Prob. 2.4

(a) $x(t)*y(t)=2\delta(t)*4u(t)=8u(t)$

(b) $x(t)*z(t)=2\delta(t)*e^{-2t}u(t)=2e^{-2t}u(t)$

(c) $y(t)*z(t)=4u(t)*e^{-2t}u(t)=4\int_0^t e^{-2\lambda}d\lambda=\left.\frac{4e^{-2\lambda}}{-2}\right|_0^t=2(1-e^{-2t})$

(d) $y(t)*[y(t)+z(t)]=4u(t)*[4u(t)+e^{-2t}u(t)]=4\int[4u(\lambda)+e^{-2\lambda}u(\lambda)]d\lambda$

$$=4\int_0^t[4+e^{-2\lambda}]d\lambda=4[4t+\frac{e^{-2\lambda}}{-2}]\Big|_0^t=16t-2e^{-2t}+2$$

Prob. 2.14

$x(t)=u(t)$

$$y(t)=x(t)*h(t)=\int_0^t e^{-\tau}u(\tau)d\tau=(1-e^{-t})u(t)$$

Method 2:

$$\int_{-\infty}^{+\infty} e^{-(t-\tau)}u(t-\tau)u(\tau)\,d\tau$$

$$=\int_0^t e^{-(t-\tau)}u(t)d\tau$$

$$=e^{-t}u(t)\int_0^t e^{\tau}d\tau$$

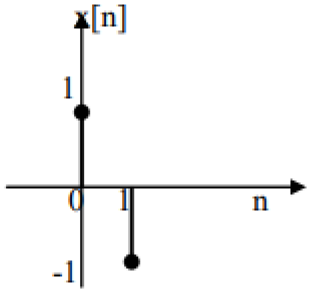
$$=e^{-t}\times\left(e^{\tau}\Big|_0^t\right)u(t) \qquad =e^{-t}\times(e^{t}-e^{0})\,u(t) \qquad =(1-\ e^{-t})u(t)$$

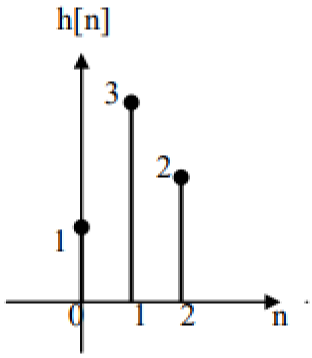
Prob. 2.24

$h(t)=h_5(t)*\left[h_4(t)+h_2(t)*h_3(t)-h_1(t)\right]$

Prob. 2.30

(a) x[n] and h[k] are sketched below.





(b)This can be done in three ways.

<u>Method 1:</u> Using MATLAB,

x = [1 -1];

h = [1 3 2];

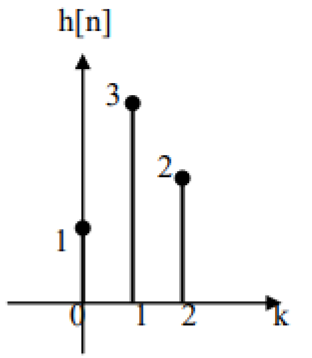
y = conv(x,h) = [1 2 -1 -2]

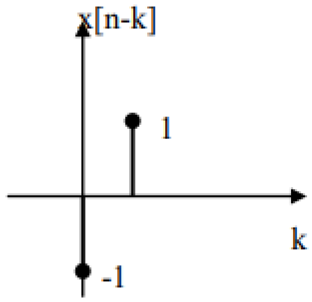
Method 2: Analytically, we can apply x[n]*δ[n-k] = x[n-k]

$$x[n] = \delta[n] - \delta[n-1]$$
$$h[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2]$$
$$y[n] = x[n] * h[n] = \delta[n] * \delta[n] + 3\delta[n] * \delta[n-1] + 2\delta[n] * \delta[n-2]$$
$$-\delta[n] * \delta[n-1] - 3\delta[n-1] * \delta[n-1] - 2\delta[n-1] * \delta[n-2]$$
$$= \delta[n] + 2\delta[n-1] - \delta[n-2] - 2\delta[n-3]$$

