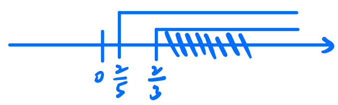
**7.5** Determine the z-transform and its ROC for

$$x[n]=2\left(\frac{2}{3}\right)^n u[n]-\left(\frac{2}{5}\right)^n u[n]$$

令 $x_1[n]=2\left(\frac{2}{3}\right)^n u[n]$ , $x_2[n]=\left(\frac{2}{5}\right)^n u[n]$

则 $X_1(z)=\frac{2z}{z-\frac{2}{3}} \Rightarrow |z|>\frac{2}{3}$

$X_2(z)=\frac{z}{z-\frac{2}{5}} \Rightarrow |z|>\frac{2}{5}$




$\therefore X(z)=X_1(z)-X_2(z)=\frac{2z}{z-\frac{2}{3}}-\frac{z}{z-\frac{2}{5}}=\frac{6z}{3z-2}-\frac{5z}{5z-2}$ , $|z|>\frac{2}{3}$ ※

**7.6** Find the z-transform of the following signals:

(a) $u[n-m]$

(b) $na^n u[n]$

(c) $a^n \cos \pi n\, u[n]$

$e^{jw}=\cos w+j\sin w$

$e^{-jw}=\cos w-j\sin w$ $\Rightarrow \cos w=\frac{e^{jw}+e^{-jw}}{2}$

(a)

$X(z)=z^{-m}\frac{z}{z-1}=\frac{z}{z^m(z-1)}$ ※

(b)

$X(z)=-z\cdot\frac{d}{dz}\mathcal{Z}\{a^n u[n]\}$

$=-z\cdot\frac{d}{dz}\left(\frac{z}{z-a}\right)$

$=\frac{az}{(z-a)^2}$ ※

(c)

$X(z)=\frac{1}{2}\left(\frac{e^{j\pi}z}{e^{j\pi}z-a}+\frac{e^{-j\pi}z}{e^{-j\pi}z-a}\right)$

$=\frac{1}{2}\left(\frac{z^2-ae^{-j\pi}z+z^2-ae^{j\pi}z}{z^2-(e^{j\pi}z+e^{-j\pi}z)a+a^2}\right)$

$=\frac{1}{2}\left(\frac{2z^2-az\cdot 2\cos\pi}{z^2-az\cdot 2\cos\pi+a^2}\right)$ 且 $\cos\pi=-1$

$=\frac{z^2+az}{z^2+2az+a^2}$ ※

**7.20** The z-transform of a discrete-time signal is

$$X(z)=\frac{2z}{z^2+3z+1}$$

Find the z-transform of the following signals:

(a) $y[n] = x[n-1]u[n-1]$

(b) $y[n] = \sin(\pi n/4)\, x[n]$ $\Omega=\frac{\pi}{4}$

(c) $y[n] = n^2 x[n]$

(d) $y[n] = 2x[n]*x[n]$

(a)

$Y(z)=z^{-1}X(z)$

$=\frac{2}{z^2+3z+1}$ ※

(b)

$$y(z) = \frac{j}{2}\left[X(e^{\frac{\pi}{4}j}z) - X(e^{-\frac{\pi}{4}j}z)\right]$$

$$= \frac{j}{2}\left[\frac{2e^{\frac{\pi}{4}j}z}{e^{\frac{\pi}{2}j}z^2 + 3e^{\frac{\pi}{4}j}z + 1} - \frac{2e^{-\frac{\pi}{4}j}z}{e^{-\frac{\pi}{2}j}z^2 + 3e^{-\frac{\pi}{4}j}z + 1}\right] \#$$

(c)

$$y(z) = -z\frac{d}{dz}\left(-z\frac{d}{dz}X(z)\right)$$

$$= -z\left(-\frac{d}{dz}X(z) + (-z)\frac{d^2}{dz^2}X(z)\right)$$

$$= z\frac{d}{dz}X(z) + z^2\frac{d^2}{dz^2}X(z)$$

$$= z\cdot\frac{-2z^2+2}{(z^2+3z+1)^2} + z^2\cdot\frac{4z^3-12z-12}{(z^2+3z+1)^3}$$

$$= \frac{2z^5-6z^4-12z^3-6z^2+2z}{(z^2+3z+1)^3} \#$$

(d)

