2.4 Given the following signals

$$x(t) = 2\delta(t), \quad y(t) = 4u(t), \quad z(t) = e^{-2t}u(t),$$

$$* \; u(\tau)\cdot u(\tau - t) = \begin{cases} 1, & 0<\tau<t \\ 0, & \text{otherwise} \end{cases}$$

Evaluate the following operations.

(a) $x(t)*y(t)$

(b) $x(t)*z(t)$

(c) $y(t)*z(t)$

(d) $y(t)*[y(t) + z(t)]$

$$* \; \delta(t) * u(t) = u(t)$$

(a) 所求 $= 2\delta(t) * 4u(t)$

$= 8u(t)$ #

(b) 所求 $= 2\delta(t) * e^{-2t}u(t)$

$= 2e^{-2t}u(t)$ #

(c) 所求 $= \int_{-\infty}^{\infty} 4u(\tau)\cdot e^{-2(t-\tau)}u(t-\tau)\,d\tau$

$= 4e^{-2t}\int_0^t e^{2\tau}\,d\tau$

$= 4\cdot e^{-2t}\cdot \frac{1}{2}(e^{2t}-1)$

$= 2(1-e^{-2t}), \; (t>0)$ #

(d) 所求 $= \int_{-\infty}^{\infty} 4u(\tau)\cdot(4+e^{-2(t-\tau)})\,u(t-\tau)\,d\tau$

$= 4\int_0^t (4+e^{-2t}\cdot e^{2\tau})\,d\tau$

$= 4\cdot\left(4\tau + \frac{e^{-2t}}{2}e^{2\tau}\right)\Big|_0^t$

$= 4\cdot\left[\left(4t+\frac{1}{2}\right)-\left(0+\frac{e^{-2t}}{2}\right)\right]$

$= 16t+2-2e^{-2t}, \; (t>0)$ #

2.14 The impulse response of a low-pass filter is $h(t) = e^{-t}u(t)$. Determine its step response, that is, the output when the input is a unit step.

$x(t) = u(t)$

所求 $= u(t) * e^{-t}u(t) = \int_{-\infty}^{\infty} u(\tau)\cdot e^{-(t-\tau)}u(t-\tau)\,d\tau = e^{-t}\cdot\int_0^t e^{\tau}\,d\tau$

$= e^{-t}(e^{t}-1) = 1-e^{-t}, \; (t>0)$ #

2.24 Determine the overall impulse response for the system shown in Figure 2.34.

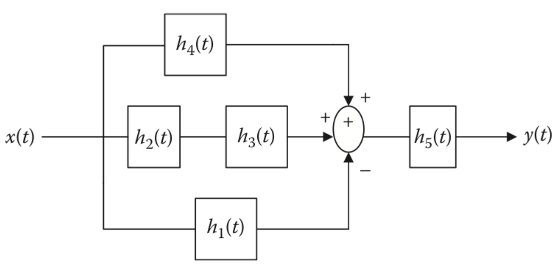


FIGURE 2.34 For Problem 2.24.

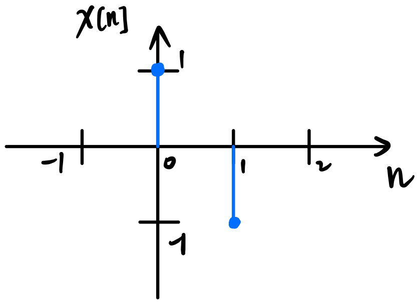
$h(t) = \{h_4(t) + [h_2(t) * h_3(t)] - h_1(t)\} * h_5(t)$

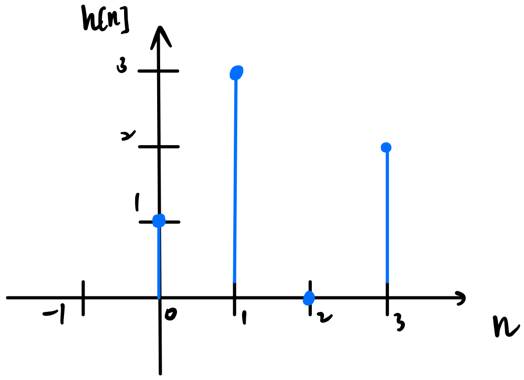
#

2.30 Given that $x[n]=\begin{cases}1, & n=0\\ -1, & n=1\\ 0, & \text{otherwise}\end{cases}$, $h[n]=\begin{cases}1, & n=0\\ 3, & n=1\\ 2, & n=3\\ 0, & \text{otherwise}\end{cases}$

(a) Sketch $x[n]$ and $h[n]$.
(b) Find $x[n]*h[n]$.

