Prob. 7.5

Let $x_1[n] = 2\left(\frac{2}{3}\right)^n u[n] \longrightarrow X_1(z) = \frac{2z}{z-2/3} = \frac{6z}{3z-2}, |z| > 2/3$

Let $x_2[n] = \left(\frac{2}{5}\right)^n u[n] \longrightarrow X_2(z) = \frac{z}{z-2/5} = \frac{5z}{5z-2}, |z| > 2/5$

Hence,

$$X(z) = X_1(z) - X_2(z) = \frac{6z}{3z-2} - \frac{5z}{5z-2}, \qquad |z| > 2/3$$

Prob. 7.6

(a) $X(z) = z^{-m}\frac{z}{z-1} = \frac{z}{z^m(z-1)}$

(b) $X(z) = \frac{az}{(z-a)^2}$

(c)
$$X(z) = \frac{z^2 za\cos\pi}{z^2 - 2za\cos\pi + a^2}, \qquad \cos\pi = -1$$
$$= \frac{z^2 + za}{z^2 + 2za + a^2}$$

Prob. 7.19

$$X(z) = \frac{z-2}{z(z-1)} = \frac{A}{z} + \frac{B}{z-1}$$

$$A = zX(z)\Big|_{z=0} = \frac{-2}{-1} = 2$$

$$B = (z-1)X(z)\Big|_{z=1} = \frac{-1}{1} = -1$$

$$X(z) = \frac{2}{z} - z^{-1}\frac{z}{z-1}$$

$x[n] = 2\delta[n-1] - u[n-1]$

$x[0] = 0, x[1] = 2 - 1 = 1, x[10^5] = x[\infty] = -1$

Prob. 7.21

$$X(z) = 1 - z^{-1} + 3z^{-2} + 2z^{-3}$$

$$H(z) = 1 + 0z^{-1} + 2z^{-2} + z^{-3} - 3z^{-4}$$

$$\begin{aligned} Y(z) = X(z)H(z) = 1 + 0z^{-1} &+ 2z^{-2} + z^{-3} - 3z^{-4} \\ -z^{-1} &+ 0 - 2z^{-3} - z^{-4} + 3z^{-5} \\ &+3z^{-2} + 0 + 6z^{-4} + 3z^{-5} - 9z^{-6} \\ &+ \quad 2z^{-3} + 0 + 4z^{-5} + 2z^{-6} - 6z^{-7} \end{aligned}$$

$$Y(z) = 1 - z^{-1} + 5z^{-2} + z^{-3} + 2z^{-4} + 10z^{-5} - 7z^{-6} - 6z^{-7}$$

Thus,

y[n] = [1, -1, 5, 1, 2, 10, -7, -6]

Prob. 7.25

$$\text{Let } X_1(z) = \frac{X(z)}{z} = \frac{x^2 + 2z - 10}{z(z-1)(z+2)(z+3)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z+2} + \frac{D}{z+3}$$

$$A = zX_1(z)\Big|_{z=0} = \frac{-10}{(1)(-2)(-3)} = \frac{5}{3}$$

$$B = (z-1)X_1(z)\Big|_{z=1} = \frac{-7}{(1)(3)(4)} = \frac{-7}{12}$$

$$C = (z+2)X_1(z)\Big|_{z=-2} = \frac{-10}{(-2)(-3)(1)} = \frac{-5}{3}$$

$$D = (z+3)X_1(z)\Big|_{z=-3} = \frac{-7}{(-3)(-4)(-1)} = \frac{7}{12}$$

$$X(z) = \frac{5}{3} - \frac{7}{12}\left(\frac{z}{z-1}\right) - \frac{5}{3}\left(\frac{z}{z+2}\right) + \frac{7}{12}\left(\frac{z}{z+3}\right)$$

$$x[n] = \frac{5}{3}\delta[n] - \frac{7}{12}u[n] - \frac{5}{3}(-2)^n u[n] + \frac{7}{12}(-3)^n u[n]$$

