Prob. 3.3

(a) 2t=2(t-4) + 8

$$x(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)$$

$$X(s) = \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s} = \left(\frac{2}{s^2} + \frac{8}{s}\right)e^{-4s}$$

(b) $$X(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty 5\cos t\,\delta(t-2)e^{-st}dt = 5\cos t e^{-st}\Big|_{t=2} = 5\cos 2e^{-2s}$$

(c) $$e^{-t} = e^{-(t-\tau)}e^{-\tau}$$

$$x(t) = e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$X(s) = e^{-\tau}e^{-\tau s}\frac{1}{s+1} = \frac{e^{-\tau(s+1)}}{s+1}$$

(d) $$\sin 2t = \sin[2(t-\tau)+2\tau] = \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$$

$$x(t) = \cos 2\tau \sin 2(t-\tau)u(t-\tau) + \sin 2\tau\cos 2(t-\tau)u(t-\tau)$$

$$X(s) = \cos 2\tau e^{-\tau s}\frac{2}{s^2+4} + \sin 2\tau e^{-\tau s}\frac{s}{s^2+4}$$

Prob. 3.13

(a) $X(s)=\dfrac{3s+1}{(s+1)^2+4}=\dfrac{3(s+1)-2}{(s+1)^2+2^2}=\dfrac{3(s+1)}{(s+1)^2+2^2}\dfrac{-2}{(s+1)^2+2^2}$

$$x(t)=\left(3e^{-t}\cos 2t-e^{-t}\sin 2t\right)u(t)$$

(b) $Y(s)=\dfrac{3s+7}{(s+1)(s+2)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}$

$$A=Y(s)(s+1)\Big|_{s=-1}=4$$

$$B=Y(s)(s+2)\Big|_{s=-2}=-1$$

$$Y(s)=\frac{4}{s+1}-\frac{1}{s+2}$$

$$y(t)=\left(4e^{-t}-e^{-2t}\right)u(t)$$

(c) Express Z(s) as a proper fraction

$$Z(s)=1+\frac{-4}{(s+2)(s-2)}=1+\frac{A}{s+2}+\frac{B}{s-2}$$

$$A=\frac{-4}{-4}=1,\qquad B=\frac{-4}{4}=-1$$

$$Z(s)=1+\frac{1}{s+2}-\frac{1}{s-2}$$

$$z(t)=\delta(t)+\left(e^{-2t}-e^{2t}\right)u(t)$$

(d) $\dfrac{n!}{(s+a)^{n+1}} \quad \Leftrightarrow \quad t^n e^{-at}$

Let $n=3, a=2$

$$\frac{6}{(s+2)^4} \quad \Leftrightarrow \quad t^3 e^{-2t}$$

$$H(s)=\frac{12}{(s+2)^4} \quad \longrightarrow \quad h(t)=2t^3e^{-2t}u(t)$$

Prob. 3.14

(a) $F(s) = \frac{20(s+2)}{s(s^2+6s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+6s+25}$

$20(s+2) = A(s^2+6s+25) + Bs^2 + Cs$

Equating components,

s^2 : $0 = A + B$ or $B = -A$

s: $20 = 6A + C$

constant: $40 = 25A$ or $A = 8/5$, $B = -8/5$, $C = 20 - 6A = 52/5$

$$F(s) = \frac{8}{5s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{(s+3)^2+4^2} = \frac{8}{5s} + \frac{-\frac{8}{5}(s+3) + \frac{24}{5} + \frac{52}{5}}{(s+3)^2+4^2}$$

$$f(t) = \frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t$$

(b) $P(s) = \frac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$

$$A = \frac{6-36+20}{(-1+2)(-1+3)} = -5$$

$$B = \frac{24-72+20}{(-1)(1)} = 28$$

$$C = \frac{54-108+20}{(-2)(-1)} = -17$$

$$P(s) = \frac{-5}{s+1} + \frac{28}{s+2} - \frac{17}{s+3}$$

$$p(t) = (-5e^{-t} + 28e^{-2t} - 17e^{-3t})u(t)$$

Prob. 3.18

(a) $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5(1+1/s)}{(1+2/s)(1+3/s)} = 5$

$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

(b) $F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$$A = \frac{5(-1)}{1} = -5, \qquad B = \frac{5(-2)}{-1} = 10$$

$$F(s) = \frac{-5}{s+2} + \frac{10}{s+3} \qquad \longrightarrow \qquad f(t) = -5e^{-2t} + 10e^{-3t}$$

f(0) = -5 + 10 = 5
f(∞)= -0 + 0 = 0.

