2023

Theory of Computation

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Outline

Deterministic Finite Accepters (DFA)

Nondeterministic Finite Accepters (NFA)

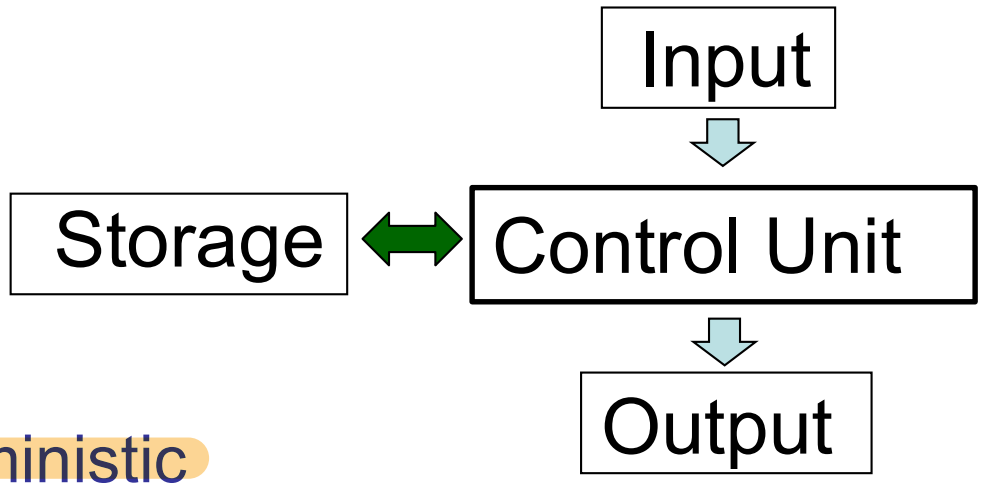
Equivalence of DFA and NFA

Reduction of the Number of States in FA*

Automata

Automaton:

An abstract model of a digital computer



- Deterministic V.S. Nondeterministic

  與明確只有一種走法

- An automaton whose output is YES or NO

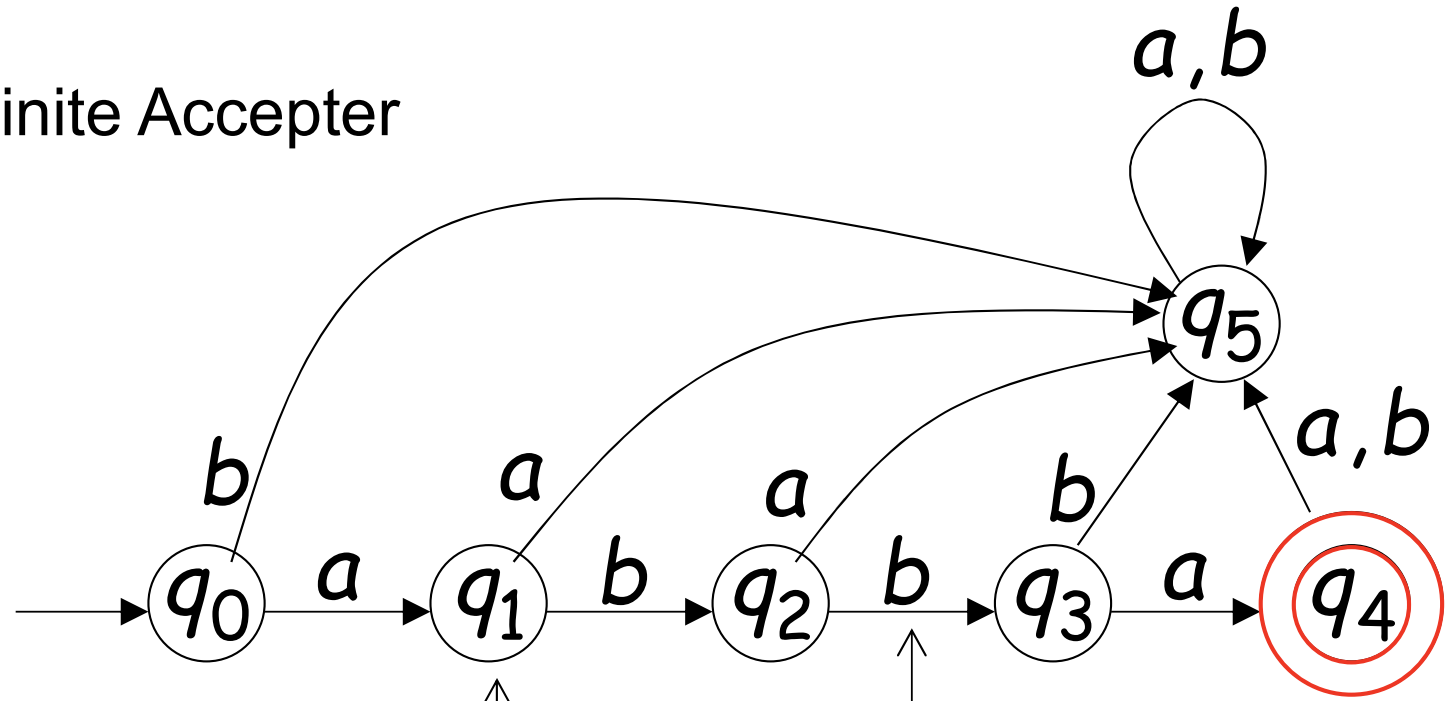
Accepter

- An automaton whose output are strings of symbols

Transducer

Transition Graph

Finite Accepter



initial state

state

transition

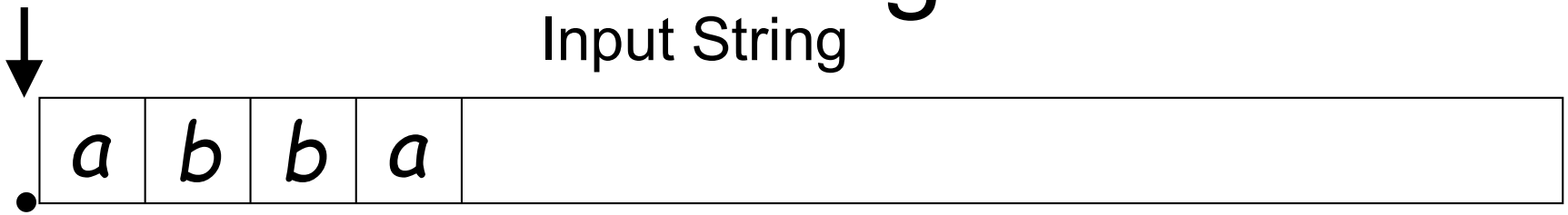
final state "accept"

