Homework One

Theory of Computation 2023

Important Note:

Please remember that you should return your answer at 10/11 (Wednesday) 15:10 and your HW should be handwritten. We will take your HW during the class. After 10/11 15:10, you must upload your HW to moodle. But remember penalty for late submission: 20% per day.

Q1: For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of

(a) all strings of even length.

(b) all strings of length greater than 5.

(c) all strings with an even number of a's and an odd number of b's.

(d) all the strings with exactly two a's and more than three b's.

(e) all strings with no more than two a's.

Q2: Find dfa's for the following languages on $\Sigma = \{a, b\}$.

(a) $L = \{w : |w| \bmod 5 = 0\ \}$.

(b) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 0\}$.

Q3: Let L be the language accepted by automaton in Figure 1. Find a dfa that accepts L^2.

Figure 1: Automaton diagram.

Q4: Find a dfa that accepts the language defined by the nfa in Figure 2.

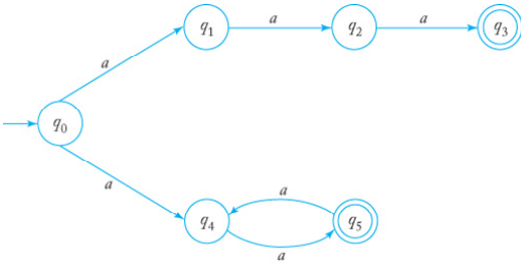

Figure 2: NFA diagram.

Q5: Design an nfa with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$

Q6: Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

Q7: Convert the nfa defined by

$\delta(q_0, a) = \{q_0, q_1\}$

$\delta(q_1, b) = \{q_1, q_2\}$

$\delta(q_2, a) = \{q_2\}$

$\delta(q_0, \lambda) = \{q_2\}$

with initial state q_0 and final state q_2 into an equivalent dfa.

Q8: Find a minimal dfa for the following language, and prove that the result is minimal.

$L = \{a^n : n \geq 0, n \neq 2\}$.

Q1: For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of

(a) all strings of even length.

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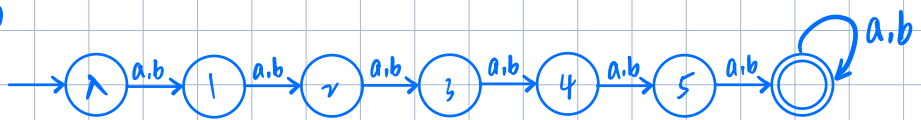
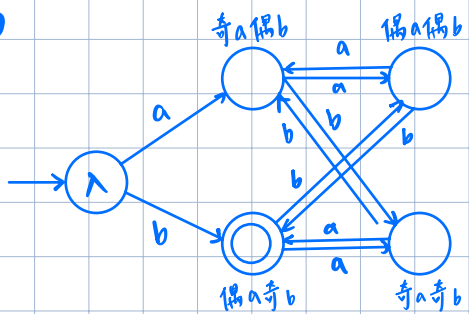
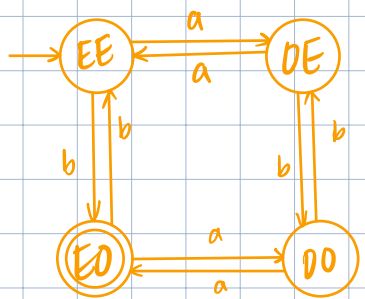
(a)

(b)

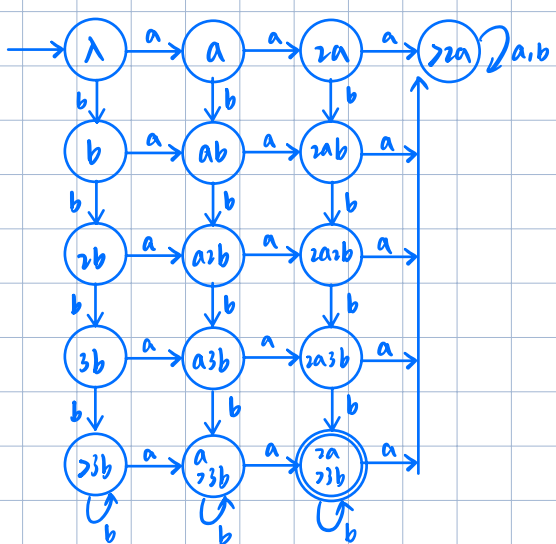


(c)

