6.1 (a) Show that $X(\Omega)$ is periodic with period 2π, that is, $X(\Omega + 2\pi) = X(\Omega)$.

(b) Specifically show that $X(\Omega) = \dfrac{1}{1-ae^{-j\Omega}}$ is periodic.

(a) Given that $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

$$X(\Omega+2\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-j(\Omega+2\pi)n} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}e^{-jn2\pi}$$

But $e^{-jn2\pi} = \cos(2n\pi) - j\sin(2n\pi) = 1 - j0 = 1$

Hence,

$$X(\Omega+2\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} = X(\Omega)$$

(b) $X(\Omega+2\pi) = \dfrac{1}{1-ae^{-j(\Omega+2\pi)}} = \dfrac{1}{1-a^{-j\Omega}e^{-j2\pi}}$

But $e^{-j2\pi} = \cos(2\pi) - j\sin(2\pi) = 1$

$$X(\Omega+2\pi) = \frac{1}{1-a^{-j\Omega}} = X(\Omega)$$

6.10 The DTFT of a signal $x[n]$ is

$$X(\Omega) = \frac{2}{3+e^{-j\Omega}}$$

Find the DTFT of the following signals:

(a) $y[n] = x[-n]$

(b) $z[n] = nx[n]$

(c) $w[n] = x[n] + x[n-1]$

(d) $v[n] = x[n]\cos(n\pi)$

(a) $y[n] = x[-n] \quad \Leftrightarrow \quad X(-\Omega)$

$$Y(\Omega) = \frac{2}{3+e^{j\Omega}}$$

(b) $z[n] = nx[n] \quad \leftrightarrow \quad jX'(\Omega)$

$$Z(\Omega) = j\frac{d}{d\Omega}\left(\frac{2}{3+e^{-j\Omega}}\right) = 2j(-1)(-je^{-j\Omega})(3+e^{-j\Omega})^{-2}$$

$$= \frac{-2e^{-j\Omega}}{(3+e^{-j\Omega})^2}$$

(c) $W(\Omega) = \dfrac{2}{3+e^{-j\Omega}} + \dfrac{2}{3+e^{-j\Omega}}e^{-j\Omega} = \dfrac{2(1+e^{-j\Omega})}{3+e^{-j\Omega}}$

(d) $v[n] = x[n]\cos(n\pi) = \dfrac{1}{2}x[n]\left[e^{j\pi n} + e^{-jn\pi}\right]$

$$V(\Omega) = \frac{1}{2}X(\Omega+\pi) + \frac{1}{2}X(\Omega-\pi) = \frac{1/2}{3+e^{-j(\Omega-\pi)}} + \frac{1/2}{3+e^{-j(\Omega+\pi)}}$$

$$= \frac{1/2}{3+e^{-j\Omega}e^{j\pi}} + \frac{1/2}{3+e^{-j\Omega}e^{-j\pi}}$$

But $e^{j\pi} = \cos\pi + j\sin\pi = -1, \qquad e^{-j\pi} = \cos(-\pi) + j\sin(-\pi) = -1$

$$V(\Omega = \frac{1}{3-e^{-j\Omega}}$$

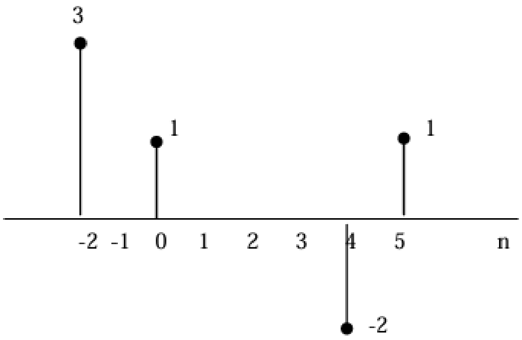
6.13 Determine the signal $x[n]$ corresponding to each of the following Fourier transforms:

(a) $X(\Omega) = 1 + 3e^{-j2\Omega} - 2e^{j4\Omega} + e^{j5\Omega}$

(b) $$X(\Omega) = \frac{e^{-j\Omega} - \frac{1}{2}}{1 - \frac{1}{2}e^{-j\Omega}}$$

(c) $X(\Omega) = 3\pi[\delta(\Omega - 2) + \delta(\Omega + 2)]$

(a) x[n] is obtained from X(Ω) and shown below.



