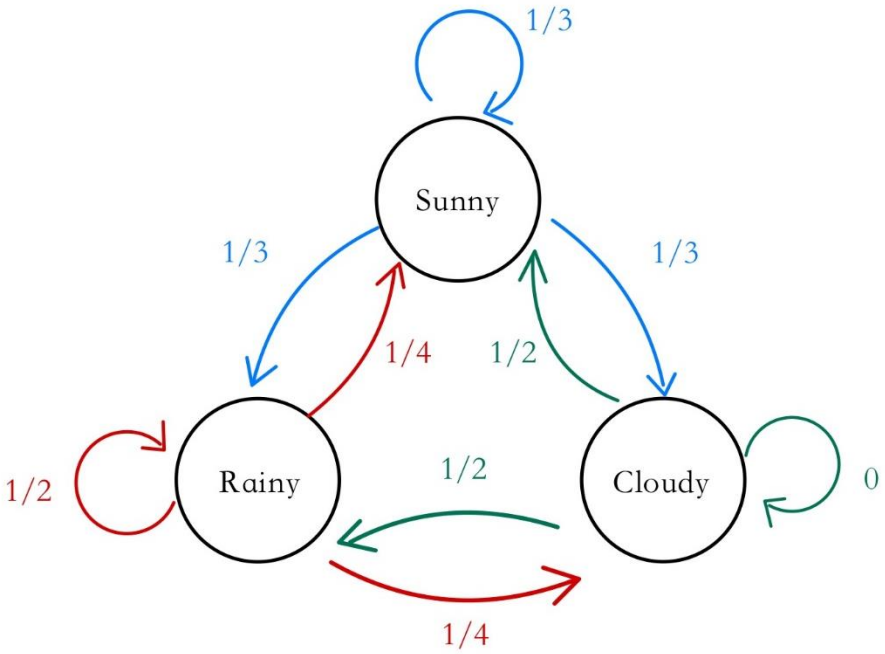
無線通訊網路作業一

系級：_____ 學號：_____ 姓名：_____



I. On average, how many consecutive sunny days?

1. 設 $p=\frac{2}{3}$ (sunny → rainy / cloudy)

$q=\frac{1}{3}$ (sunny → sunny)

$b^*(x;1,p)=pq^{x-1}$

$E(x)=\sum_{i=0}^{\infty} i\cdot pq^{i-1}=p+2pq+3pq^2+4pq^3+\cdots$ — ①

(×q) $qE(x)=\sum_{i=0}^{\infty} i\cdot pq^{i}=pq+2pq^2+3pq^3+4pq^4+\cdots$ — ②

$p+q=1$

① - ② : $pE(x)=p+pq+pq^2+pq^3+\cdots$

$E(x)=1+q+q^2+\cdots$ → 無窮等比級數

$=\frac{1}{1-q}=\frac{3}{2}$ (天)

II. If today is cloudy, how many days can be expected before next cloudy day arrives?

2、先求得穩定狀態：

$$[x, y, z] \begin{matrix} (S) & (C) & (R) \\ \end{matrix} \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} = [x, y, z]$$

$z = (1-x-y)$

$x = \frac{1}{3}$, $y = \frac{2}{9}$, $z = \frac{4}{9}$

sunny　cloudy　rainy

令今天為cloudy，n天後出現cloudy的機率為：

$$p(n) = (1-\frac{2}{9})^{n-1} \cdot \frac{2}{9} = \frac{2}{9}(\frac{7}{9})^{n-1}$$

$$E[p(n)] = \sum_{n=1}^{\infty} \frac{2}{9} n (\frac{7}{9})^n = \frac{2}{9} \cdot \frac{9}{7} \sum_{n=1}^{\infty} n(\frac{7}{9})^n$$

$$= \frac{2}{7} \cdot \frac{x}{(1-x)^2}\Big|_{x=\frac{7}{9}} = \frac{9}{2} \text{ (天)}$$

∵題意不清，故答案為4.5或3.5天皆為正確.