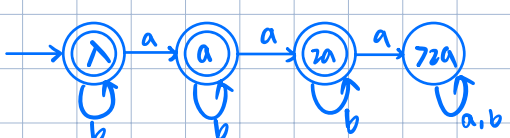




(d)



(e)



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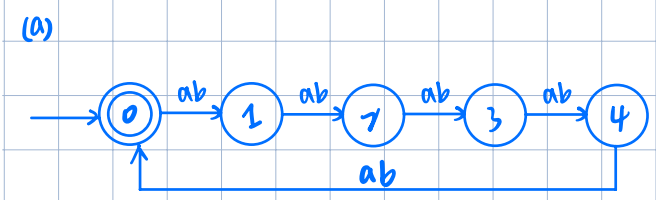

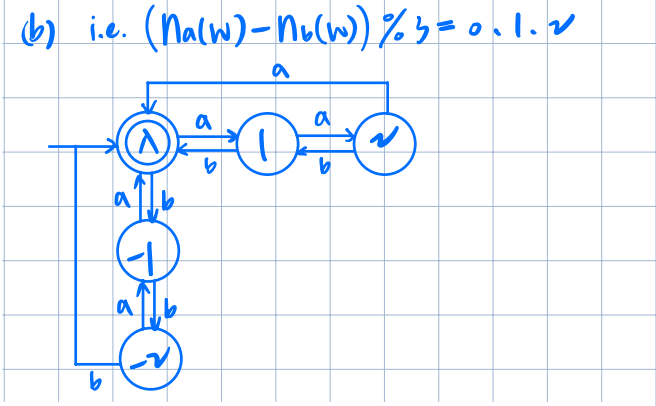

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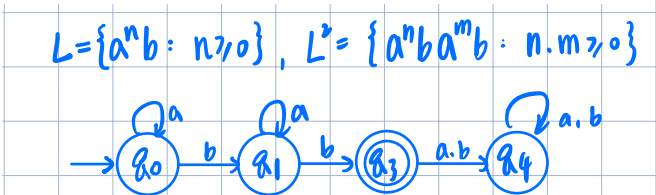
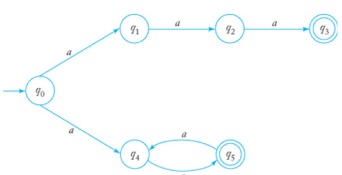
Figure 1: Automaton diagram.



Q4: Find a dfa that accepts the language defined by the nfa in Figure 2.



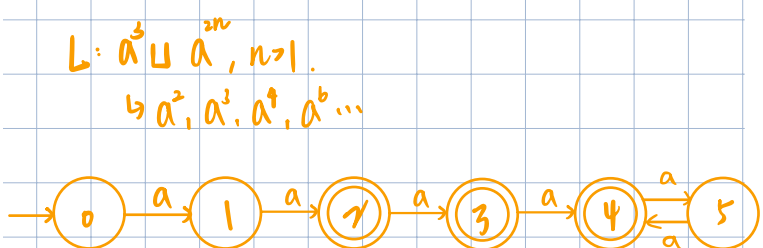
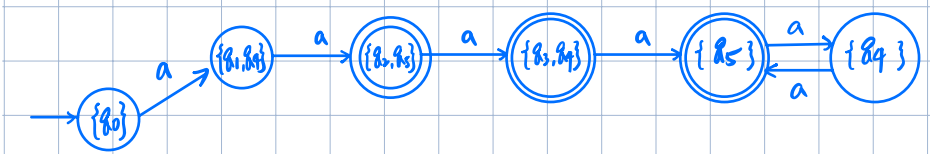


Figure 2: NFA diagram.