$$y(z) = 2\cdot\frac{2z}{z^2+3z+1}\cdot\frac{2z}{z^2+3z+1}$$

$$= \frac{8z^2}{(z^2+3z+1)^2} \#$$

**7.21** Using the z-transform, determine the convolution of these sequences:

$$x[n] = [1,\ -1,\ 3,\ 2],\quad h[n] = [1,\ 0,\ 2,\ 1,\ -3].$$

$$X(z) = 1 - z^{-1} + 3z^{-2} + 2z^{-3}$$

$$H(z) = 1 + 0z^{-1} + 2z^{-2} + 1z^{-3} - 3z^{-4}$$

$$Y(z) = X(z)H(z) = 1 + 2z^{-2} + 1z^{-3} - 3z^{-4} - z^{-1} - 2z^{-3} - z^{-4} + 3z^{-5} + 3z^{-2} + 6z^{-4} + 3z^{-5} - 9z^{-6} + 2z^{-3} + 4z^{-5} + 2z^{-6} - 6z^{-7}$$

$$= 1 - z^{-1} + 5z^{-2} + z^{-3} + 2z^{-4} + 10z^{-5} - 7z^{-6} - 6z^{-7}$$

$$\Rightarrow y[n] = [1, -1, 5, 1, 2, 10, -7, -6] \#$$

**7.24** Find the inverse z-transform of the following functions:

(a) $X(z)=\dfrac{2z}{z^2-z+1}$

(b) $Y(z)=\dfrac{z(z+2)}{(z+1)(z-1)}$

(c) $H(z)=\dfrac{4z}{(z^2+z+1)(z+1/2)}$

$$Z\{a^n\sin(wn)u[n]\}=\frac{az\sin w}{z^2-2az\cos w+a^2}$$

(a) $a=1 \quad 2\cos w=1 \Rightarrow w=60^\circ \quad \sin 60^\circ \doteq 0.866$

$$X(z)=\frac{0.866z}{z^2-z+1}\cdot\frac{2}{0.866} \Rightarrow x[n]=2.309\sin(60^\circ n)u[n]_{\#}$$

(b)

$$Y_1(z)=\frac{Y(z)}{z}=\frac{A}{z+1}+\frac{B}{z-1}$$

$$A=(z+1)Y_1(z)\Big|_{z=-1}=\frac{1}{-2}=-\frac{1}{2}$$

$$B=(z-1)Y_1(z)\Big|_{z=1}=\frac{3}{2}$$

$$Y(z)=\frac{-1}{2}\frac{z}{z+1}+\frac{3}{2}\frac{z}{z-1} \Rightarrow y[n]=-\frac{1}{2}(-1)^n u[n]+\frac{3}{2}u[n]_{\#}$$

(c)

$$H_1(z)=\frac{H(z)}{z}=\frac{4}{(z^2+z+1)(z+\frac{1}{2})}=\frac{Az+B}{z^2+z+1}+\frac{C}{z+\frac{1}{2}} \Rightarrow A=-\frac{16}{3},\ B=-\frac{8}{3},\ C=\frac{16}{3}$$

$$\therefore H(z)=-\frac{8}{3}\left(\frac{2z^2+z}{z^2+z+1}\right)+\frac{16}{3}\left(\frac{z}{z+\frac{1}{2}}\right)$$

$$=-\frac{16}{3}\left(\frac{z^2+\frac{1}{2}z}{z^2+z+1}\right)+\frac{16}{3}\left(\frac{z}{z+\frac{1}{2}}\right)$$

$$Z\{a^n\cos(\Omega n)u[n]\}=\frac{z^2-az\cos\Omega}{z^2-2az\cos\Omega+a^2}$$

$$\Rightarrow a=1 \quad \cos\Omega=-\frac{1}{2} \Rightarrow \Omega=120^\circ$$

$$\Rightarrow h[n]=-\frac{16}{3}\cos(120^\circ n)u[n]+\frac{16}{3}\left(-\frac{1}{2}\right)^n u[n]_{\#}$$

**7.27** Invert each of the following z-transform:

(a) $X_1(z) = \dfrac{1 - z^{-1}}{1 - z^{-1} - 0.75z^{-2}}$

(b) $X_2(z) = \dfrac{1 + z^{-1}}{1 - 0.8z^{-1} + 0.64z^{-2}}$

$$\mathcal{Z}\{a^n \cos(\Omega n) u[n]\} = \frac{z^2 - az\cos\Omega}{z^2 - 2az\cos\Omega + a^2}$$