Thus, x[-2] =3, x[0] = 1, x[4] = -2, x[5] = 1

(b) $$X(\Omega) = \frac{e^{-j\Omega}}{1 - 0.5e^{-j\Omega}} + \frac{0.5}{1 - 0.5e^{-j\Omega}}$$

(b) From Table 6.2,

$$x[n] = \left(\frac{1}{2}\right)^n u[n-1] - \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] = \left(\frac{1}{2}\right)^n u[n-1] - \left(\frac{1}{2}\right)^{n+1} u[n]$$

(c) From Table 3.2,

x[n] = 3cos(2n)

6.18 Find the convolution $y[n] = h[n] * x[n]$ of the following pairs of signals:

(a) $x[n] = \left(\frac{1}{4}\right)^n u[n], \quad h[n] = 1$

(b) $x[n] = \left(\frac{1}{3}\right)^n u[n], \quad h[n] = \delta[n] + \delta[n-1]$

(c) $x[n] = \left(\frac{1}{2}\right)^n u[n], \quad h[n] = \left(\frac{1}{3}\right)^n u[n]$

(a) $X(\Omega) = \dfrac{1}{1-\frac{1}{4}e^{-j\Omega}}, \qquad H(\Omega) = 2\pi\delta(\Omega)$

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{2\pi\delta(\Omega)}{1-\frac{1}{4}e^{-j\Omega}}$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(\Omega)e^{j\Omega n}d\Omega = \int_{-\pi}^{\pi}\frac{\delta(\Omega)}{1-\frac{1}{4}e^{-j\Omega}}e^{j\Omega n}d\Omega = \left.\frac{e^{j\Omega n}}{1-\frac{1}{4}e^{-j\Omega}}\right|_{\Omega=0} = \frac{4}{3}$$

(b) $X(\Omega) = \dfrac{1}{1-\frac{1}{3}e^{-j\Omega}}, \qquad H(\Omega) = 1 + e^{-j\Omega}$

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{1+e^{-j\Omega}}{1-\frac{1}{3}e^{-j\Omega}}$$

$$y[n] = \left(\frac{1}{3}\right)^n \left[u[n] + u[n-1]\right]$$

(c) $X(\Omega) = \dfrac{1}{1-\frac{1}{2}e^{-j\Omega}}, \qquad H(\Omega) = \dfrac{1}{1-\frac{1}{3}e^{-j\Omega}}$

$$Y(\Omega) = H(\Omega)X(\Omega) = \frac{1}{1-\frac{1}{4}e^{-j\Omega}} \cdot \frac{1}{1-\frac{1}{3}e^{-j\Omega}} = \frac{3}{1-\frac{1}{2}e^{-j\Omega}} - \frac{2}{1-\frac{1}{3}e^{-j\Omega}}$$

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$$

6.20 Prove the following DFT properties:

(a) $X[0] = \sum_{n=0}^{N-1} x[n]$

(b) $X[N/2] = \sum_{n=0}^{N-1} (-1)^n x[n]$

(a) $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$

When $k = 0$,

$$X[0] = \sum_{n=0}^{N-1} x[n] e^{-j0} = \sum_{n=0}^{N-1} x[n]$$

(b) When $k = N/2$,

$$X[N/2] = \sum_{n=0}^{N-1} x[n] e^{-jn\pi}$$

But $e^{-jn\pi} = \cos n\pi - j\sin n\pi = (-1)^n$

Hence,

$$X[N/2] = \sum_{n=0}^{N-1} (-1)^n x[n]$$

6.21 Find the DFT of the following sequences:

(a) $x[n] = \{0, 1, 2, 3\}$

(b) $y[n] = \{1, 1, -1, -1, 1, 1, -1, -1\}$

(a) Using FFT, X[k] = [6 -2+j2 -2 -2-j2]

(b) X[k] = [0 0 4-j4 0 0 0 4+j4 0]

6.23題目更正

(a)

$$x[n]cos(\frac{2\pi mn}{N}) \leftrightarrow \frac{1}{2}[X(k-m)+X(k+m)]$$

(b)

$$x[n]sin(\frac{2\pi mn}{N}) \leftrightarrow \frac{1}{2j}[X(k-m)-X(k+m)]$$

(a) $X[k]=\sum_{n=0}^{N-1} x[n]e^{-j2\pi nk/N}=\sum_{n=0}^{N-1} x[n]W_N^{kn}$

$W_N^{-km}x[n] \quad \Leftrightarrow \quad x[m-k] \qquad (1)$

$W_N^{km}x[n] \quad \Leftrightarrow \quad x[m+k] \qquad (2)$

Adding (1) and (2) and multiplying by ½, We get

$$\frac{\left(W_N^{-km}+W_N^{km}\right)}{2}x[n]=\frac{e^{j2\pi km/N}+e^{-j2\pi km/N}}{2}x[n]=x[n]\cos\left(\frac{2\pi km}{N}\right)$$