Prob. 7.27

(a) $a^2 = 0.75 \longrightarrow a = 0.86$

$2a\cos\Omega = 1 \longrightarrow \Omega = \cos^{-1} 0.5774 = 54.73^o$

$a\sin\Omega = 0.707$

$$X_1(z) = \frac{z^2 - 0.5z}{z^2 - z + 0.75} - \frac{0.5}{0.707} \cdot \frac{0.707z}{z^2 - z + 0.75}$$

$x_1[n] = (0.866)^n \cos 54.73^o nu[n] - 0.7072(0.866)^n \sin 54.73^o nu[n]$

(b) $a^2 = 0.64 \longrightarrow a = 0.8$

$2a\cos\Omega = 0.8 \longrightarrow \Omega = \cos^{-1} 0.5 = 60^o$

$a\sin\Omega = 0.8\cos 60^o = 0.6928$

$$X_2(z) = \frac{z^2 - 0.4z}{z^2 - 0.8z + 0.64} + \frac{1.4z}{z^2 - 0.8z + 0.65}$$

$$= \frac{z^2 - 0.4z}{z^2 - 0.8z + 0.64} + \frac{1.4}{0.6928} \frac{0.6928z}{z^2 - 0.8z + 0.65}$$

$x_2[n] = (0.8)^n \cos 60^o nu[n] + 2.02(0.8)^n \sin 60^o nu[n]$

Prob. 7.30

$$Y(z) + 6\left(z^{-1}Y(z) + y[-1]\right) + 15\left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]\right) = 0$$

$$Y(z) = \frac{-6 - 15z^{-1}}{1 + 6z^{-1} + 15z^{-2}} = \frac{-6z^2 - 15z}{z^2 + 6z + 15}$$

We compare the denominator with the entries 14 and 15 in Table 7.2

$z^2 - 2az\cos\Omega + a^2 \equiv z^2 + 6z + 15$

$a = \sqrt{15} = 3.873, -2a\cos\Omega = 6 \longrightarrow \Omega = 140.77$

$a\sin\Omega = 2.449$

$$Y(z) = \frac{-6(z^2 + 3z\cos\Omega)}{z^2 + 6z + 15} + \frac{3z}{z^2 + 6z + 15} \frac{2.449}{2.449}$$

$y[n] = -6(\sqrt{15})^n \cos(140.77°n)u[n] + 1.225\,(\sqrt{15})^n \sin(140.77°n)u[n]$

Prob. 7.36

$$Y(z) = H(z)X(z) = \left(\frac{z}{z-1}\right)\left(\frac{1+2z^{-1}}{1-z^{-1}+z^{-2}}\right) = \frac{z(z^2+2z)}{(z-1)(z^2-z+1)}$$

Let $\quad Y_1(z) = \dfrac{Y(z)}{z} = \dfrac{(z^2+2z)}{(z-1)(z^2-z+1)} = \dfrac{A}{z-1} + \dfrac{Bz+C}{z^2-z+1}$

$$A = Y_1(z)(z-1)\Big|_{z=1} = 3/1 = 3$$

$$z^2 + 2z = A(z^2 - z + 1) + B(z^2 - z) + C(z-1)$$

Equating coefficients,

$z^2 : 1 = A + B \quad \longrightarrow \quad B = 1 - A = -2$

$z : 2 = A - B + C$

constant: $\quad 0 = A - C \quad \longrightarrow \quad C = A = 3$

$$Y(z) = \frac{3z}{z-1} + \frac{z(-2z+3)}{z^2-z+1} = \frac{3z}{z-1} - \frac{2(z^2-1.5z)}{z^2-z+1}$$

