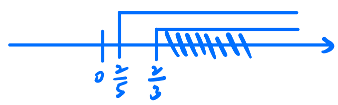
7.5 Determine the z-transform and its ROC for

$$x[n] = 2\left(\frac{2}{3}\right)^n u[n] - \left(\frac{2}{5}\right)^n u[n]$$

令 $x_1[n] = 2\left(\frac{2}{3}\right)^n u[n]$, $x_2[n] = \left(\frac{2}{5}\right)^n u[n]$

則 $X_1(z) = \dfrac{2z}{z-\frac{2}{3}} \Rightarrow |z| > \frac{2}{3}$

$X_2(z) = \dfrac{z}{z-\frac{2}{5}} \Rightarrow |z| > \frac{2}{5}$



$\therefore X(z) = X_1(z) - X_2(z) = \dfrac{2z}{z-\frac{2}{3}} - \dfrac{z}{z-\frac{2}{5}} = \dfrac{6z}{3z-2} - \dfrac{5z}{5z-2}$, $|z| > \frac{2}{3}$ #

7.6 Find the z-transform of the following signals:

(a) $u[n-m]$

(b) $na^n u[n]$

(c) $a^n \cos \pi n\, u[n]$

$e^{j\omega} = \cos\omega + j\sin\omega$

$e^{-j\omega} = \cos\omega - j\sin\omega$ $\Rightarrow \cos\omega = \dfrac{e^{j\omega}+e^{-j\omega}}{2}$

(a)

$X(z) = z^{-m}\dfrac{z}{z-1} = \dfrac{z}{z^m(z-1)}$ #

(b)

$X(z) = -z\cdot\dfrac{d}{dz}\, Z\{a^n u[n]\}$

$= -z\cdot\dfrac{d}{dz}\left(\dfrac{z}{z-a}\right)$

$= \dfrac{az}{(z-a)^2}$ #

(c)

$X(z) = \frac{1}{2}\left(\dfrac{e^{j\pi}z}{e^{j\pi}z-a} + \dfrac{e^{-j\pi}z}{e^{-j\pi}z-a}\right)$

$= \frac{1}{2}\left(\dfrac{z^2-ae^{-j\pi}z+z^2-ae^{j\pi}z}{z^2-(e^{j\pi}z+e^{-j\pi}z)a+a^2}\right)$

$= \frac{1}{2}\left(\dfrac{2z^2-az\cdot 2\cos\pi}{z^2-az\cdot 2\cos\pi+a^2}\right)$ 且 $\cos\pi = -1$

$= \dfrac{z^2+az}{z^2+2az+a^2}$ #

7.20 The z-transform of a discrete-time signal is

$$X(z) = \frac{2z}{z^2+3z+1}$$

Find the z-transform of the following signals:

(a) $y[n] = x[n-1]u[n-1]$

(b) $y[n] = \sin(\pi n/4)\, x[n]$ $\Omega = \frac{\pi}{4}$

(c) $y[n] = n^2 x[n]$

(d) $y[n] = 2x[n]*x[n]$

(a)

$Y(z) = z^{-1}X(z)$

$= \dfrac{2}{z^2+3z+1}$ #

(b)

$$y(z) = \frac{j}{2}\left[X(e^{\frac{\pi}{4}j} z) - X(e^{-\frac{\pi}{4}j} z) \right]$$

$$= \frac{j}{2}\left[\frac{2e^{\frac{\pi}{4}j} z}{e^{\frac{\pi}{2}j} z^2 + 3e^{\frac{\pi}{4}j} z + 1} - \frac{2e^{-\frac{\pi}{4}j} z}{e^{-\frac{\pi}{2}j} z^2 + 3e^{-\frac{\pi}{4}j} z + 1} \right] \#$$

(c)

$$y(z) = -z\frac{d}{dz}\left(-z\frac{d}{dz}X(z)\right)$$

$$= -z\left(-\frac{d}{dz}X(z) + (-z)\frac{d^2}{dz^2}X(z)\right)$$

$$= z\frac{d}{dz}X(z) + z^2\frac{d^2}{dz^2}X(z)$$

$$= z \cdot \frac{-2z^2+2}{(z^2+3z+1)^2} + z^2 \cdot \frac{4z^3-12z-12}{(z^2+3z+1)^3}$$

$$= \frac{2z^5-6z^4-12z^3-6z^2+2z}{(z^2+3z+1)^3} \#$$

(d)