Thus, $x[n]\cos\left(\frac{2\pi km}{N}\right) \quad \Leftrightarrow \quad \frac{1}{2}\left[X(m-k)+X(m+k)\right]$

(b) Subtracting (2) from (1) and multiplying by 1/j2 gives

$$\frac{\left(W_N^{-km}-W_N^{km}\right)}{j2}x[n]=\frac{e^{j2\pi km/N}-e^{-j2\pi km/N}}{j2}x[n]=x[n]\sin\left(\frac{2\pi km}{N}\right)$$

Thus, $x[n]\sin\left(\frac{2\pi km}{N}\right) \quad \Leftrightarrow \quad \frac{1}{2j}\left[X(m-k)-X(m+k)\right]$

6.25 The DFT of a signal $x[n]$ is

$$X(0)=1, \quad X(1)=1+j2, \quad X(2)=1-j, \quad X(3)=1+j, \quad X(4)=1-j2$$

Compute $x[n]$.

$$x[n]=\frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi nk/N}=\frac{1}{5}\sum_{k=0}^{4} X[k]e^{j2\pi nk/5}$$

$$=\frac{1}{5}\left[1+(1+j2)e^{j2\pi n/5}+(1-j)e^{j6\pi n/5}+(1-2j)e^{j8\pi n/5}\right]$$

$x[0]=1, x[1]=-0.5257, x[2]=-0.8507, x[3]=0.8507, x[4]=0.5257$

MATLAB

6.28 Use MATLAB to compute the FFT of the following signals. For each signal, plot $|X(k)|$.

(a) $x[n] = 1, 0 \le n \le 12$

(b) $x[n] = n, 0 \le n \le 10$

(c) $x[n] = \begin{cases} 1, & n = 0 \\ 1/n, & n = 1, 2, \ldots, 10 \\ 0, & \text{otherwise} \end{cases}$

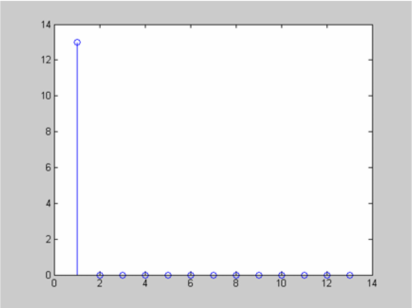
(d) $x[n] = n(0.8)^n, \quad 0 \le n \le 10$

(a) We use the following MATLAB script to find X[k]. The result and the plot are shown next.

```
x = [ 1 1 1 1 1 1 1 1 1 1 1 1 1];
X = fft(x)
stem(abs(X))
```

X =

13 0 0 0 0 0 0 0 0 0 0 0 0



(b) The MATLAB code and the result are shown below.

```
x = [ 0 1 2 3 4 5 6 7 8 9 10];
X = fft(x)
stem(abs(X))
```

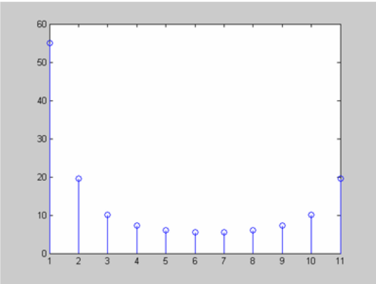
X =

Columns 1 through 7

55.0000 -5.5000 +18.7313i -5.5000 + 8.5582i -5.5000 + 4.7658i -5.5000 + 2.5118i -5.5000 + 0.7908i -5.5000 - 0.7908i

Columns 8 through 11

-5.5000 - 2.5118i -5.5000 - 4.7658i -5.5000 - 8.5582i -5.5000 -18.7313i



(c)

The MATLAB code and the result are presented below.

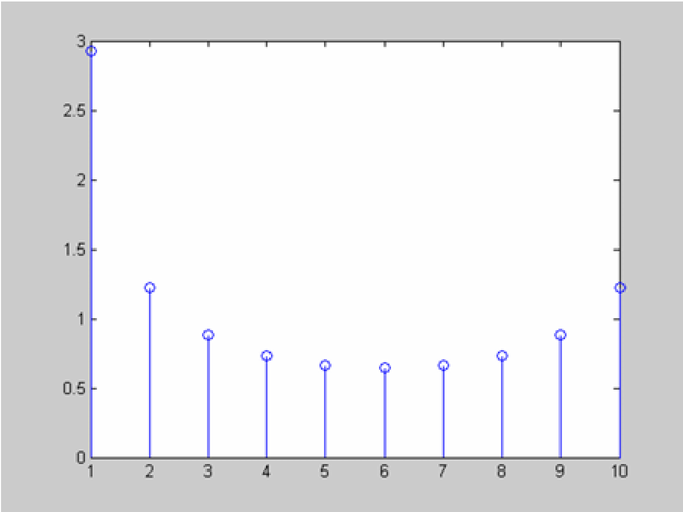
```
x = [ 1 1/2 1/3 1/4 1/5 1/6 1/7 1/8 1/9 1/10];
X = fft(x)
stem(abs(X))
```