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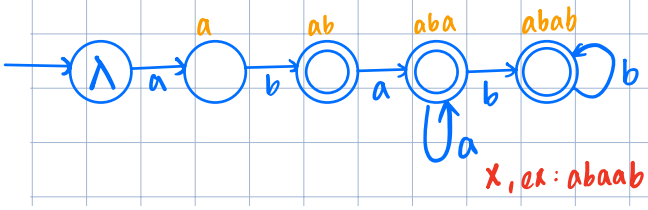
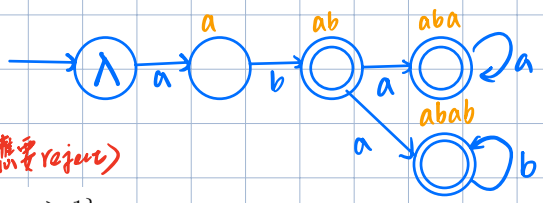
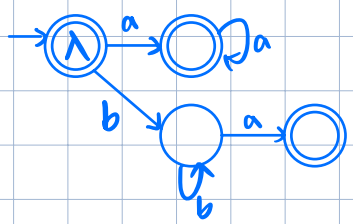


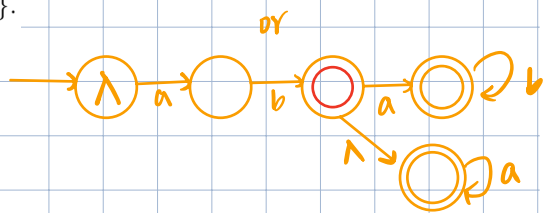
X, ex: abaab不行(應要reject)



Q6: Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.



or



Q7: Convert the nfa defined by
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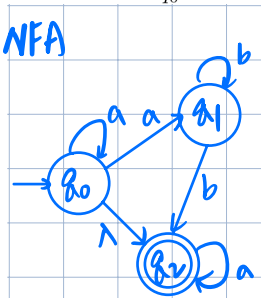
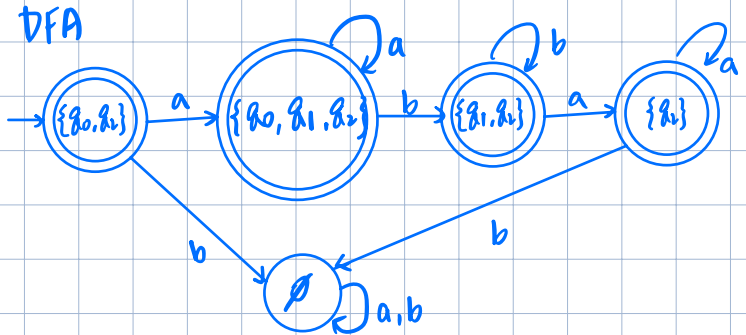
NFA



DFA



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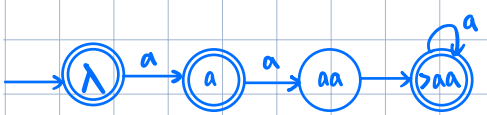

$\{q_0, q_1, q_3\} \in F$, $\{q_2\} \notin F$

找 $(q_0$ vs $q_2)$、$(q_0$ vs $q_1)$、$(q_1$ vs $q_3)$

④ ② ③

<pf>

$q_2 \notin F$, $q_3 \in F \rightarrow q_2, q_3$ 為 distinguishable

③ $\delta(q_1, a) = q_2 \notin F, \delta(q_2, a) = q_3 \in F \rightarrow q_1, q_2$ 為 distinguishable

② $\delta^*(q_0, aa) = q_2 \notin F, \delta^*(q_1, aa) = q_3 \in F \rightarrow q_0, q_1$ 為 distinguishable

④ $\delta^*(q_0, aa) = q_2 \notin F, \delta(q_3, aa) = q_3 \in F \rightarrow q_0, q_2$ 為 distinguishable

由以上可知，4狀態互相為distinguishable

∴ DFA為最小