$$y(z) = 2 \cdot \frac{2z}{z^2+3z+1} \cdot \frac{2z}{z^2+3z+1}$$

$$= \frac{8z^2}{(z^2+3z+1)^2} \#$$

7.21 Using the z-transform, determine the convolution of these sequences:

$$x[n] = [1,\ -1,\ 3,\ 2], \quad h[n] = [1,\ 0,\ 2,\ 1,\ -3].$$

$$X(z) = 1 - z^{-1} + 3z^{-2} + 2z^{-3}$$

$$H(z) = 1 + 0z^{-1} + 2z^{-2} + 1z^{-3} - 3z^{-4}$$

$$Y(z) = X(z)H(z) = 1 + 2z^{-2} + 1z^{-3} - 3z^{-4} - z^{-1} - 2z^{-3} - z^{-4} + 3z^{-5} + 3z^{-2} + 6z^{-4} + 3z^{-5} - 9z^{-6}$$
$$+ 2z^{-3} + 4z^{-5} + 2z^{-6} - 6z^{-7}$$

$$= 1 - z^{-1} + 5z^{-2} + z^{-3} + 2z^{-4} + 10z^{-5} - 7z^{-6} - 6z^{-7}$$

$$\Rightarrow y[n] = [1, -1, 5, 1, 2, 10, -7, -6] \#$$

7.24 Find the inverse z-transform of the following functions:

(a) $X(z) = \dfrac{2z}{z^2 - z + 1}$

(b) $Y(z) = \dfrac{z(z+2)}{(z+1)(z-1)}$

(c) $H(z) = \dfrac{4z}{(z^2 + z + 1)(z + 1/2)}$

$$Z\{a^n \sin(wn)u[n]\} = \frac{az\sin w}{z^2 - 2az\cos w + a^2}$$

(a) $a = 1$ $\quad 2\cos w = 1 \Rightarrow w = 60^\circ$ $\quad \sin 60^\circ \doteq 0.866$

$$X(z) = \frac{0.866z}{z^2 - z + 1} \cdot \frac{2}{0.866} \Rightarrow x[n] = 2.309 \sin(60^\circ n) u[n]_{\#}$$

(b)

$$Y_1(z) = \frac{Y(z)}{z} = \frac{A}{z+1} + \frac{B}{z-1}$$

$$A = (z+1)Y_1(z)\Big|_{z=-1} = \frac{1}{-2} = -\frac{1}{2}$$

$$B = (z-1)Y_1(z)\Big|_{z=1} = \frac{3}{2}$$

$$Y(z) = \frac{-1}{2}\frac{z}{z+1} + \frac{3}{2}\frac{z}{z-1} \Rightarrow y[n] = -\frac{1}{2}(-1)^n u[n] + \frac{3}{2}u[n]_{\#}$$

(c)

$$H_1(z) = \frac{H(z)}{z} = \frac{4}{(z^2+z+1)(z+\frac{1}{2})} = \frac{Az+B}{z^2+z+1} + \frac{C}{z+\frac{1}{2}} \Rightarrow A = -\frac{16}{3},\ B = -\frac{8}{3},\ C = \frac{16}{3}$$

$$\therefore H(z) = -\frac{8}{3}\left(\frac{2z^2+z}{z^2+z+1}\right) + \frac{16}{3}\left(\frac{z}{z+\frac{1}{2}}\right)$$

$$= -\frac{16}{3}\left(\frac{z^2+\frac{1}{2}z}{z^2+z+1}\right) + \frac{16}{3}\left(\frac{z}{z+\frac{1}{2}}\right)$$

$$Z\{a^n \cos(\Omega n)u[n]\} = \frac{z^2 - az\cos\Omega}{z^2 - 2az\cos\Omega + a^2}$$

$$\Rightarrow a = 1 \quad \cos\Omega = -\frac{1}{2} \Rightarrow \Omega = 120^\circ$$

$$\Rightarrow h[n] = -\frac{16}{3}\cos(120^\circ n)u[n] + \frac{16}{3}\left(-\frac{1}{2}\right)^n u[n]_{\#}$$

7.27 Invert each of the following z-transform:

(a) $X_1(z) = \dfrac{1 - z^{-1}}{1 - z^{-1} - 0.75z^{-2}}$

(b) $X_2(z) = \dfrac{1 + z^{-1}}{1 - 0.8z^{-1} + 0.64z^{-2}}$

$$\mathcal{Z}\{a^n \cos(\Omega n)u[n]\} = \frac{z^2 - az\cos\Omega}{z^2 - 2az\cos\Omega + a^2}$$