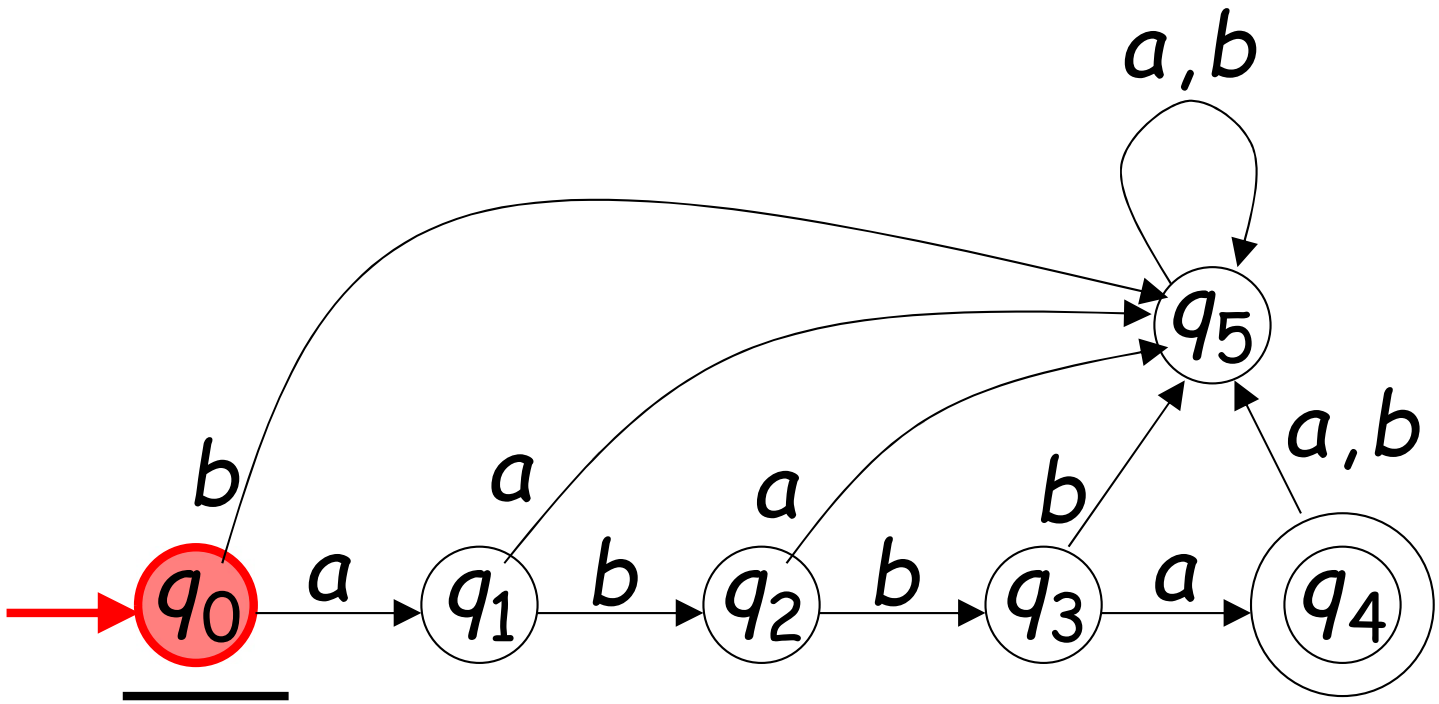
↳ 走得到 final ⇒ accept

走不到 final ⇒ reject

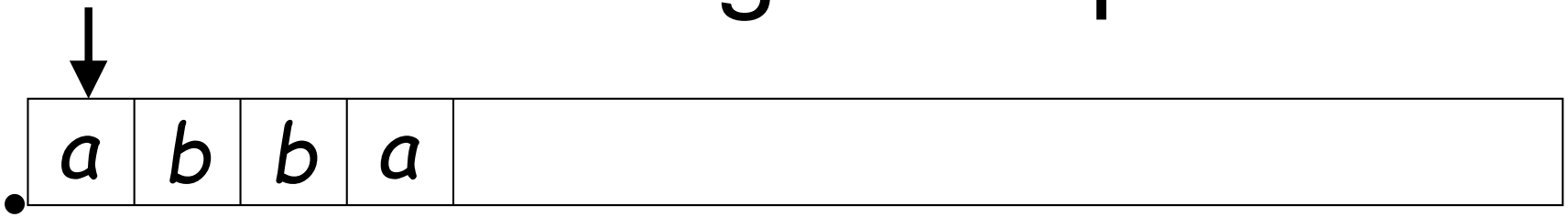
Initial Configuration

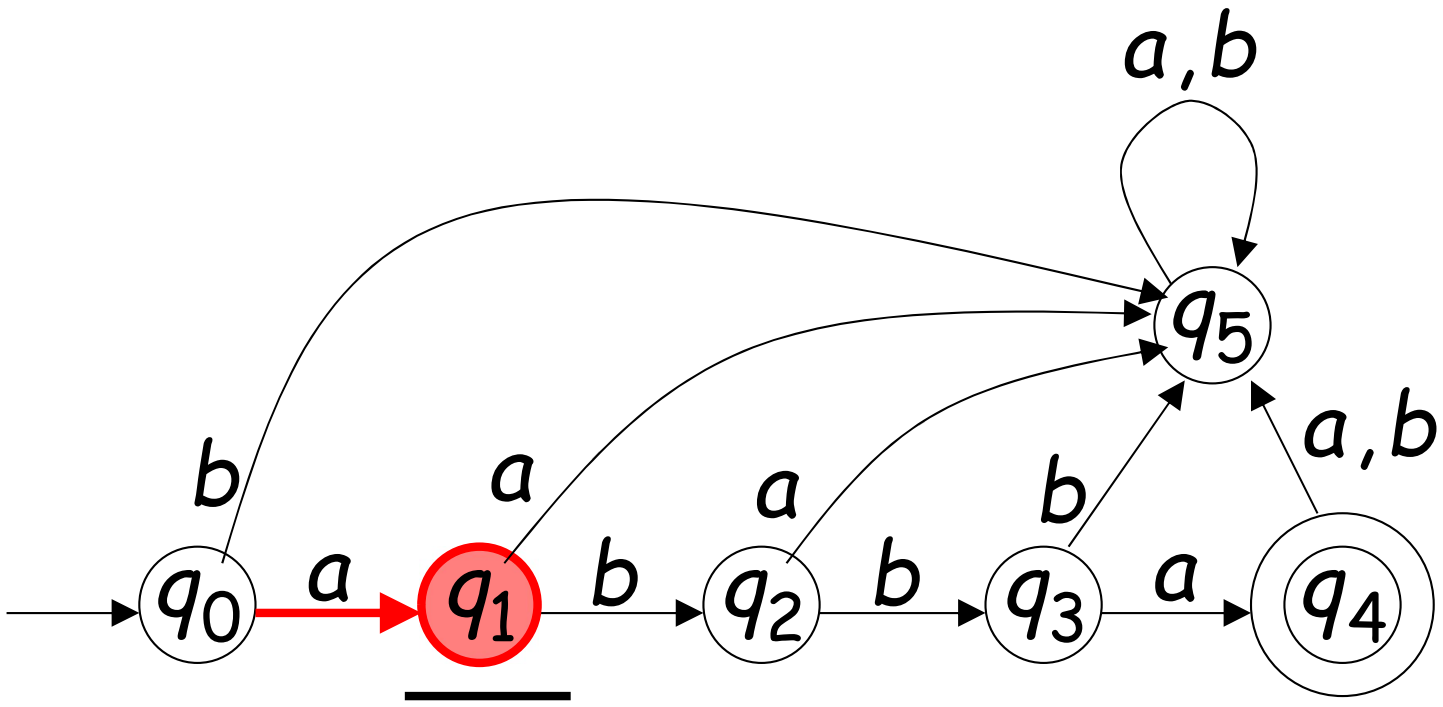
Input String



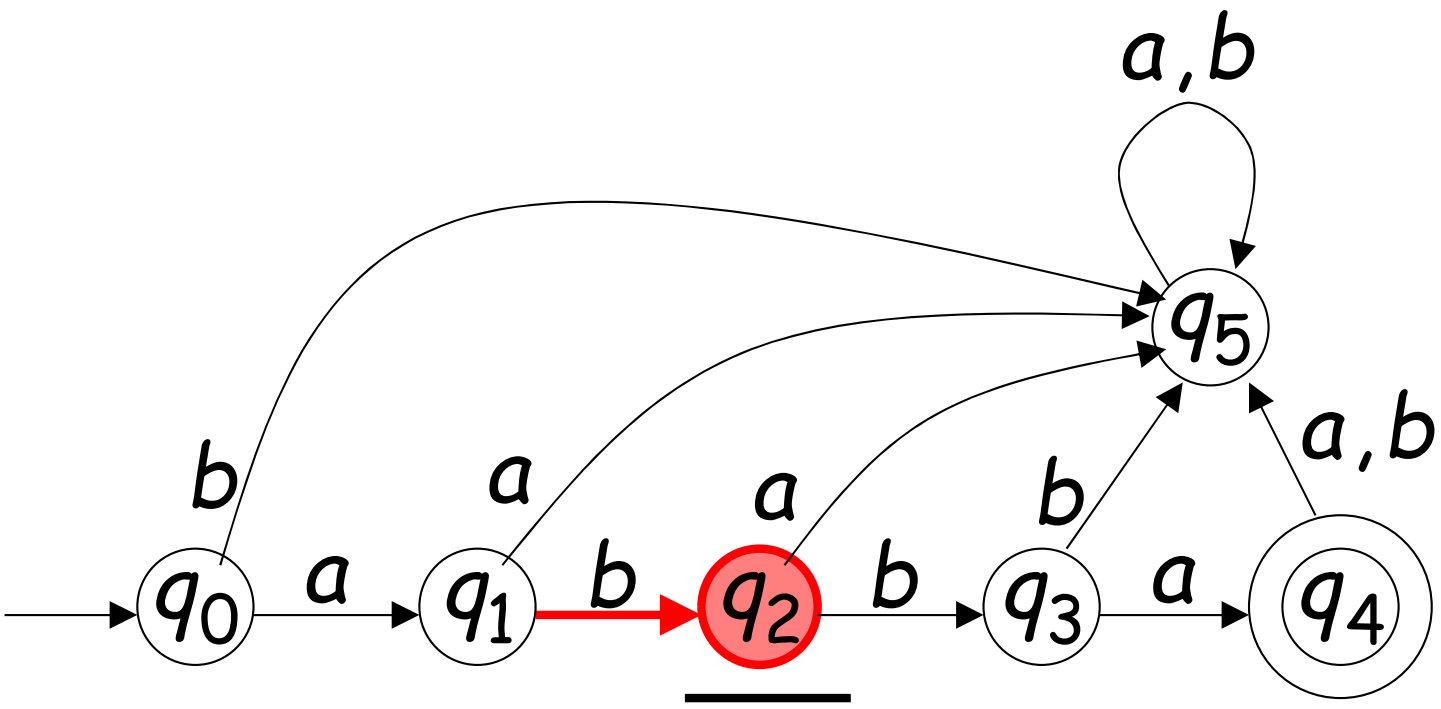


Reading the Input

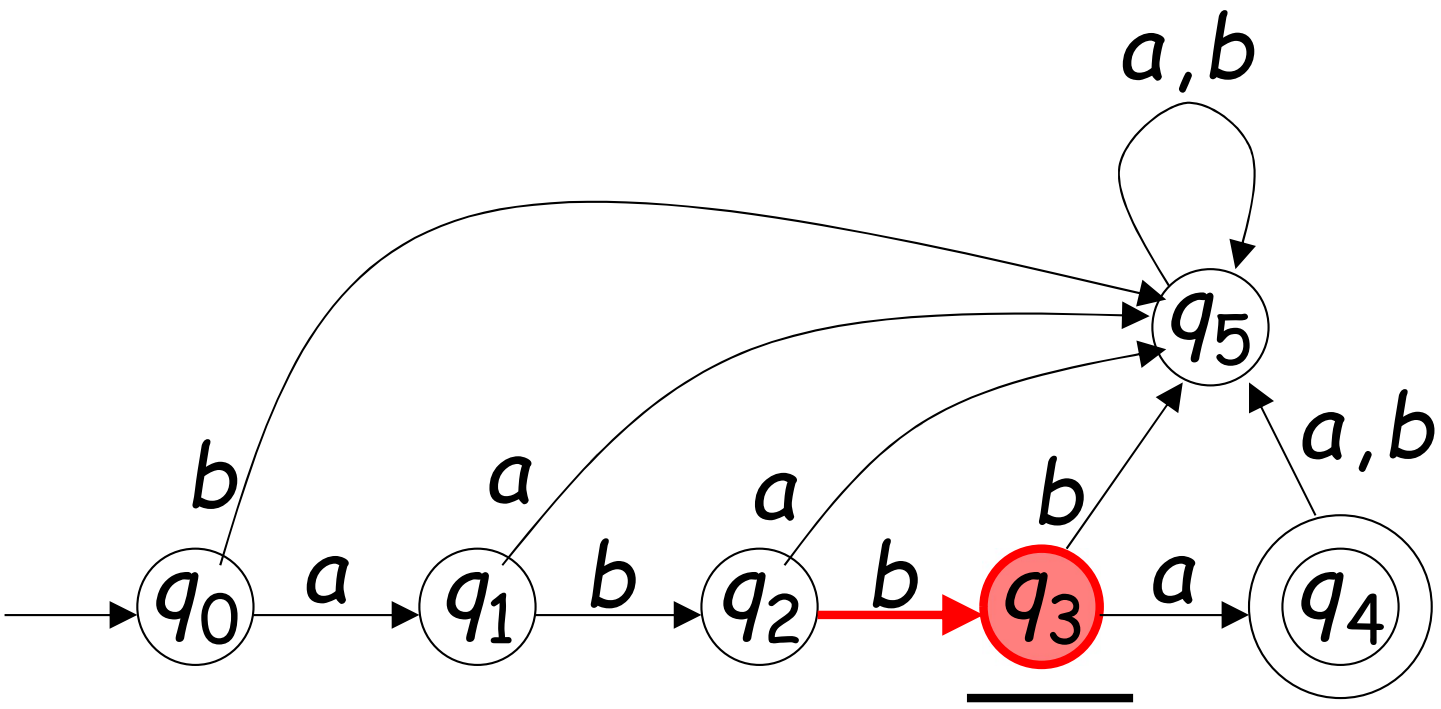


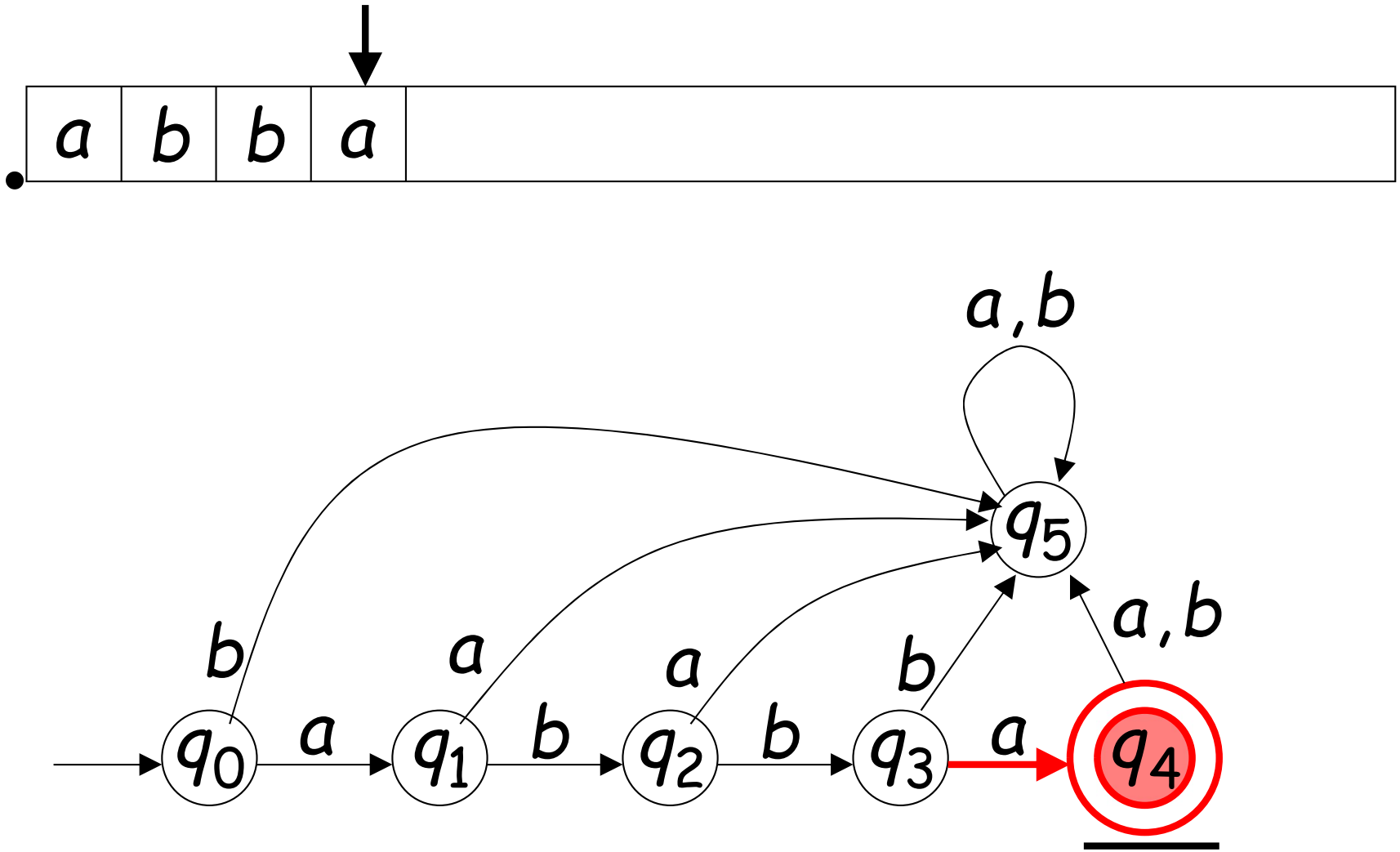



a
b
b
a

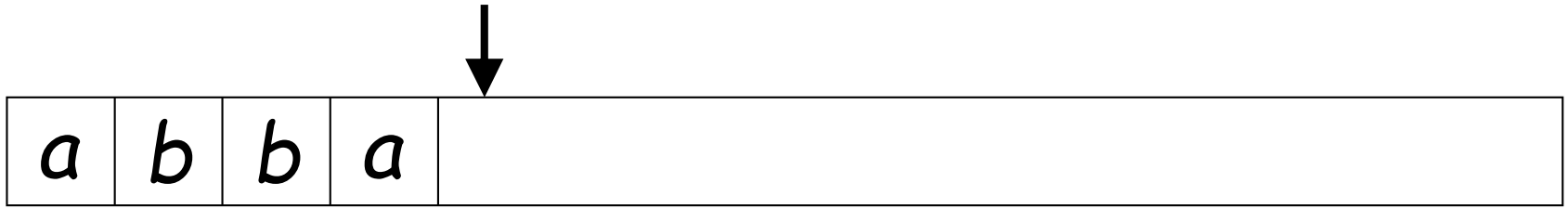

a, b
q5
b
a
a
b
a, b
q0
a
q1
b
q2
b
q3
a
q4

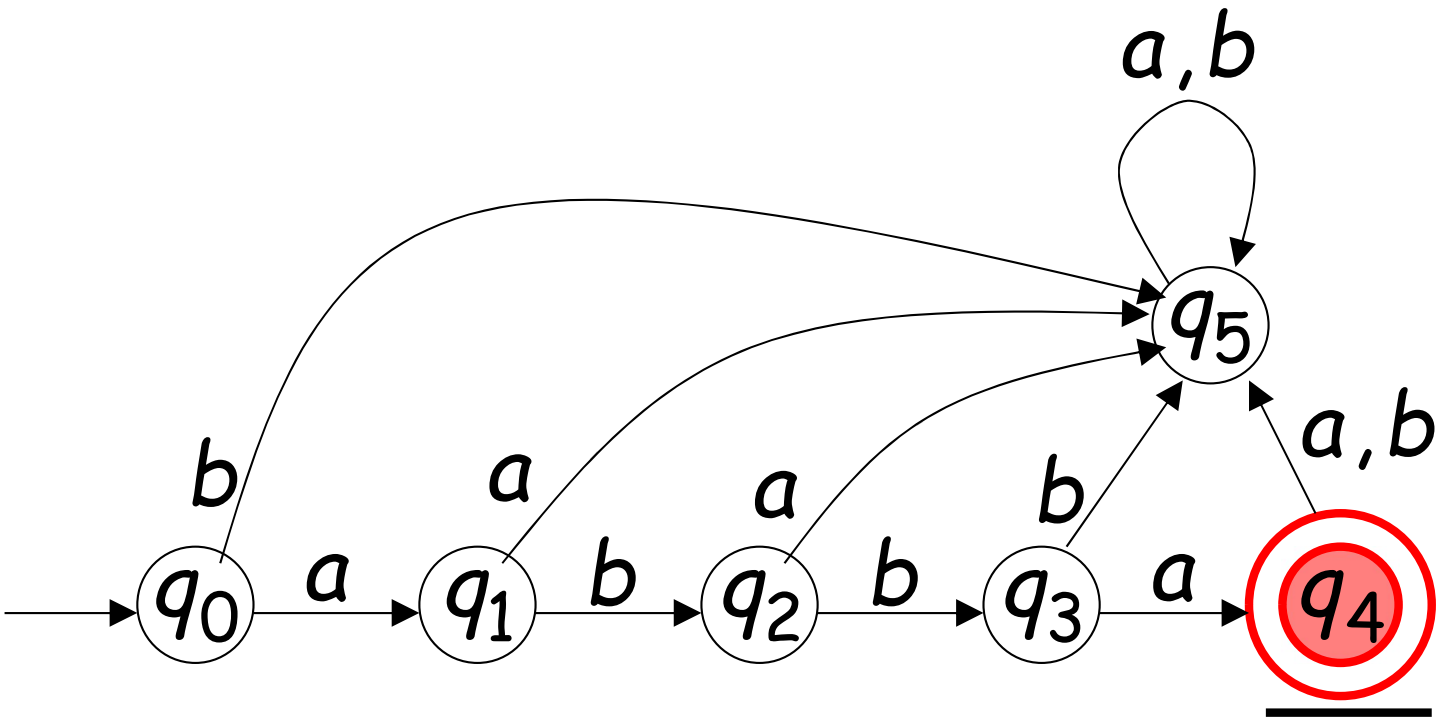

a
b
b
a


a, b
q5
b
a
a
b
a, b
q0
a
q1
b
q2
b
q3
a
q4


a b b a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

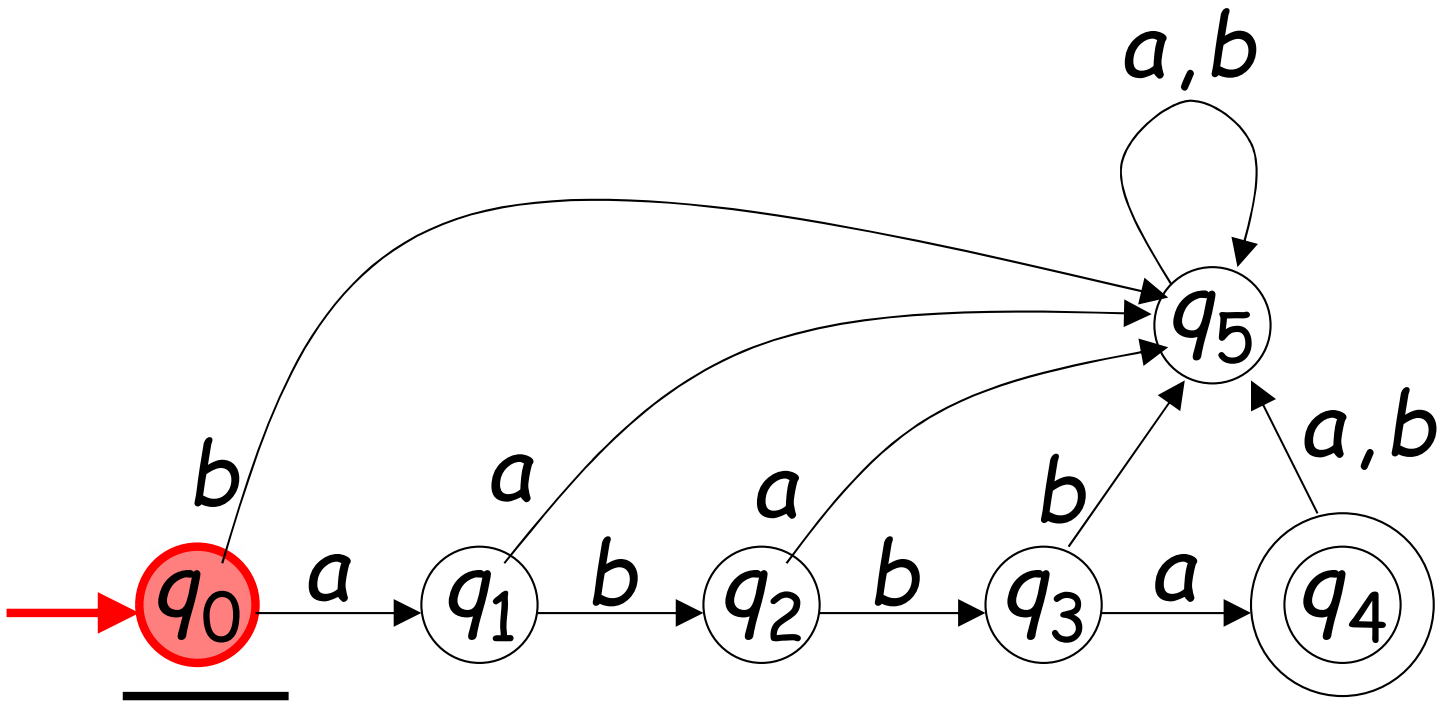
Input finished



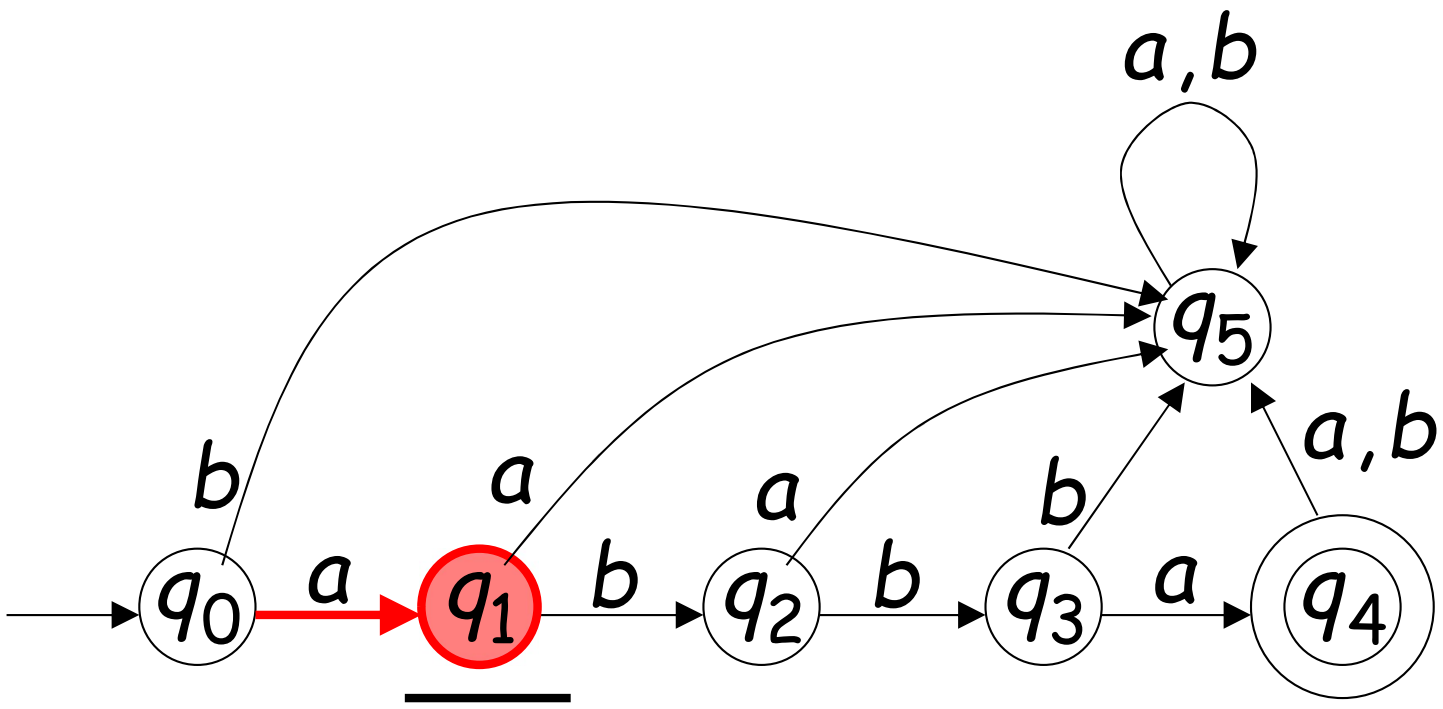


Output: “accept”