(a)





(b) $x[n]*h[n]=\sum_{k=0}^{n}x[k]h[n-k]=y[n]$

$*\ x*\delta[n]=x$

$*\ x[n]*\delta[n-1]=x[n-1]$

$x[n]=\delta[n]-\delta[n-1]$

$h[n]=\delta[n]+3\delta[n-1]+2\delta[n-3]$

$y[n]=x[n]*h[n]=(\delta[n]*\delta[n]+3\delta[n]*\delta[n-1]+2\delta[n]*\delta[n-3])-(\delta[n-1]*\delta[n]$
$+3\delta[n-1]*\delta[n-1]+2\delta[n-1]\delta[n-3])$
$=(\delta[n]+3\delta[n-1]+2\delta[n-3])-(\delta[n-1]+3\delta[n-2]+2\delta[n-4])$
$=\delta[n]+2\delta[n-1]-3\delta[n-2]+2\delta[n-3]-2\delta[n-4]$ ※

2.33 Two systems are described by

$$h_1[n]=(0.4)^n u[n],\qquad h_2[n]=\delta[n]+0.5\delta[n-1]$$

Determine the response to the input $x[n]=(0.4)^n u[n]$ if
(a) The two systems are connected in parallel
(b) The two systems are connected in cascade

(a) 並聯 $h[n]=h_1[n]+h_2[n]$
$=(0.4)^n u[n]+\delta[n]+0.5\delta[n-1]$

$y[n]=x[n]*h[n]$
$=(0.4)^n u[n]*[(0.4)^n u[n]+\delta[n]+0.5\delta[n-1]]$
$=(n+1)(0.4)^n u[n]+(0.4)^n u[n]+0.5(0.4)^{n-1}u[n-1]$
$=(n+2)(0.4)^n u[n]+0.5(0.4)^{n-1}u[n-1]$ ※

(b) 串聯 $h=h_1*h_2$
$=(0.4)^n u[n]+0.5(0.4)^{n-1}u[n-1]$

$y[n]=x[n]*h[n]$
$=(0.4)^n u[n]*[(0.4)^n u[n]+0.5(0.4)^{n-1}u[n-1]]$

$\because z[n]=a^n u[n]*a^n u[n]=(n+1)a^n u[n]$
$=r[n]a^n$
$\Rightarrow a^n u[n]*a^{n-1}u[n-1]=z[n-1]$
$=(n-1+1)a^{n-1}u[n-1]=na^{n-1}u[n-1]$