(a)

$$X_1(z) = \frac{z^2 - z}{z^2 - z - 0.75} = \frac{4z^2 - z}{(2z+1)(2z-3)}$$

$$\frac{X_1(z)}{z} = \frac{A}{2z+1} + \frac{B}{2z-3} \Rightarrow A = \frac{3}{2},\ B = \frac{1}{2}$$

$$X_1(z) = \frac{\frac{3}{2}z}{2z+1} + \frac{\frac{1}{2}z}{2z-3} = \frac{3}{2}\cdot\frac{1}{2}\left(\frac{z}{z+\frac{1}{2}}\right) + \frac{1}{2}\cdot\frac{3}{2}\left(\frac{z}{z-\frac{3}{2}}\right)$$

$$X_1[n] = \frac{3}{4}\left(-\frac{1}{2}\right)^n u[n] + \frac{3}{4}\left(\frac{3}{2}\right)^n u[n] \ \#$$

(b) $a^2 = 0.64 \Rightarrow a = 0.8$

$2a\cos\Omega = 0.8 \Rightarrow \Omega = \cos^{-1}\frac{1}{2} = 60^\circ$

$a\sin\Omega = 0.8\sin 60^\circ \doteq 0.6928$

$$X_2(z) = \frac{z^2 - 0.4z}{z^2 - 0.8z + 0.64} + \frac{1.4z}{z^2 - 0.8z + 0.64} \times \frac{0.6928}{0.6928}$$

$$\Rightarrow x_2[n] = (0.8)^n \cos(60^\circ n) u[n] + 2.02\,(0.8)^n \sin(60^\circ n) u[n] \ \#$$

**7.31** Using the z-transform, solve the following difference equation:

$$y[n+1] - 2y[n] = (1.5)^n, \quad y[0] = 1.$$

$$\mathcal{Z}\{y[n+1]\} = zY(z) - zy[0]$$

$$(zY(z) - zy[0]) - 2Y(z) = \frac{z}{z-1.5}$$

$$Y(z)(z-2) = \frac{z}{z-1.5} + z = \frac{z^2 - 0.5z}{z-1.5}$$

$$Y(z) = \frac{z(z-0.5)}{(z-1.5)(z-2)}$$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{A}{z-1.5} + \frac{B}{z-2}$$

$$A = (z-1.5)Y_1(z)\Big|_{z=1.5} = \frac{1}{-0.5} = -2$$

$$B = (z-2)Y_1(z)\Big|_{z=2} = \frac{1.5}{0.5} = 3$$

$$\therefore Y(z) = \frac{-2z}{z-1.5} + \frac{3z}{z-2}$$

$$\Rightarrow y[n] = -2(1.5)^n u[n] + 3(2)^n u[n] \ \#$$

**7.36** The transfer function of a discrete-time system is

$$H(z) = \frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}$$

Find the system response $y[n]$ when the input is a unit step function $u[n]$.

$$Y(z) = H(z)X(z) = \frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}\left(\frac{z}{z-1}\right) = \frac{z(z^2+2z)}{(z-1)(z^2-z+1)}$$

令 $Y_1(z) = \frac{Y(z)}{z} = \frac{z^2+2z}{(z-1)(z^2-z+1)} = \frac{A}{z-1} + \frac{Bz+C}{z^2-z+1}$

$$A = (z-1)Y_1(z)\Big|_{z=1} = 3$$

通分比較係數 $\Rightarrow B=-2,\ C=3$

$$\therefore Y(z) = \frac{3z}{z-1} - \frac{2(z^2-1.5z)}{z^2-z+1}$$

令 $-z = -2z\cos\Omega \Rightarrow \Omega = \cos^{-1}(0.5) = 60^\circ,\ a=1,\ a\sin\Omega \doteq 0.866$

$$= \frac{3z}{z-1} - \frac{2(z^2-0.5z)}{z^2-z+1} + \frac{2}{0.866}\cdot\frac{0.866z}{z^2-z+1}$$

$$\Rightarrow y[n] = 3u[n] - 2\cos(60^\circ n)u[n] + 2.309\sin(60^\circ n)u[n] \ \#$$

**7.38** Determine the transfer function of the feedback system represented in Figure 7.14.

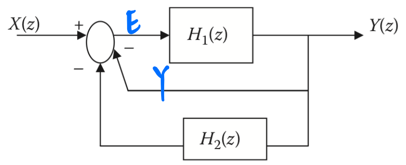



**FIGURE 7.14** For Problem 7.38.

$E = X - Y - H_2Y$

$Y = H_1E = H_1X - H_1Y - H_1H_2Y$

$(1 + H_1 + H_1H_2)Y = H_1X$

$$\therefore H = \frac{Y}{X} = \frac{H_1(z)}{1 + H_1(z) + H_1(z)H_2(z)} \quad \#$$