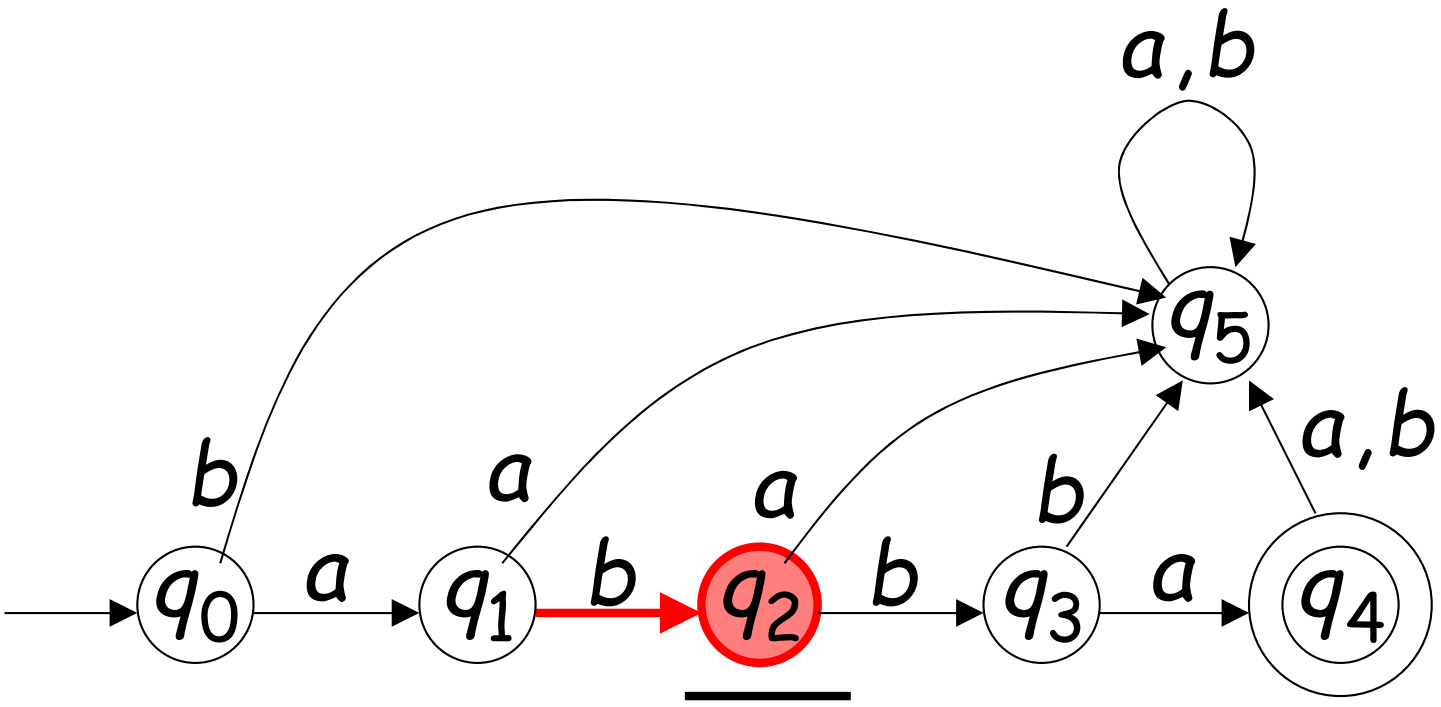
Rejection



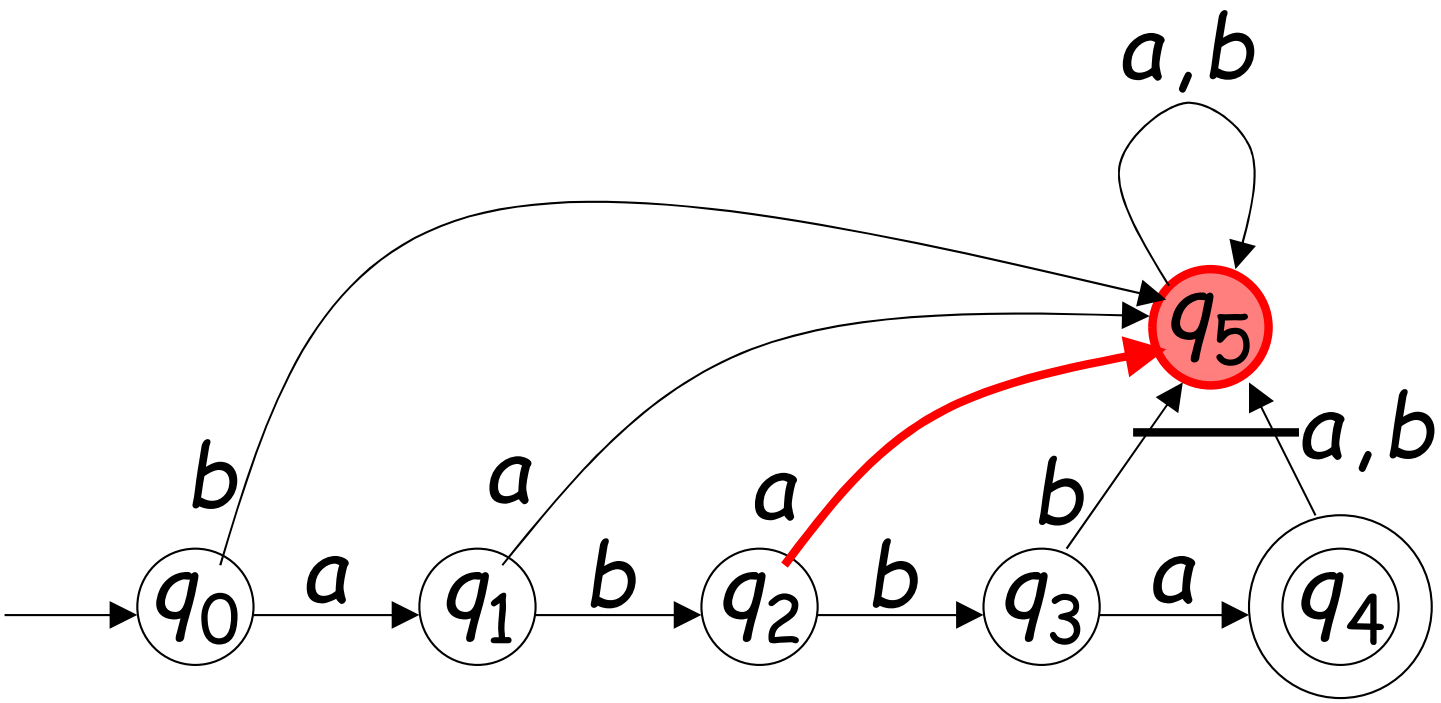

a
b
a


a, b
q5
b
a
a
b
a, b
a
b
b
a
q0
q1
q2
q3
q4

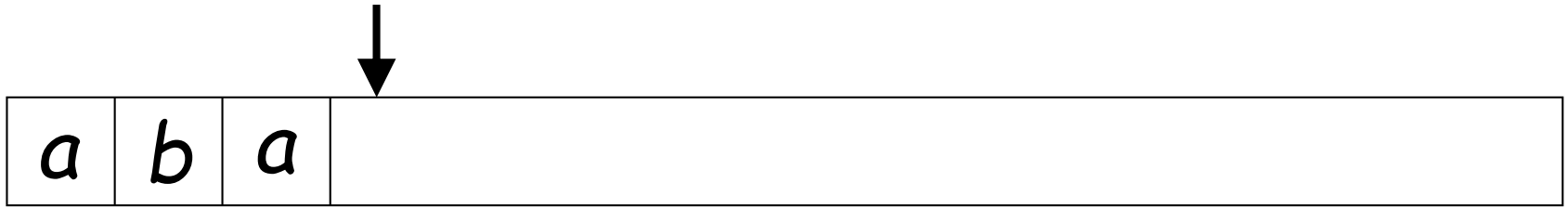

a
b
a

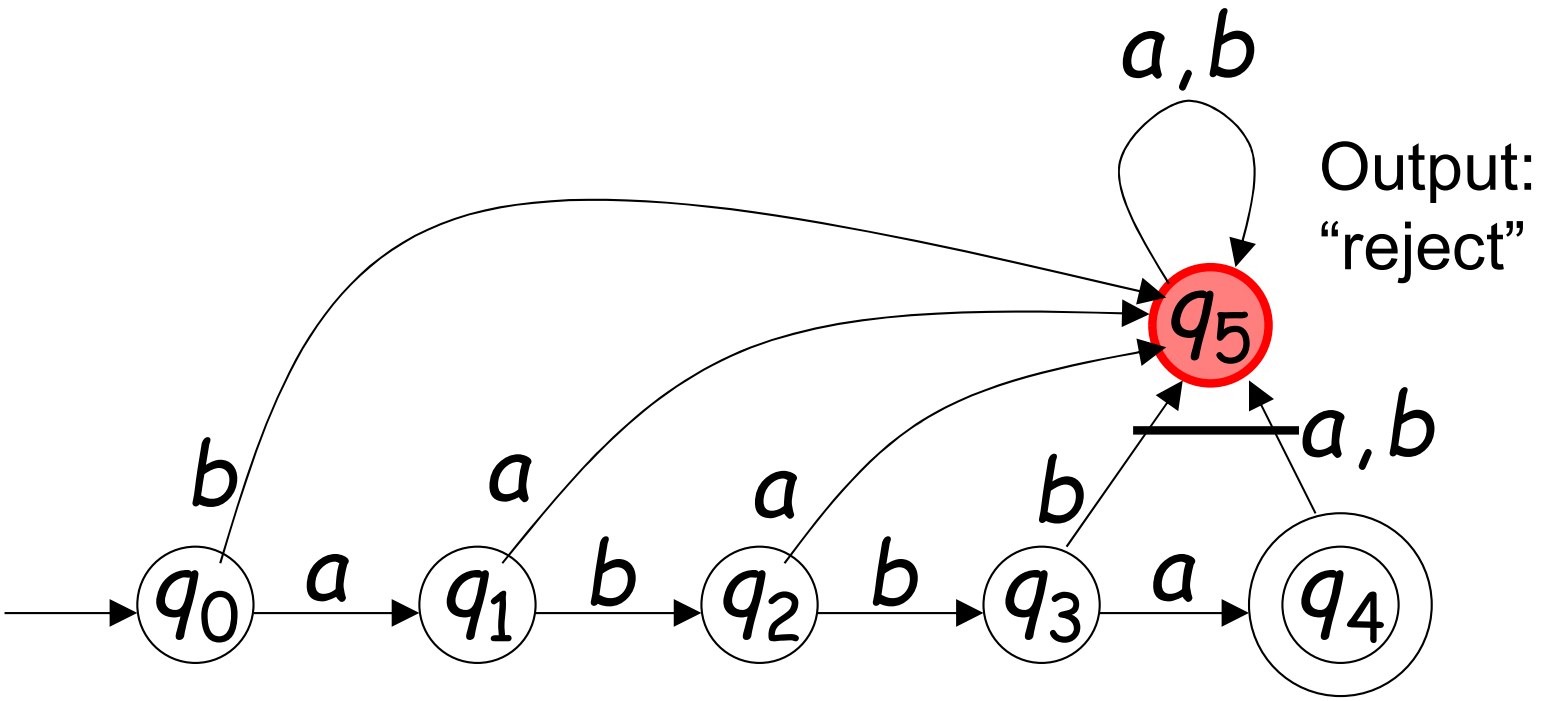

a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4


a
b
a

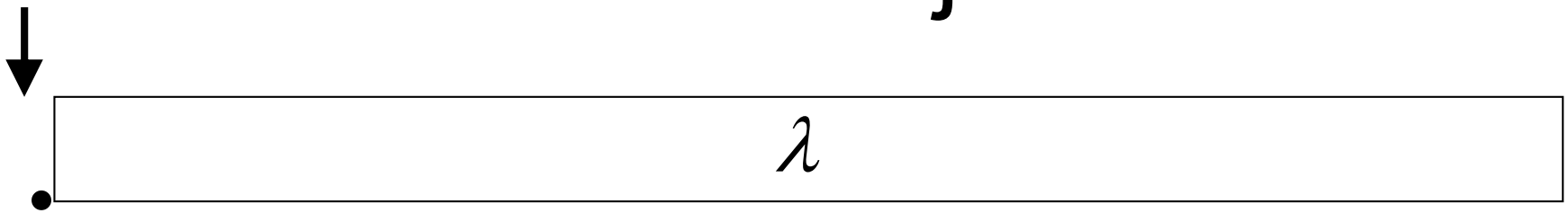

a, b
q5
a, b
b
a
a
b
q0
a
q1
b
q2
b
q3
a
q4

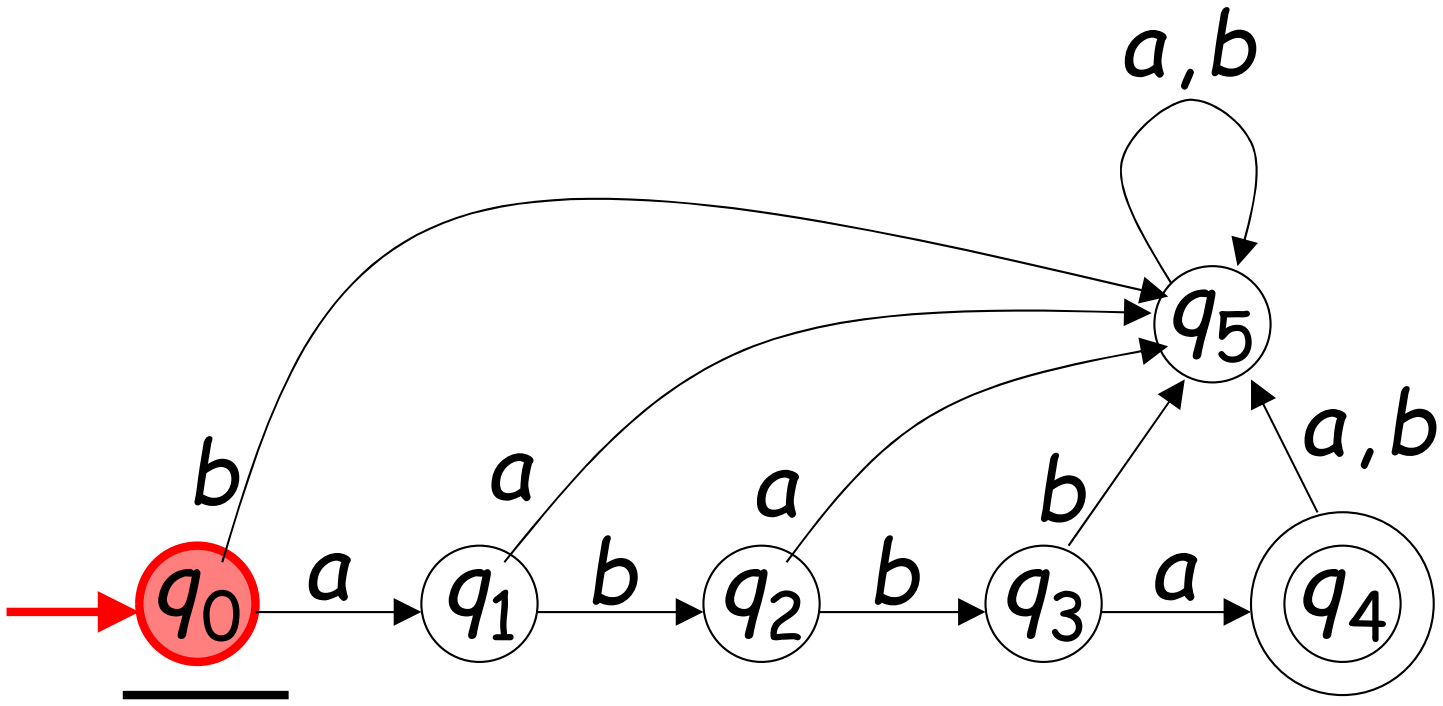
Input finished

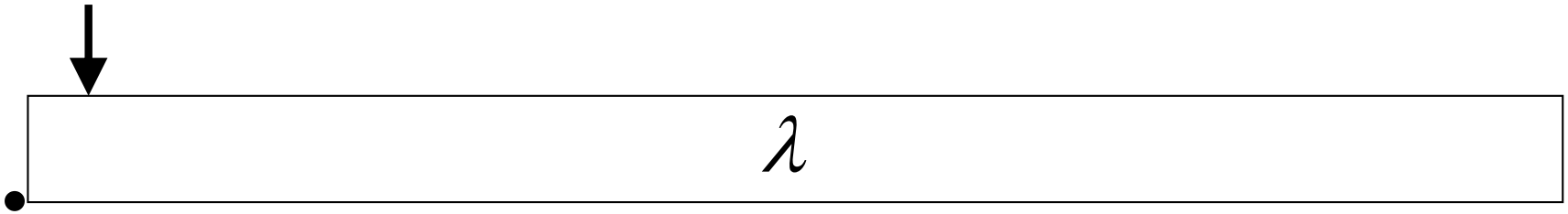


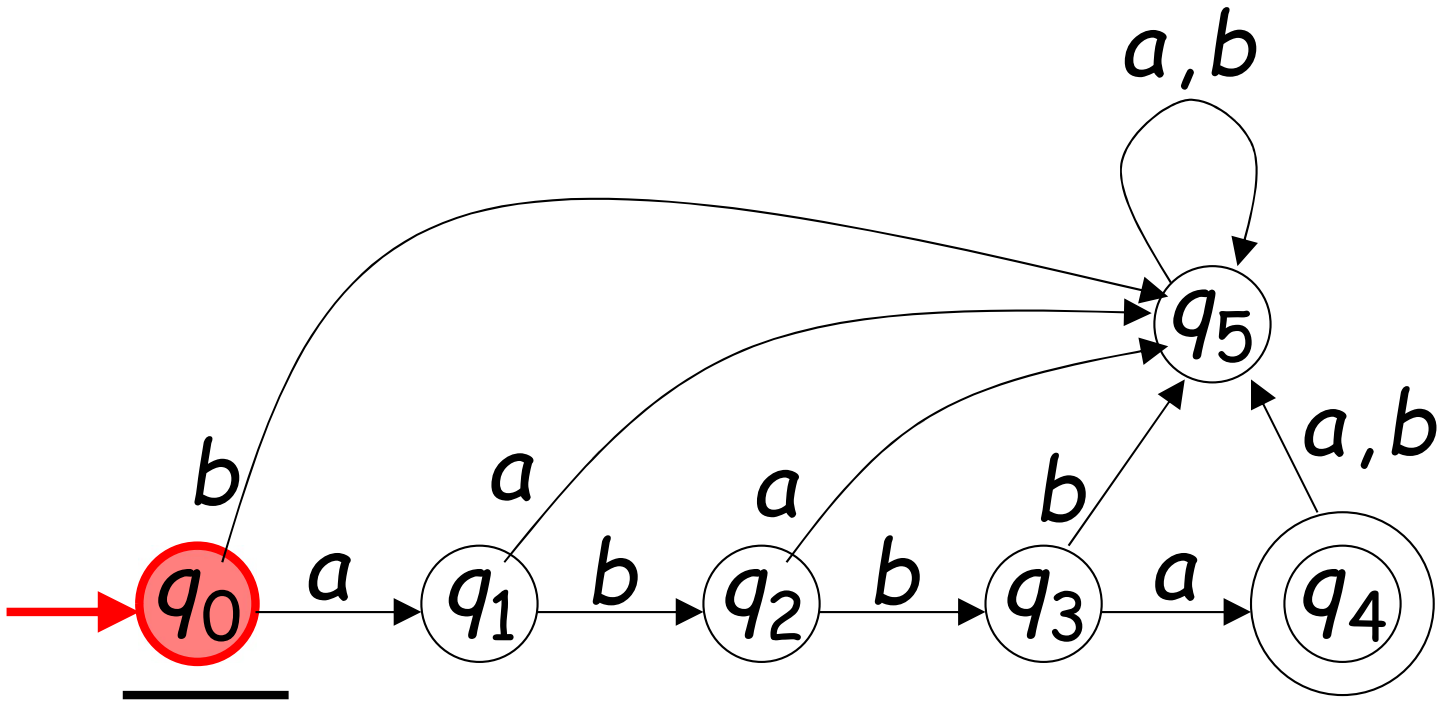


Another Rejection



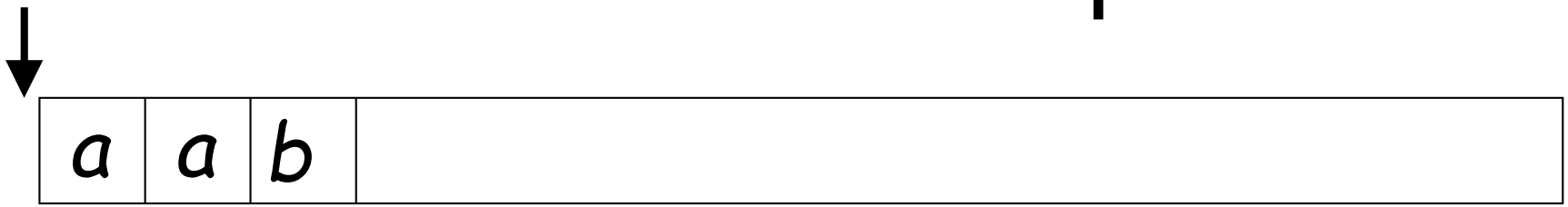


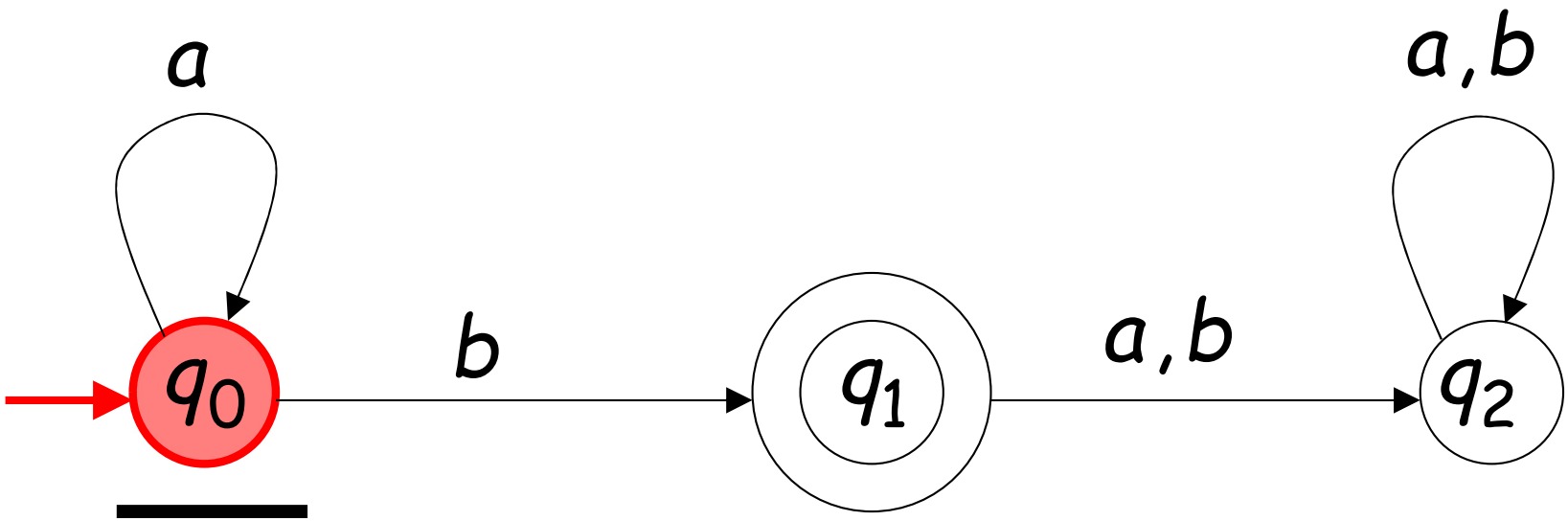


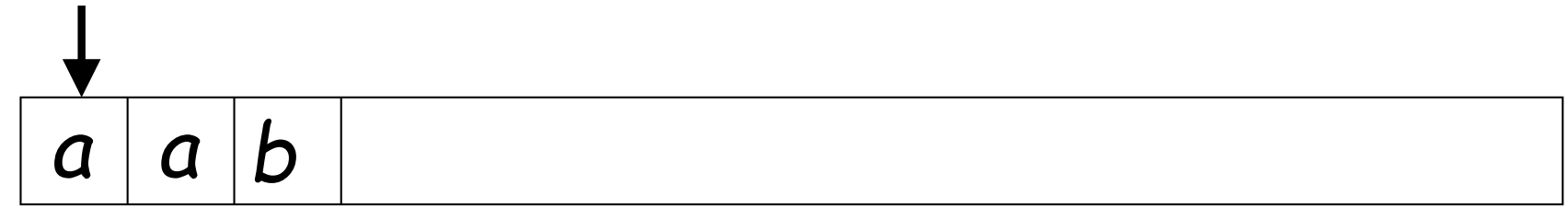


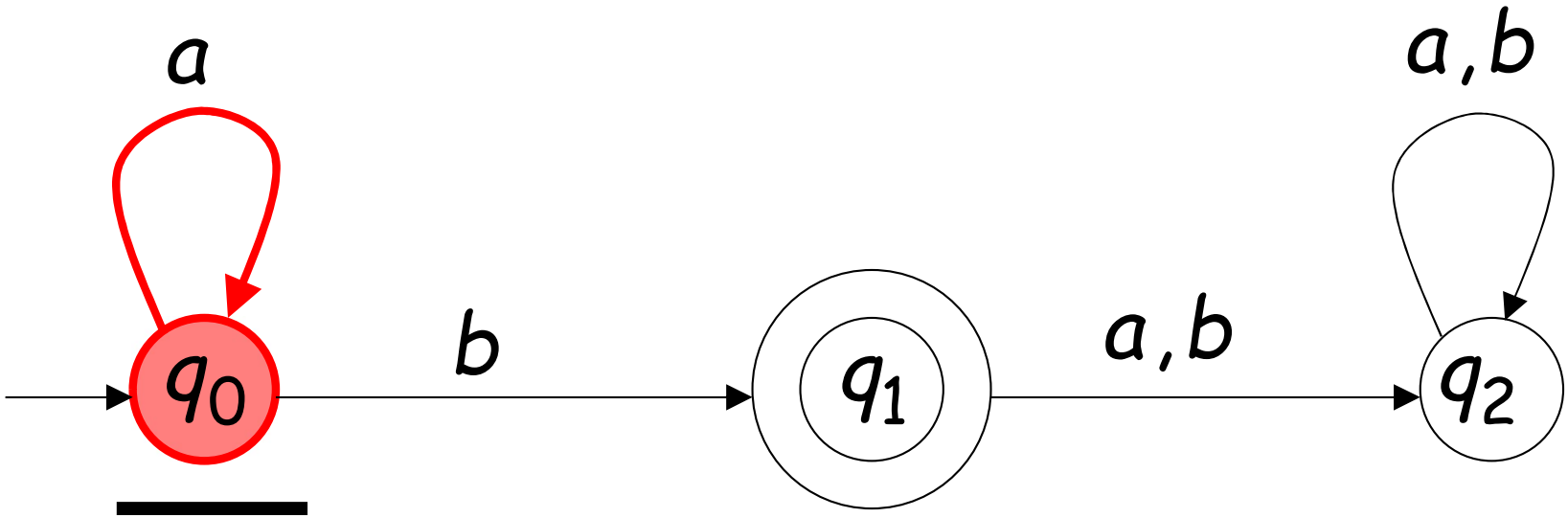
Output:
"reject" ⇒ λ：一開始就停

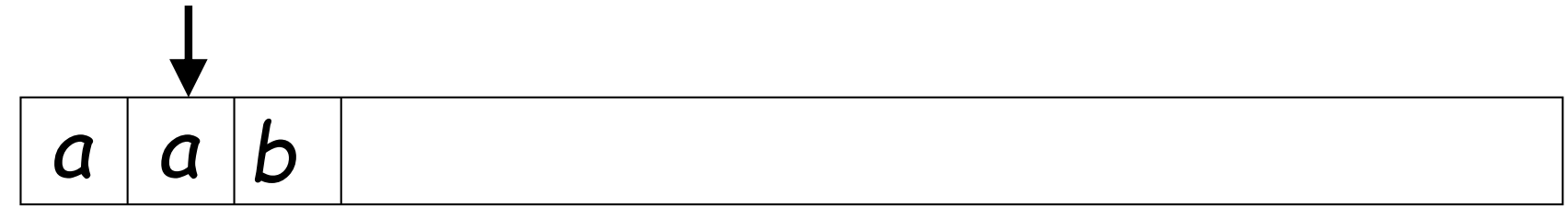
Another Example

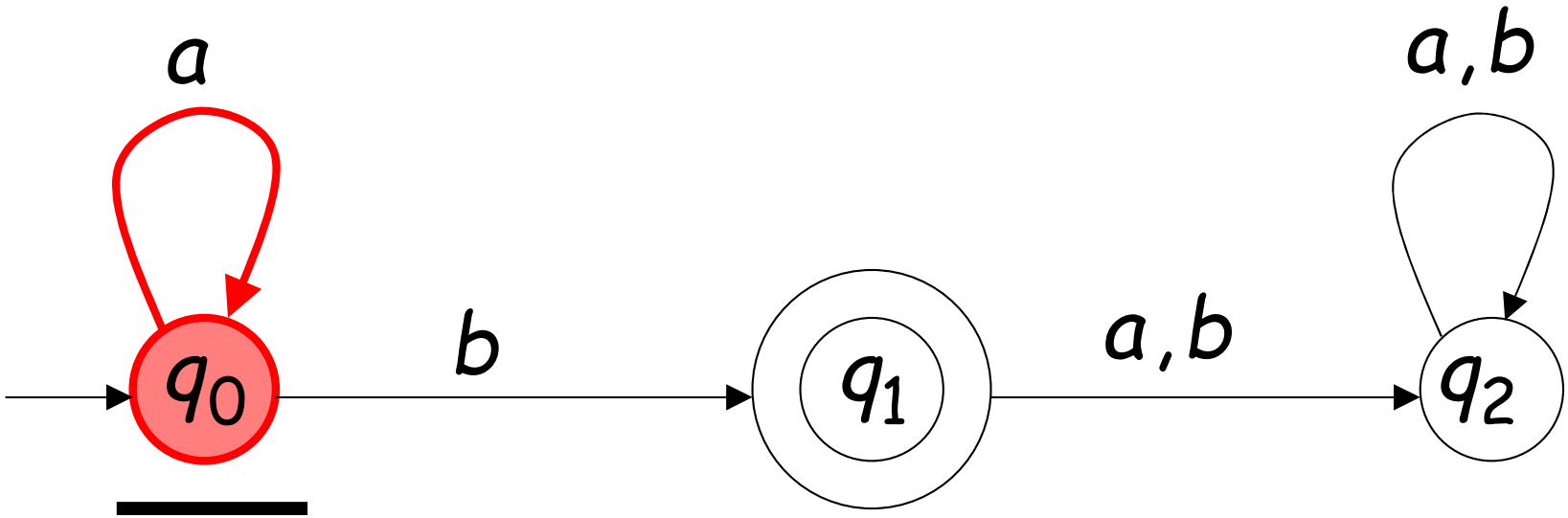


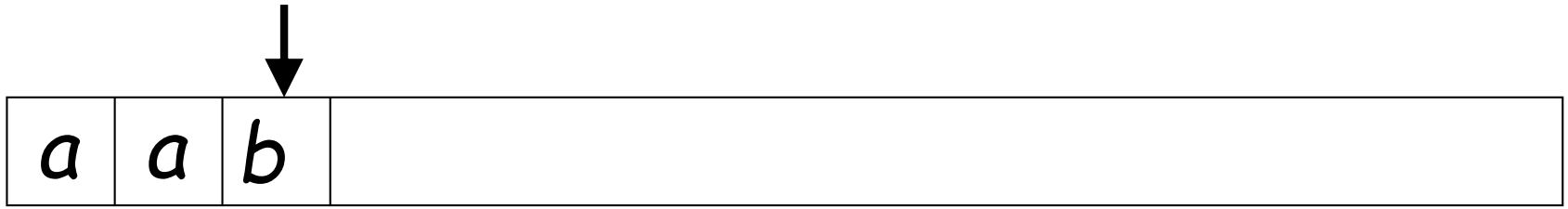


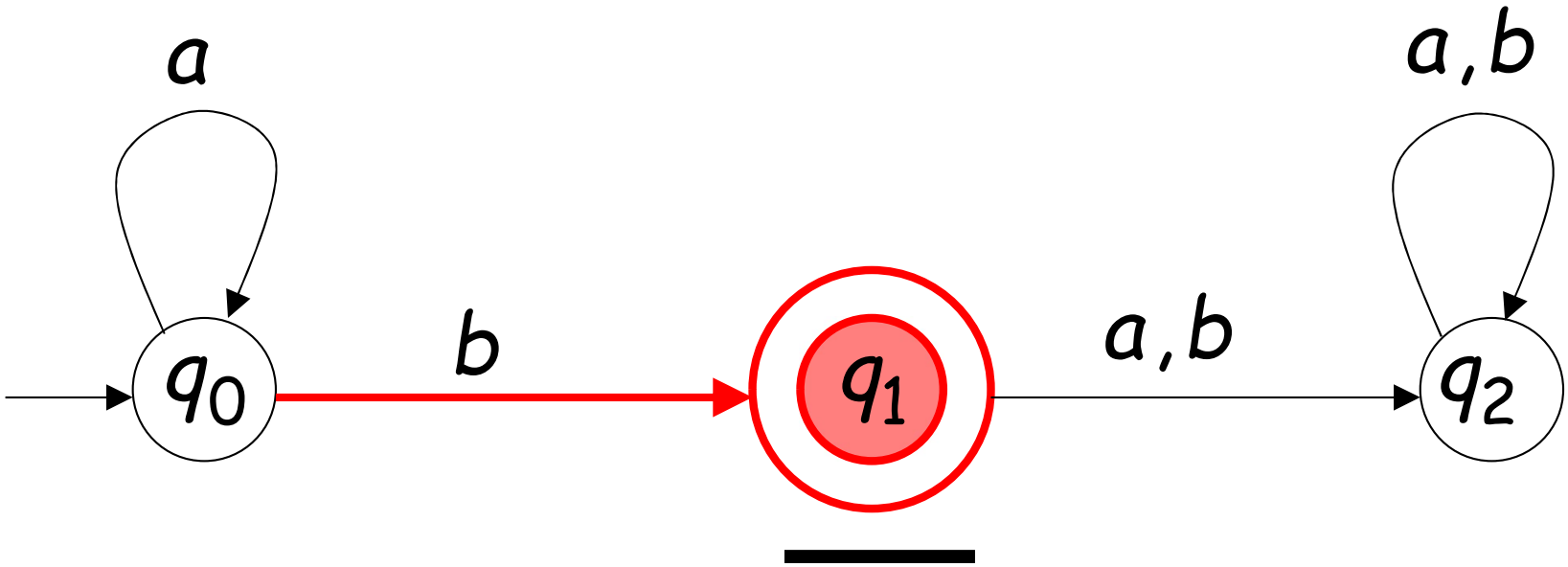

a
a
b

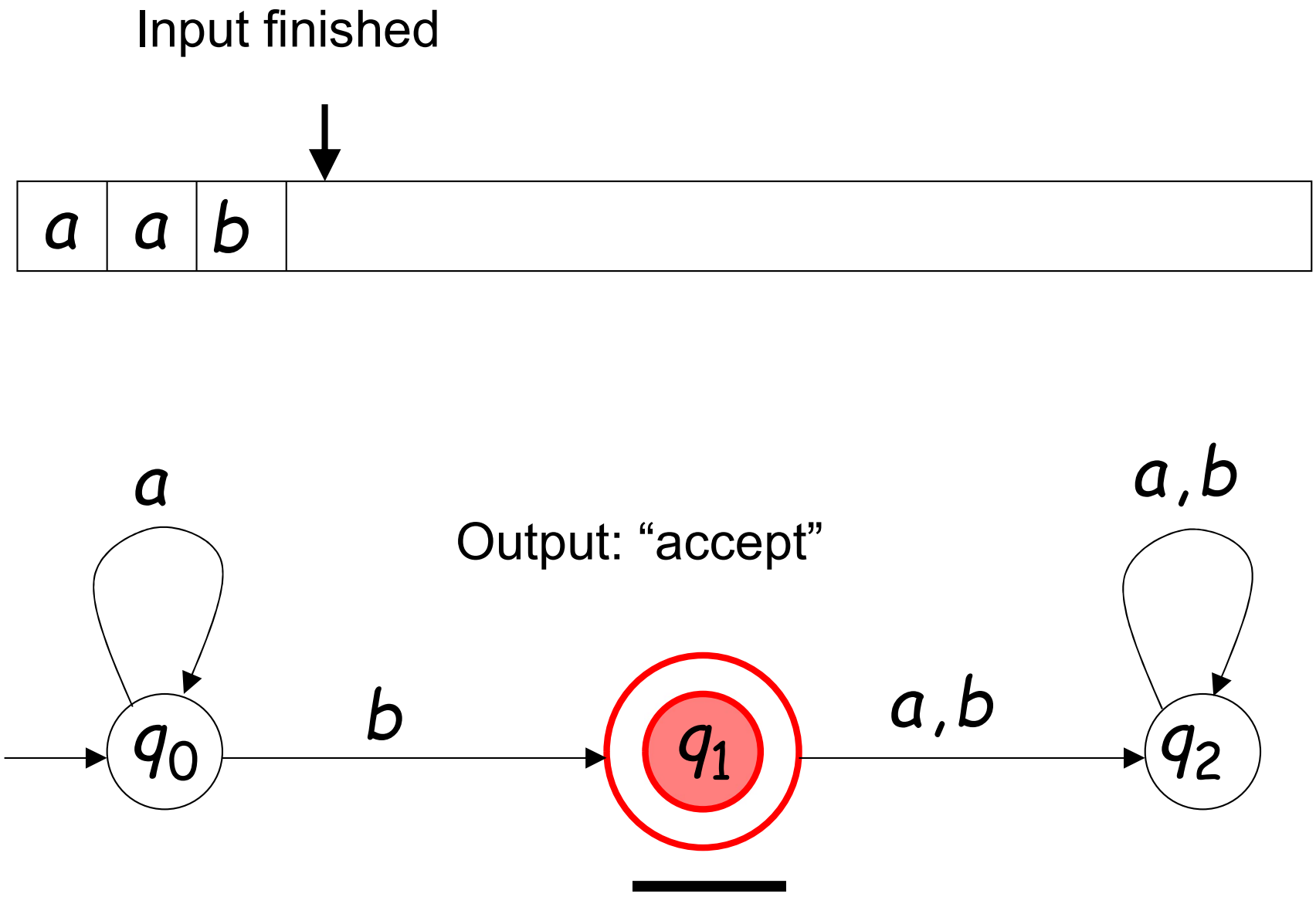

a
a,b
b
a,b
q_0
q_1
q_2


a
a
b

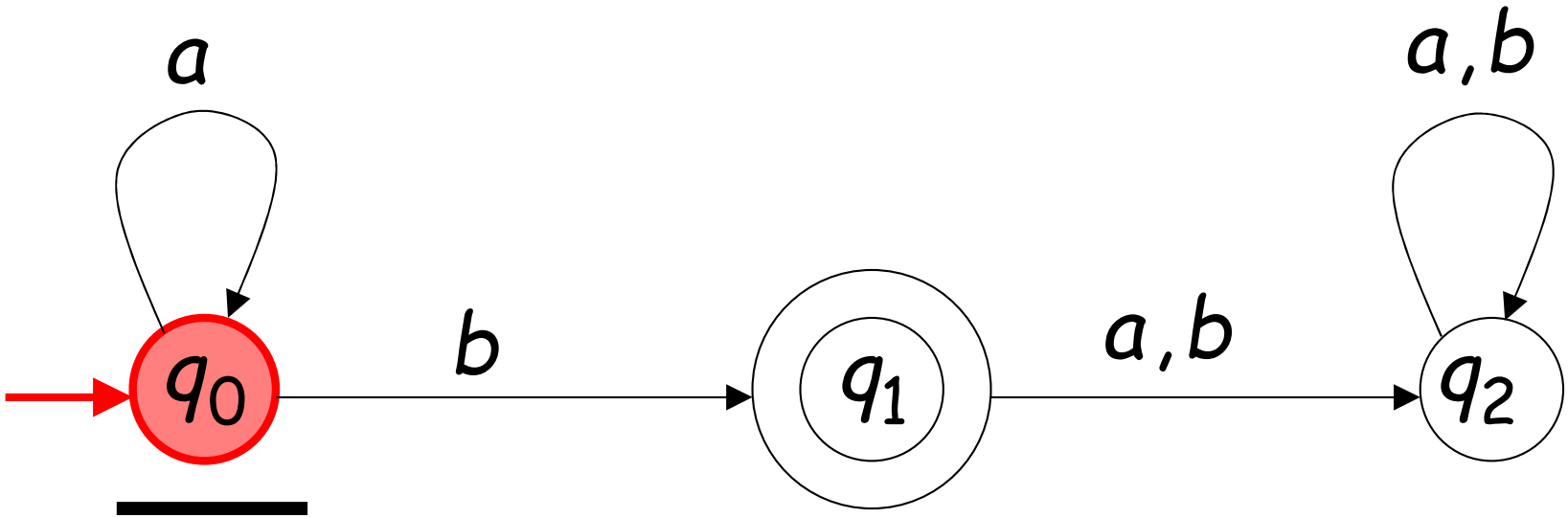

a
a,b
b
a,b
q_0
q_1
q_2