$\therefore y[n]=(n+1)(0.4)^n u[n]+0.5n(0.4)^{n-1}u[n-1]$ ※

2.36 The input $x[n] = [1\ -1]$ to a system produces the output $y[n] = [4\ 2\ 5\ 1]$. Determine the impulse response.

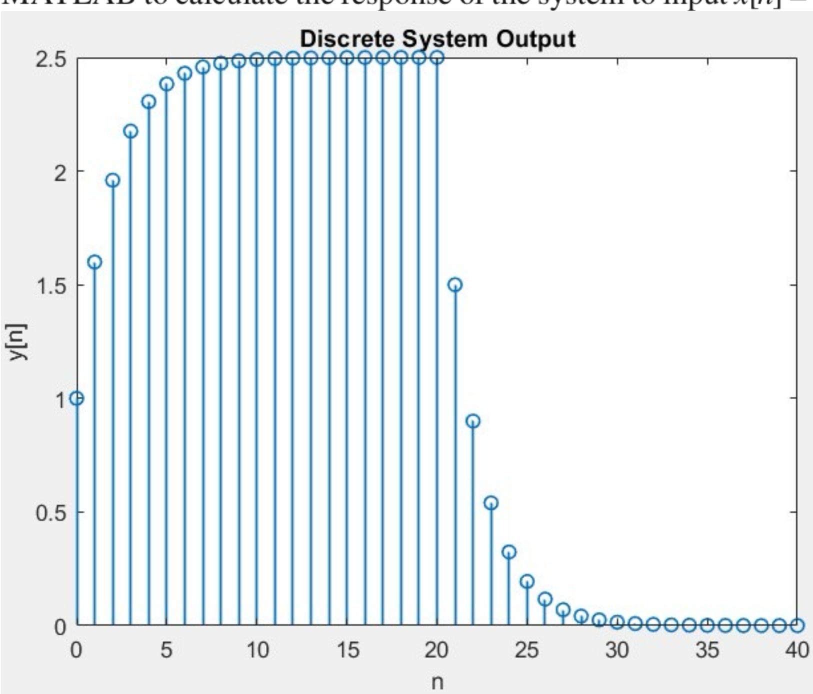
令 $y[z] = 4z^3+2z^2+5z+1$, $x[z] = z-1$

$$
\begin{array}{r|l}
 & 4+6+11 \\
\hline
1-1 & 4+2+5+1 \\
 & 4-4 \\
\hline
 & 6+5 \\
 & 6-6 \\
\hline
 & 11+1 \\
 & 11-11 \\
\hline
 & 12
\end{array}
$$

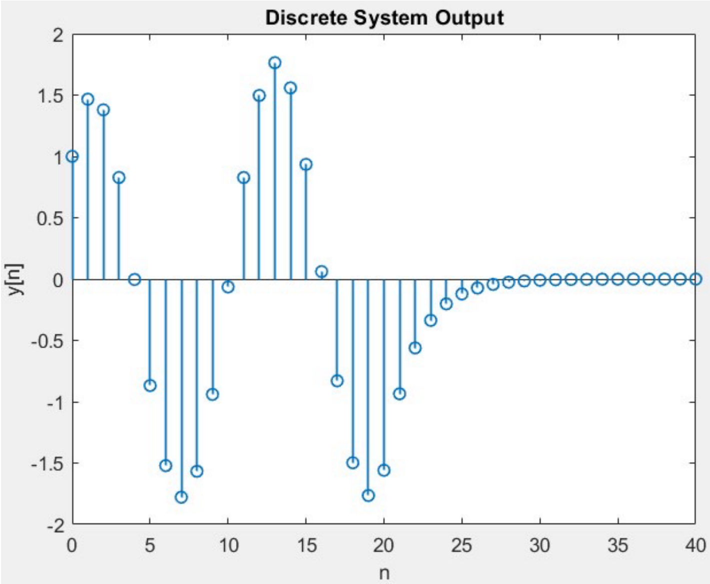
✓

$\therefore h[n] = [4\ 6\ 11]$ #

2.39 An LTI discrete system has the impulse response $h[n] = (0.6)^n\ u[n]$. Use MATLAB to calculate the response of the system to input $x[n] = u[n]$ and plot it.



2.40 Repeat the previous problem for $x[n] = \cos(n\pi/6)u[n]$.



2.41 Given that $x[n] = [1\ -1\ 2\ 4]$ and $y[n] = [2\ 6\ 4\ 0\ 8\ 5\ 12]$, use MATLAB to find $h[n]$.

(a)

```
>> HW2_41a
h(1) = 2
h(2) = 8
h(3) = 8
h(4) = -16
```

```
function value = h(n)
    x = [1 -1 2 4];
    y = [2 6 4 0 8 5 12];

    if n==1
        value = y(1)/x(1); %常數項
        return;
    end

    temp = 0;

    for i = 1 : (n-1)
        temp = temp + h(i)*x(n-i+1);
    end

    value = (y(n)-temp);
end


x = [1 -1 2 4];
y = [2 6 4 0 8 5 12];

for i = 1:(length(y)+1-length(x))
    disp(['h(', num2str(i), ') = ', num2str(h(i))]);
end
```

(b) deconv() ⇒

```
h =

     2     8     8   -16
```

2+8+8-16

1-1+2+4) 2+6+4+0+8+5+12

2-2+4+8

8+0-8+8

8-8+16+32

8-24-24+5

8-8+16+32

-16-40-27+12

-16+16-32-64

-64+5+76