Columns 1 through 7

2.9290 1.0628 - 0.5989i 0.7951 - 0.3832i 0.6977 - 0.2307i 0.6571 - 0.1091i 0.6456 0.6571 + 0.1091i

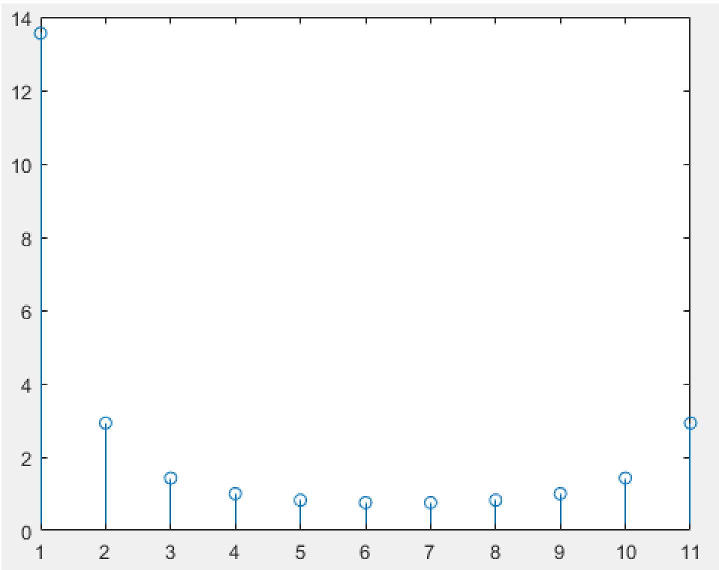
Columns 8 through 10

0.6977 .2307i 0.7951 + 0.3832i 1.0628 + 0.5989i



(d) The MATLAB script and the result arte given below.

```
x = [0 0.8 2*(0.8^2) 3*(0.8^3) 4*(0.8^4) 5*(0.8^5) 6*(0.8^6) 7*(0.8^7) 8*(0.8^8) 9*(0.8^9) 10*(0.8^10)];
X = fft(x);
stem(abs(X))
```



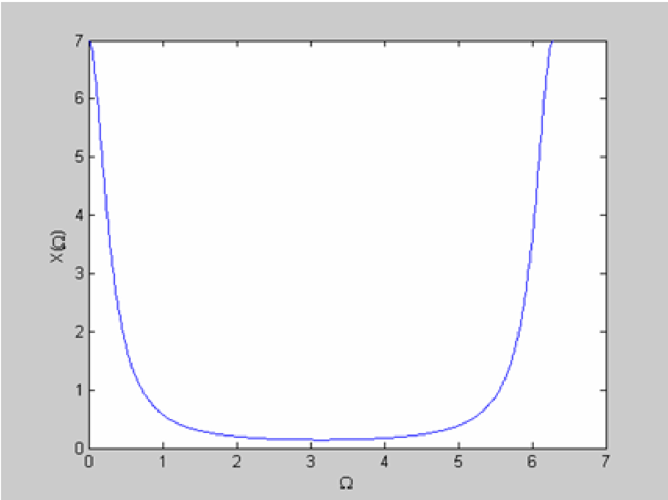
6.29 In Example 6.3, the DTFT of the signal $x[n] = a^{|n|}$ is

$$X(\Omega) = \frac{1-a^2}{1-2a\cos\Omega + a^2}$$

For $a = 0.75$ and $0 < \Omega < 2\pi$, plot $|X(\Omega)|$.

The MATLAB code and the plot are shown below.

```
a=0.75;
Omega= 0:0.01:2*pi;
X =(1-a^2)./(1- 2*a*cos(Omega)+ a^2);
plot(Omega,abs(X));
xlabel('\Omega')
ylabel('X(\Omega)')
```



6.30 Use MATLAB to find the DFT of the discrete signal

$$x[n] = \{1, 2, 0, -1, -2, 1, 5, 4\}$$

The MATLAB code with the result is shown below.

```
x = [1  2  0  -1  -2  1  5  4 ];
X = fft(x)


X =

  Columns 1 through 7

  10.0000            7.2426 + 7.8284i  -6.0000            -1.2426 - 2.1716i  -2.0000
1.2426 + 2.1716i  -6.0000

  Column 8

   7.2426 - 7.8284i
```