2.4 Given the following signals

$$x(t) = 2\delta(t), \quad y(t) = 4u(t), \quad z(t) = e^{-2t}u(t),$$

$* \; u(\tau)\cdot u(\tau - t) = \begin{cases} 1, & 0<\tau<t \\ 0, & \text{otherwise} \end{cases}$

Evaluate the following operations.

(a) $x(t)*y(t)$

(b) $x(t)*z(t)$

(c) $y(t)*z(t)$

(d) $y(t)*[y(t) + z(t)]$

$* \; \delta(t) * u(t) = u(t)$

(a) 所求 $= 2\delta(t) * 4u(t)$

$= 8u(t)$ #

(b) 所求 $= 2\delta(t) * e^{-2t}u(t)$

$= 2e^{-2t}u(t)$ #

(c) 所求 $= \int_{-\infty}^{\infty} 4u(\tau)\cdot e^{-2(t-\tau)}u(t-\tau)d\tau$

$= 4e^{-2t}\int_0^t e^{2\tau}d\tau$

$= 4\cdot e^{-2t}\cdot \frac{1}{2}(e^{2t}-1)$

$= 2(1-e^{-2t}), \; (t>0)$ #

(d) 所求 $= \int_{-\infty}^{\infty} 4u(\tau)\cdot(4+e^{-2(t-\tau)})u(t-\tau)d\tau$

$= 4\int_0^t (4+e^{-2t}\cdot e^{2\tau})d\tau$

$= 4\cdot\left(4\tau + \frac{e^{-2t}}{2}e^{2\tau}\right)\Big|_0^t$

$= 4\cdot\left[\left(4t+\frac{1}{2}\right)-\left(0+\frac{e^{-2t}}{2}\right)\right]$

$= 16t+2-2e^{-2t}, \; (t>0)$ #

2.14 The impulse response of a low-pass filter is $h(t) = e^{-t}u(t)$. Determine its step response, that is, the output when the input is a unit step.

$x(t) = u(t)$

所求 $= u(t) * e^{-t}u(t) = \int_{-\infty}^{\infty} u(\tau)\cdot e^{-(t-\tau)}u(t-\tau)d\tau = e^{-t}\cdot\int_0^t e^{\tau}d\tau$

$= e^{-t}(e^{t}-1) = 1-e^{-t}, \; (t>0)$ #

2.24 Determine the overall impulse response for the system shown in Figure 2.34.

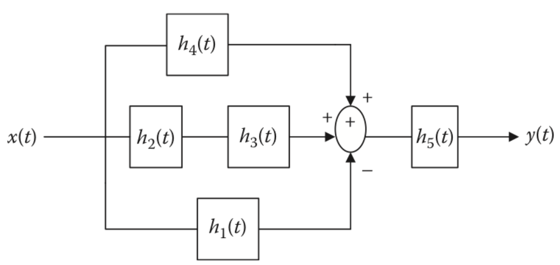


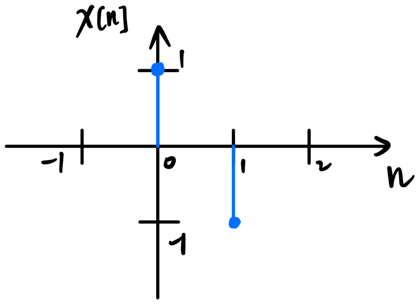
FIGURE 2.34 For Problem 2.24.

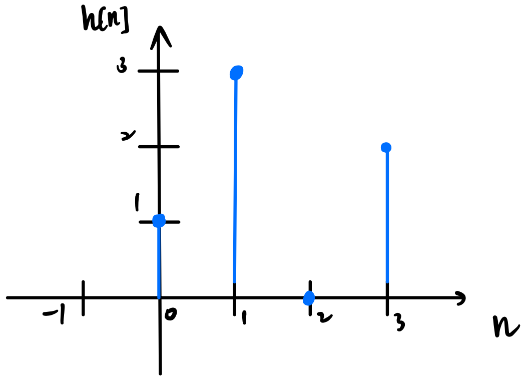
$h(t) = \{h_4(t) + [h_2(t) * h_3(t)] - h_1(t)\} * h_5(t)$ #

2.30 Given that $x[n] = \begin{cases} 1, & n=0 \\ -1, & n=1 \\ 0, & \text{otherwise} \end{cases}$, $\quad h[n] = \begin{cases} 1, & n=0 \\ 3, & n=1 \\ 2, & n=3 \\ 0, & \text{otherwise} \end{cases}$

(a) Sketch $x[n]$ and $h[n]$.
(b) Find $x[n]*h[n]$.

