Homework Two

Theory of Computation 2023

Important Note:

Please remember that you should return your answer at 12/06 (Wednesday) 15:00 and your HW should be handwritten. We will take your HW during the class. After 12/06 15:00, you must upload your HW to moodle. But remember penalty for late submission: 20% per day.

Q1: Find context-free grammars for the following languages.

(a) $L = a^n b^n$, n is even.

(b) $L = a^n b^n$, n is not a multiple of three.

Q2: Transform the grammar with productions

$S \to baAB$,

$A \to bAB|\lambda$,

$B \to BAa|A|\lambda$.

into Chomsky normal form.

Q3: Given a grammar G:

$S \to AB$,

$A \to BB|a$,

$B \to AB|b$.

Use the CYK algorithm to determine whether the strings abb, bbb, and $aabba$ are in the language generated by the grammar.

Q4: Find a pda that accepts the language $L = \{a^n b^{2n} : n \geq 0\}$.

Q5: Construct a pda that accepts the language generated by the grammar $S \to abSb|\lambda$.

Q6: Show that $L = \{a^n b^m, n > m\}$ is a deterministic context-free language.

#1 (a) ① 不含0: $S \to aaAbb$

$A \to aaAbb \mid A$

② 含0: $S \to aaSbb \mid \lambda$

(b) $S \to aAb \mid aaAbbb \mid \lambda$ = b 含0才有

$A \to aaaAbbb$

CYK

#2.

① 拿掉 λ (I. $A \to \lambda$, II. $B \to \lambda$)

$S \to baAB \mid baB \mid baA \mid ba$

$A \to bAB \mid bB \mid bA \mid b$

$B \to BAa \mid Ba \mid A \mid Aa \mid a$

② 拿掉 unit production ($B \to A$)

$S \to baAB \mid baB \mid baA \mid ba$

$A \to bAB \mid bB \mid bA \mid b$

$B \to BAa \mid Ba \mid \cancel{A} \mid Aa \mid a$

$bAB \mid bB \mid bA \mid b$

③

$V_a \to a$

$V_b \to b$

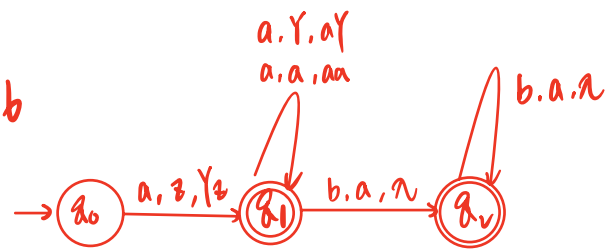
#3.

a	b	b
A	B	B
ab	bb	
S,B	A	
abb		
A		

↳ 不含S ⇒ ∉ L

a	a	b	b	a
A	A	B	B	A
aa	ab	bb	ba	
∅	S,B	A	∅	
aab	abb	bba		
S,B	A	∅		
aabb	abba			
A	∅			
aabba				
∅				

#b



Q1: Find context-free grammars for the following languages.
(a) $L = a^n b^n$, n is even.
(b) $L = a^n b^n$, n is not a multiple of three.

(a)

$S \to aaSbb \mid \lambda$

(b) $\alpha = 1.2$

$S \to aAb \mid aaAbb$
$A \to aaaAbbb \mid \lambda$

Q2: Transform the grammar with productions
$S \to baAB$,
$A \to bAB|\lambda$,
$B \to BAa|A|\lambda$.
into Chomsky normal form.

① 消除 $A \to \lambda$

$S \to baAB \mid baB$
$A \to bAB \mid bB$
$B \to BAa \mid A \mid Ba \mid \lambda$

② 消除 $B \to \lambda$

$S \to baAB \mid baB \mid baA \mid ba$
$A \to bAB \mid bB \mid bA \mid b$
$B \to BAa \mid A \mid Ba \mid Aa \mid a$

③ 移除 unit production

$S \to baAB \mid baB \mid baA \mid ba$
$A \to bAB \mid bB \mid bA \mid b$
$B \to BAa \mid Ba \mid Aa \mid a$
$\mid bAB \mid bB \mid bA \mid b$

$T_a \to a$
$T_b \to b$
$V_1 \to T_b T_a$
$V_2 \to V_1 A$
$V_3 \to T_b A$
$V_4 \to BA$
~~$V_5 \to T_b A$~~
$S \to V_2 B \mid V_1 B$
$\mid V_1 A \mid T_b T_a$
$A \to V_3 B \mid T_b B$
$\mid T_b A \mid b$
$B \to V_4 T_a \mid BTA$
$\mid AT_a \mid a$
$\mid V_3 B \mid T_b B$
$\mid T_b A \mid b$

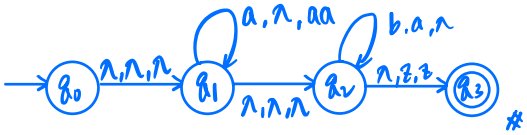
Q3: Given a grammar G:
$S \to AB$,
$A \to BB|a$,
$B \to AB|b$.
Use the CYK algorithm to determine whether the strings *abb*, *bbb*, and *aabba* are in the language generated by the grammar.

① a b b
A B B
ab bb
S,B A
abb
① a+bb ⇒ AA ⇒ X
② ab+b ⇒ S,B+B = SB, BB ⇒ A
A
↳ not ok! 非S開始

② b b b
B B B
bb bb
A A
bbb
① A+B ⇒ S,B
② B+A ⇒ S,B
S,B
↳ ok!

③ a a b b a
A A B B A
aa ab bb ba
S,B A
aab abb bba
S,B A
aabb abba
A
aabba
↳ not ok

Q4: Find a pda that accepts the language $L = \{a^n b^{2n} : n \geq 0\}$.



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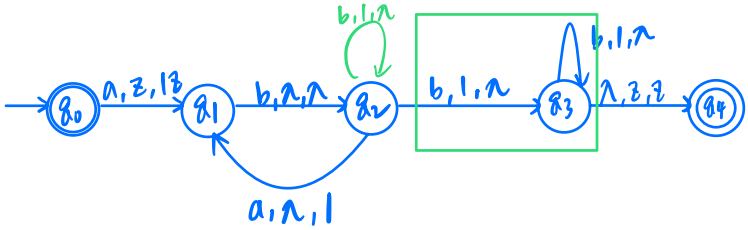
Q5: Construct a pda that accepts the language generated by the grammar $S \rightarrow abSb|\lambda$.



λ

abb

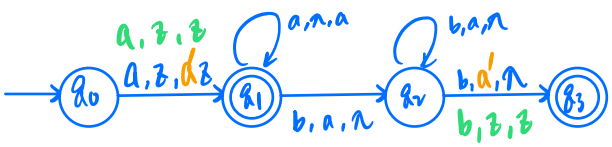
ababbb

abababbbbb

Q6: Show that $L = \{a^n b^m, n > m\}$ is a deterministic context-free language.

aab

m=0



aabb

∵ DPDA exist

∴ L is deterministic CFL #