For the second term, let

$-z = -2z\cos\Omega \quad \longrightarrow \quad \Omega = \cos^{-1}(0.5) = 60^o$

$$Y(z) = \frac{3z}{z-1} - \frac{2(z^2-0.5z)}{z^2-z+1} + \frac{2}{0.866}\frac{0.866z}{z^2-z+1}$$

$$y[n] = 3u[n] - 2\cos(60^o n)u[n] + 2.309\sin(60^o n)u[n]$$

Prob. 7.38

Let E by the input to H_1. From the figure,

$$E = X - Y - H_2 Y$$

$$Y = EH_1 = H_1 X - H_1 Y - H_1 H_2 Y$$

$$Y\left(1 + H_1 + H_1 H_2\right) = H_1 X$$

$$H = \frac{Y}{X} = \frac{H_1}{1 + H_1 + H_1 H_2}$$

Prob. 7.41

(a) $Y(z) = H(z)X(z) = \dfrac{z(z-0.6)}{(z-1)(z+0.2)(z-0.8)}$

Let $\quad Y_1(z) = \dfrac{Y(z)}{z} = \dfrac{z-0.6}{(z-1)(z+0.2)(z-0.8)} = \dfrac{A}{z-1} + \dfrac{B}{z+0.2} + \dfrac{C}{z-0.8}$

$$A = Y_1(z)(z-1)\Big|_{z=1} = \frac{0.4}{1.2(0.2)} = 1.667$$

$$B = Y_1(z)(z+0.2)\Big|_{z=-0.2} = \frac{-0.8}{(-1.2)(-1)} = -0.6667$$

$$C = Y_1(z)(z-0.8)\Big|_{z=0.8} = \frac{0.2}{(-0.2)(1)} = -1$$

$$Y(z) = \frac{1.667z}{z-1} - \frac{0.6667z}{z+0.2} - \frac{z}{z-0.8}$$

$$y[n] = 1.667u[n] - 0.667(-0.2)^n u[n] - (0.8)^n u[n]$$

(b)

$$X(z) = \frac{z}{z-2}$$

$$Y(z) = H(z)X(z) = \frac{z(z-0.6)}{(z-2)(z+0.2)(z-0.8)}$$

Let $\quad Y_1(z) = \dfrac{Y(z)}{z} = \dfrac{z-0.6}{(z-2)(z+0.2)(z-0.8)} = \dfrac{A}{z-2} + \dfrac{B}{z+0.2} + \dfrac{C}{z-0.8}$