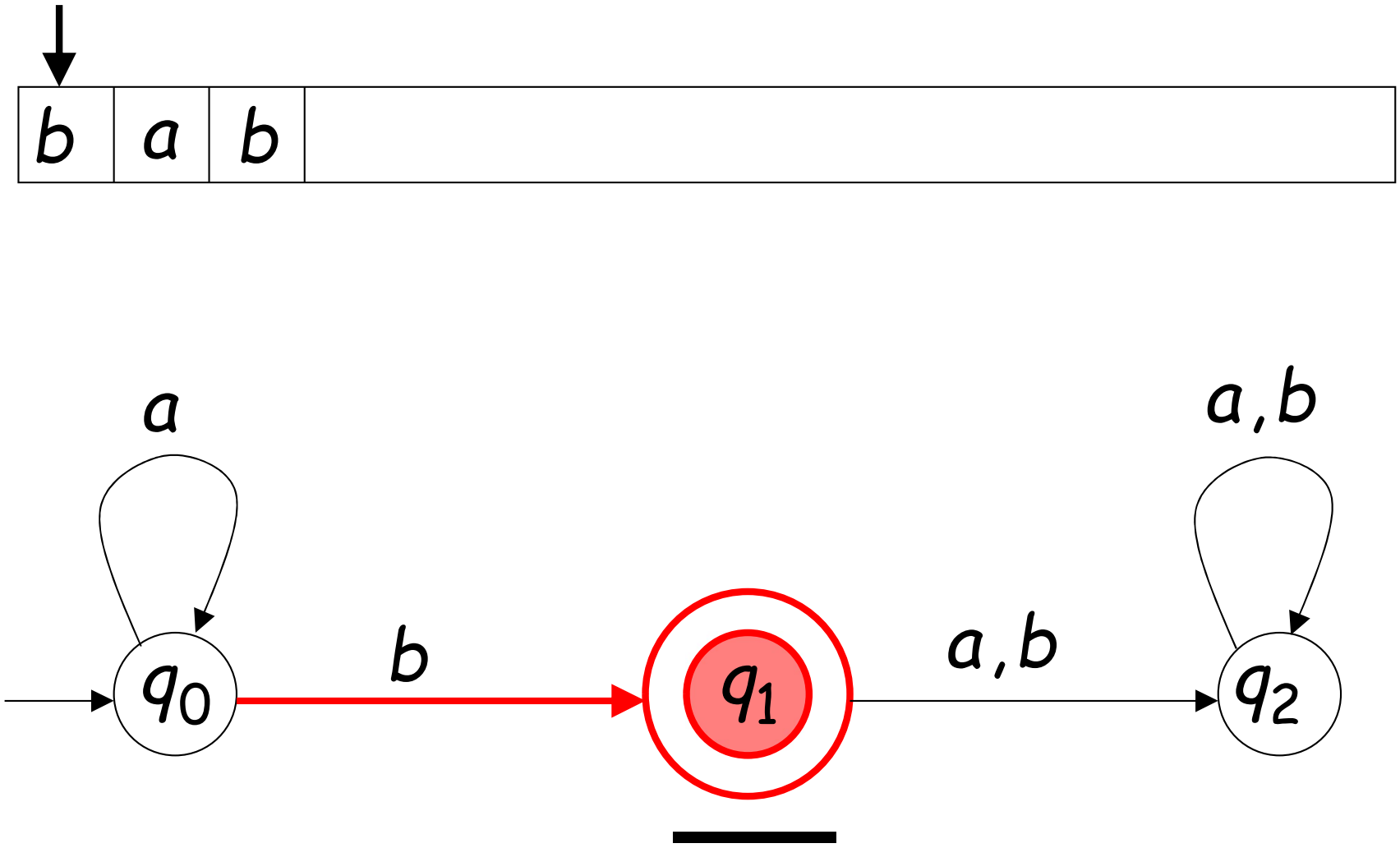

a
a
b

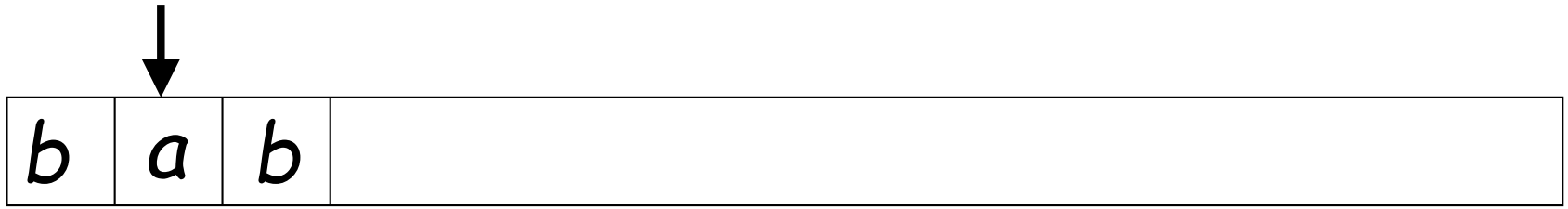

a
a,b
b
a,b
q_0
q_1
q_2

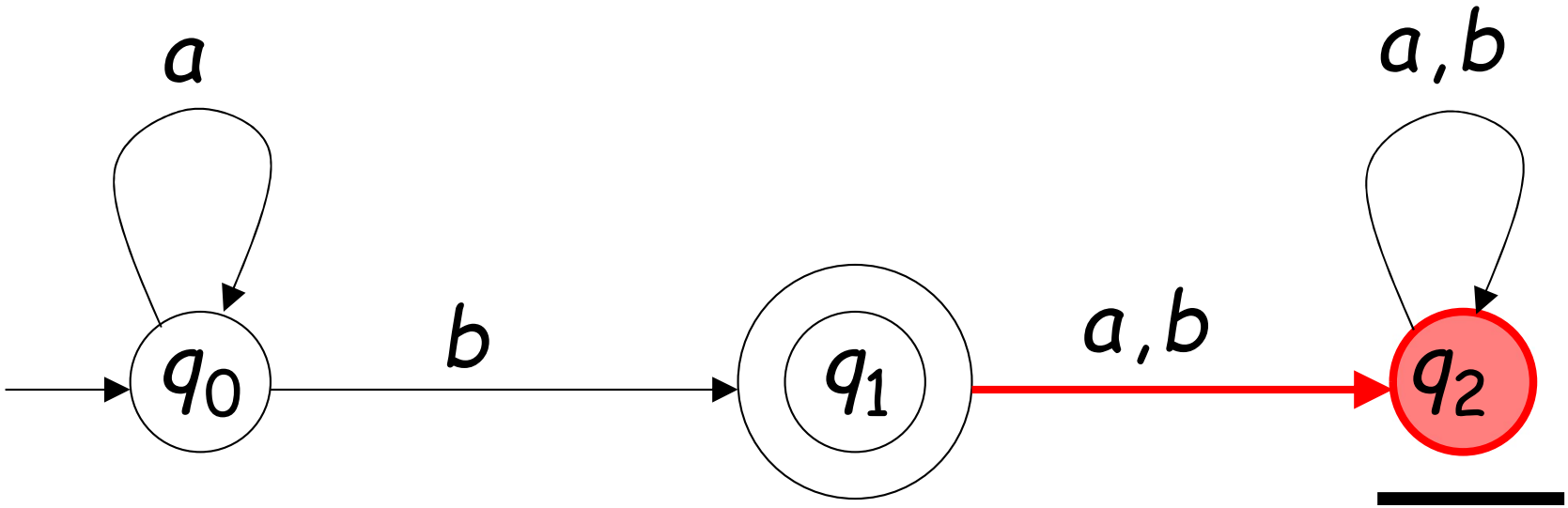

Input finished
a
a
b
a
a, b
Output: “accept”
b
a, b
q_0
q_1
q_2

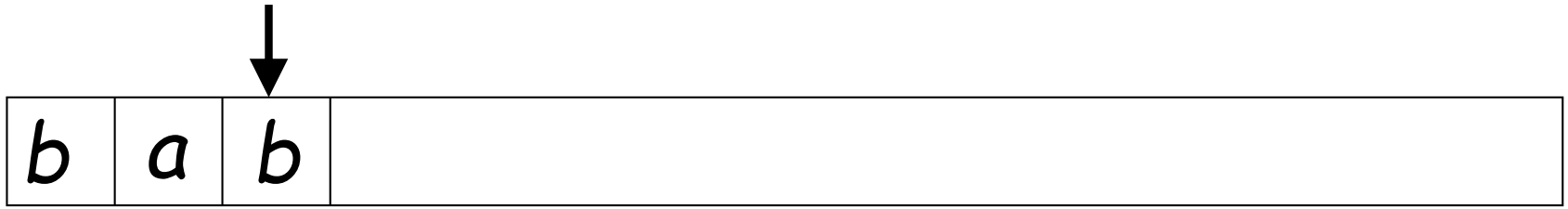
Rejection

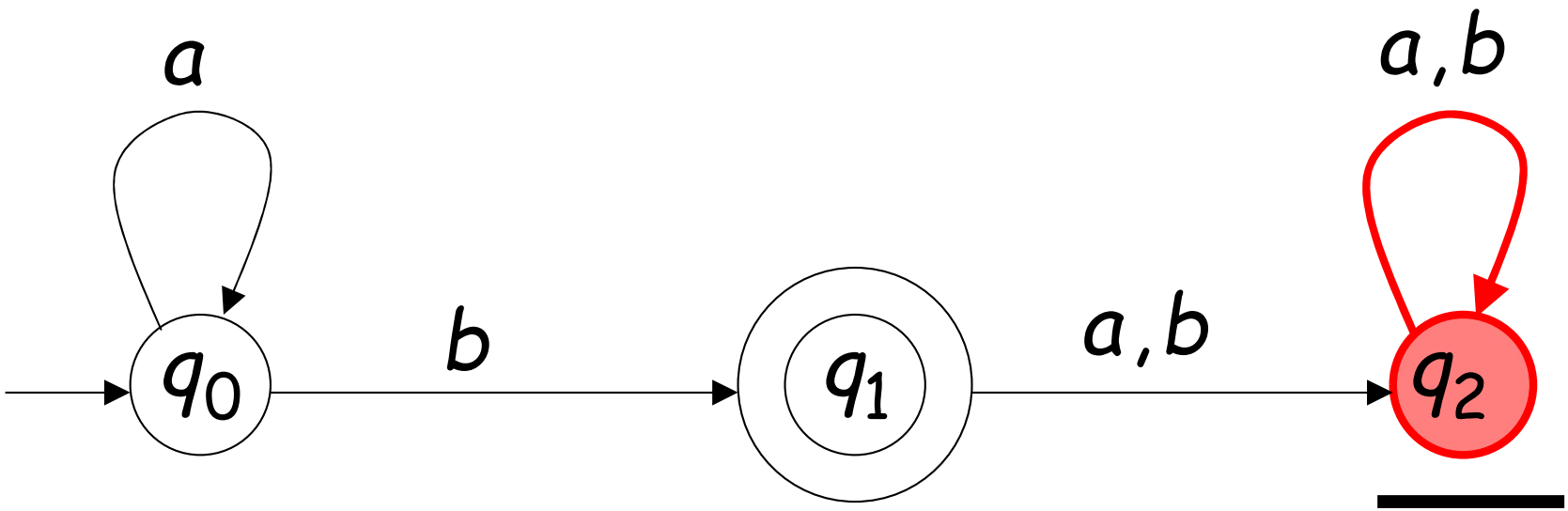



b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

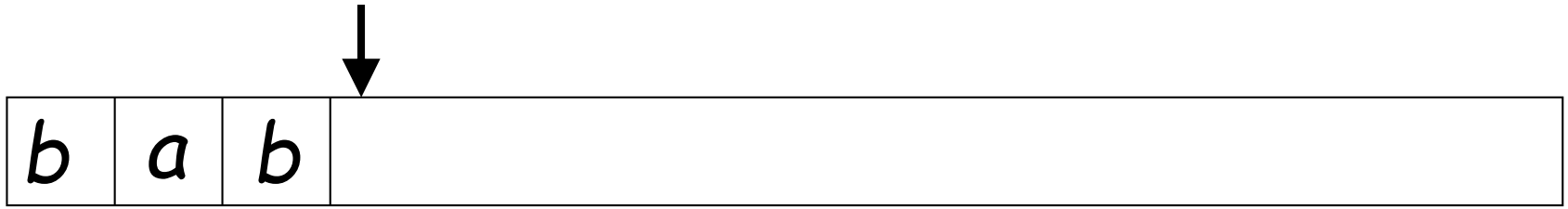

b
a
b

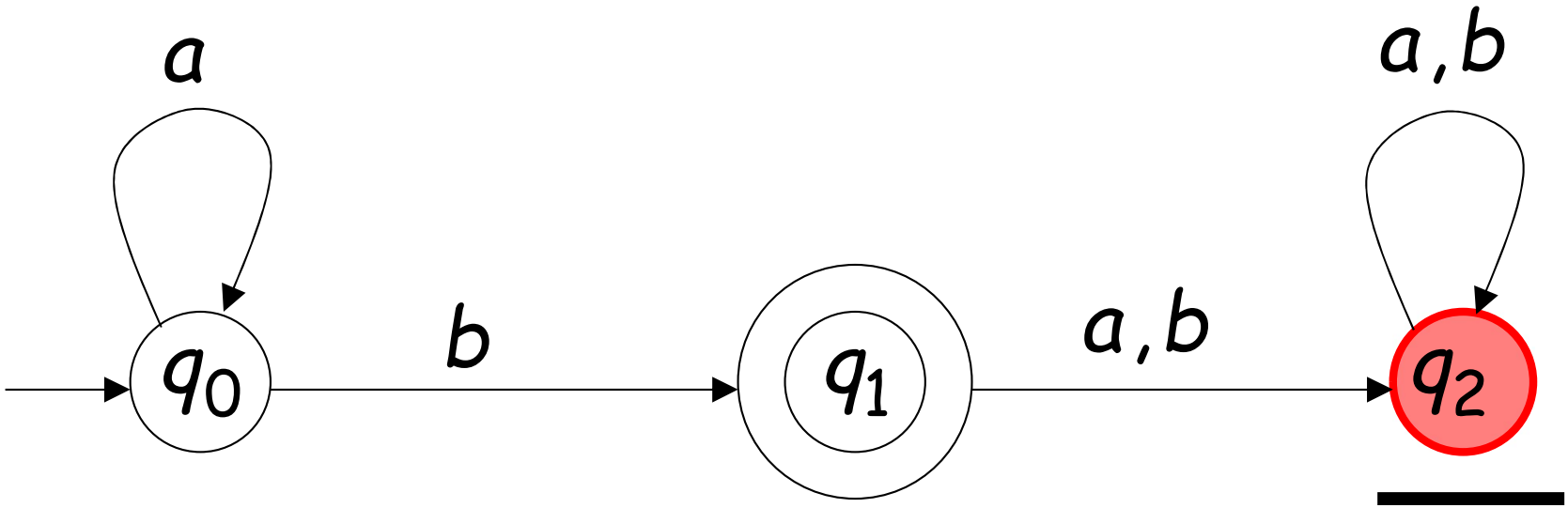

a
a,b
b
a,b
q_0
q_1
q_2


b
a
b


a
a,b
b
a,b
q_0
q_1
q_2

Input finished





Output: “reject”

Trap state

死路

Definition 2.1

Deterministic Finite Accepter (DFA) is define by the 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : a finite set of **internal states** ex: q_0, q_1 ...

Σ : a finite set of symbols called **input alphabet**

δ : $Q \times \Sigma \rightarrow Q$ called **transition function** (Total function)

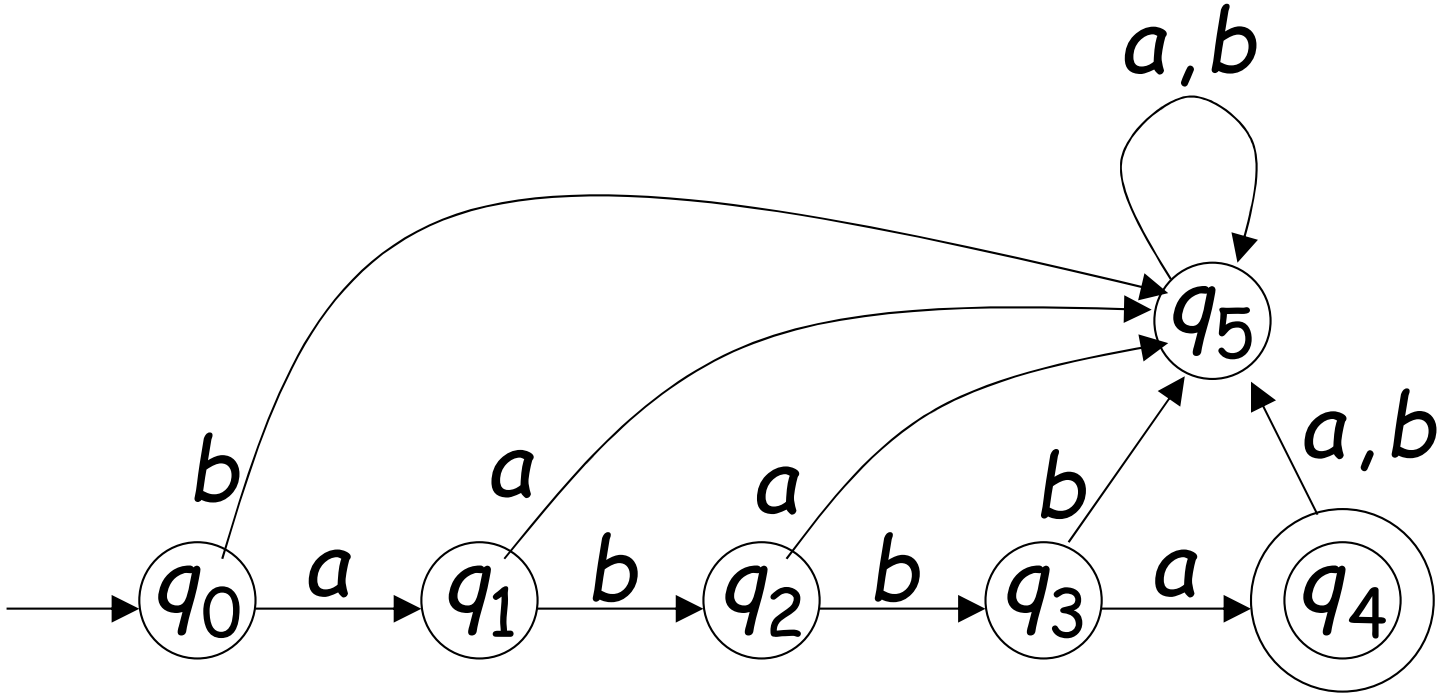
(state; alphabet; new state)

q_0 : $q_0 \in Q$ is the **initial state** $\in Q$ (只有一個)

F : $F \subseteq Q$ is a set of **final states** → 集合 (可有多個)

