Prob. 2.4

(a) $x(t) * y(t) = 2\delta(t) * 4u(t) = 8u(t)$

(b) $x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = 2e^{-2t}u(t)$

(c) $y(t) * z(t) = 4u(t) * e^{-2t}u(t) = 4\int_0^t e^{-2\lambda} d\lambda = \left.\frac{4e^{-2\lambda}}{-2}\right|_0^t = 2(1-e^{-2t})$

(d) $y(t) * [y(t)+z(t)] = 4u(t) * [4u(t) + e^{-2t}u(t)] = 4\int [4u(\lambda) + e^{-2\lambda}u(\lambda)]d\lambda$

$$= 4\int_0^t [4+e^{-2\lambda}]d\lambda = 4[4t + \frac{e^{-2\lambda}}{-2}]\Big|_0^t = 16t - 2e^{-2t} + 2$$

Prob. 2.14

$x(t) = u(t)$

$$y(t) = x(t) * h(t) = \int_0^t e^{-\tau}u(\tau)d\tau = (1-e^{-t})u(t)$$

Method 2:

$$\int_{-\infty}^{+\infty} e^{-(t-\tau)}u(t-\tau)u(\tau)\, d\tau$$

$$= \int_0^t e^{-(t-\tau)}u(t)d\tau$$

$$= e^{-t}u(t)\int_0^t e^{\tau}d\tau$$

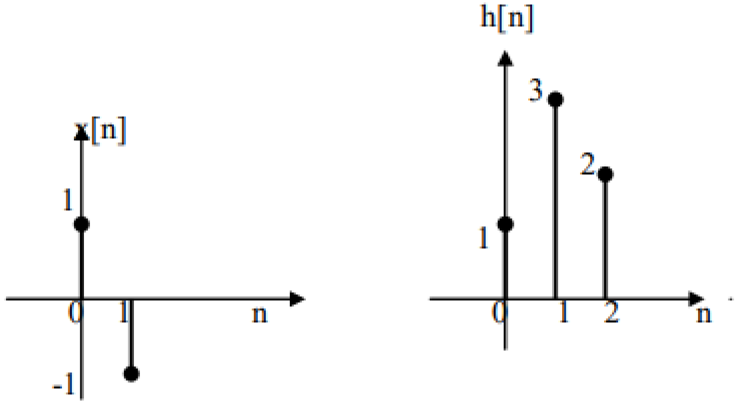
$$= e^{-t} \times \left(\left. e^{\tau}\right|_0^t\right) u(t) \qquad = e^{-t} \times (e^t - e^0)\, u(t) \qquad = (1 - e^{-t})u(t)$$

Prob. 2.24

$$h(t) = h_5(t) * [h_4(t) + h_2(t) * h_3(t) - h_1(t)]$$

Prob. 2.30

(a) x[n] and h[k] are sketched below.



(b)This can be done in three ways.

Method 1: Using MATLAB,

x = [1 -1];

h = [1 3 2];

y = conv(x,h) = [1 2 -1 -2]

Method 2: Analytically, we can apply x[n]*δ[n-k] = x[n-k]

$$x[n] = \delta[n] - \delta[n-1]$$

$$h[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2]$$

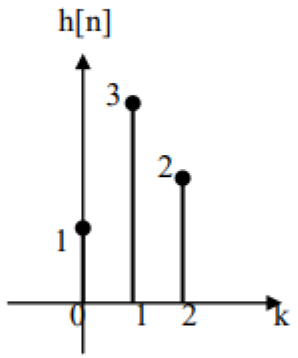
$$y[n] = x[n] * h[n] = \delta[n] * \delta[n] + 3\delta[n] * \delta[n-1] + 2\delta[n] * \delta[n-2]$$