(a)





(b) $x[n]*h[n] = \sum_{k=0}^{n} x[k]h[n-k] = y[n]$

$* \; x * \delta[n] = x$

$* \; x[n] * \delta[n-1] = x[n-1]$

$x[n] = \delta[n] - \delta[n-1]$

$h[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-3]$

$y[n] = x[n]*h[n] = (\delta[n]*\delta[n] + 3\delta[n]*\delta[n-1] + 2\delta[n]*\delta[n-3]) - (\delta[n-1]*\delta[n] + 3\delta[n-1]*\delta[n-1] + 2\delta[n-1]\delta[n-3])$

$= (\delta[n] + 3\delta[n-1] + 2\delta[n-3]) - (\delta[n-1] + 3\delta[n-2] + 2\delta[n-4])$

$= \delta[n] + 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3] - 2\delta[n-4]$ ※

2.33 Two systems are described by

$$h_1[n] = (0.4)^n u[n], \qquad h_2[n] = \delta[n] + 0.5\delta[n-1]$$

Determine the response to the input $x[n] = (0.4)^n u[n]$ if
(a) The two systems are connected in parallel
(b) The two systems are connected in cascade

(a) 並聯 $h[n] = h_1[n] + h_2[n]$

$= (0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]$

$y[n] = x[n]*h[n]$

$= (0.4)^n u[n] * [(0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]]$

$= (n+1)(0.4)^n u[n] + (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$

$= (n+2)(0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$ ※

(b) 串聯 $h = h_1 * h_2$

$= (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$

$y[n] = x[n] * h[n]$

$= (0.4)^n u[n] * [(0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]]$

$\because z[n] = a^n u[n] * a^n u[n] = (n+1)a^n u[n]$

$= r[n] a^n$

$\Rightarrow a^n u[n] * a^{n-1} u[n-1] = z[n-1]$

$= (n-1+1)a^{n-1} u[n-1] = n a^{n-1} u[n-1]$

$\therefore y[n] = (n+1)(0.4)^n u[n] + 0.5n(0.4)^{n-1} u[n-1]$ ※

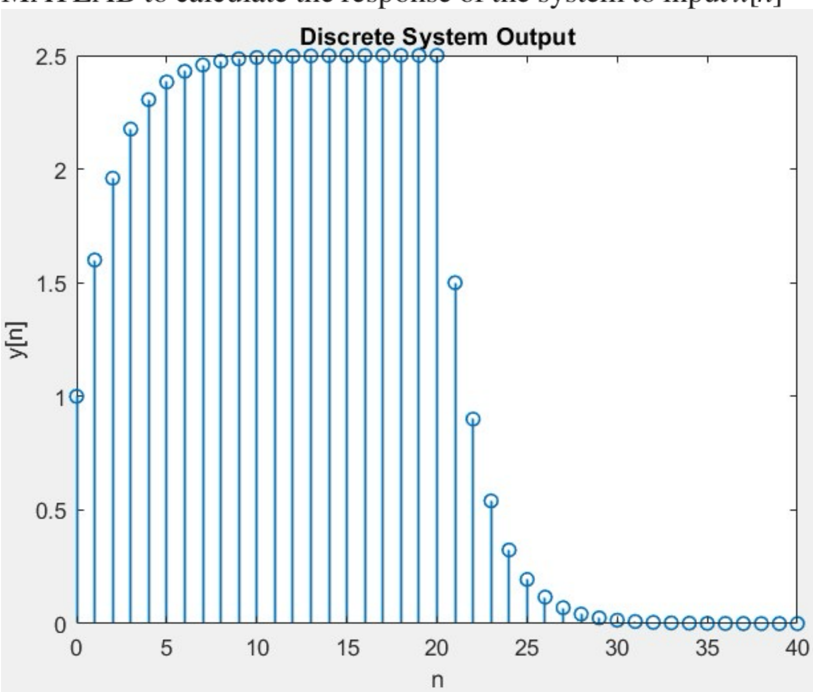
2.36 The input $x[n] = [1\ \ -1]$ to a system produces the output $y[n] = [4\ \ 2\ \ 5\ \ 1]$. Determine the impulse response.