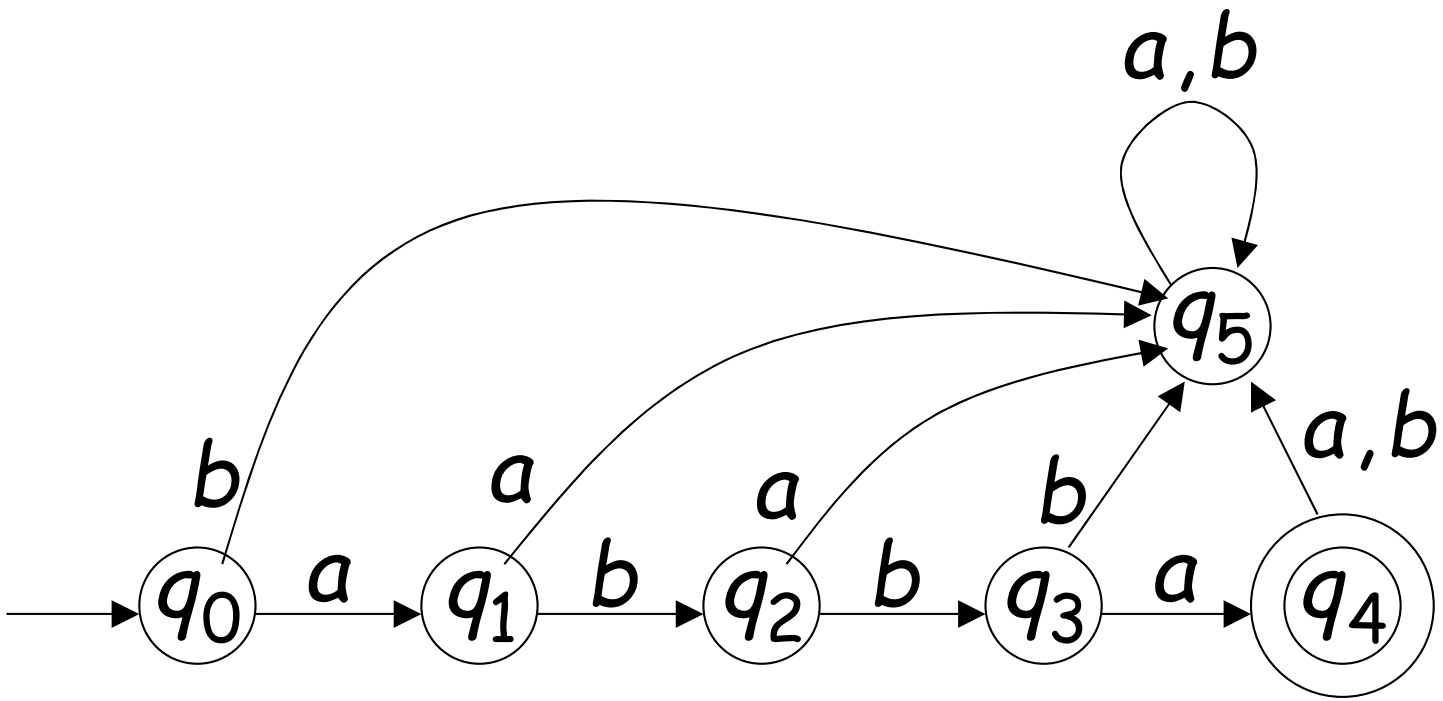
Input Alphabet Σ

$$\Sigma = \{a, b\}$$

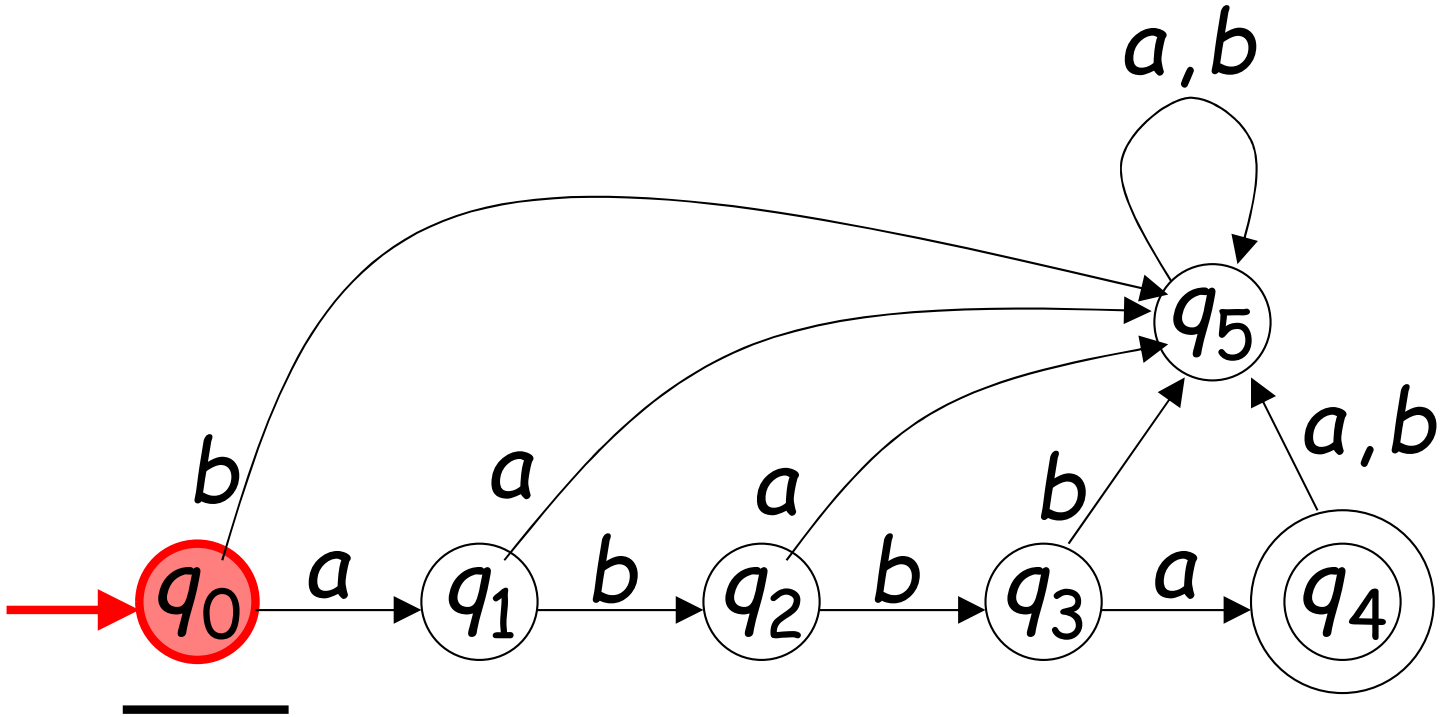


Set of States Q

$$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$$

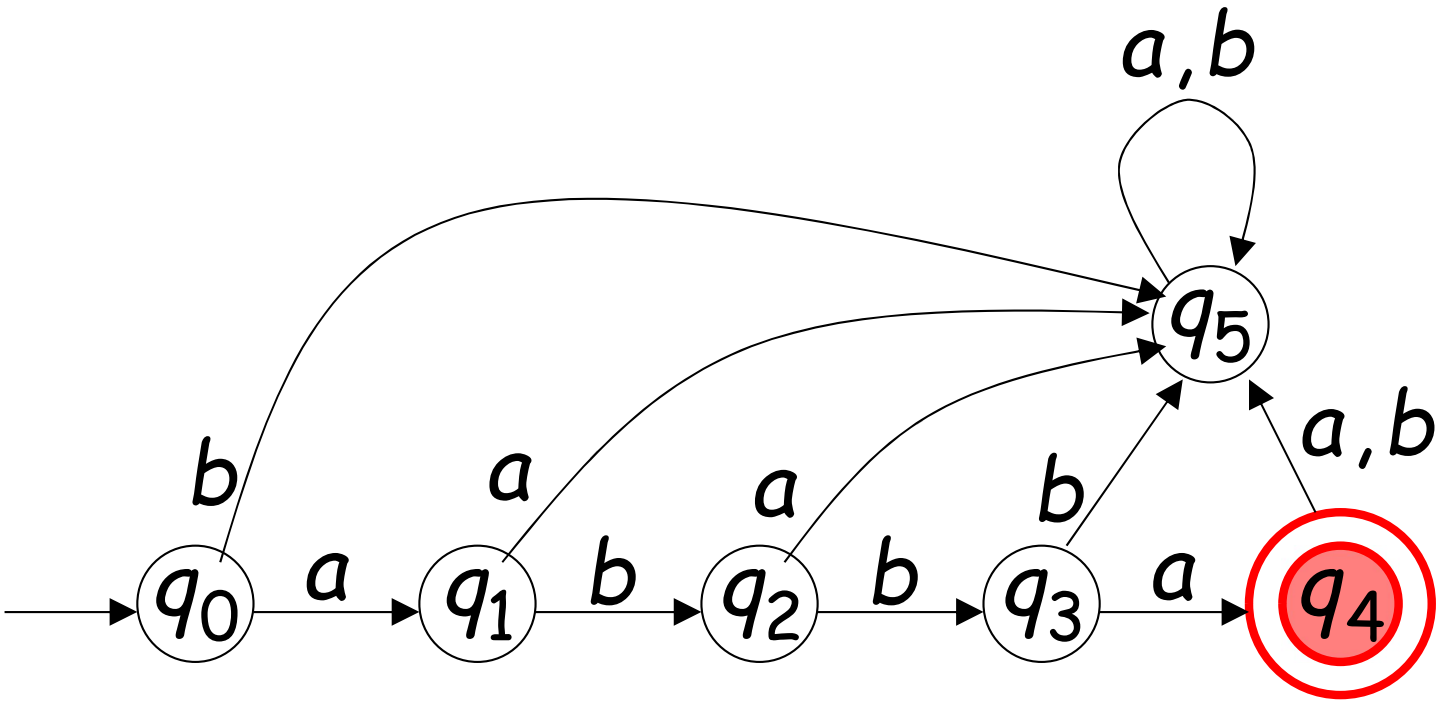


Initial State q_0



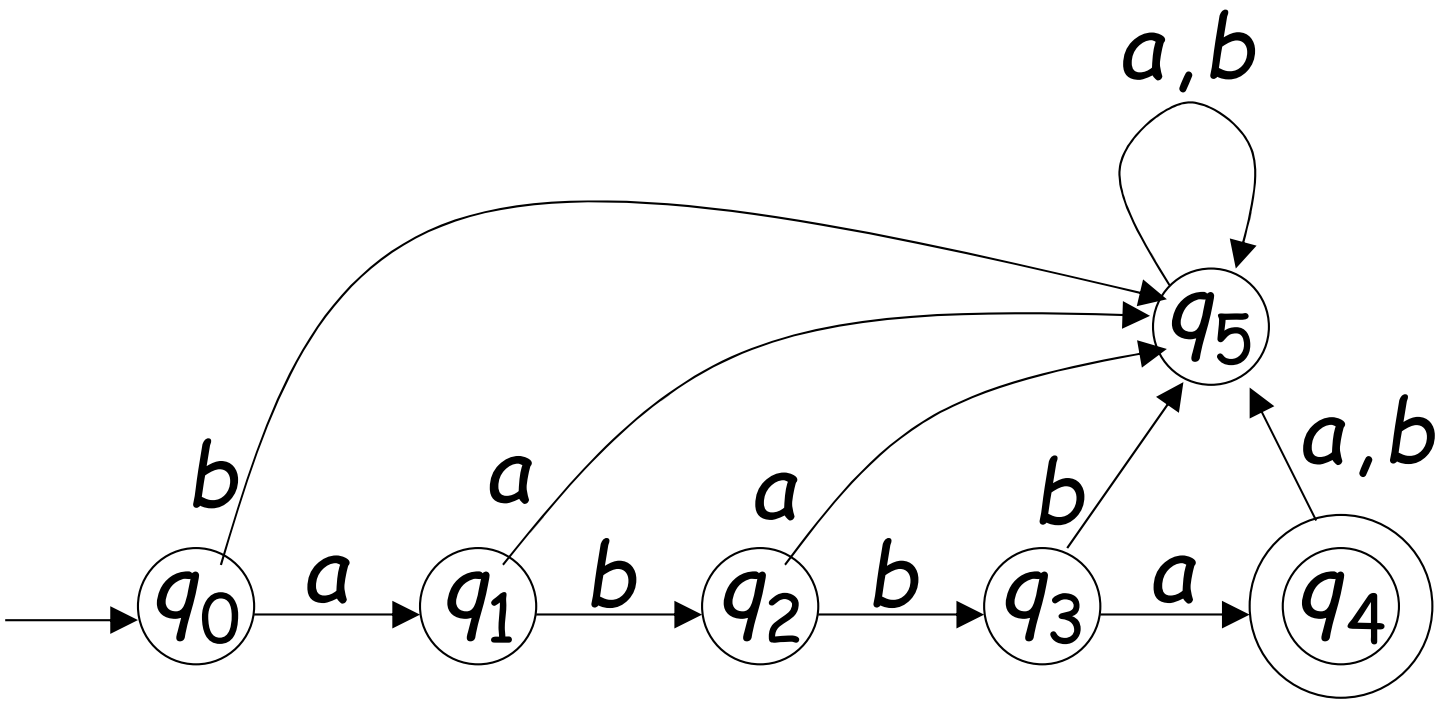
Set of Final States F

$$F = \{q_4\}$$

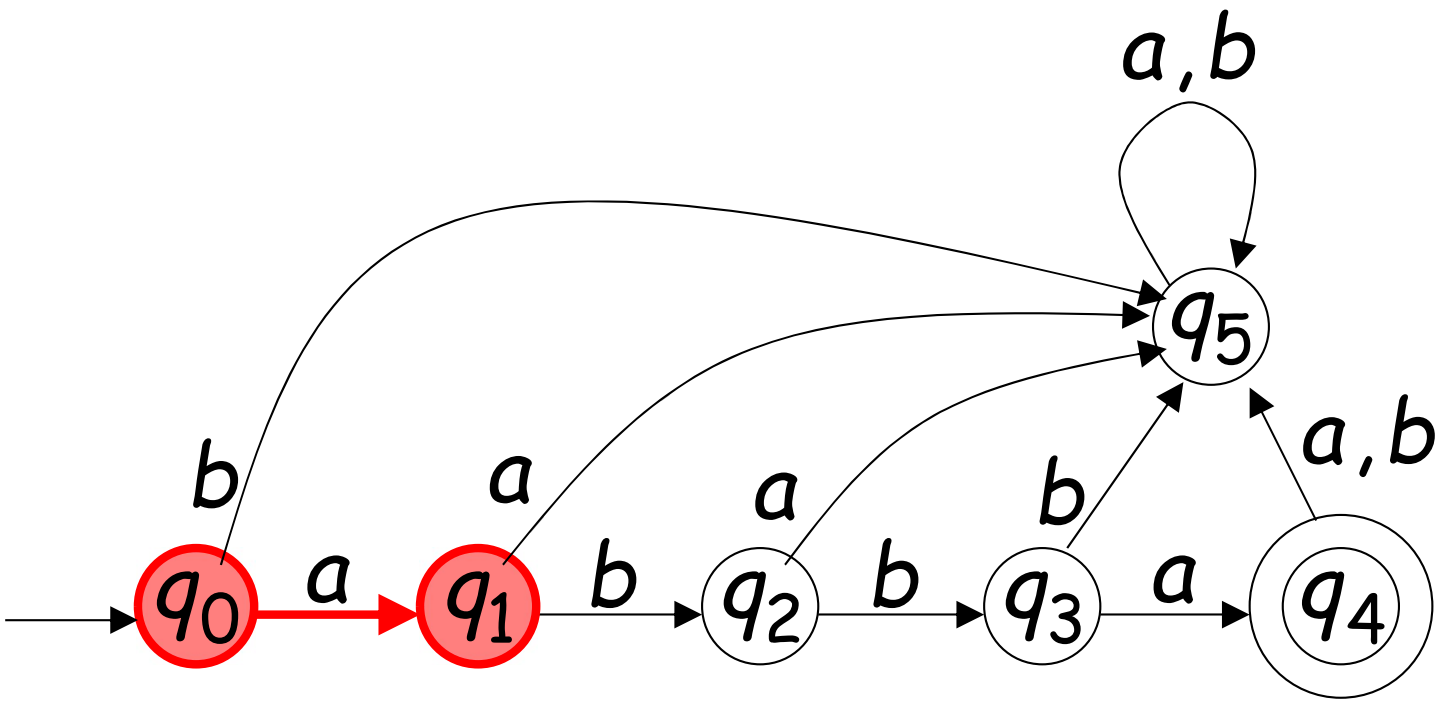


Transition Function δ

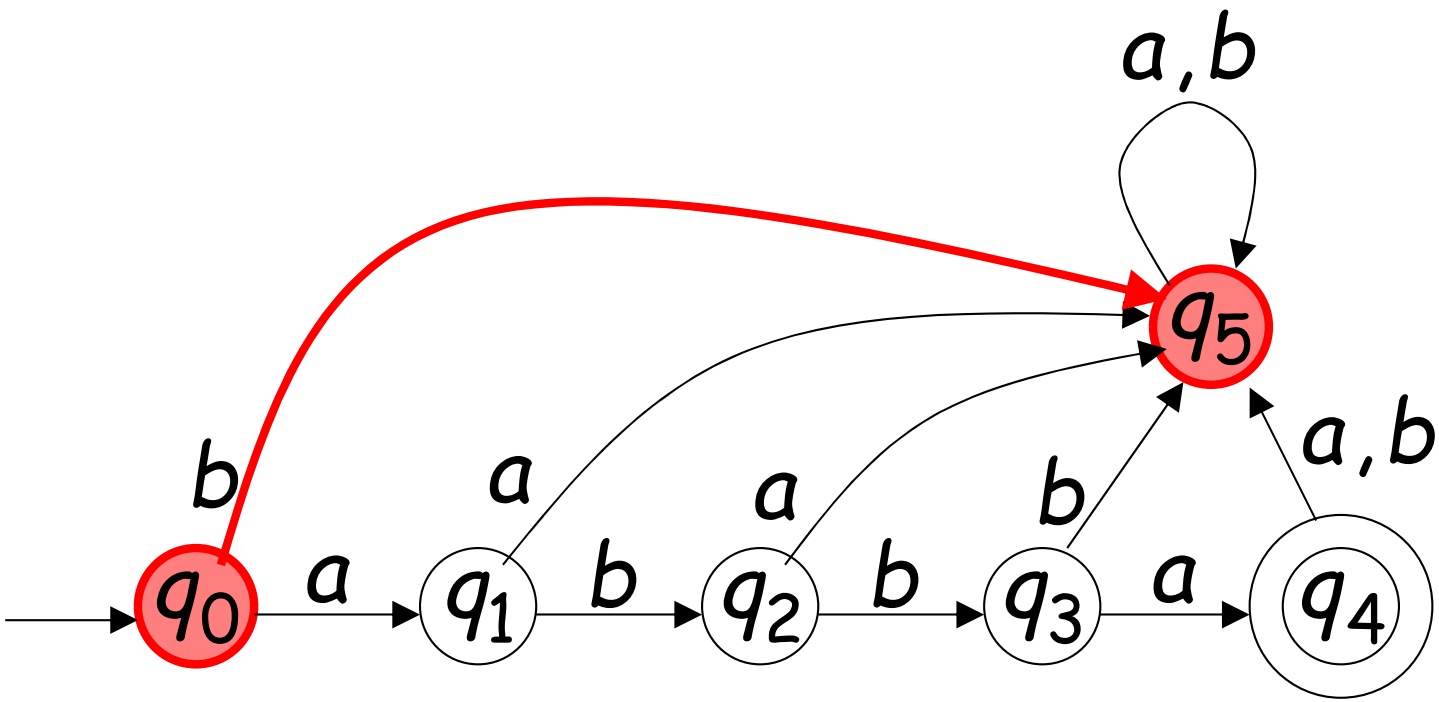
$$\delta : Q \times \Sigma \to Q$$



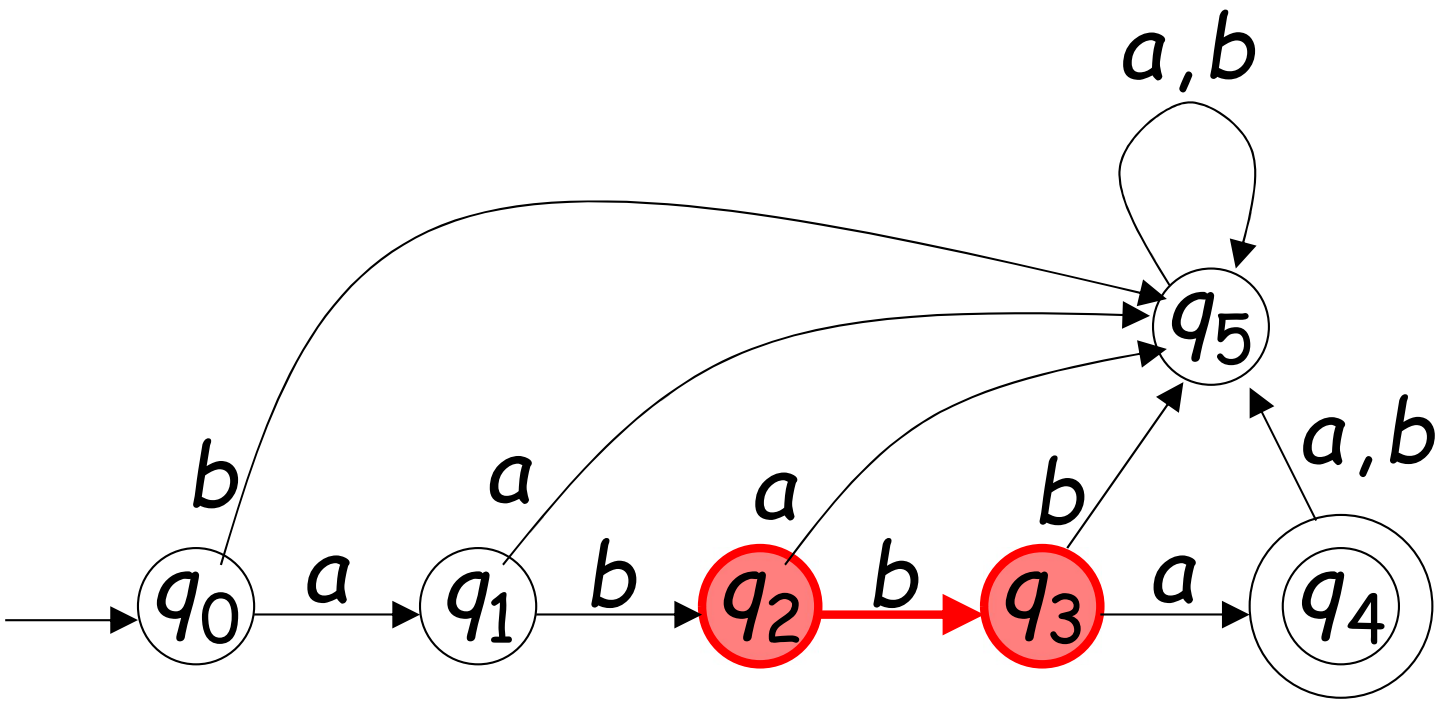
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$