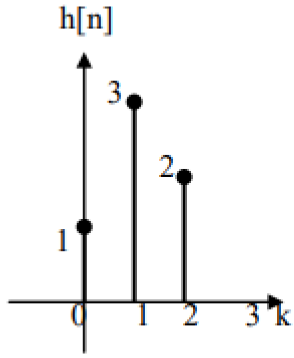
Method 3: Graphically, $y[n] = x[n] * h[n] = \sum_{k=0}^{n} h[k]x[n-k]$

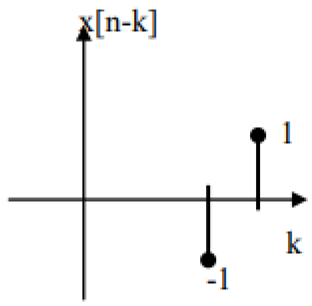
For n = 0, y]0] = (1)(1)=1. For n=1, we have the figure below





y[1] = (1)(-1)+(1)(3)=2
For n=2, y[2] = (3)(-1) + (1)(2)= -1
For n=3, we have the figure shown below.





y[3]=(2)(-1)= -2. Thus,

$$y[n]=\begin{cases} 1, & n=0 \\ 2, & n=1 \\ -1, & n=2 \\ -2, & n=3 \\ 0, & \text{otherwise} \end{cases}$$

Prob. 2.33

(a) When they are connected in parallel,

$$h = h_1 + h_2 = (0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]$$

$$y[n] = h[n] * x[n] = (0.4)^n u[n] * \left[(0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]\right]$$

$$= (n+1)(0.4)^n u[n] + (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$$

$$= (n+2)(0.4)^n u[n] + 1.25(0.4)^n u[n-1]$$

(b) When they are cascaded

$$h = h_1 * h_2 = (0.4)^n u[n] * \left[\delta[n] + 0.5\delta[n-1]\right]$$

$$= (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$$

$$y[n] = h[n] * x[n] = (0.4)^n u[n] * \left[(0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]\right]$$

But if $z[n] = a^n u[n] * a^n u[n] = (n+1)a^n u[n] = r[n]a^n$

$$a^n u[n] * a^{n-1} u[n-1] = z[n-1] = (n-1+1)\,a^{n-1} u[n-1] = na^{n-1} u[n-1]$$

