Prob. 7.5

Let $x_1[n] = 2\left(\frac{2}{3}\right)^n u[n] \longrightarrow X_1(z) = \frac{2z}{z-2/3} = \frac{6z}{3z-2}, |z| > 2/3$

Let $x_2[n] = \left(\frac{2}{5}\right)^n u[n] \longrightarrow X_2(z) = \frac{z}{z-2/5} = \frac{5z}{5z-2}, |z| > 2/5$

Hence,

$$X(z) = X_1(z) - X_2(z) = \frac{6z}{3z-2} - \frac{5z}{5z-2}, \qquad |z| > 2/3$$

Prob. 7.6

(a) $X(z) = z^{-m}\frac{z}{z-1} = \frac{z}{z^m(z-1)}$

(b) $X(z) = \frac{az}{(z-a)^2}$

(c)
$$X(z) = \frac{z^2 za\cos\pi}{z^2 - 2za\cos\pi + a^2}, \qquad \cos\pi = -1$$
$$= \frac{z^2 + za}{z^2 + 2za + a^2}$$

Prob. 7.20

(a) $Y(z) = z^{-1}X(z) = \frac{2}{(z^2+3z+1)}$

(b)
$$Y(z) = \frac{j}{2}\left[X\left(e^{j\pi/4}z\right) - X\left(e^{-j\pi/4}z\right)\right]$$
$$= \frac{j}{2}\left[\frac{2e^{j\pi/4}z}{e^{j\pi/2}z^2 + 3e^{j\pi/4}z + 1} - \frac{2e^{-j\pi/4}z}{e^{-j\pi/2}z^2 + 3e^{-j\pi/4}z + 1}\right]$$

(c)
$$Y(z) = z\frac{dX(z)}{dz} + z^2\frac{d^2X(z)}{dz^2} = \frac{2z\left[(z^2+3z+1)(1) - z(2z+3)\right]}{(z^2+3z+1)^2} + z^2\frac{d^2X(z)}{dz^2}$$
$$= z\frac{(2-2z^2)}{(z^2+3z+1)^2} + z^2\frac{\left[(z^2+3z+1)^2(-4z) - (2-2z^2)(z^2+3z+1)(2)(2z+3)\right]}{(z^2+3z+1)^4}$$
$$= \frac{2z(1-z^2)}{(z^2+3z+1)^2} - \frac{4(z^3+z^2-4z-3)}{(z^2+3z+1)^3}$$

(d) $Y(z) = 2X(z)X(z) = \frac{8z^2}{(z^2+3z+1)^2}$

Prob. 7.21

$X(z) = 1 - z^{-1} + 3z^{-2} + 2z^{-3}$

$H(z) = 1 + 0z^{-1} + 2z^{-2} + z^{-3} - 3z^{-4}$

$$
\begin{aligned}
Y(z) = X(z)H(z) = 1 + 0z^{-1} + 2z^{-2} + z^{-3} - 3z^{-4} & \\
-z^{-1} + 0 - 2z^{-3} - z^{-4} + 3z^{-5} & \\
+3z^{-2} + 0 + 6z^{-4} + 3z^{-5} - 9z^{-6} & \\
+ \quad 2z^{-3} + 0 + 4z^{-5} + 2z^{-6} - 6z^{-7} &
\end{aligned}
$$

$Y(z) = 1 - z^{-1} + 5z^{-2} + z^{-3} + 2z^{-4} + 10z^{-5} - 7z^{-6} - 6z^{-7}$

Thus,

y[n] = [1, -1, 5, 1, 2, 10, -7, -6]