**7.43** Obtain the impulse and step responses of the discrete-time system with transfer function

$$H(z) = \frac{0.8z}{(z-0.6)(z-2)}$$

(1) impulse response: $\delta[n] \xrightarrow{z} 1$

$Y(z) = H(z)X(z) = H(z)$

$Y_1(z) = \frac{Y(z)}{z} = \frac{A}{z-0.6} + \frac{B}{z-2}$

$A: (z-0.6)Y_1(z)\Big|_{z=0.6} = \frac{0.8}{-1.4} \doteq -0.5714$

$B: (z-2)Y_1(z)\Big|_{z=2} = \frac{0.8}{1.4} \doteq 0.5714$

$$\therefore Y(z) = \frac{-0.5714z}{z-0.6} + \frac{0.5714z}{z-2} \Rightarrow y[n] = -0.5714(0.6)^n u[n] + 0.5714(2)^n u[n] \quad \#$$

(2) step response: $u[n] \xrightarrow{z} \frac{z}{z-1}$

$$Y(z) = H(z)X(z) = \frac{0.8z^2}{(z-0.6)(z-2)(z-1)}$$

$$Y_1(z) = \frac{Y(z)}{z} = \frac{0.8z}{(z-0.6)(z-2)(z-1)} = \frac{A}{z-0.6} + \frac{B}{z-2} + \frac{C}{z-1}$$

$A: (z-0.6)Y_1(z)\Big|_{z=0.6} = \frac{0.48}{(-1.4)(-0.4)} = 0.8571$

$B: (z-2)Y_1(z)\Big|_{z=2} = \frac{1.6}{1.4 \times 1} \doteq 1.143 \Rightarrow y[n] = \left(0.8571(0.6)^n + 1.143(2)^n - 2\right)u[n] \quad \#$

$C: (z-1)Y_1(z)\Big|_{z=1} = \frac{0.8}{0.4(-1)} = -2$

**7.45** Use MATLAB to find the inverse z-transform of

$$X(z) = \frac{z}{z - 0.6}$$

```
syms z;  % 聲明為符號，而不是数值
X = z / (z - 0.6);

x = iztrans(X);  % reverse z-transform

disp(x);  % 打印结果
```

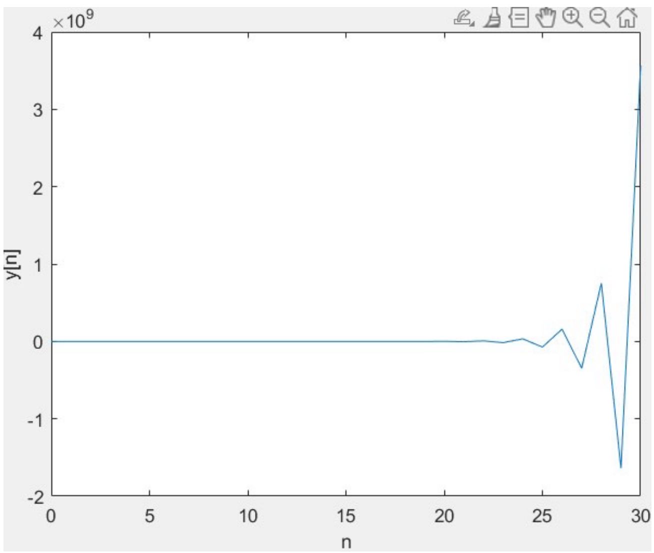
Command Window

```
>> HW4_45
(3/5)^n
```

**7.46** A linear discrete-time system is represented by the transfer function

$$H(z) = \frac{z+1}{z^3 + 2z^2 + z + 3}$$

Use MATLAB to plot the step response of the system.



**7.47** Determine the poles and zeros of the transfer function

$$H(z) = \frac{z^2 + 6z + z}{z^4 + 3z^3 + 4z + 10}$$

```
zeros =
   -5.8284
   -0.1716

poles =
  -3.0794 + 0.0000i
   0.7401 + 1.3305i
   0.7401 - 1.3305i
  -1.4009 + 0.0000i
```

**7.50** Check the stability of a system described by the following transfer function:

$$H(z) = \frac{z^{-3} - 2z^2 + 6z + 1}{z^5 - 2z^4 + 5z^2 - z + 4}$$

```
p =

   1.6969 + 1.1485i
   1.6969 - 1.1485i
  -1.4722 + 0.0000i
   0.0391 + 0.8035i
   0.0391 - 0.8035i
```

長度超過單位圓之半徑

⇒ unstable ※