(a)

$$X_1(z) = \frac{z^2 - z}{z^2 - z - 0.75} = \frac{4z^2 - z}{(2z+1)(2z-3)}$$

$$\frac{X_1(z)}{z} = \frac{A}{2z+1} + \frac{B}{2z-3} \Rightarrow A = \frac{3}{2},\ B = \frac{1}{2}$$

$$X_1(z) = \frac{\frac{3}{2}z}{2z+1} + \frac{\frac{1}{2}z}{2z-3} = \frac{3}{2}\cdot\frac{1}{2}\left(\frac{z}{z+\frac{1}{2}}\right) + \frac{1}{2}\cdot\frac{3}{2}\left(\frac{z}{z-\frac{3}{2}}\right)$$

$$x_1[n] = \frac{3}{4}\left(-\frac{1}{2}\right)^n u[n] + \frac{3}{4}\left(\frac{3}{2}\right)^n u[n] \quad \#$$

(b) $a^2 = 0.64 \Rightarrow a = 0.8$

$2a\cos\Omega = 0.8 \Rightarrow \Omega = \cos^{-1}\frac{1}{2} = 60^\circ$

$a\sin\Omega = 0.8\sin 60^\circ \doteqdot 0.6928$

$$X_2(z) = \frac{z^2 - 0.4z}{z^2 - 0.8z + 0.64} + \frac{1.4z}{z^2 - 0.8z + 0.64} \times \frac{0.6928}{0.6928}$$

$$\Rightarrow x_2[n] = (0.8)^n \cos(60^\circ n)u[n] + 2.02\,(0.8)^n \sin(60^\circ n)\,u[n] \quad \#$$

7.31 Using the z-transform, solve the following difference equation:

$$y[n+1] - 2y[n] = (1.5)^n, \quad y[0] = 1.$$

$$\mathcal{Z}\{y[n+1]\} = zY(z) - zy[0]$$

$$(zY(z) - zy[0]) - 2Y(z) = \frac{z}{z - 1.5}$$

$$Y(z)(z-2) = \frac{z}{z-1.5} + z = \frac{z^2 - 0.5z}{z - 1.5}$$

$$Y(z) = \frac{z(z-0.5)}{(z-1.5)(z-2)}$$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{A}{z-1.5} + \frac{B}{z-2}$$

$$A = (z-1.5)Y_1(z)\Big|_{z=1.5} = \frac{1}{-0.5} = -2$$

$$B = (z-2)Y_1(z)\Big|_{z=2} = \frac{1.5}{0.5} = 3$$

$$\therefore Y(z) = \frac{-2z}{z-1.5} + \frac{3z}{z-2}$$

$$\Rightarrow y[n] = -2(1.5)^n u[n] + 3(2)^n u[n] \#$$

7.36 The transfer function of a discrete-time system is

$$H(z) = \frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}$$

Find the system response $y[n]$ when the input is a unit step function $u[n]$.

$$Y(z) = H(z)X(z) = \frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}\left(\frac{z}{z-1}\right) = \frac{z(z^2+2z)}{(z-1)(z^2-z+1)}$$

令 $Y_1(z) = \frac{Y(z)}{z} = \frac{z^2+2z}{(z-1)(z^2-z+1)} = \frac{A}{z-1} + \frac{Bz+C}{z^2-z+1}$

$$A = (z-1)Y_1(z)\Big|_{z=1} = 3$$

通分比較係數 $\Rightarrow B=-2,\ C=3$

$$\therefore Y(z) = \frac{3z}{z-1} - \frac{2(z^2-1.5z)}{z^2-z+1}$$

令 $-z = -2z\cos\Omega \Rightarrow \Omega = \cos^{-1}(0.5) = 60^\circ,\ a=1,\ a\sin\Omega \doteq 0.866$

$$= \frac{3z}{z-1} - \frac{2(z^2-0.5z)}{z^2-z+1} + \frac{2}{0.866}\cdot\frac{0.866z}{z^2-z+1}$$

$$\Rightarrow y[n] = 3u[n] - 2\cos(60^\circ n)u[n] + 2.309\sin(60^\circ n)u[n] \#$$

7.38 Determine the transfer function of the feedback system represented in Figure 7.14.

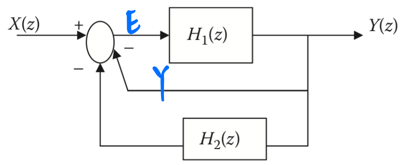


FIGURE 7.14 For Problem 7.38.

