectors

$$\xi = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad q = \begin{bmatrix} \xi \\ \eta \end{bmatrix}. \tag{1}$$

The origin of the body reference (body frame) is in the center of mass of the quadcopter. In the body frame, the linear velocities are determined by *JB* and the angular velocities by ***ω***.

$$JB = \begin{bmatrix} Jx,B \\ Jy,B \\ Jz,B \end{bmatrix}, \quad \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix}. \tag{2}$$

The rotation matrix from the body frame to the inertial frame is

$$R = \begin{pmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{pmatrix} \tag{3}$$

in which $S_x$ = *sin(x)* and $C_x$ = *cos(x)*. The rotation matrix *R* is orthogonal thus $R^{-1} = R^T$ which is the rotation matrix from the inertial frame to the body frame.

There are 3 types of angular speeds which can describe as the derivative of (*φ, θ, ψ*) with respect to time,

$\dot{\phi}$=Roll rate , $\dot{\theta}$=Pitch rate, $\dot{\psi}$= Yaw rate.

Considering the hovering condition of quadcopter gives 4 equations of forces, directions, moments and rotation speeds. Those are described by following,

Equilibrium of forces : $\sum_i^4 = 1\, T_i = -mg$

Equilibrium of directions: $T_{1,2,3,4} \parallel g$

Equilibrium of moments: $\sum_i^4 = 1\, M_i = 0$

Equilibrium of rotation speeds: $(\omega_1 + \omega_3) - (\omega_2 + \omega_4) = 0$,

And the consequence is: $\dot{\phi} = 0$, $\dot{\theta} = 0$, $\dot{\psi} = 0$.

By increasing/decreasing the rotation speed of all the propellers, the quadcopter can make movements flying up and down,

Flying up: $\sum_i^4 = 1\, T_i > -mg$,

Flying down: $\sum_i^4 = 1\, T_i < -mg$, Euler angles and rates remain 0.

Changing the equilibrium of propellers speed, directions and moments gives the following equations of yaw, roll and pitch of quadcopter.

$$\text{Yaw: } \dot{\psi} = k_Y((\omega_1 + \omega_3) - (\omega_2 + \omega_4)) \quad \phi = \int \dot{\psi}\, dt \tag{4}$$

$$\text{Roll: } \dot{\phi} = k_R((\omega_1 + \omega_4) - (\omega_2 + \omega_3)) \quad \phi = \int \dot{\phi}\, dt \tag{5}$$

$$\text{Pitch: } \dot{\theta} = k_P((\omega_1 + \omega_2) - (\omega_3 + \omega_4)) \quad \theta = \int \dot{\theta}\, dt \tag{6}$$

Thus, decreasing the 2nd rotor velocity and increasing the 4th rotor velocity acquires the roll movement. Similarly, decreasing the 1st rotor velocity and increasing the 3th rotor velocity acquire the pitch movement. Increasing the angular velocities of two opposite rotors and decreasing the velocities of the other two acquire yaw movement.

### B. Equation of Movement

Assume a common factor of proportionality $k$ and $F=\sqrt{T}$, each equation of movement for quadcopter has written down below:

$$\dot{\phi}=k((\omega_1+\omega_4)-(\omega_2+\omega_3))=k\omega_1-k\omega_2-k\omega_3+k\omega_4$$
$$\dot{\theta}=k((\omega_1+\omega_2)-(\omega_3+\omega_4))=k\omega_1+k\omega_2-k\omega_3-k\omega_4$$
$$\dot{\psi}=k((\omega_1+\omega_3)-(\omega_2+\omega_4))=k\omega_1-k\omega_2+k\omega_3-k\omega_4$$
$$F=k((\omega_1+\omega_2+\omega_3+\omega_4))=k\omega_1+k\omega_2+k\omega_3+k\omega_4$$

by using matrices:

$$\begin{pmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ F\end{pmatrix}=\begin{pmatrix}k & -k & -k & k\\ k & k & -k & -k\\ k & -k & k & -k\\ k & k & k & k\end{pmatrix}\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{pmatrix} \tag{7}$$

$$\begin{pmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ F\end{pmatrix}=\begin{pmatrix}k & -k & -k & k\\ k & k & -k & -k\\ k & -k & k & -k\\ k & k & k & k\end{pmatrix}\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{pmatrix}=\mathrm{K}\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{pmatrix} \tag{8}$$