$$A = Y_1(z)(z-2)\Big|_{z=2} = \frac{1.4}{(2.2)(1.2)} = 0.5303$$

$$B = Y_1(z)(z+0.2)\Big|_{z=-0.2} = \frac{-0.8}{(-2.2)(-1)} = -0.3636$$

$$C = Y_1(z)(z-0.8)\Big|_{z=0.8} = \frac{0.2}{(-1.2)(1)} = -0.1667$$

$$Y(z) = \frac{0.5303z}{z-2} - \frac{0.3636z}{z+0.2} - \frac{0.1667z}{z-0.8}$$

$$y[n] = 0.5303(2)^n u[n] - 0.3636(-0.2)^n u[n] - 0.1667(0.8)^n u[n]$$

Prob. 7.45

```
syms  X x n z
X = z/(z-0.6);
x = iztrans(X)
```

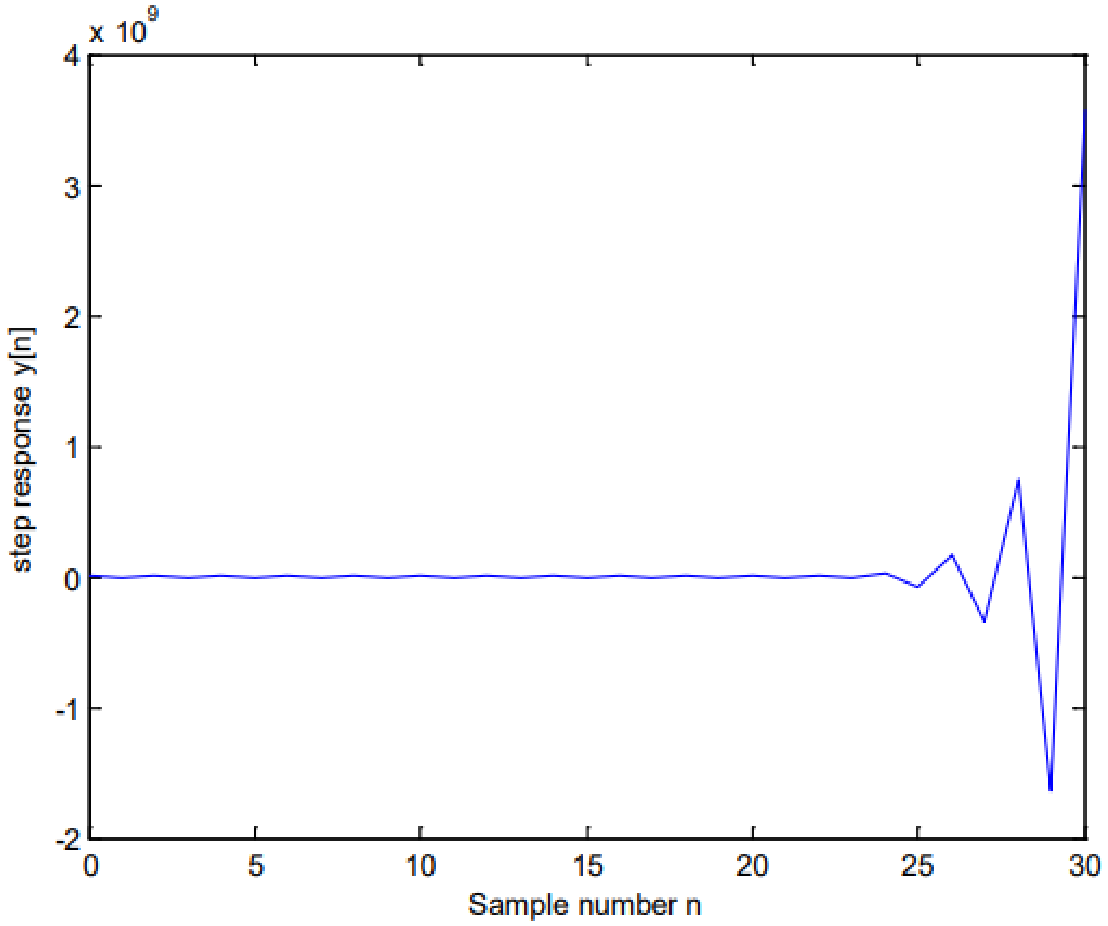
x =

(3/5)^n

Prob. 7.46

The MATLAB script and the plot are presented below.

```
num = [1  1];
den = [ 1  2  1  3];
n = 0:1:30;
x = [1*ones(size(n))]; % unit step input
y = filter(num, den, x);
plot(n,y);
xlabel('Sample number n');
ylabel('step response y[n]')
```



Prob. 7.47

(a) The MATLAB code with the result is shown below.

```
num = [1 6 1];
den = [ 1   3   4   10];
z = roots(num)
p=roots(den)
```

Or den = [1 3 0 4 10]

z =

-5.8284
-0.1716

p =

-2.8338 + 0.0000i
-0.0831 + 1.8767i
-0.0831 - 1.8767i

or

P =
-3.0794,
-1.4009,
$0.7401 \pm 1.3305i$

Prob. 7.49

```
den = [ 1  1.25  0.5  -0.375  -0.2];
p=roots(den)
```

p =

-0.6598 + 0.6299i
-0.6598 - 0.6299i
0.5263 + 0.0000i
-0.4567 + 0.0000i

Since the absolute value of each pole is less than unity, we conclude that all poles lie within a unit circle. Hence, the system is stable.