Prob. 7.24

(a) Compare this with

$$\sin\Omega n u[n] \quad \Leftrightarrow \quad \frac{z\sin\Omega}{z^2 - 2z\cos\Omega + 1}$$

$$2\cos\Omega = 1 \quad \longrightarrow \quad \Omega = \cos^{-1} 0.5 = 60^o, \sin\Omega = 0.866$$

$$X(z) = \frac{1}{0.866}\frac{2(z)(0.866)}{z^2 - z + 1} = \frac{2.309(z0.866)}{z^2 - z + 1}$$

$$x[n] = 2.309\sin 60^o n u[n]$$

(b) $Y_1(z) = \dfrac{Y(z)}{z} = \dfrac{z+2}{(z+1)(z-1)} = \dfrac{A}{z+1} + \dfrac{B}{z-1}$

$$A = Y_1(z)(z+1)\Big|_{z=-1} = -1/2$$

$$B = Y_1(z)(z-1)\Big|_{z=1} = 3/2$$

$$Y(z) = \frac{-1/2\, z}{z+1} + \frac{3/2\, z}{z-1}$$

$$y[n] = -\frac{1}{2}(-1)^n u[n] + \frac{3}{2}u[n]$$

(c) Let $H_1(z) = \dfrac{H(z)}{z} = \dfrac{4}{(z+1/2)(z^2+z+1)} = \dfrac{A}{z+1/2} + \dfrac{Bz+C}{z^2+z+1}$

$$4 = A(z^2+z+1) + B(z^2 + \frac{1}{2}z) + C(z+\frac{1}{2})$$

We equate coefficients:

$z^2 : 0 = A + B \longrightarrow B = -A$

$z : 0 = A + 0.5B + C = 0.5A + C \longrightarrow C = -0.5A$

constant: 4 = A + 0.5C = 0.75A $\longrightarrow$ A=16/3, B=-16/3, C=-8/3

$H(z) =$ 16/3*z/(z+1/2) – 8/3*(2z^2+z)/(z^2+z+1)

For the second term, $-2\cos\Omega = 1 \longrightarrow \Omega = \cos^{-1}(-0.5) = 120^o$

$h[n] =$ -16/3 * cos(120°n) u[n] + 16/3 *(-1/2)^n u[n]

Prob. 7.27

(a) $a^2 = 0.75 \longrightarrow a = 0.86$

$2a\cos\Omega = 1 \longrightarrow \Omega = \cos^{-1} 0.5774 = 54.73^o$

$a\sin\Omega = 0.707$

$$X_1(z) = \frac{z^2 - 0.5z}{z^2 - z + 0.75} - \frac{0.5}{0.707}\cdot\frac{0.707z}{z^2 - z + 0.75}$$

$$x_1[n] = (0.866)^n \cos 54.73^o nu[n] - 0.7072(0.866)^n \sin 58.74^o nu[n]$$

或是

$$X_1(z)=\frac{z^2-z}{z^2-z-0.75}=\frac{z^2-z}{(z^2-z+\frac{1}{4})-1}=\frac{z^2-z}{(z-\frac{1}{2})^2-1^2}=\frac{z^2-z}{(z-\frac{3}{2})(z+\frac{1}{2})}$$

$$\frac{X_1(z)}{z}=\frac{z-1}{(z-\frac{3}{2})(z+\frac{1}{2})}=\frac{A}{z-\frac{3}{2}}+\frac{B}{z+\frac{1}{2}},\ A=\frac{1}{4},\ B=\frac{3}{4}$$

$$\frac{X_1(z)}{z}=\frac{1}{4}\cdot\frac{1}{z-\frac{3}{2}}+\frac{3}{4}\cdot\frac{1}{z+\frac{1}{2}}$$

$$X_1(z)=\frac{1}{4}\cdot\frac{z}{z-\frac{3}{2}}+\frac{3}{4}\cdot\frac{z}{z+\frac{1}{2}}$$

$$x_1[n]=(\frac{1}{4}\cdot(\frac{3}{2})^n+\frac{3}{4}\cdot(-\frac{1}{2})^n)u[n]$$

(b) $a^2=0.64 \longrightarrow a=0.8$

$2a\cos\Omega=0.8 \longrightarrow \Omega=\cos^{-1}0.5=60^o$

$a\sin\Omega=0.8\cos 60^o=0.6928$

$$X_2(z)=\frac{z^2-0.4z}{z^2-0.8z+0.64}+\frac{1.4z}{z^2-0.8z+0.65}$$

$$=\frac{z^2-0.4z}{z^2-0.8z+0.64}+\frac{1.4}{0.6928}\frac{0.6928z}{z^2-0.8z+0.65}$$