According to equation (8), controlling the four input forces (roll, pitch, yaw, thrust) can be write down as below,

$$\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\\ \omega_4\end{pmatrix}=K^{-1}\begin{pmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ F\end{pmatrix}=\begin{pmatrix}k & -k & -k & k\\ k & k & -k & -k\\ k & -k & k & -k\\ k & k & k & k\end{pmatrix}\begin{pmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ F\end{pmatrix} \tag{9}$$

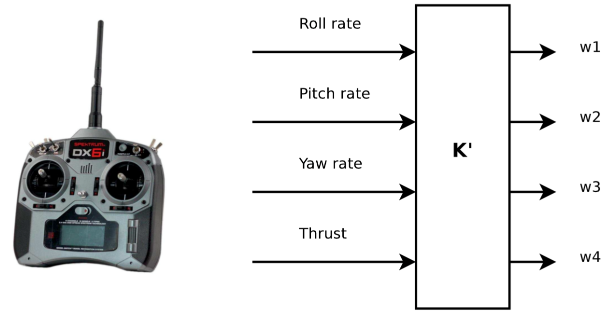


Fig. 3. Controlling the Roll, Pitch, Yaw and total thrust forces.

## III. L1 Adaptive Control Algorithm for Quadcopter Flight Control

Figure 4 shows the closed-loop system with L1 adaptive controller. The controller includes a reference model and a lowpass filter C(s). Adding the low-pass filter C(s) does two important things. First, it limits the bandwidth of the control signal u being sent to the plant. Second, the portion of σ that gets sent into the reference model is the high-frequency portion.

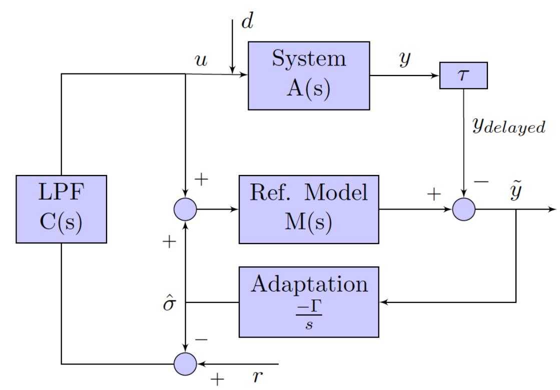


Fig. 4. L1 adaptive feedback control block diagram.

Closed-loop response

$$y(s) = \underbrace{H(s)C(s)r(s)}_{\text{Response to reference r(s)}} - \underbrace{H(s)(1-C(s))d(s)}_{\text{Response to disturbance d(s)}} \tag{10}$$

$$\text{where } H(s) = \frac{A(s)M(s)}{C(s)A(s)+(1-C(s))M(s)} \tag{11}$$

Adaptive function and controller:
$C := xxx\_rate\_controller(e);$
That is:

$$c(t) := K_p e(t) + K_i\int_0^t e(\tau)d(\tau) + K_d\frac{de(t)}{dt} \tag{12}$$

In a discrete world (at $k^{th}$ sampling instant):

$$C(k) := K_p e(k) + K_i\sum_{j=0}^{k} e(j)\Delta T + K_d\frac{e(k)-e(k-1)}{\Delta T} \tag{13}$$

On the other hand, the L1 adaptive control system can be algorithmically described as following,

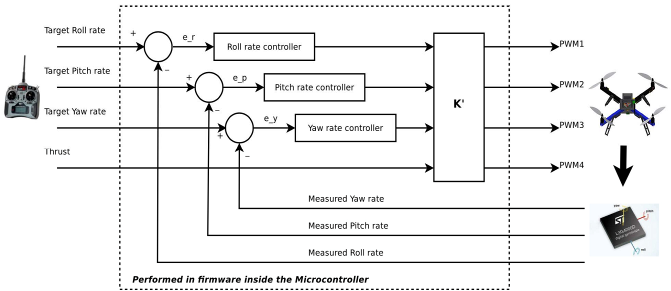


Fig. 5. Full block diagram of the L1 adaptive control system of quadcopter.