Prob. 3.26

(a) $y(t) = \delta(t) * 4e^{-2t}u(t) + \delta(t) * te^{-t}u(t)$

$\quad = \left(4e^{-2t} + te^{-t}\right)u(t)$

(b) $y(t) = e^{-t}u(t) * u(t) + e^{-2t}u(t) * u(t) + \delta(t) * u(t)$

$\quad = \frac{e^{-t} - 1}{-1}u(t) + \frac{e^{-2t} - 1}{-2} + u(t)$

$\quad = (1.5 - e^{-t} - 0.5e^{-2t})u(t)$

Prob. 3.43

(a) $s^2Y(s) + 2sY(s) + 2Y(s) = sX(s) - 3X(s)$

$(s^2 + 2s + 2)Y(s) = (s-3)X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s-3}{s^2 + 2s + 2}$

(b) $h(t) = \mathcal{L}^{-1}\left(H(s)\right)$

$H(s) = \frac{s+1}{(s+1)^2 + 1} - \frac{4}{(s+1)^2 + 1}$

$h(t) = \left(e^{-t}\cos t - 4e^{-t}\sin t\right)u(t)$

Prob. 3.46

```
num = [ 1 6  10];
den = [ 1  7  11  5];
[r,p,k] = residue(num,den)
```

r =

    0.3125
    0.6875
    1.2500

p =

   -5.0000
   -1.0000
   -1.0000

k =

     []

$$H(s) = \frac{0.3125}{s+5} + \frac{0.6875}{s+1} + \frac{1.25}{(s+1)^2}$$

The inverse Laplace transform of this is:

$$x(t) = 0.3125e^{-5t} + 0.6875e^{-t} + 1.25te^{-t}, t > 0$$

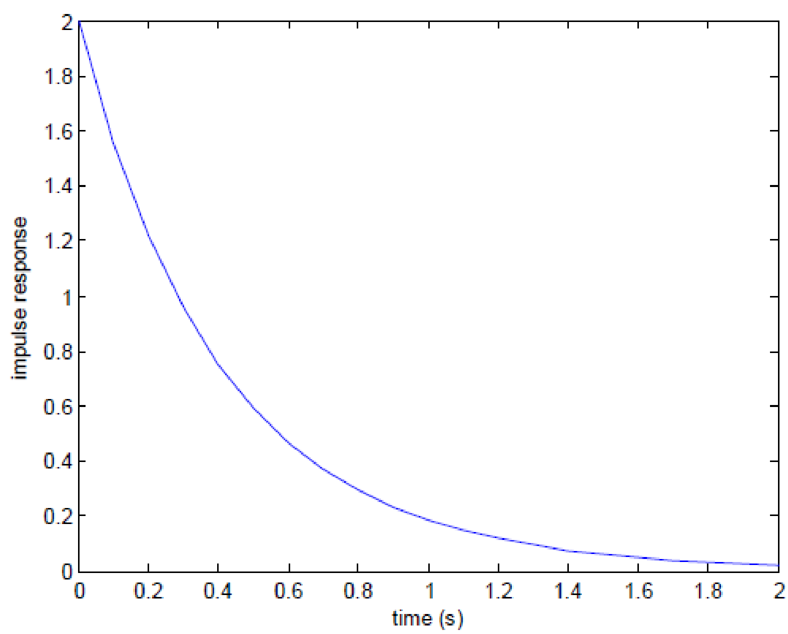
Prob. 3.49

(a) Expand the denominator to obtain

$$H(s) = \frac{2s+5}{s^2+5s+6}$$

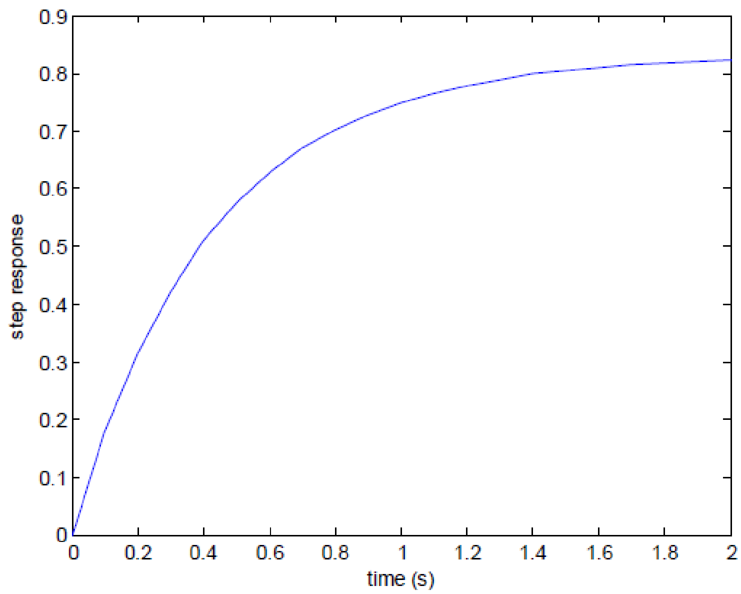
We now use MATLAB code shown below to obtain the impulse response shown below.

```
num = [ 2  5 ];
den = [1  5   6];
t = 0:0.1:2;
y=impulse(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('impulse response')
```