$$x_2[n]=(0.8)^n\cos 60^o nu[n]+2.02(0.8)^n\sin 60^o nu[n]$$

Prob. 7.31

$$zY(z)-zy[0]-2Y(z)=\frac{z}{z-1.5}$$

$$Y(z)(z-2)=\frac{z}{z-1.5}+z=\frac{z+z^2-1.5z}{z-1.5}$$

$$Y(z)=\frac{z^2-0.5z}{(z-2)(z-1.5)}$$

Let $Y_1(z)=\frac{Y(z)}{z}=\frac{z-0.5}{(z-2)(z-1.5)}=\frac{A}{z-2}+\frac{B}{z-1.5}$

$$A=Y_1(0)(z-2)\Big|_{z=2}=1.5/0.5=3$$

$$B=Y_1(0)(z-1.5)\Big|_{z=1.5}=1./(-0.5)=-2$$

$$Y(z)=\frac{3z}{z-2}-\frac{2z}{z-1.5}$$

$$y[n]=3(2)^n u[n]-2(1.5)^n u[n]$$

Prob. 7.36

$$Y(z)=H(z)X(z)=\left(\frac{z}{z-1}\right)\left(\frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}\right)=\frac{z(z^2+2z)}{(z-1)(z^2-z+1)}$$

Let $\quad Y_1(z)=\dfrac{Y(z)}{z}=\dfrac{(z^2+2z)}{(z-1)(z^2-z+1)}=\dfrac{A}{z-1}+\dfrac{Bz+C}{z^2-z+1}$

$$A=Y_1(z)(z-1)\Big|_{z=1}=3/1=3$$

$$z^2+2z=A(z^2-z+1)+B(z^2-z)+C(z-1)$$

Equating coefficients,

$z^2:1=A+B \quad \longrightarrow \quad B=1-A=-2$

$z:2=A-B+C$

constant: $\quad 0=A-C \quad \longrightarrow \quad C=A=3$

$$Y(z)=\frac{3z}{z-1}+\frac{z(-2z+3)}{z^2-z+1}=\frac{3z}{z-1}-\frac{2(z^2-1.5z)}{z^2-z+1}$$

For the second term, let

$-z=-2z\cos\Omega \quad \longrightarrow \quad \Omega=\cos^{-1}(0.5)=60^\circ$

$$Y(z)=\frac{3z}{z-1}-\frac{2(z^2-0.5z)}{z^2-z+1}+\frac{2}{0.866}\frac{0.866z}{z^2-z+1}$$

$$y[n]=3u[n]-2\cos(60^\circ n)u[n]+2.309\sin(60^\circ n)u[n]$$

Prob. 7.38

Let E by the input to H_1. From the figure,

$$E=X-Y-H_2Y$$

$$Y=EH_1=H_1X-H_1Y-H_1H_2Y$$

$$Y\left(1+H_1+H_1H_2\right)=H_1X$$

$$H=\frac{Y}{X}=\frac{H_1}{1+H_1+H_1H_2}$$

Prob. 7.43

Y(z) = H(z) X(z)

For the impulse response,

$X(z) = \mathcal{Z}\{\delta[n]\} = 1$

-

$Y(z)H(z) =$ 0.8z / (z-0.6)(z-2)

Let $Y_1(z) = \frac{Y(z)}{z} =$ 0.8 / (z-0.6)(z-2) = A/(z-0.6) + B/(z-2)

A + B = 0

-2A-0.6B = 0.8

-2A+0.6B = 0.8

-1.4A = 0.8, A = -4/7, B = 4/7

Y(z) = -4/7 * z/(z-0.6) + 4/7 * z/(z+2)

y[n] = (-4/7*(0.6)^n+4/7*(2)^n) u[n]

For the step response,

$X(z) = \mathcal{Z}(u[n]) = \frac{z}{z-1}$

$$Y(z) = H(z)X(z) = \frac{0.8z^2}{(z-1)(z-0.6)(z+2)}$$

Let $Y_2(z) = \frac{Y(z)}{z} = \frac{0.8z}{(z-1)(z-0.6)(z+2)} = \frac{A}{z-1} + \frac{B}{z-0.6} + \frac{C}{z+2}$

