3.3 Find the Laplace $X(s)$ given that $x(t)$ is

(a) $2tu(t-4)$

(b) $5\cos t\ \delta(t-2)$

(c) $e^{-t}u(t-\tau)$

(d) $\sin 2t\ u(t-\tau)$

(a) $\mathcal{L}\{2tu(t-4)\} = \mathcal{L}\{2(t-4)u(t-4) + 8u(t-4)\}$

$= 2e^{-4s}\cdot\left(-\frac{d}{ds}s^{-1}\right) + 8e^{-4s}\cdot\frac{1}{s}$

$= \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s}$

$= \left(\frac{2}{s^2}+\frac{8}{s}\right)e^{-4s}$ ※

(b) $\mathcal{L}\{5\cos t\,\delta(t-2)\} = \int_0^\infty 5\cos t\,\delta(t-2)e^{-st}dt$

$= 5\cos t\, e^{-st}\Big|_{t=2} = 5\cos 2\, e^{-2s}$ ※

(c) $\mathcal{L}\{e^{-t}u(t-\tau)\} = \mathcal{L}\{e^{-(t-\tau)}\cdot e^{-\tau}\cdot u(t-\tau)\}$

$= e^{-\tau}\cdot e^{-\tau s}\cdot\frac{1}{s+1} = \frac{e^{-\tau(1+s)}}{s+1}$ ※

(d) $\mathcal{L}\{\sin 2t\, u(t-\tau)\}$

$\sin(a+b) = \sin a\cos b + \cos a\sin b$

$= \mathcal{L}\{\sin[\underline{2(t-\tau)}+\underline{2\tau}]\, u(t-\tau)\}$

$= \mathcal{L}\{\sin 2(t-\tau)\cos 2\tau\, u(t-\tau)$

$+ \cos 2(t-\tau)\sin 2\tau\, u(t-\tau)\}$

$= e^{-\tau s}\cos 2\tau\cdot\frac{2}{s^2+4} + e^{-\tau s}\sin 2\tau\cdot\frac{s}{s^2+4}$ ※

3.13 Find the inverse Laplace transform for

(a) $X(s) = \dfrac{3s+1}{s^2+2s+5}$

(b) $Y(s) = \dfrac{3s+7}{s^2+3s+2}$

(c) $Z(s) = \dfrac{s^2-8}{s^2-4}$

(d) $H(s) = \dfrac{12}{(s+2)^2}$

(a)

$X(s) = \frac{3(s+1)-2}{(s+1)^2+2^2}$

$\mathcal{L}^{-1}\{X(s)\} = (3e^{-t}\cos 2t - e^{-t}\sin 2t)u(t)$ ※

(b)

$X(s) = \frac{3s+7}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$

$A = (s+2)X(s)\Big|_{s=-2} = \frac{1}{-1} = -1$

$B = (s+1)X(s)\Big|_{s=-1} = \frac{4}{1} = 4$

$\mathcal{L}^{-1}\{X(s)\} = (-e^{-2t} + 4e^{-t})u(t)$ ※

(c)

$$\begin{array}{r|l} & 1 \\ 1+0-4 & 1+0-8 \\ & 1+0-4 \\ \hline & -4 \end{array}$$

$Z(s) = 1 + \frac{-4}{s^2-4} = 1 + \frac{-4}{(s+2)(s-2)}$

$\frac{-4}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$

$A = (s-2)\frac{-4}{(s-2)(s+2)}\Big|_{s=2} = -1$

$B = (s+2)\frac{-4}{(s-2)(s+2)}\Big|_{s=-2} = 1$

$\mathcal{L}^{-1}\{Z(s)\} = \delta(t) + (-e^{2t} + e^{-2t})u(t)$ ※

(d)

$\mathcal{L}^{-1}\{H(s)\} = 12\cdot\frac{t^{2-1}}{(2-1)!}e^{-2t}u(t)$

$= 12te^{-2t}u(t)$ ※

3.14 Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

(a)

$$F(s) = \frac{20(s+2)}{s(s^2+6s+25)} = \frac{A}{s} + \frac{Bs+C}{s^2+6s+25}$$

$$A = sF(s)\Big|_{s=0} = \frac{40}{25} = \frac{8}{5}$$

$$Bs^2 + Cs + \frac{8}{5}(s^2+6s+25) = 20s+40$$

$$B = -\frac{8}{5},\quad C + \frac{48}{5} = \frac{100}{5} \Rightarrow C = \frac{52}{5}$$

$$\therefore F(s) = \frac{8}{5}\cdot\frac{1}{s} + \left(-\frac{8}{5}\right)\frac{(s+3)}{(s+3)^2+4^2} + \frac{\overset{4\times\frac{19}{5}}{\boxed{\frac{24}{5}+\frac{52}{5}}}}{(s+3)^2+4^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t \quad (t \geq 0)\ \#$$

(b)

$$P(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = (s+1)P(s)\Big|_{s=-1} = \frac{6-36+20}{1\times 2} = -5$$

$$B = (s+2)P(s)\Big|_{s=-2} = \frac{24-72+20}{(-1)\times 1} = 28$$

$$C = (s+3)P(s)\Big|_{s=-3} = \frac{54-108+20}{(-2)(-1)} = -17$$

$$\mathcal{L}^{-1}\{P(s)\} = (-5e^{-t} + 28e^{-2t} - 17e^{-3t})u(t)\ \#$$

3.18 Let $F(s) = \dfrac{5(s+1)}{(s+2)(s+3)}$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.
(b) Verify your answer in part (a) by finding $f(t)$ using partial fractions.