Hence,

$$y[n] = (n+1)(0.4)^n u[n] + 0.5n(0.4)^{n-1} u[n-1]$$

Prob. 2.36

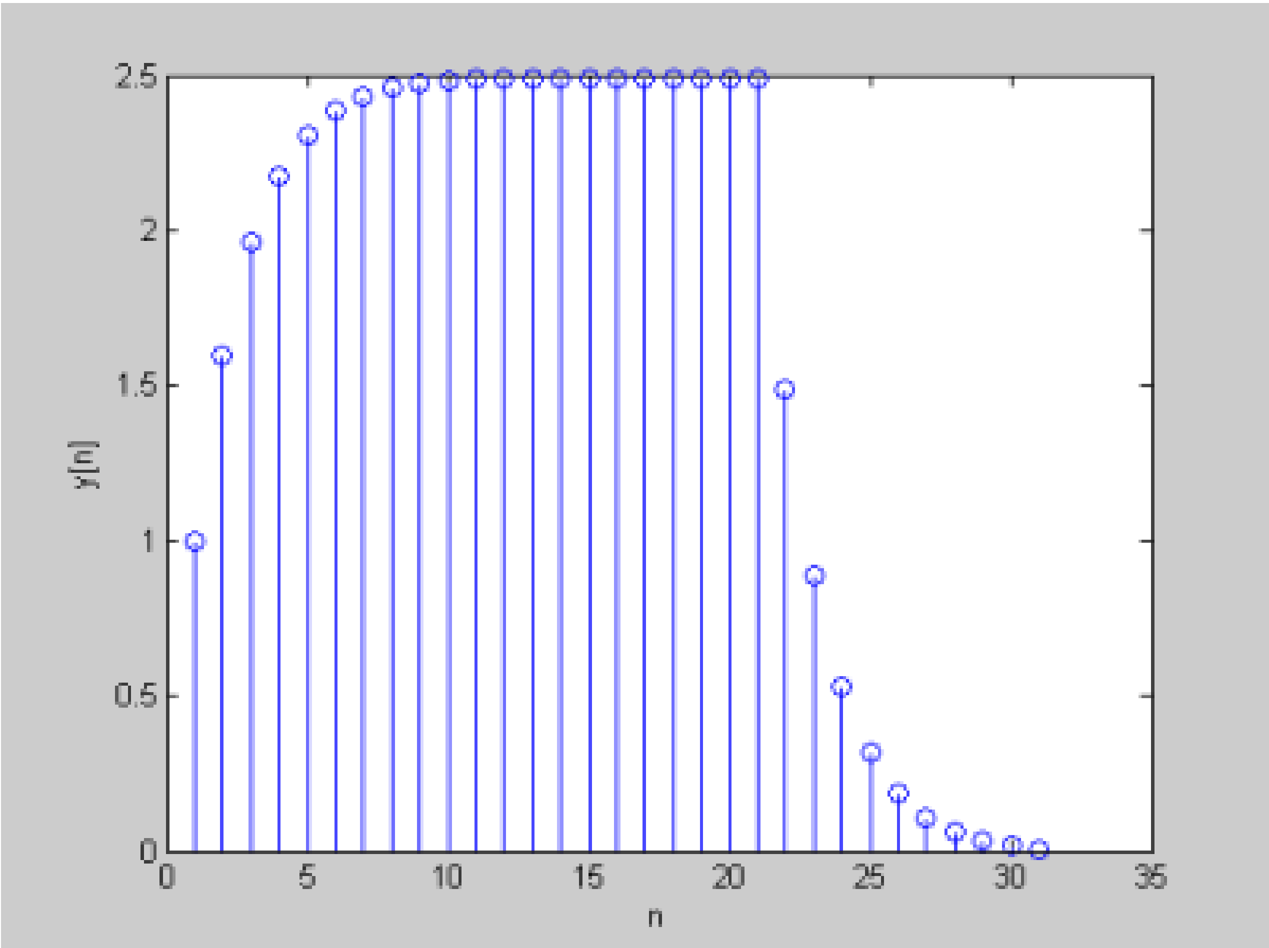
h = deconv(y,x)
= [4 6 11]

Remainder = 12

Prob. 2.39

The MATLAB code and the plot of y are provided below.