$$\delta(q_2, b) = q_3$$

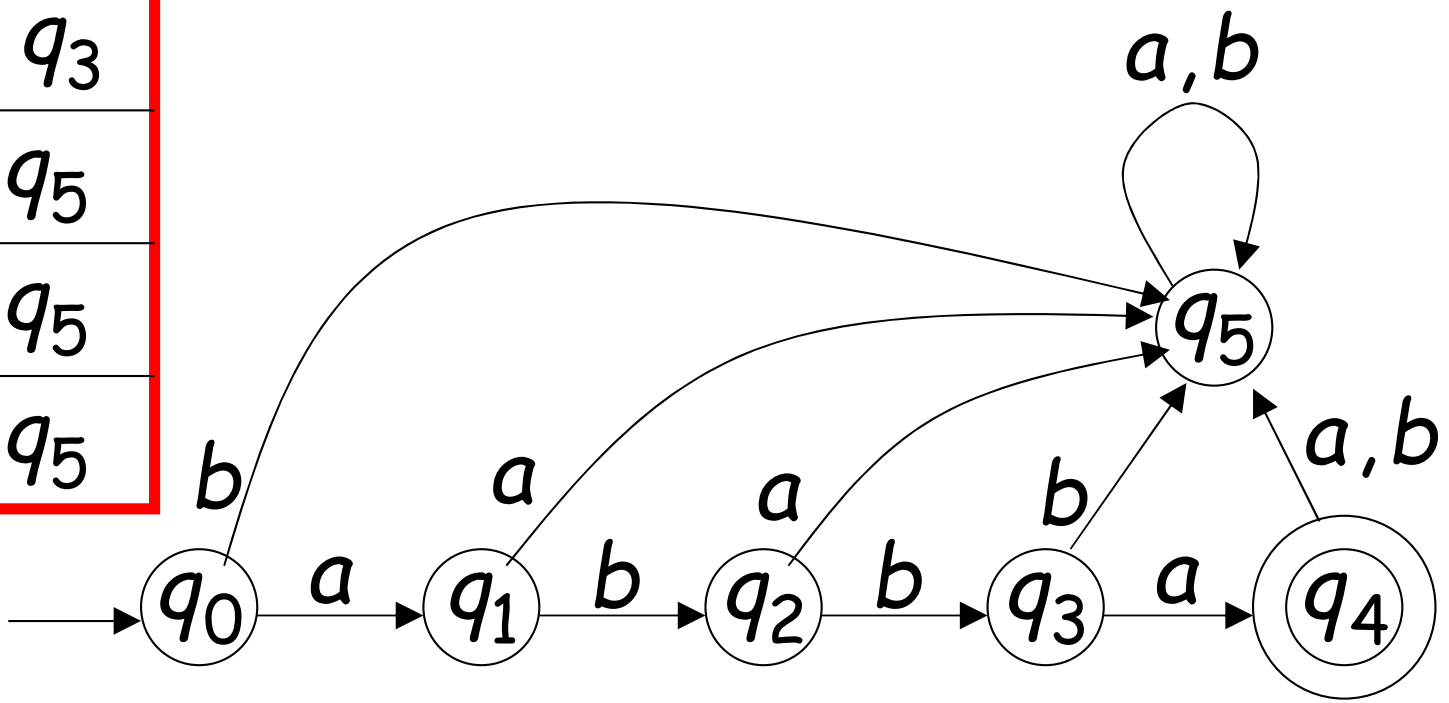


Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

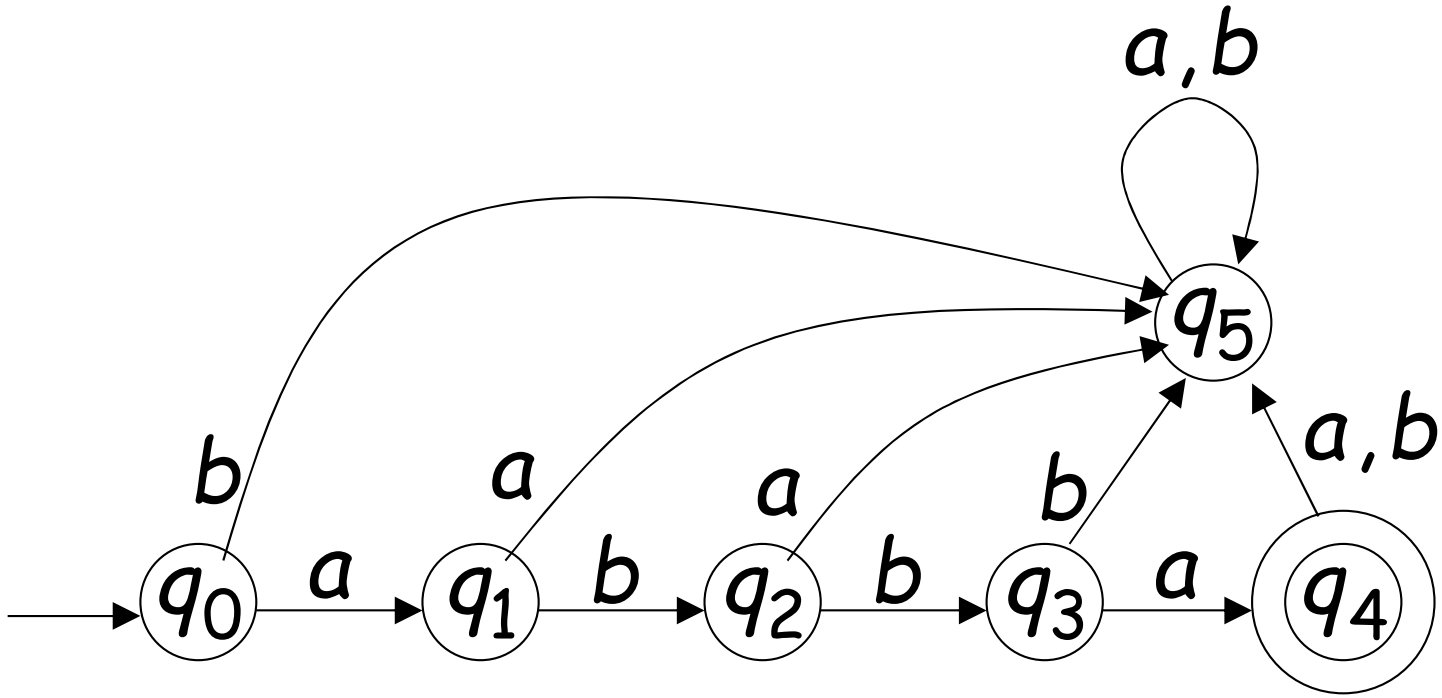
Next_state

Transition Table

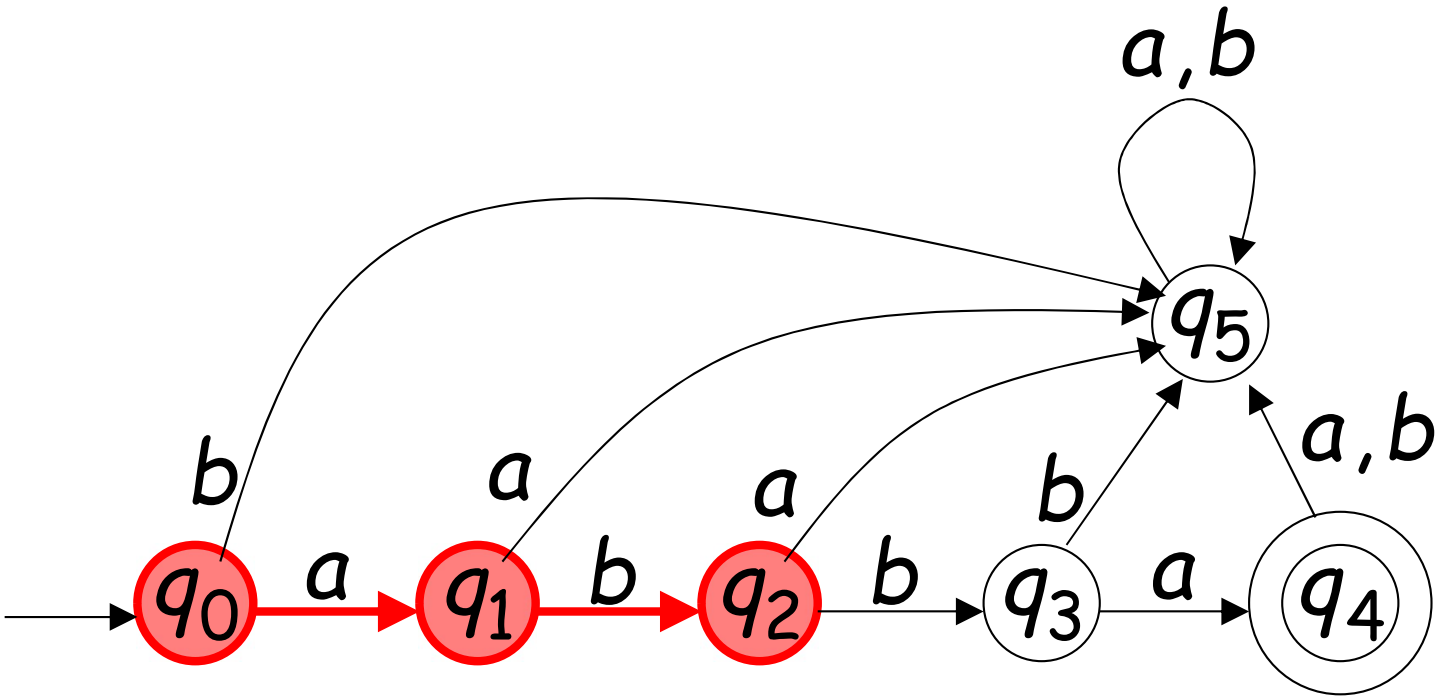


Extended Transition Function δ^*

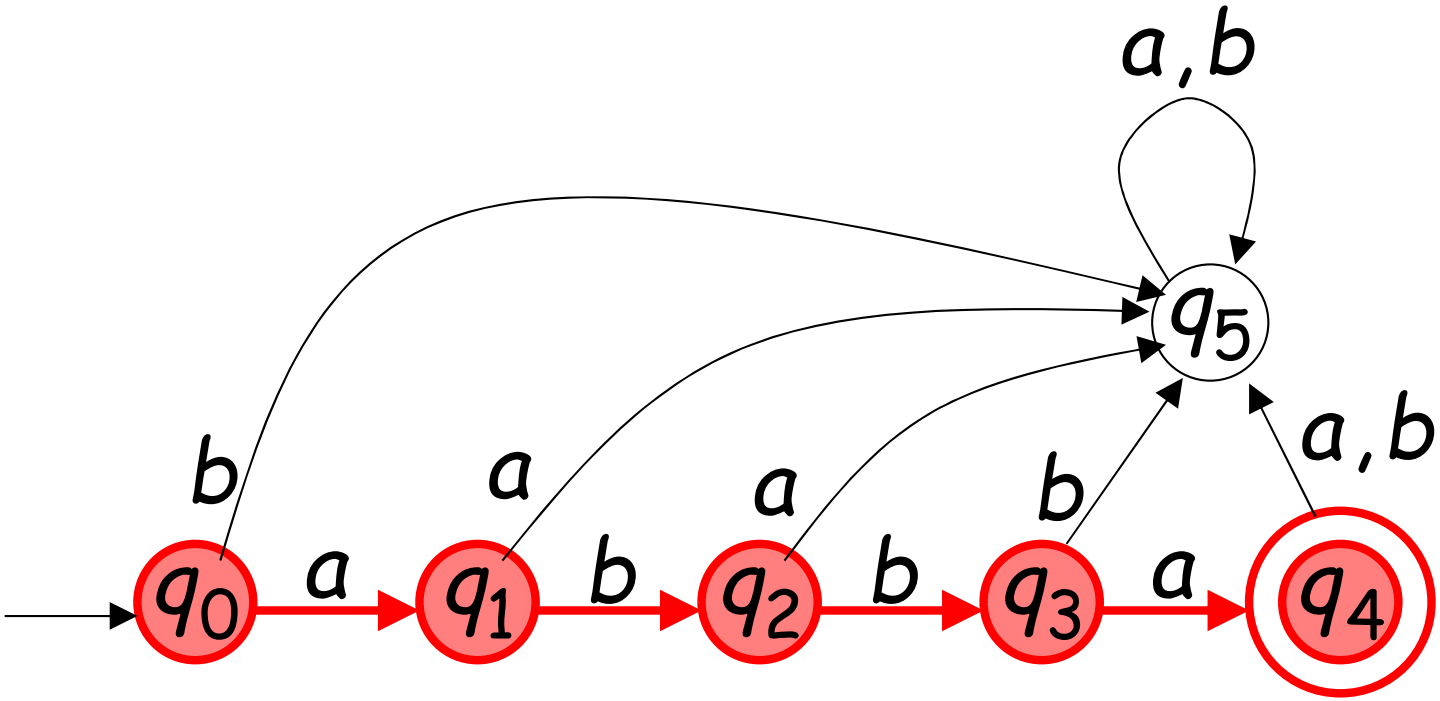
$$\delta^* : Q \times \Sigma^* \to Q$$



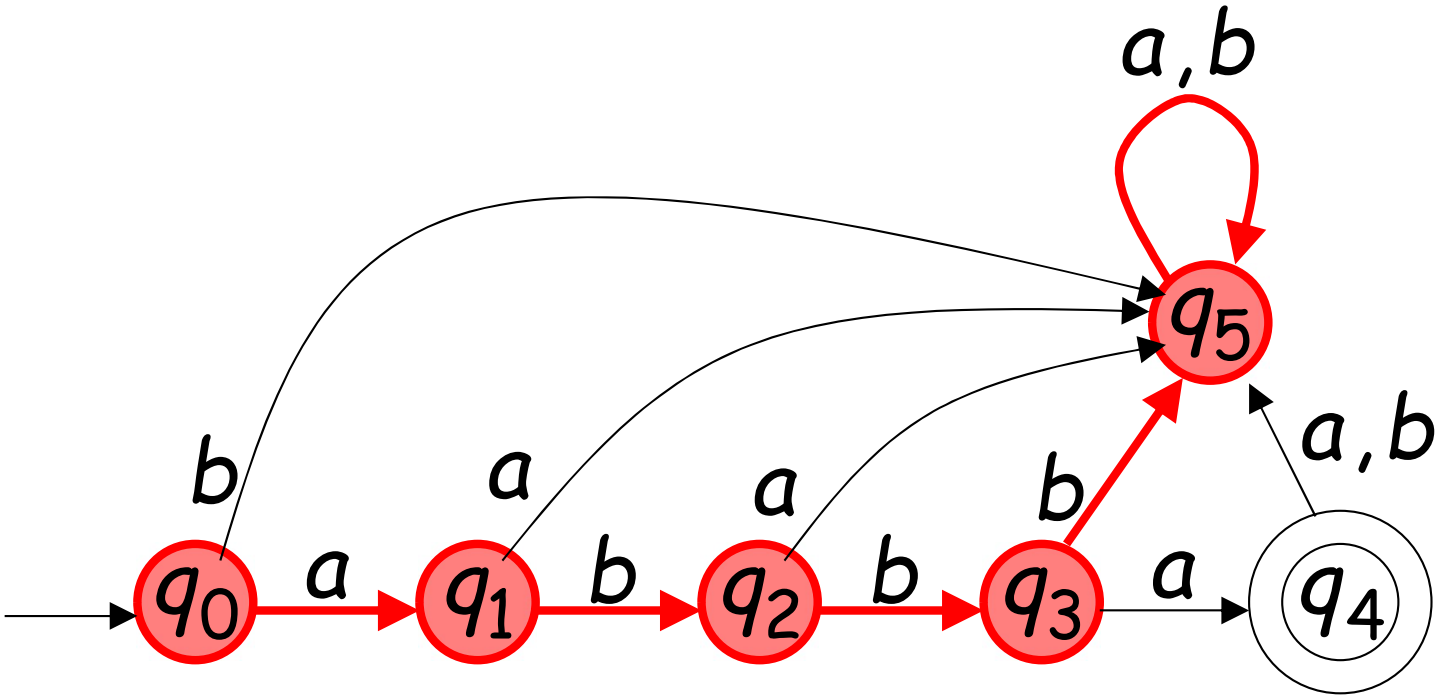
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$



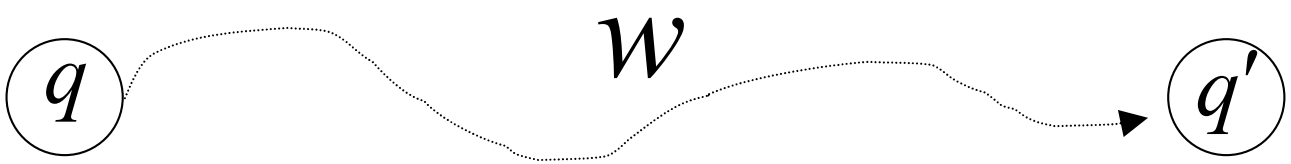
$$\delta^*(q_0, abbbaa) = q_5$$



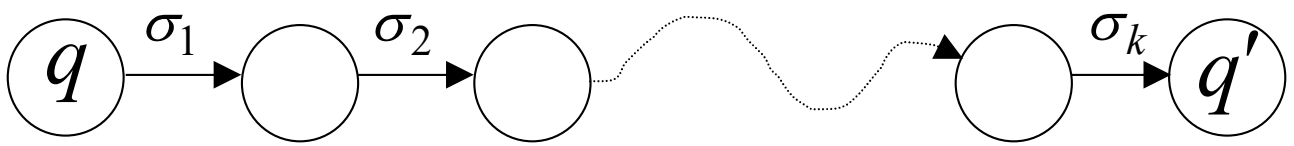
Observation: If there is a walk from q to q' with label w

■ Theorem 2.1

iff $\delta^*(q,w)=q'$

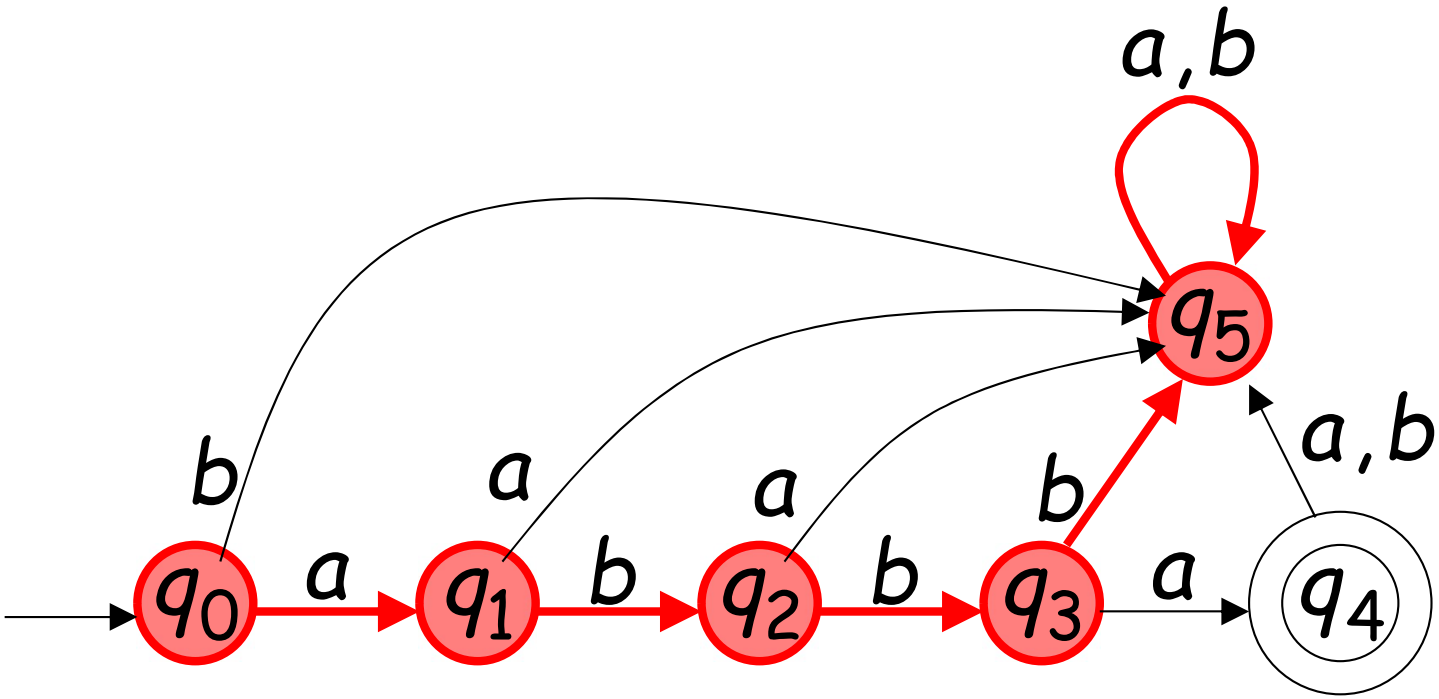


$$w=\sigma_1\sigma_2\cdots\sigma_k$$



Example: There is a walk from q_0 to q_5 with label $abbbaa$

$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q,\lambda) = q$$

$$\delta^*(q,w\sigma) = \delta(\delta^*(q,w),\sigma)$$

$$\delta^*(q,w\sigma) = q'$$

$$\delta(q_1,\sigma) = q'$$

$$\Rightarrow \delta^*(q,w\sigma) = \delta(q_1,\sigma)$$

$$\delta^*(q,w) = q_1$$

$$\Rightarrow \delta^*(q,w\sigma) = \delta(\delta^*(q,w),\sigma)$$

$$\delta^*(q_0, ab) =$$

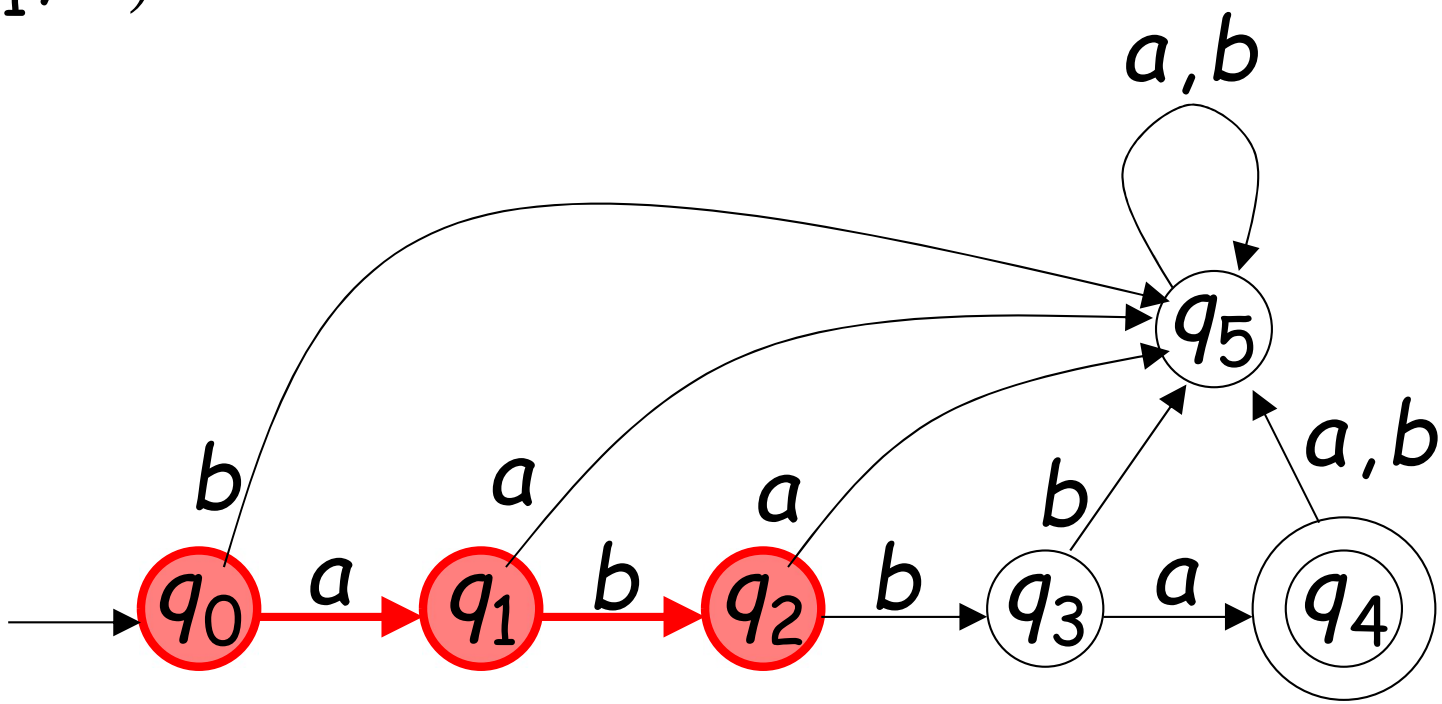
$$\delta(\delta^*(q_0, a), b) =$$

$$\delta(\delta(\delta^*(q_0, \lambda), a), b) =$$

$$\delta(\delta(q_0, a), b) =$$

$$\delta(q_1, b) =$$

$$q_2$$



Languages Accepted by DFAs

Take DFA M

Definition:

- The language $L(M)$ contains all input strings accepted by M

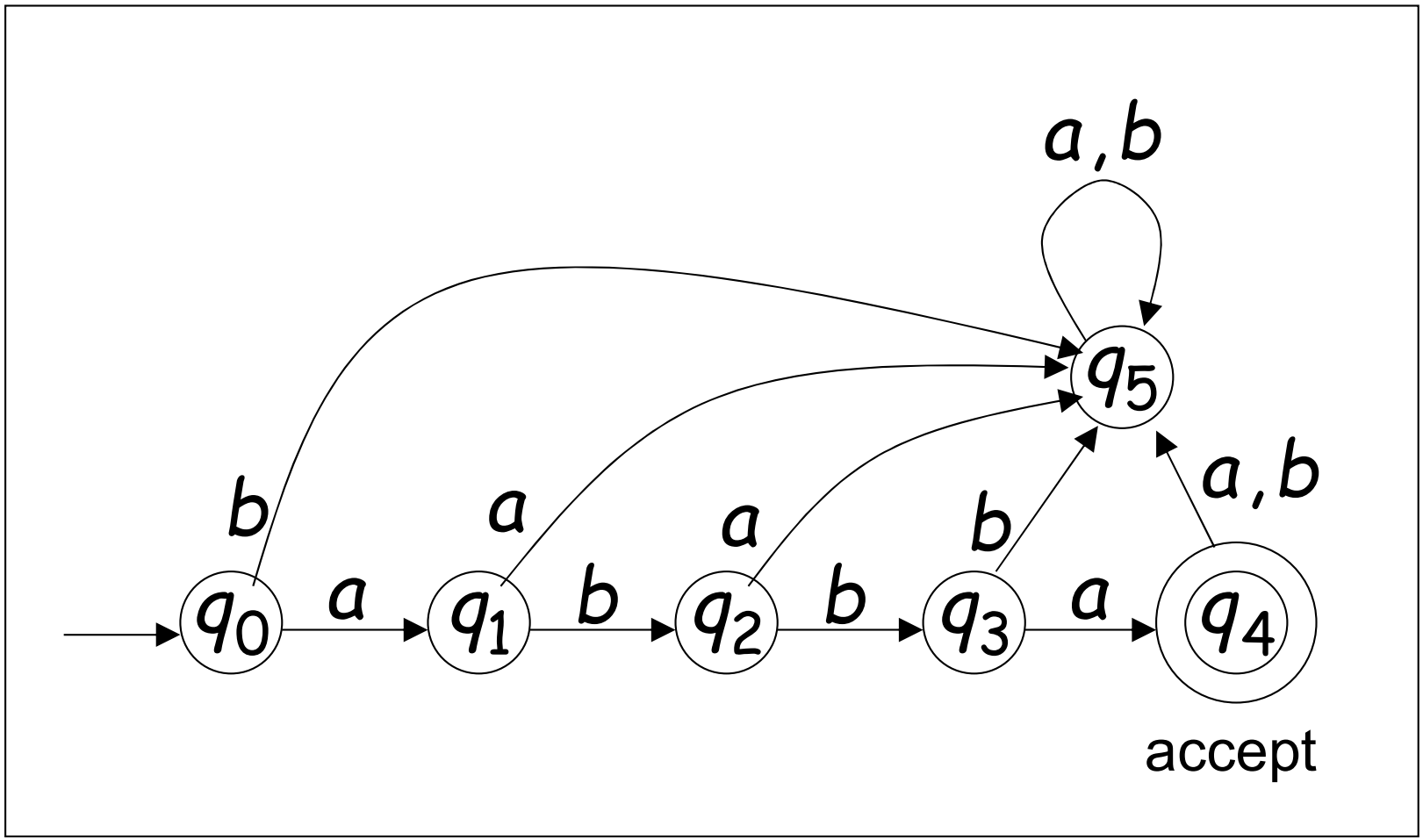
- $L(M)$ = { strings that drive M to a final state}

↳ accept case

Example

$L(M) = \{abba\}$

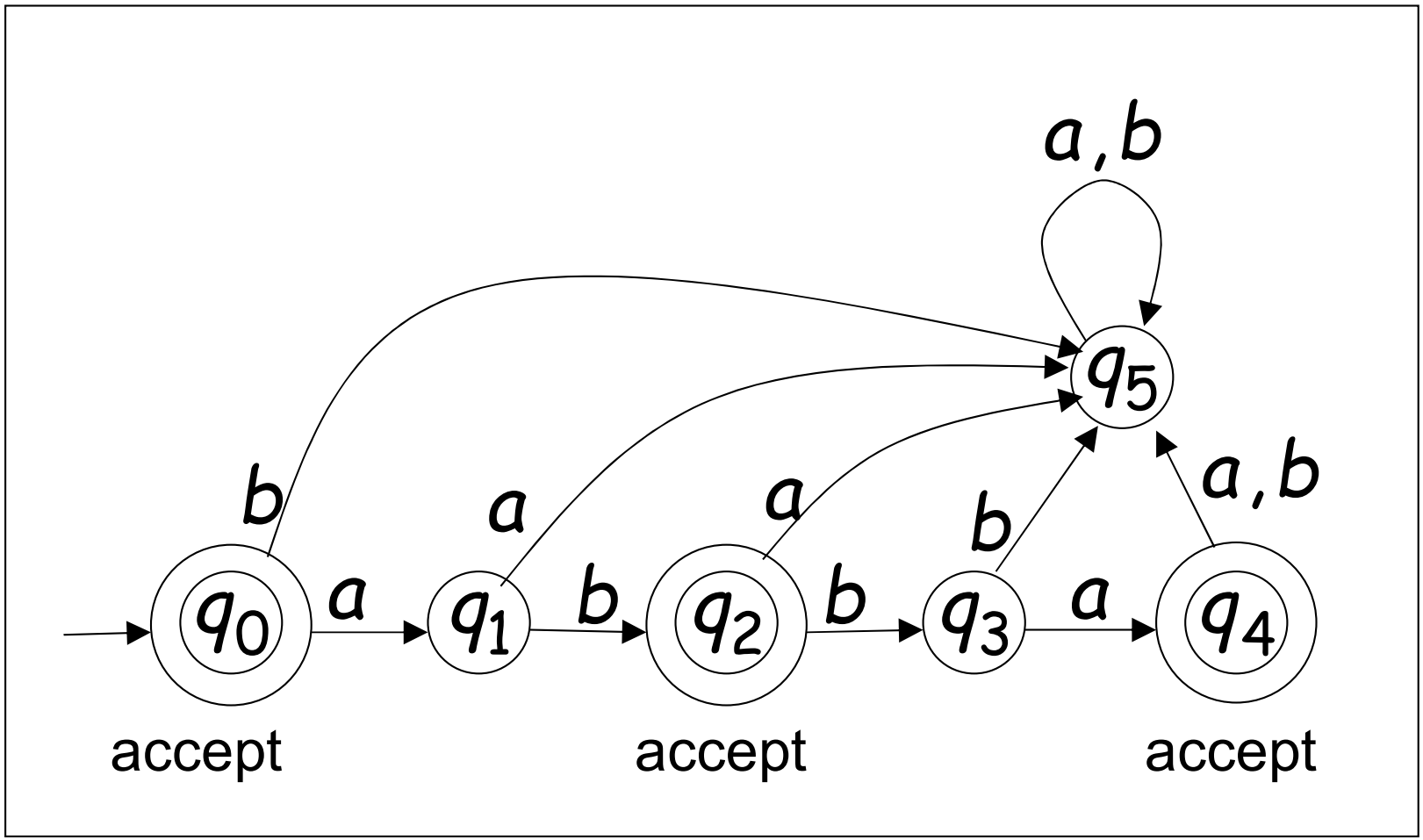
M



Another Example

$L(M) = \{\lambda, ab, abba\}$

M



Formally

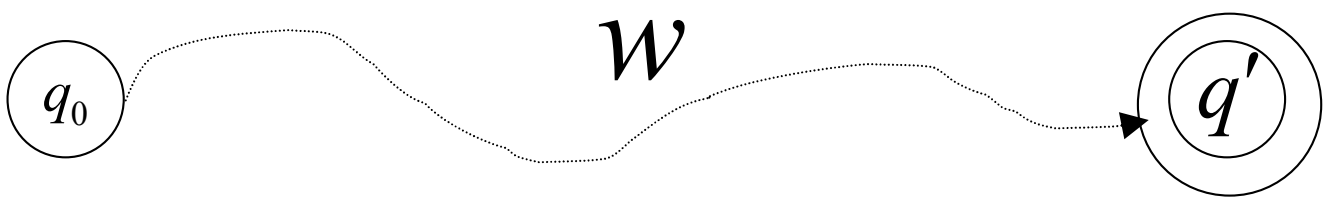
For a DFA $M = (Q, \Sigma, \delta, q_0, F)$

Language accepted by M :