A = -2

B = 6/7

C = 8/7

Y(z) = -2 * z/(z-1) + 6/7 * z/(z-0.6) + 8/7 * z/(z-2)

y[n] = (-2 + 6/7 * (0.6)^n + 8/7 * (2)^n) u[n]

Prob. 7.45

```
syms  X x n z
X = z/(z-0.6);
x = iztrans(X)
```

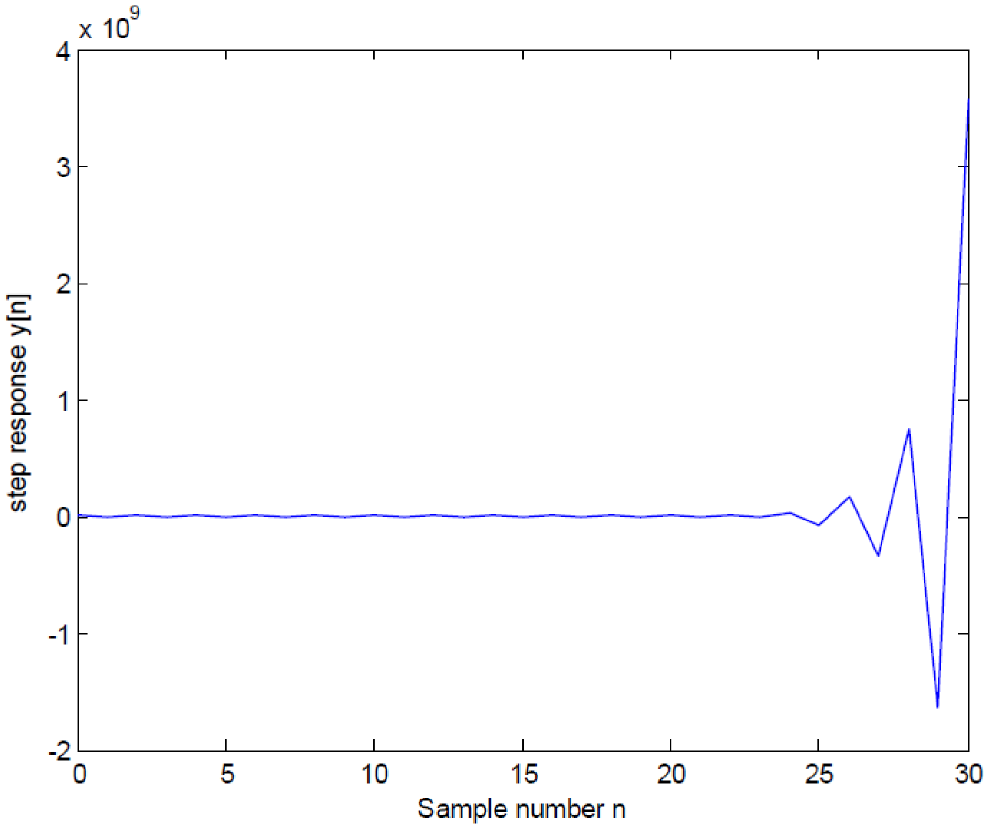
x =

(3/5)^n

Prob. 7.46

The MATLAB script and the plot are presented below.

```
num = [1  1];
den = [ 1  2  1  3];
n = 0:1:30;
x = [1*ones(size(n))]; % unit step input
y = filter(num, den, x);
plot(n,y);
xlabel('Sample number n');
ylabel('step response y[n]')
```



Prob. 7.47

(a) The MATLAB code with the result is shown below.

```
>> num = [1 6 1];
>> den = [1 3 0 4 10];
>> z = roots(num);
>> p = roots(den);
>> z

z =

   -5.8284
   -0.1716

>> p

p =

  -3.0794 + 0.0000i
   0.7401 + 1.3305i
   0.7401 - 1.3305i
  -1.4009 + 0.0000i
```

Prob. 7.50

```
den = [ 1  -2   0   5  -1    4];
p=roots(den)
```

p =

1.6969 + 1.1485i
1.6969 - 1.1485i
-1.4722 + 0.0000i
0.0391 + 0.8035i
0.0391 - 0.8035i

The first three poles lie outside a unit circle. Hence, the system is unstable.