(b) For the step response, we use the following MATLAB code.

```
num = [ 2   5 ];
den = [1   5    6];
t = 0:0.1:2;
y=step(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('step response')
```



Prob. 3.51

(a) We first expand the denominator to get

$$H(s) = \frac{s-2}{s^2+2s+10}$$

```
num = [ 1   -2 ];
den = [1   2   10];
z = roots(num)
p = roots(den)
```

z =

2

p =

-1.0000 + 3.0000i
-1.0000 - 3.0000i

(b)

```
num = [ 1   2   5 ];
den = [1   4   13   0];
z = roots(num)
p = roots(den)
```

z =

-1.0000 + 2.0000i
-1.0000 - 2.0000i

p =

0.0000 + 0.0000i
-2.0000 + 3.0000i
-2.0000 - 3.0000i

(c)

```
num = [1  10  5 ];
den = [1  4  10  6];
z = roots(num)
p = roots(den)
```

z =

-9.4721
-0.5279

p =

-1.5956 + 2.2075i
-1.5956 - 2.2075i
-0.8087 + 0.0000i

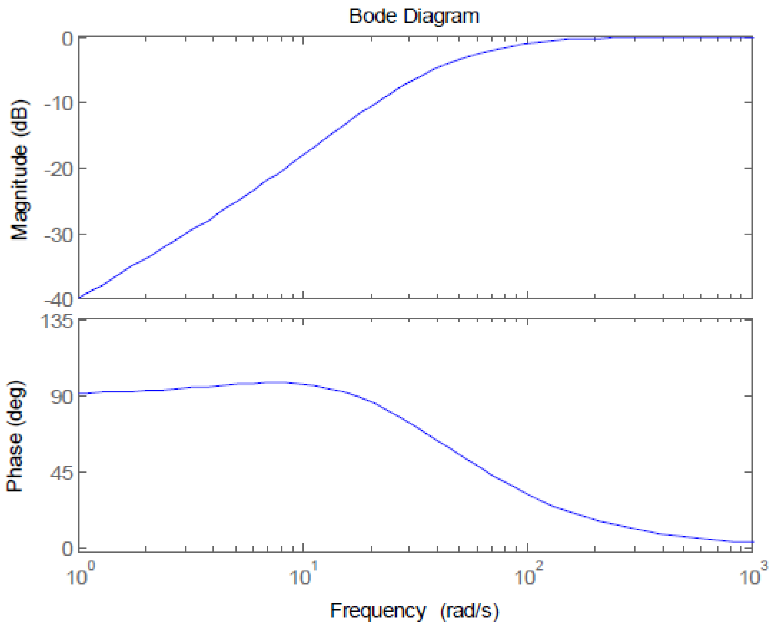
Prob. 3.53

(a) We expand H(s) as

$$H(s) = \frac{s^2 + 10s}{s^2 + 70s + 1000}$$

The MATLAB code with the Bode plot is presented below.

```
num = [ 1  10  0];
den = [ 1 70 1000];
bode(num,den)
```

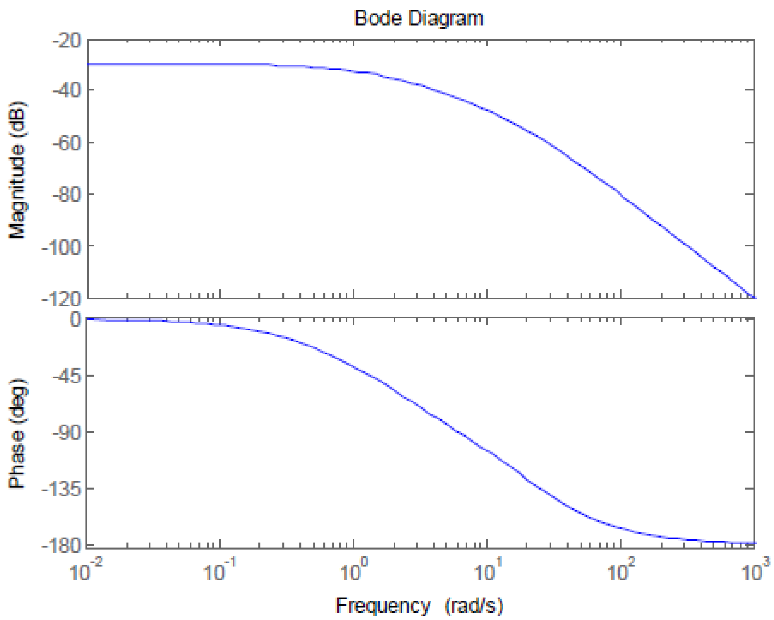


(b) We first expand the denominator of H(s) as follows.

$$H(s) = \frac{s+1}{s^3 + 24.5s^2 + 61s + 32}$$

The MATLAB code with the result is shown below.

```
num = [ 1  1 ];
den = [ 1  24.5  61  32 ];
bode(num,den)
```


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
-20
-40
-60
-80
-100
-120
0
-45
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
10^3