$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \in F\}$$

與的有可能string

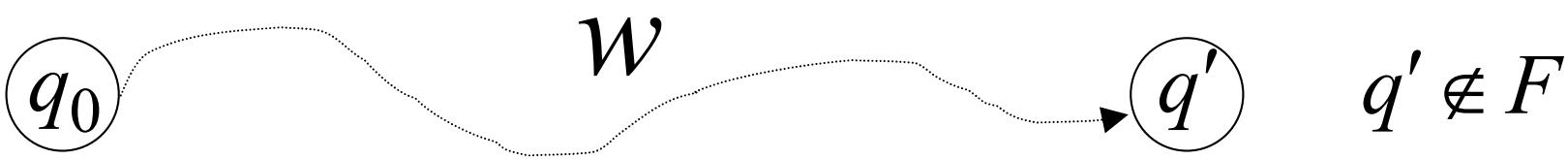
$q' \in F$



Observation

- Language rejected by M :

$$\overline{L(M)} = \{w \in \Sigma^* : \delta^*(q_0, w) \notin F\}$$

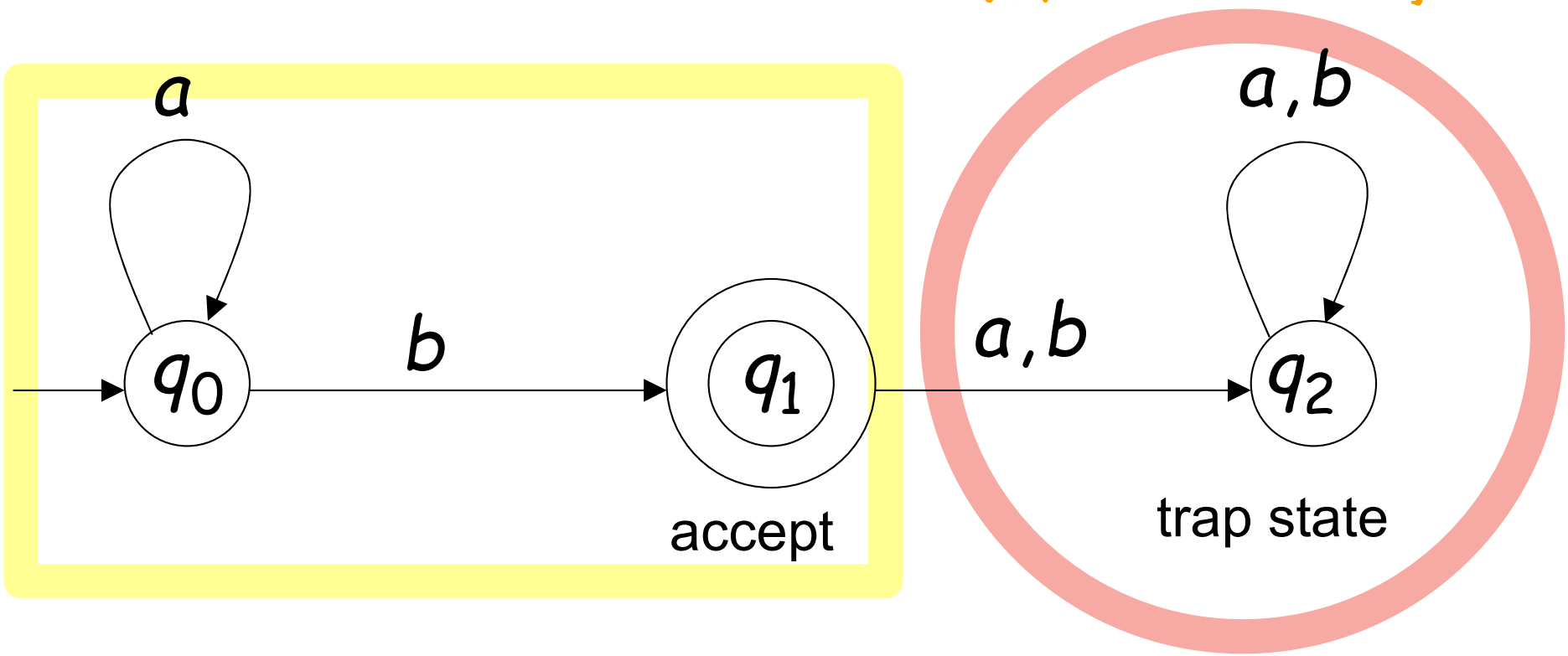


Example 2.2

$$M = (Q, \Sigma, \delta, q_0, F)$$

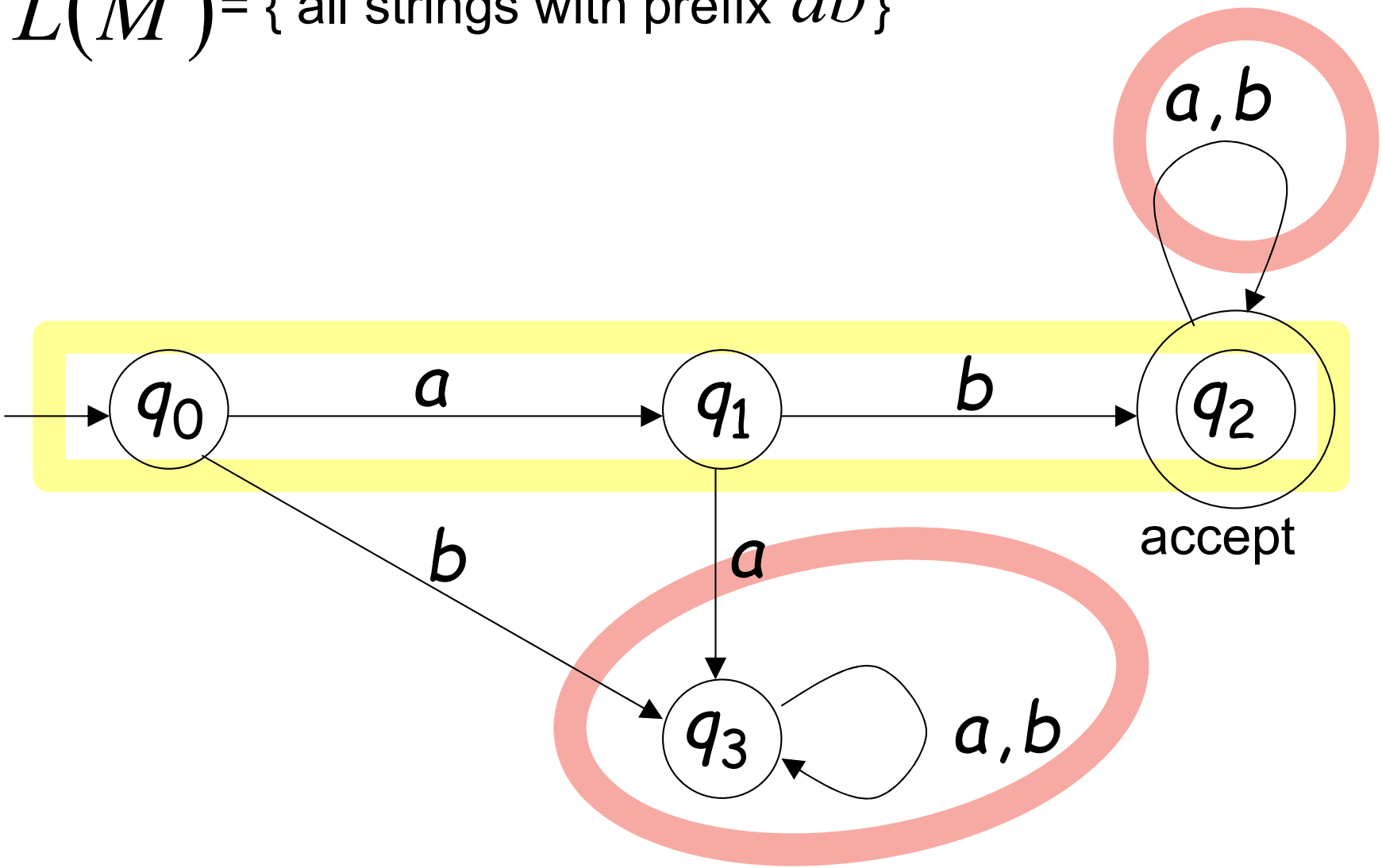
$$L(M) = \{a^n b : n \geq 0\}$$

*所有狀態都要吃參數到下一狀態



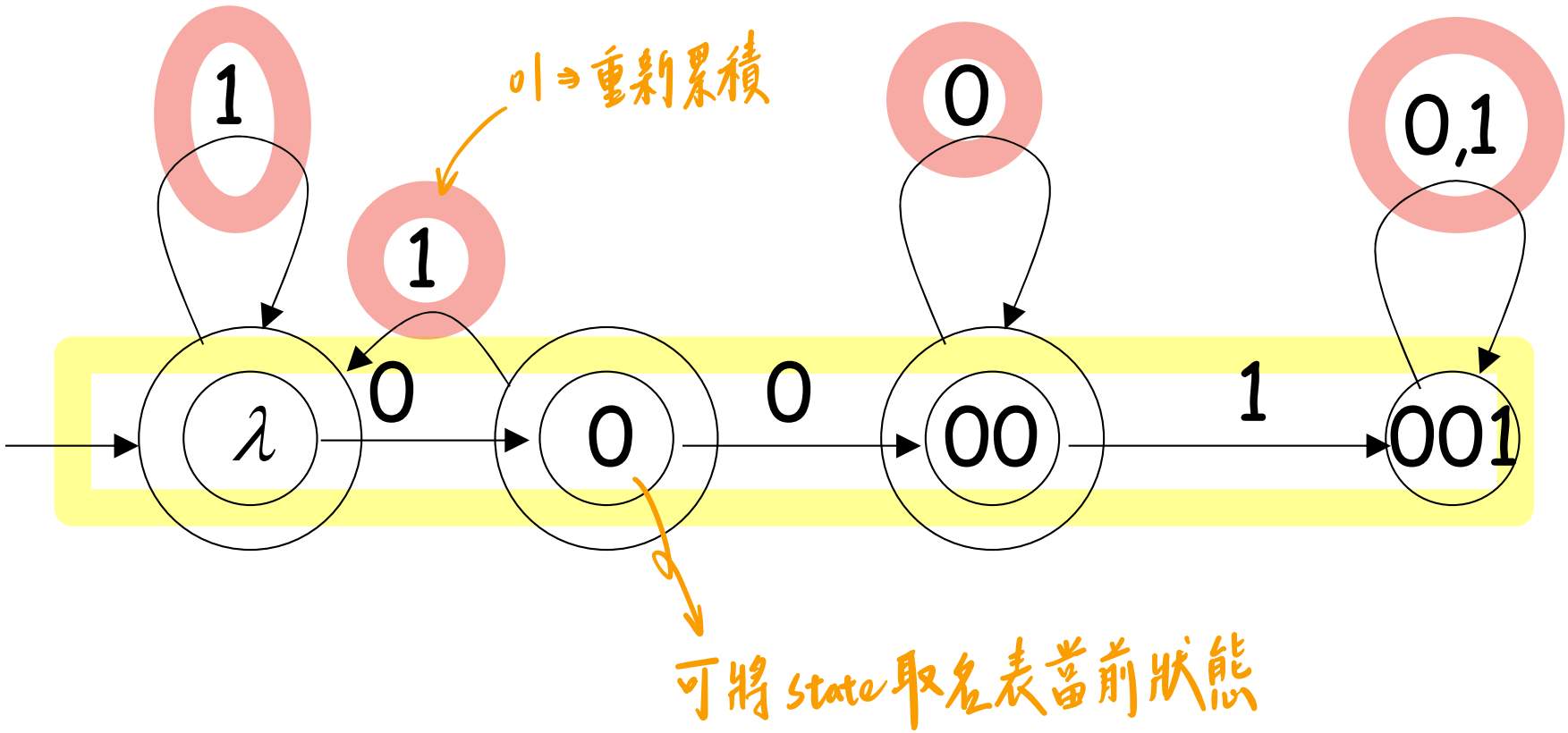
Example 2.3

$L(M)$ = { all strings with prefix ab }



Example 2.4

$L(M)$ = { all strings without substring 001 }



Regular Languages

A language L is regular iff there exists some DFA M such that $L = L(M)$

All regular languages form a language family

有無窮個

Examples of regular languages:

$$\{abba\} \qquad \{\lambda, ab, abba\} \qquad \{a^n b : n \geq 0\}$$

{ all strings with prefix ab }

{ all strings without substring 001 }

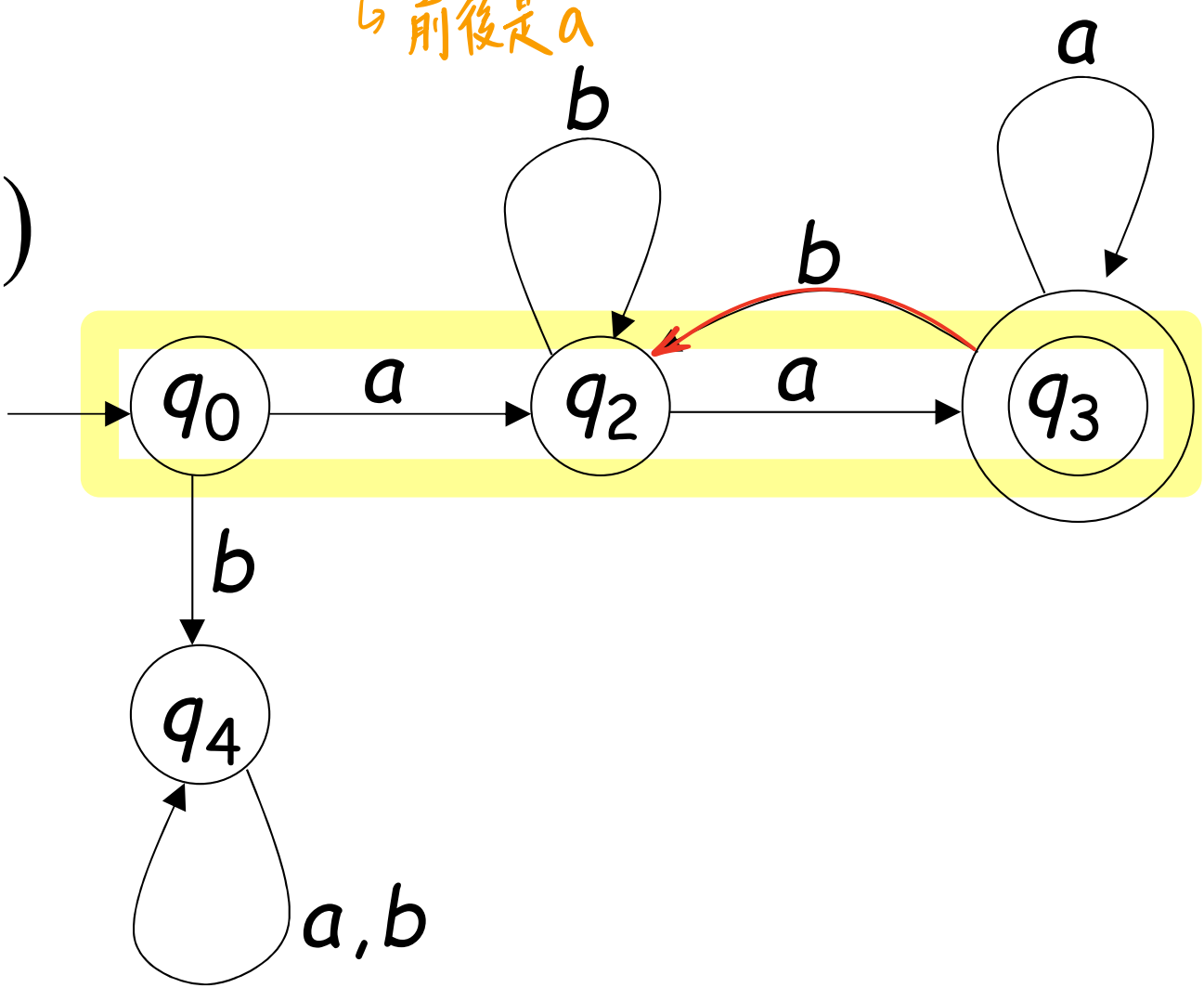
There exists DFA that accept these Languages

Example 2.5

The language $L = \{awa : w \in \{a,b\}^*\}$ is regular:

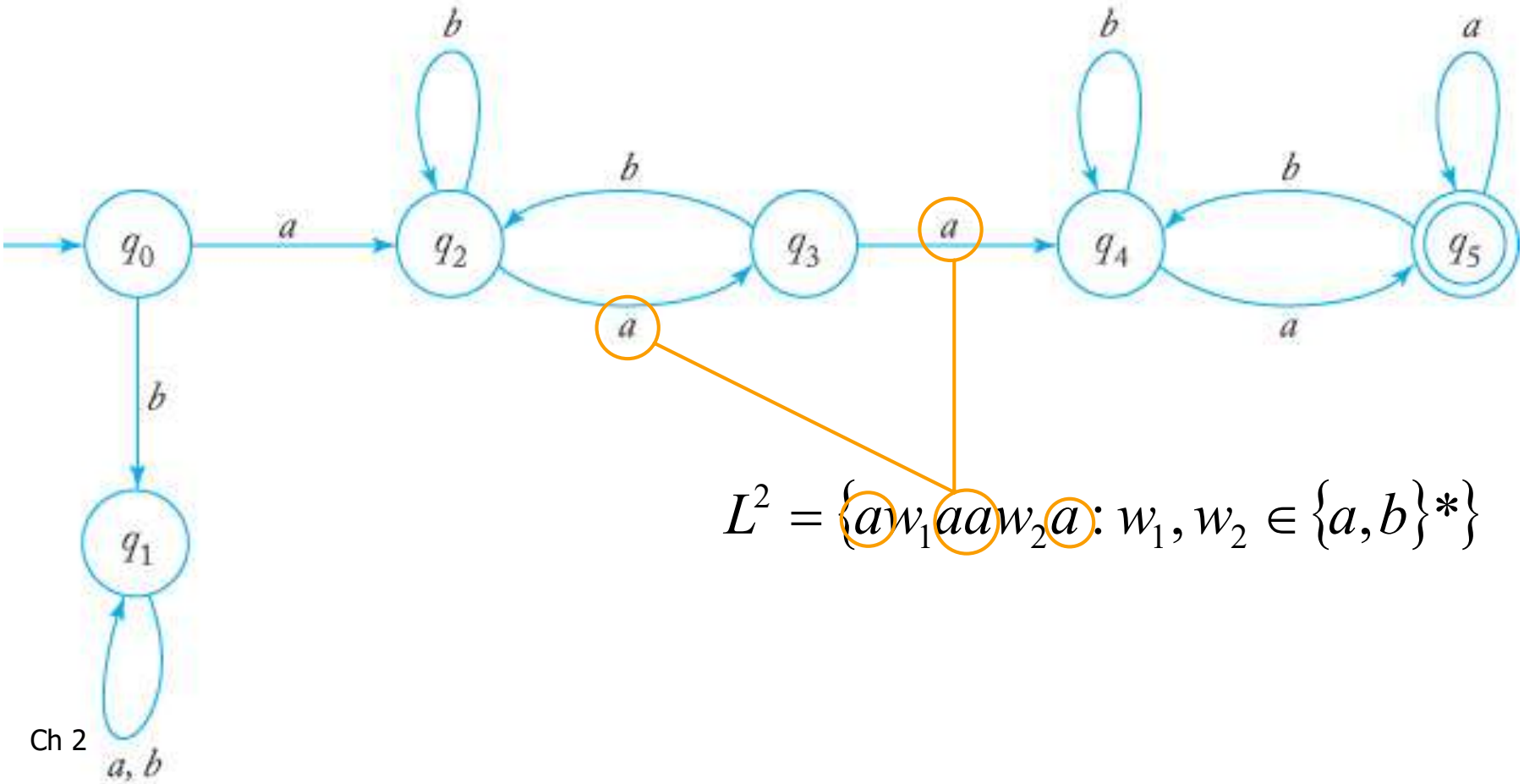
↳ 前後是a

$$L = L(M)$$



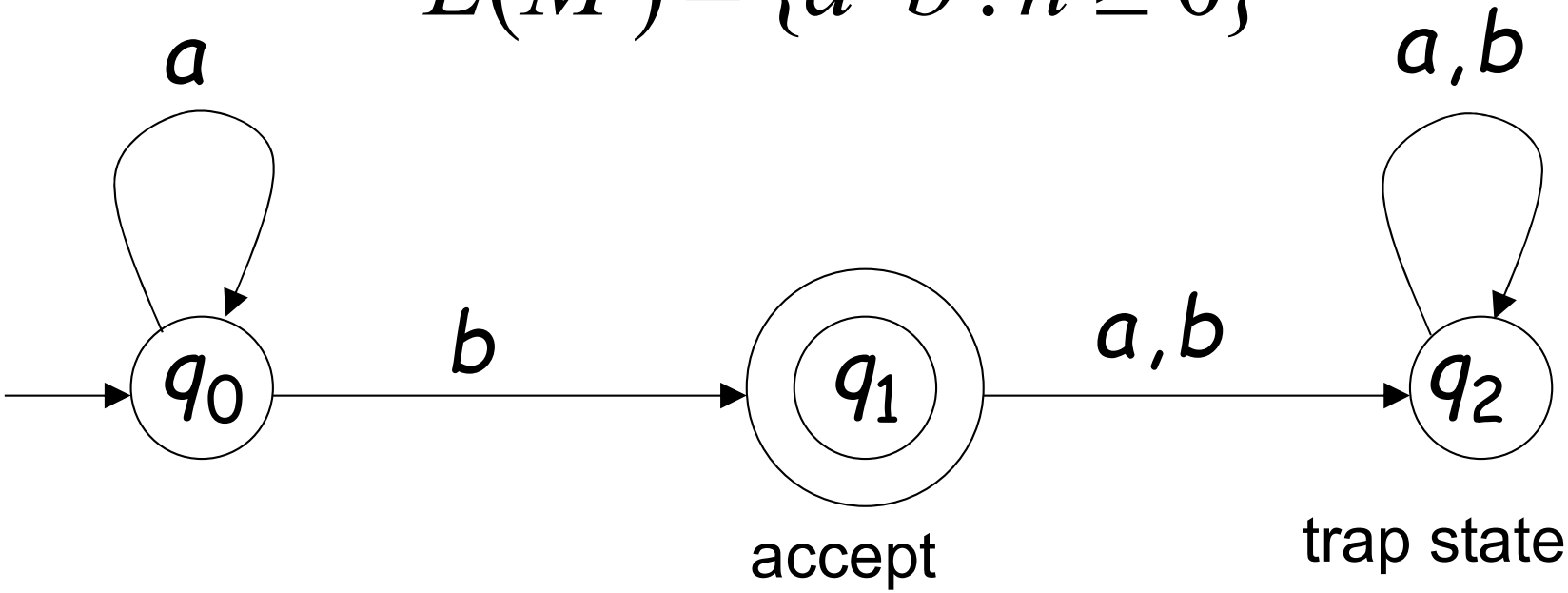
Example 2.6

The language $L = \{awa : w \in \{a,b\}^*\}$ is regular, how about L^2?



$$L^2 = \{awaawa : w_1, w_2 \in \{a,b\}^*\}$$

$$L(M) = \{a^n b : n \geq 0\}$$



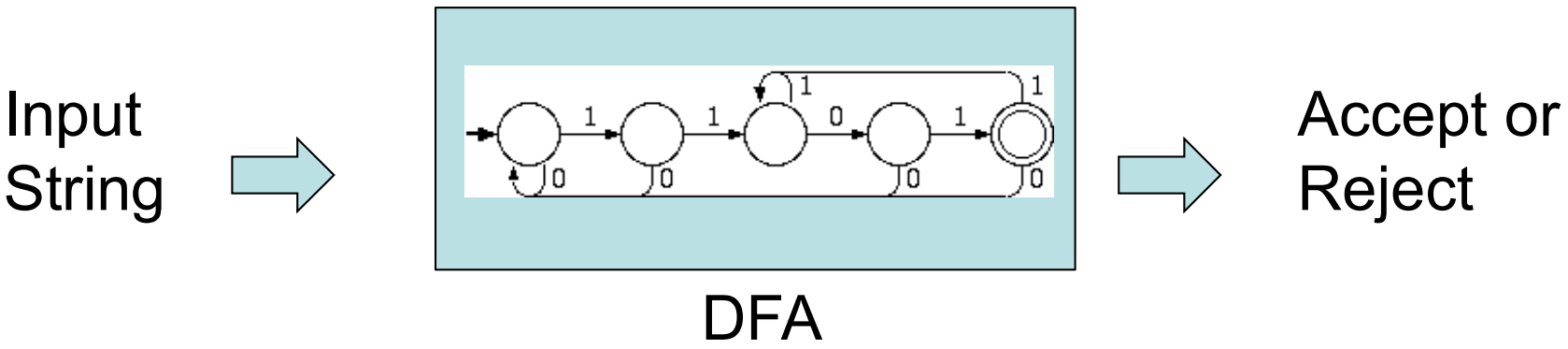
$$L = \{a^n b^n : n \geq 0\}$$?

n有無窮個情況
⇒DFA做不到

There exist languages which are not Regular:

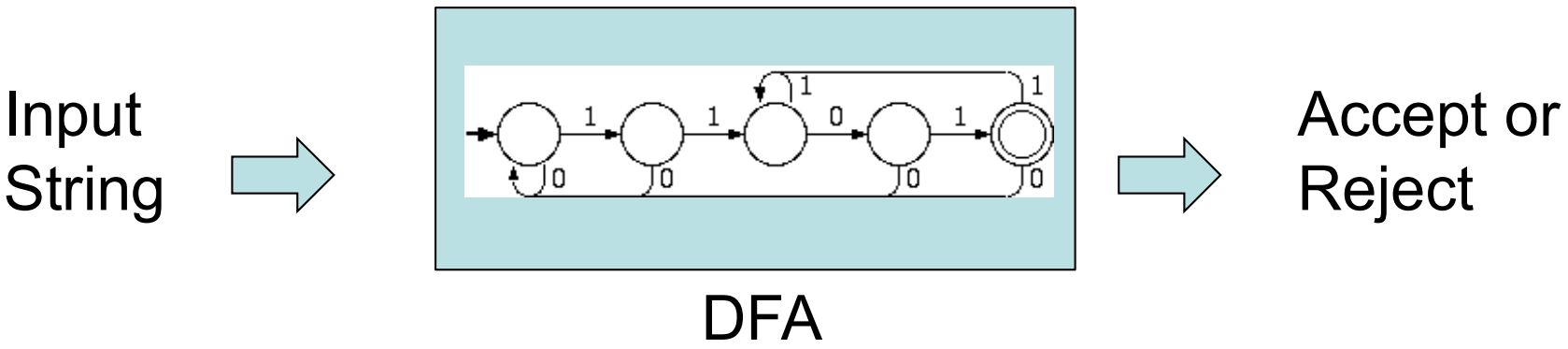
There is no DFA that accepts such a language
(we will prove this later in the class)

DFA Recap



- A machine with finite number of states, some states are accepting states, others are rejecting states
- At any time, it is in one of the states 在任意時間，處在某state內
- It reads an input string, one character at a time

DFA Recap



- After reading each character, it moves to another state depending on what is read and what is the current state
- If reading all characters, the DFA is in an accepting state, the input string is accepted.
- Otherwise, the input string is rejected.