(a)

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5\cdot 1\cdot(1+\frac{1}{s})}{(1+\frac{2}{s})(1+\frac{3}{s})} = 5\ \#$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = 0\ \#$$

(b)

$$F(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = (s+2)F(s)\Big|_{s=-2} = -5$$

$$B = (s+3)F(s)\Big|_{s=-3} = 10$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = -5e^{-2t} + 10e^{-3t},\quad (t \geq 0)$$

$$f(0) = -5 + 10 = 5$$

$$f(\infty) = 0 + 0 = 0$$

3.26 For each of the systems shown in Figure 3.28, use Laplace transform to find $y(t)$.

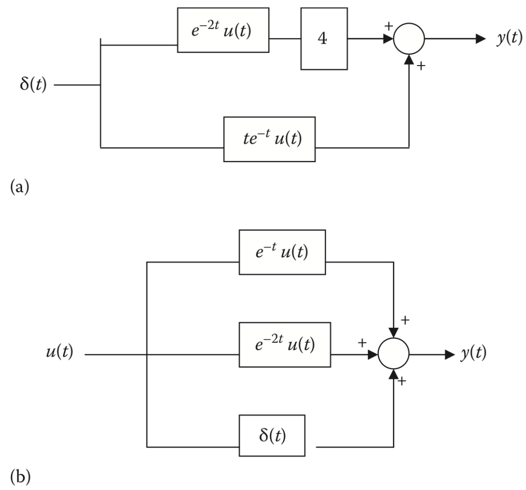


FIGURE 3.28 For Problem 3.26.

(a)
$$\begin{aligned} \delta(t) &\qquad\qquad 1 \\ e^{-2t}u(t) &\xrightarrow{\mathcal{L}} \frac{1}{s+2} \\ te^{-t}u(t) &\qquad\qquad \frac{1}{(s+2)^2} \end{aligned}$$

$$\Rightarrow Y(s) = 4\cdot\frac{1}{s+2} + \frac{1}{(s+2)^2}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = 4e^{-2t}u(t) + te^{-t}u(t) \ \#$$

(b)
$$\begin{aligned} u(t) &\qquad\qquad \frac{1}{s} \\ e^{-t}u(t) &\xrightarrow{\mathcal{L}} \frac{1}{s+1} \\ e^{-2t}u(t) &\qquad\qquad \frac{1}{s+2} \\ \delta(t) &\qquad\qquad 1 \end{aligned}$$

$$\Rightarrow Y(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2} + 1\right]$$

$$= \frac{s^2+5s+5}{s^3+3s^2+2s} = \frac{5}{2}\frac{1}{s} - \frac{1}{2}\frac{1}{s+2} - \frac{1}{s+1}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{5}{2}u(t) - \frac{1}{2}e^{-2t}u(t) - e^{-t}u(t) \ \#$$

3.43 An LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} - 3x(t)$$

(a) Determine the transfer function of the system.
(b) Obtain the impulse response of the system.

(a) $[s^2Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + 2Y(s) = [sX(s) - x(0)] - 3X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s-3}{s^2+2s+2} \ \#$$

，令 $y(0) = y'(0) = X(0) = 0$

(b) $$H(s) = \frac{s+1}{(s+1)^2+1} + \frac{-1-3}{(s+1)^2+1}$$

$$h(t) = (e^{-t}\cos t - 4e^{-t}\sin t)\,u(t) \ \#$$

3.46 Consider the function

$$H(s) = \frac{s^2+6s+10}{s^3+7s^2+11s+5}$$

Use the MATLAB residue function to obtain the inverse Laplace transform of $H(s)$.

```
r =

    0.3125
    0.6875
    1.2500

p =

   -5.0000
   -1.0000
   -1.0000

k =

     []
```