$E = X - Y - H_2 Y$

$Y = H_1 E = H_1 X - H_1 Y - H_1 H_2 Y$

$(1 + H_1 + H_1 H_2) Y = H_1 X$

$$\therefore H = \frac{Y}{X} = \frac{H_1(z)}{1 + H_1(z) + H_1(z) H_2(z)} \quad *$$

7.43 Obtain the impulse and step responses of the discrete-time system with transfer function

$$H(z) = \frac{0.8z}{(z-0.6)(z-2)}$$

(1) impulse response: $\delta[n] \xrightarrow{z} 1$

$Y(z) = H(z)X(z) = H(z)$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{A}{z-0.6} + \frac{B}{z-2}$$

A: $(z-0.6)Y_1(z)\big|_{z=0.6} = \frac{0.8}{-1.4} \doteq -0.5714$

B: $(z-2)Y_1(z)\big|_{z=2} = \frac{0.8}{1.4} \doteq 0.5714$

$$\therefore Y(z) = \frac{-0.5714z}{z-0.6} + \frac{0.5714z}{z-2} \Rightarrow y[n] = -0.5714(0.6)^n u[n] + 0.5714(2)^n u[n] \quad *$$

(2) step response: $u[n] \xrightarrow{z} \frac{z}{z-1}$

$$Y(z) = H(z)X(z) = \frac{0.8z^2}{(z-0.6)(z-2)(z-1)}$$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{0.8z}{(z-0.6)(z-2)(z-1)} = \frac{A}{z-0.6} + \frac{B}{z-2} + \frac{C}{z-1}$$

A: $(z-0.6)Y_1(z)\big|_{z=0.6} = \frac{0.48}{(-1.4)(-0.4)} = 0.8571$

B: $(z-2)Y_1(z)\big|_{z=2} = \frac{1.6}{1.4 \times 1} \doteq 1.143 \Rightarrow y[n] = \left(0.8571(0.6)^n + 1.143(2)^n - 2\right)u[n] \quad *$

C: $(z-1)Y_1(z)\big|_{z=1} = \frac{0.8}{0.4(-1)} = -2$

7.45 Use MATLAB to find the inverse z-transform of

$$X(z) = \frac{z}{z - 0.6}$$

```
syms z;  % 聲明為符號，而不是数值
X = z / (z - 0.6);

x = iztrans(X);  % reverse z-transform

disp(x);  % 打印结果
```

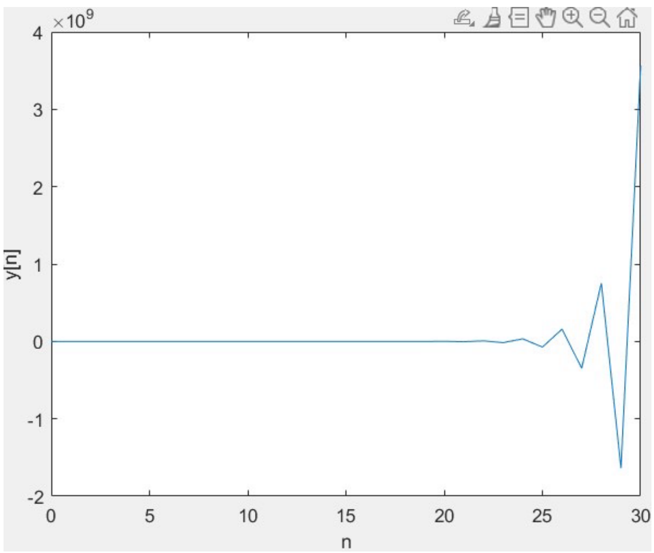
Command Window

```
>> HW4_45
(3/5)^n
```

7.46 A linear discrete-time system is represented by the transfer function

$$H(z) = \frac{z+1}{z^3 + 2z^2 + z + 3}$$

Use MATLAB to plot the step response of the system.



7.47 Determine the poles and zeros of the transfer function

$$H(z) = \frac{z^2 + 6z + z}{z^4 + 3z^3 + 4z + 10}$$

```
zeros =
   -5.8284
   -0.1716

poles =
  -3.0794 + 0.0000i
   0.7401 + 1.3305i
   0.7401 - 1.3305i
  -1.4009 + 0.0000i
```

7.50 Check the stability of a system described by the following transfer function:

$$H(z) = \frac{z^{-3} - 2z^2 + 6z + 1}{z^5 - 2z^4 + 5z^2 - z + 4}$$

```
p =

   1.6969 + 1.1485i
   1.6969 - 1.1485i
  -1.4722 + 0.0000i
   0.0391 + 0.8035i
   0.0391 - 0.8035i
```

長度超過單位圓之半徑

⇒ unstable ※