必讀完在final state才為accept

Definition 2.1

Deterministic Finite Accepter (DFA) is define by the 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q : a finite set of **internal states**

Σ : a finite set of symbols called **input alphabet**

δ : Q x Σ → Q called **transition function**

q_0 : $q_0 \in Q$ is the **initial state**

F : $F \subseteq Q$ is a set of **final states**

Regular Languages

A language L is regular iff there exists some DFA M such that $L = L(M)$

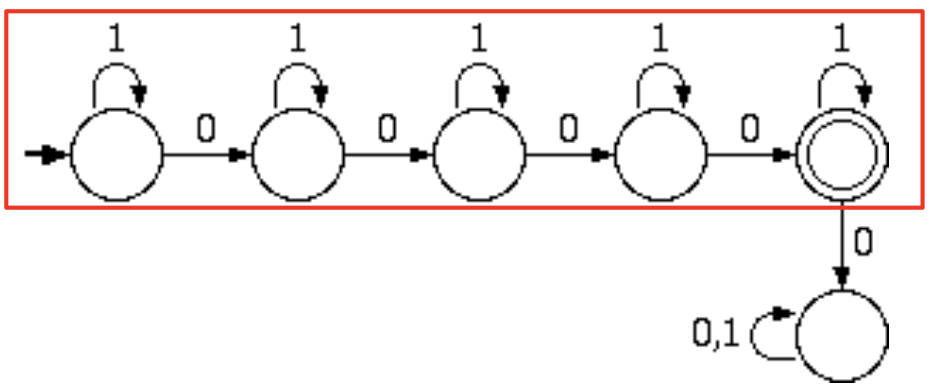
Definition:

- The language $L(M)$ contains all input strings accepted by a DFA M

- $L(M)$= { strings that drive M to a final state}

More Examples

- All strings that contain exactly 4 "0"s.
- All strings containing exactly 4 "0" s and at least 2 "1" s.
- All strings of length at most five.
- All strings ending in "1101".
- All strings whose binary interpretation is divisible by 5.
- All strings that contain the substring 0101.
- All strings that don’t contain the substring 110.



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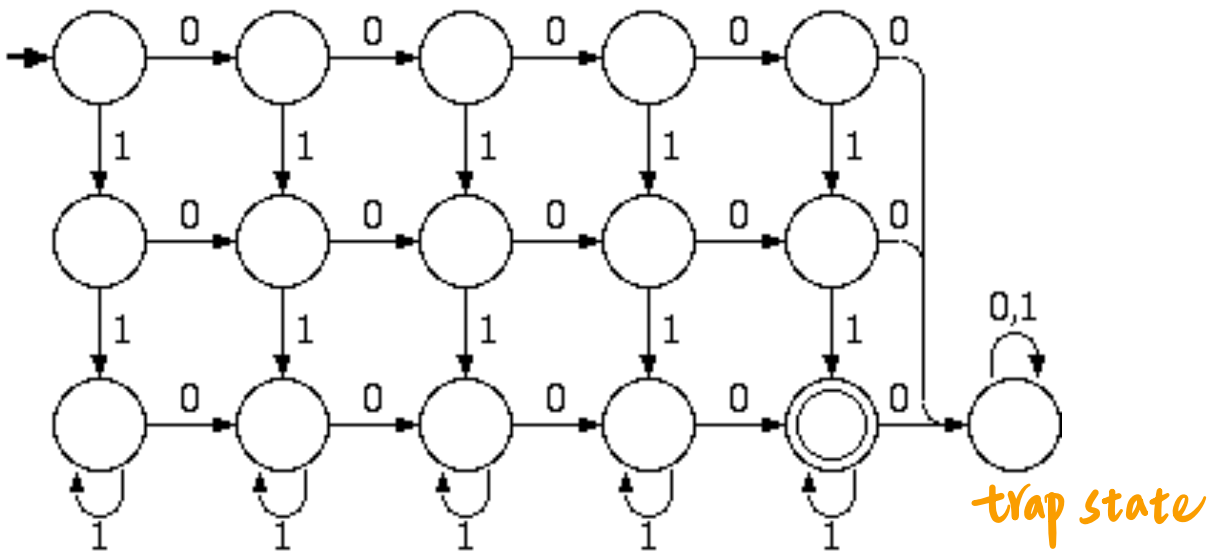

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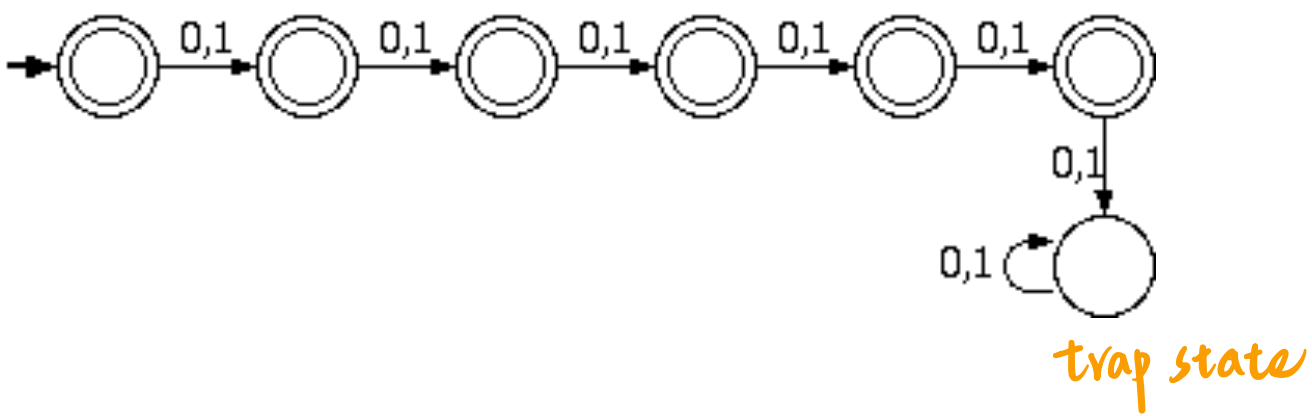

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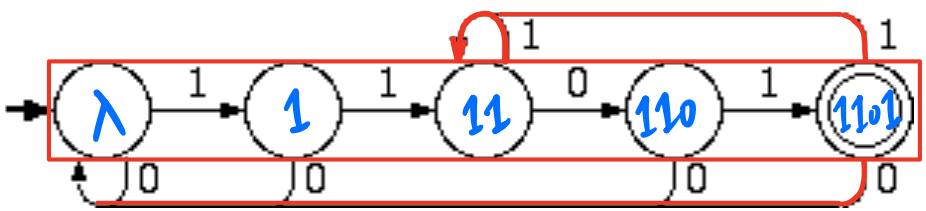

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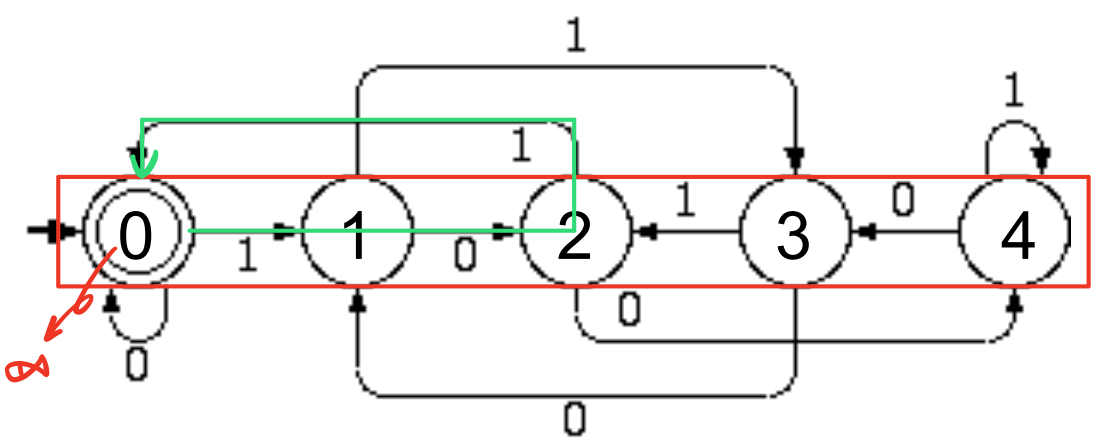

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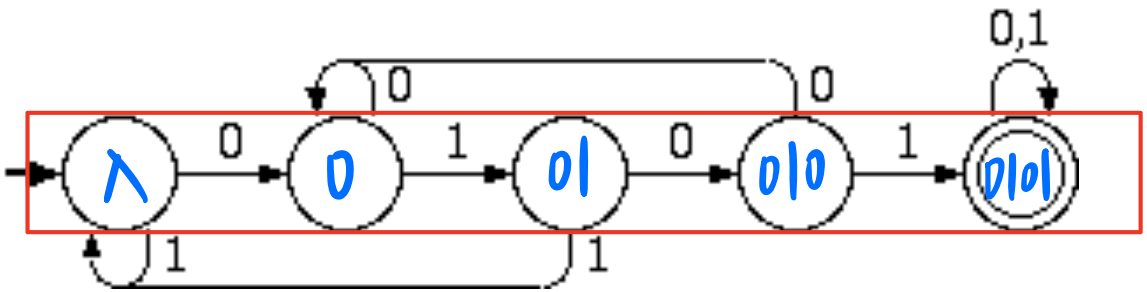

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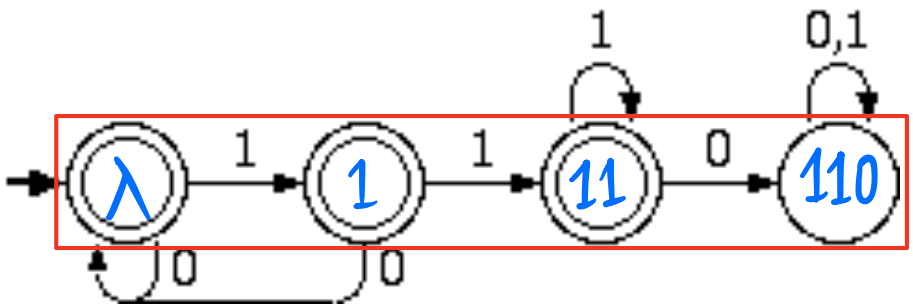
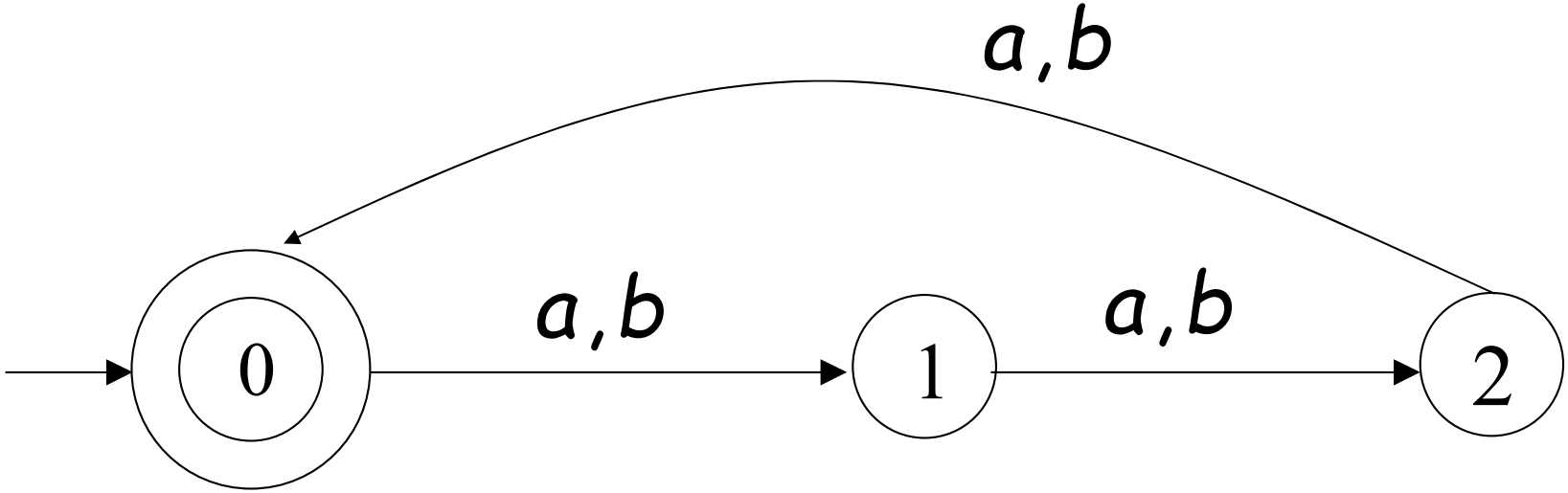


Exercise 2.1.7

Find DFAs for the following languages on ∑={a,b}

(a) L = {w: |w| mod 3 = 0} → w的長度%3=0

(b) L = {w: $n_a(w)$ mod 3 > $n_b(w)$ mod 3}



*DFA無記憶性

Exercise 2.1.7

Find DFAs for the following languages on ∑={a,b}

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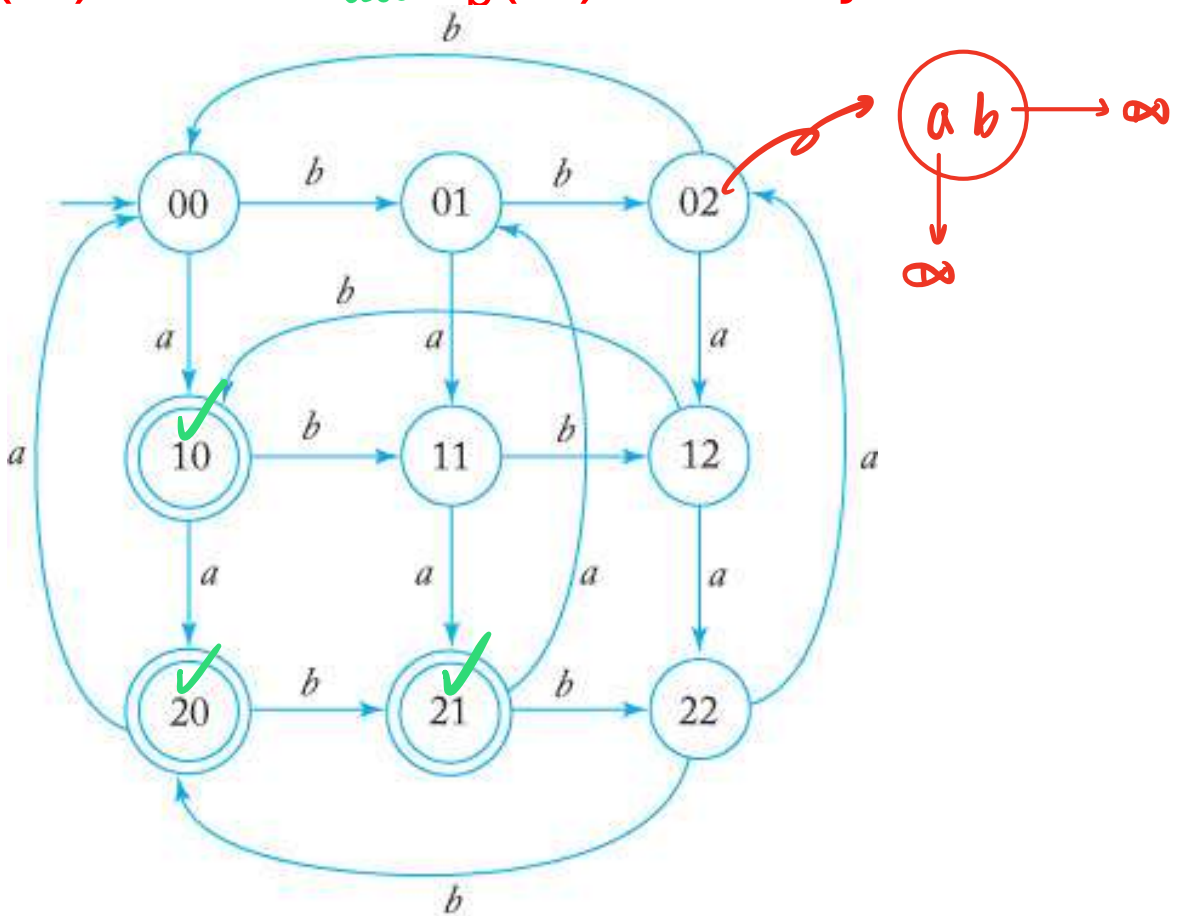


Exercise 2.1.9

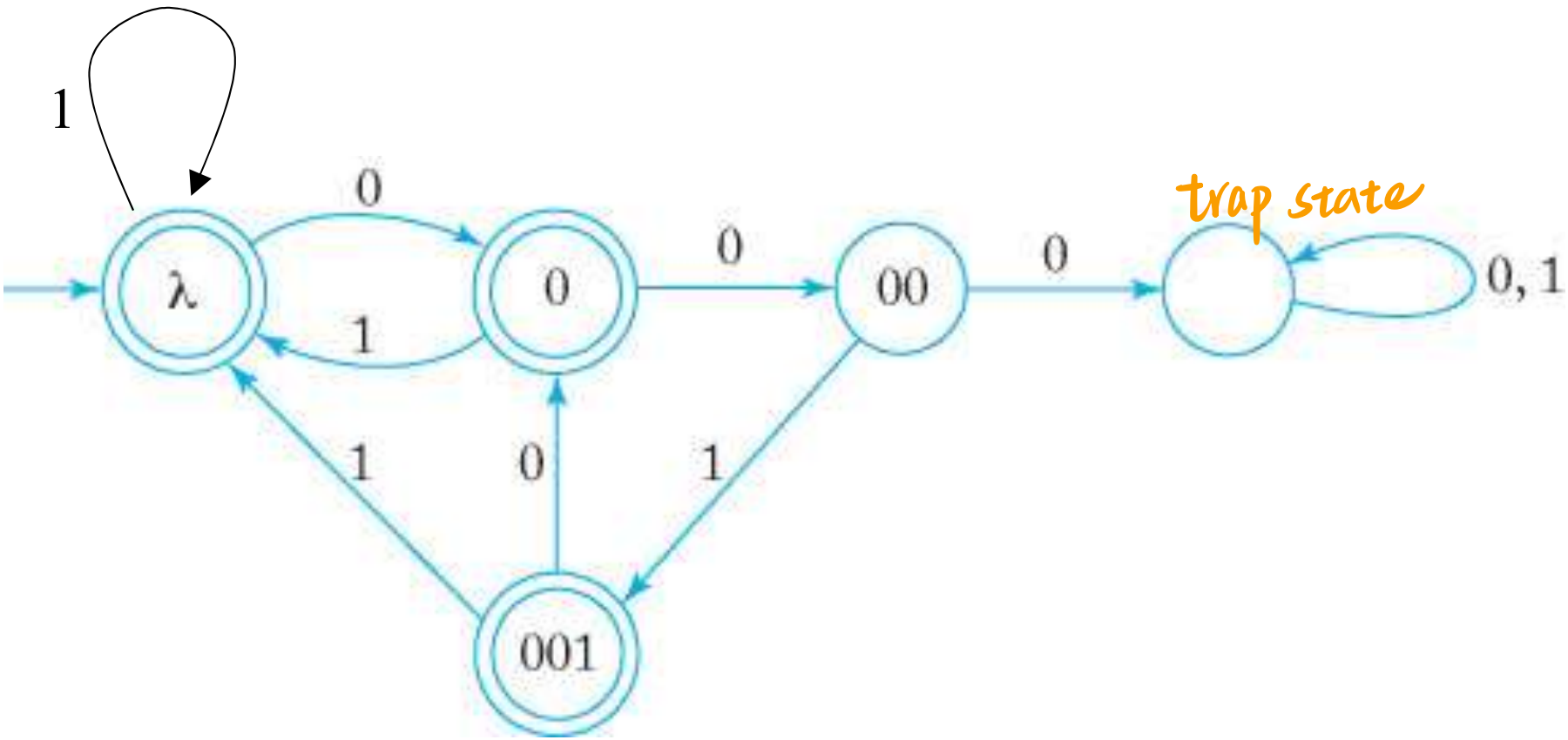
(a)Every 00 is followed immediately by a 1.

Ex: 101, 0010,0010011001 ∈ L

0001 and 00100 ∉ L

↳只要00一定要接1

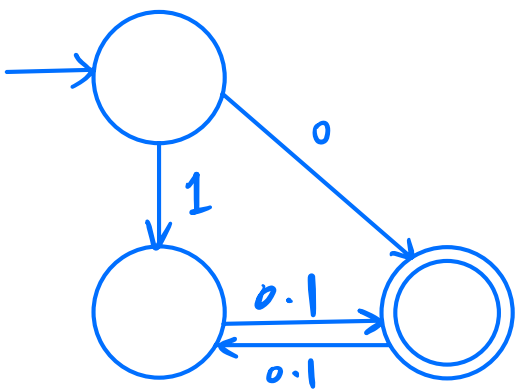
$\Sigma = \{0,1\}$

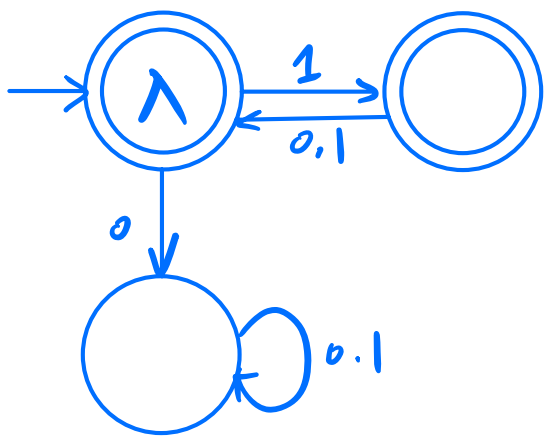


Questions?

Short Quiz

- All strings that start with 0 and have odd length or start with 1 and have even length.
- All strings where every odd position is a 1.





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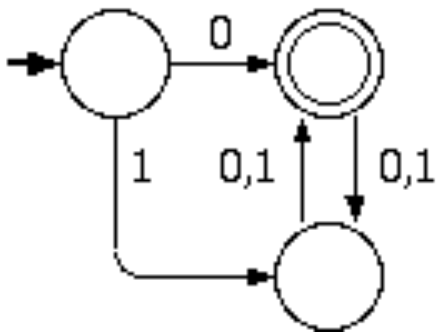

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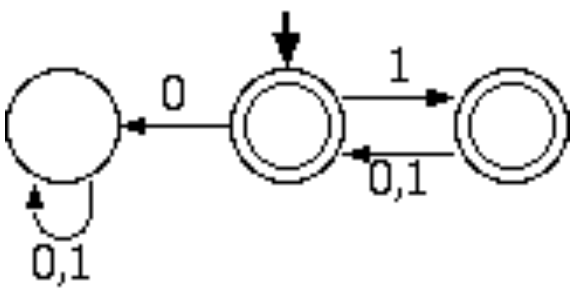
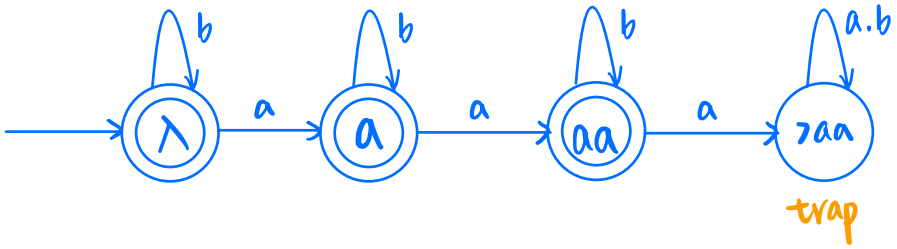
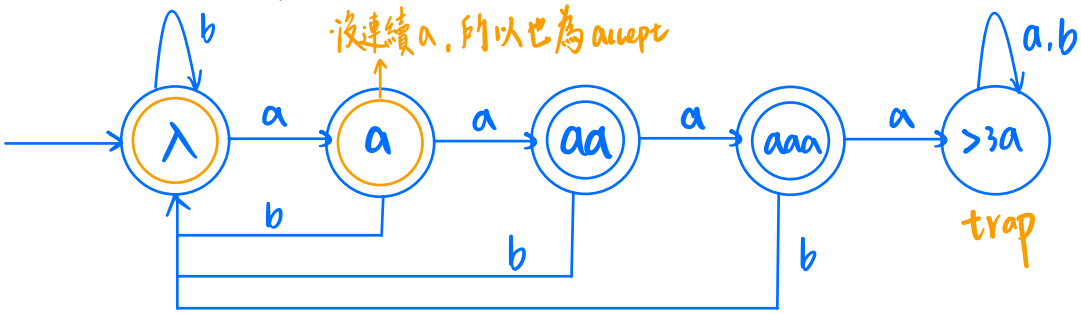


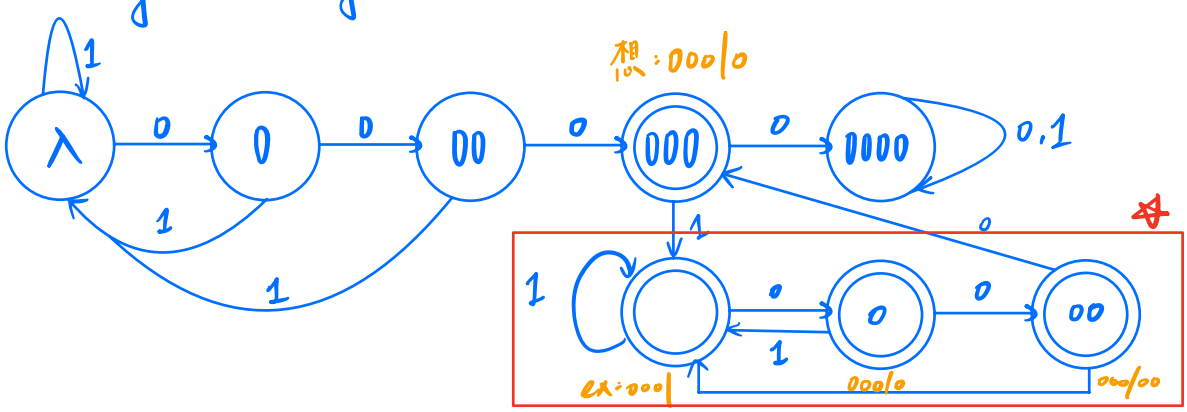
Q: all string with no more than two a's. $\Sigma\{a,b\}$



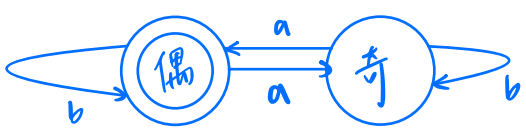
★ Q: $L = \{w:$ every consecutive a's has length either two or three. $\Sigma\{a,b\}$



★ Q: all strings containing 000 but not 0000. $\Sigma\{0,1\}$



Q: all strings with an even number of a's



☆

R: $L = \{w \in \Sigma^* : |w| \geq n,$ second-to-last symbol of w is $b\}$. $\Sigma\{a, b\}$

↳ 倒數第 n 個是 b