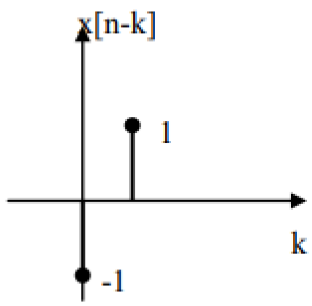
$$-\delta[n] * \delta[n-1] - 3\delta[n-1] * \delta[n-1] - 2\delta[n-1] * \delta[n-2]$$

$$= \delta[n] + 2\delta[n-1] - \delta[n-2] - 2\delta[n-3]$$

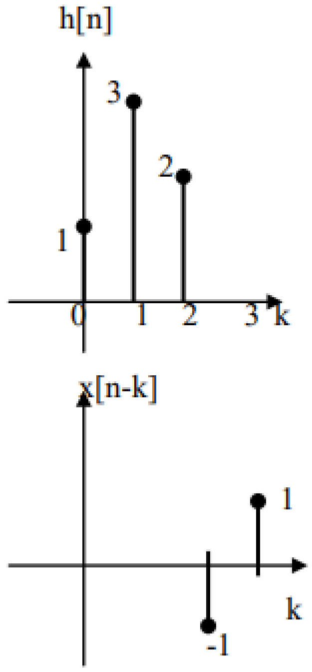
Method 3: Graphically, $y[n] = x[n] * h[n] = \sum_{k=0}^{n} h[k]x[n-k]$

For n = 0, y]0] = (1)(1)=1. For n=1, we have the figure below





y[1] = (1)(-1)+(1)(3)=2
For n=2, y[2] = (3)(-1) + (1)(2)= -1
For n=3, we have the figure shown below.



y[3]=(2)(-1)= -2. Thus,

$$y[n]=\begin{cases} 1, & n=0 \\ 2, & n=1 \\ -1, & n=2 \\ -2, & n=3 \\ 0, & \text{otherwise} \end{cases}$$

Prob. 2.33

(a) When they are connected in parallel,

$h = h_1 + h_2 = (0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]$

$$y[n] = h[n] * x[n] = (0.4)^n u[n] * \left[(0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]\right]$$

$$= (n+1)(0.4)^n u[n] + (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$$

$$= (n+2)(0.4)^n u[n] + 1.25(0.4)^n u[n-1]$$

(b) When they are cascaded

$$h = h_1 * h_2 = (0.4)^n u[n] * \left[\delta[n] + 0.5\delta[n-1]\right]$$

$$= (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$$

$$y[n] = h[n] * x[n] = (0.4)^n u[n] * \left[(0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]\right]$$

But if $z[n] = a^n u[n] * a^n u[n] = (n+1)a^n u[n] = r[n]a^n$

$$a^n u[n] * a^{n-1} u[n-1] = z[n-1] = (n-1+1)\, a^{n-1} u[n-1] = na^{n-1} u[n-1]$$

Hence,

$$y[n] = (n+1)(0.4)^n u[n] + 0.5n(0.4)^{n-1} u[n-1]$$

Prob. 2.41

a. 以recursive algorithm 求 h[n]

```
function value = h(n)
    x = [1 -1 2 4];
    y = [2 6 4 0 8 5 12];
    if n==1
        value = y(1)/x(1);
        return;
    end
    S=0;
    for k = 1 : n-1
        S = S + h(k)*x(n-k+1);
    end
    value = (y(n)-S)/x(1);
end
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> h(1), h(2), h(3), h(4)

ans =

     2


ans =

     8


ans =

     8


ans =

   -16

>>
```

Prob. 2.41

b. 手算或以deconv()驗算

```
clear
x = [ 1  -1  2  4];
y = [ 2 6 4  0 8 5 12 ];
h = deconv(y,x);
h =

     2     8     8   -16
```

				2	**8**	**8**	**-16**			
1	-1	2	4	2	6	4	0	8	5	12
				2	-2	4	8			
					8	0	-8	8		
					8	-8	16	32		
						8	-24	-24	5	
						8	-8	16	32	
							-16	-40	-27	12
							-16	16	-32	-64
								-56	5	76