由MATLAB, $H(s) = \frac{0.3125}{s+5} + \frac{0.6875}{s+1} + \frac{1.25}{(s+1)^2}$

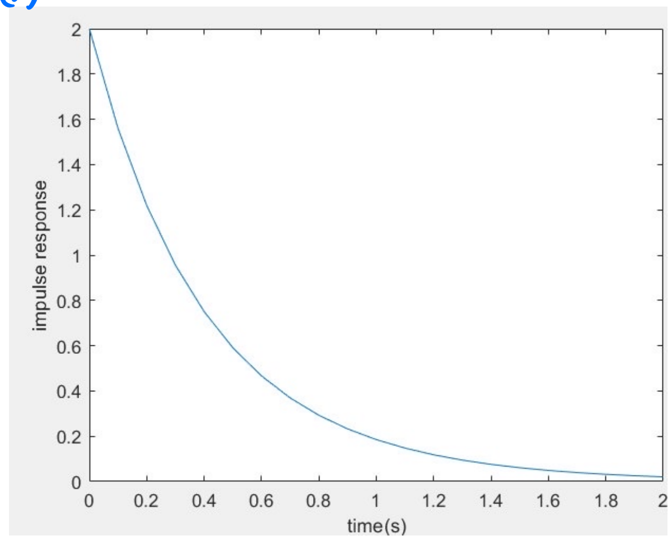
$\Rightarrow \mathcal{L}^{-1}\{H(s)\} = h(t) = 0.3125e^{-5t} + 0.6875e^{-t} + 1.25te^{-t}, \quad t>0$ #

3.49 A linear system is represented by its transfer function

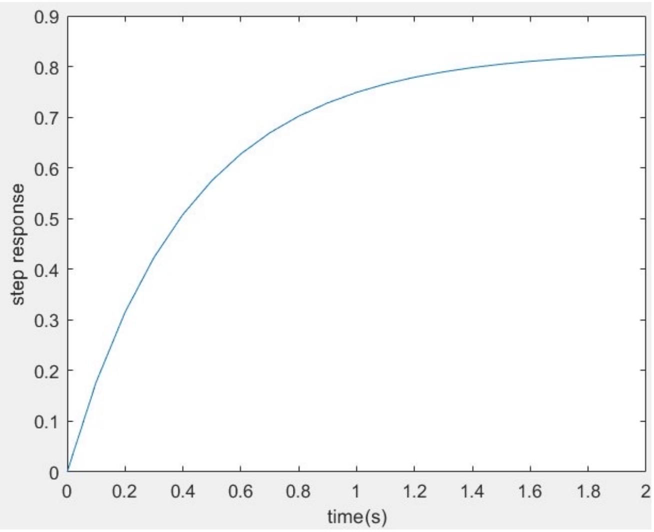
$$H(s) = \frac{2s+5}{(s+2)(s+3)}$$ s^2+5s+6

Use MATLAB to determine and plot:
(a) The impulse response
(b) The step response.

(A)



(b)



3.51 Use MATLAB to find the zeros and poles of these functions:

(a) $\dfrac{s-2}{(s+1)^2+9}$ $s^2+2s+10$

(b) $\dfrac{s^2+2s+5}{s(s^2+4s+13)}$

(c) $\dfrac{s^2+10s+5}{s^3+4s^2+10s+6}$

(a)

```
>> HW3_51a

z =

     2


p =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
```

(b)

```
>> HW3_51b

z =

  -1.0000 + 2.0000i
  -1.0000 - 2.0000i


p =

   0.0000 + 0.0000i
  -2.0000 + 3.0000i
  -2.0000 - 3.0000i
```

(c)

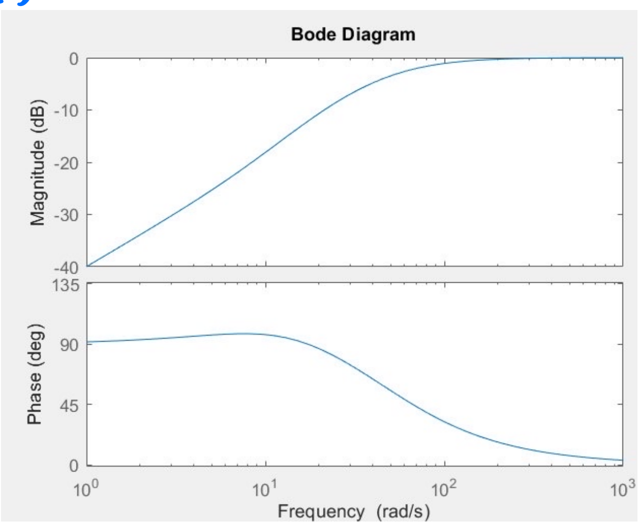
```
>> HW3_51c

z =

   -9.4721
   -0.5279


p =

  -1.5956 + 2.2075i
  -1.5956 - 2.2075i
  -0.8087 + 0.0000i
```

3.53 Obtain the Bode plots for the following transfer functions using MATLAB:

(a) $H(s) = \dfrac{s(s+10)}{(s+20)(s+50)}$ $s^2+70s+1000$

(b) $H(s) = \dfrac{s+1}{(s+2)(s^2+22.5s+16)}$ $s^3+24.5s^2+61s+32$

(a)



(b)