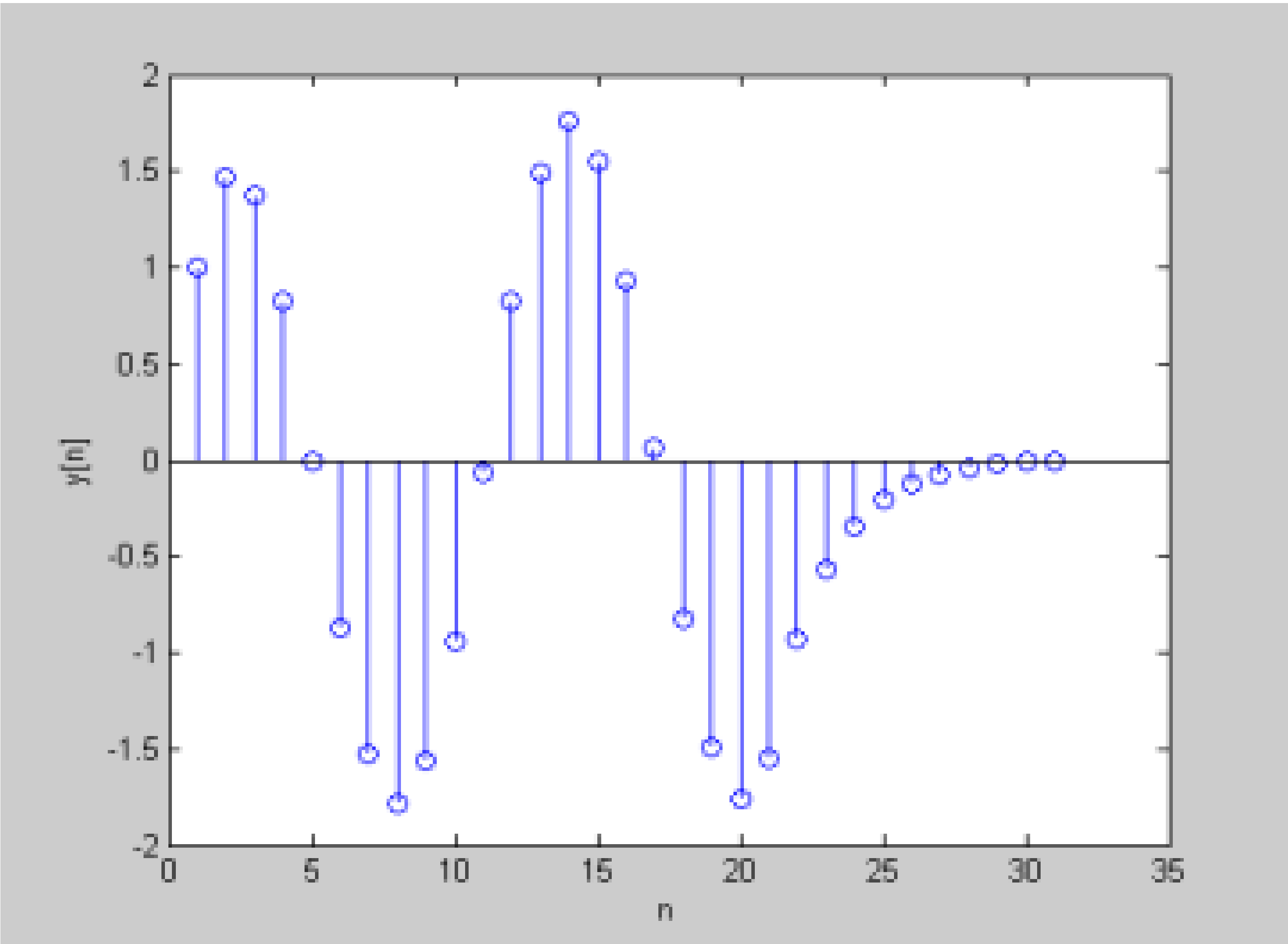
```
m=0:1:20;
len1 = max(size(m));
x =ones(len1,1); %calculates x[n]
n = 0:10;
h = (0.6).^n;  % calculates h[n]
y =  conv(x,h);
u= max(size(y));
nn=1:1:u
stem(nn,y)
xlabel('n');
ylabel('y[n]');
```



Prob. 2.40

The MATLAB code and the results are shown below.

```
m=0:1:20;
x = cos(m*pi/6); %calculates x[n]
n = 0:10;
h = (0.6).^n;  % calculates h[n]
y =  conv(x,h);
u= max(size(y));
nn=1:1:u
stem(nn,y)
xlabel('n');
ylabel('y[n]');
```



Prob. 2.41

a. 以recursive algorithm 求 h[n]

```
function value = h(n)
    x = [1 -1 2 4];
    y = [2 6 4 0 8 5 12];
    if n==1
        value = y(1)/x(1);
        return;
    end
    S=0;
    for k = 1 : n-1
        S = S + h(k)*x(n-k+1);
    end
    value = (y(n)-S)/x(1);
end
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> h(1), h(2), h(3), h(4)

ans =

     2


ans =

     8


ans =

     8


ans =

   -16

>>
```

Prob. 2.41

b. 手算或以deconv()驗算

```
clear
x = [ 1  -1  2  4];
y = [ 2 6 4  0 8 5 12 ];
h = deconv(y,x);
h =

    2    8    8   -16
```

				2	**8**	**8**	**-16**			
1	-1	2	4	2	6	4	0	8	5	12
				2	-2	4	8			
					8	0	-8	8		
					8	-8	16	32		
						8	-24	-24	5	
						8	-8	16	32	
							-16	-40	-27	12
							-16	16	-32	-64
								-56	5	76