令 $y[z] = 4z^3 + 2z^2 + 5z + 1$, $X[z] = z - 1$

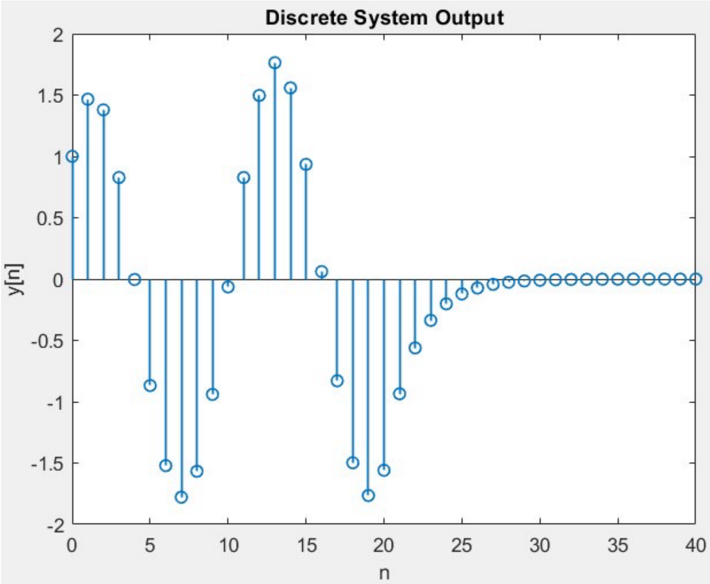
$$
\begin{array}{r|l}
 & 4+6+11 \\
\hline
1-1 & 4+2+5+1 \\
 & 4-4 \\
\hline
 & \quad 6+5 \\
 & \quad 6-6 \\
\hline
 & \quad\quad 11+1 \\
 & \quad\quad 11-11 \\
\hline
 & \quad\quad\quad 12
\end{array}
$$

$\therefore h[n] = [4\ \ 6\ \ 11]$ #

2.39 An LTI discrete system has the impulse response $h[n] = (0.6)^n\ u[n]$. Use MATLAB to calculate the response of the system to input $x[n] = u[n]$ and plot it.



2.40 Repeat the previous problem for $x[n] = \cos(n\pi/6)u[n]$.



2.41 Given that $x[n] = [1\ -1\ 2\ 4]$ and $y[n] = [2\ 6\ 4\ 0\ 8\ 5\ 12]$, use MATLAB to find $h[n]$.

(a)

```
>> HW2_41a
h(1) = 2
h(2) = 8
h(3) = 8
h(4) = -16
```

```
function value = h(n)
    x = [1 -1 2 4];
    y = [2 6 4 0 8 5 12];

    if n==1
        value = y(1)/x(1); %常數項
        return;
    end

    temp = 0;

    for i = 1 : (n-1)
        temp = temp + h(i)*x(n-i+1);
    end

    value = (y(n)-temp);
end


x = [1 -1 2 4];
y = [2 6 4 0 8 5 12];

for i = 1:(length(y)+1-length(x))
    disp(['h(', num2str(i), ') = ', num2str(h(i))]);
end
```

(b) deconv() ⇒

```
h =

     2     8     8   -16
```

$2+8+8-16$

$1-1+2+4 \,\overline{)\,2+6+4+0+8+5+12}$

$2-2+4+8$

$8+0-8+8$

$8-8+16+32$

$8-24-24+5$

$8-8+16+32$

$-16-40-27+12$

$-16+16-32-64$

$-56+5+76$