## IV. SIMULATION RESULTS

The mathematical model of the quadcopter is implemented for simulation in Matlab 2013 with Matlab programming language. Parameter values from [3] are used in the simulations and are presented in Table 1.

TABLE I. PARAMETERS OF THE SYSTEM IN SI UNITS

| Symbol | Quadcopter Parameters | | |
|---|---|---|---|
| | ***Description*** | ***Value*** | ***Unit*** |
| *g* | Weight of the quadcopter | 9.81 | $[m/s^2]$ |
| *m* | Mass of the quadcopter | 0.75 | $[kg]$ |
| *l* | Distance from center to motor | 0.26 | $[m]$ |
| *Jx* | Moment of inertia about *x* axis | 0.019688 | $[kgm^2]$ |
| *Jy* | Moment of inertia about *y* axis | 0.019688 | $[kgm^2]$ |
| *Jz* | Moment of inertia about *z* axis | 0.03938 | $[kgm^2]$ |
| Kt | Propeller Force Constant | $3.13 \times 10^{-5}$ | $[Ns^2]$ |
| Kq | Propeller Torque Constant | $7.5 \times 10^{-7}$ | $[Ns^2]$ |

Simulation results are shown in figure 6 and 7.

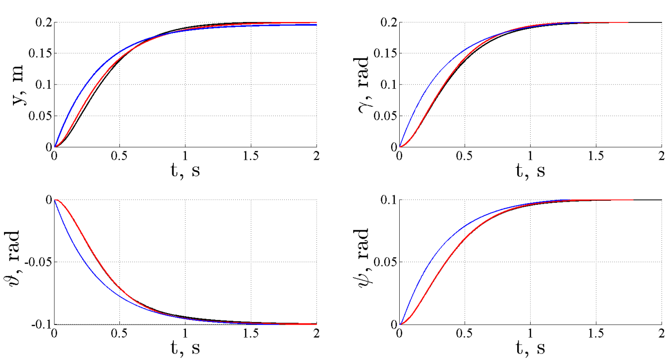


Fig. 6. Measurment changing coordinates results when using the L1 adaptive control algorithm.

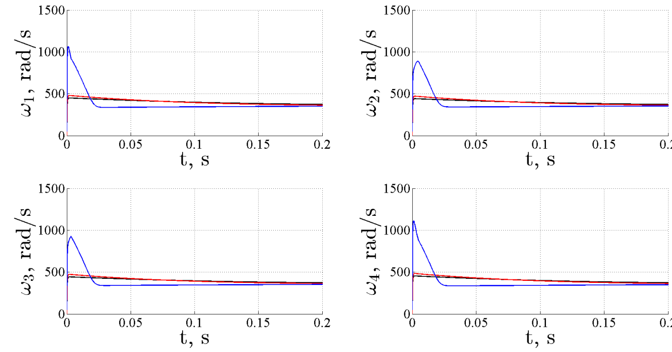


Fig. 7. Measurement changing the angular velocities when using the L1 adaptive control algorithm.

As can be seen from Figure 6, the dynamics of the quadcopter with the proposed signal-parametric algorithm change rapidly as translational speed increases from a hover configuration. From Figure 7 also shows that the signal-parametric algorithm has more accurate control ability, more spinning speed rotors that imposed the inability of the linear controller to accurately track forward velocities greater than 1.5 m/s. According to simulation results, the L1 adaptive controller shows improved performance for attitude and trajectory tracking of the quadcopter.

## V. CONCLUTIONS AND FUTURE WORK

This paper attempts to provide a systematic design and modeling process for the use of L1 adaptive feedback control in realistic flight control applications. The proposed algorithm provides the control designer with an intuitive method linking relevant performance and robustness metrics to the selection of the L1 parameters. This modeling process represents a step in the direction of more easily applying L1 adaptive control to real-world flight systems and taking advantage of its potential benefits.


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