Handwrite part

6.1 (a) Show that $X(\Omega)$ is periodic with period 2π, that is, $X(\Omega + 2\pi) = X(\Omega)$.

(b) Specifically show that $X(\Omega) = \dfrac{1}{1 - ae^{-j\Omega}}$ is periodic.

6.10 The DTFT of a signal $x[n]$ is

$$X(\Omega) = \frac{2}{3 + e^{-j\Omega}}$$

Find the DTFT of the following signals:

(a) $y[n] = x[-n]$

(b) $z[n] = nx[n]$

(c) $w[n] = x[n] + x[n-1]$

(d) $v[n] = x[n]\cos(n\pi)$

6.17 Show that if $x[n] \leftrightarrow X(\Omega)$, then

$$x[n+k] - x[n-k] \leftrightarrow 2jX(\Omega)\sin k\Omega$$

6.19 Find the DFT of the sequence $x[n] = a^n$.

6.20 Prove the following DFT properties:

(a) $X[0] = \sum_{n=0}^{N-1} x[n]$

(b) $X[N/2] = \sum_{n=0}^{N-1} (-1)^n x[n]$

6.22 Calculate the DFT of the following discrete-time signals:

(a) $x[0] = -1, \quad x[1] = 1, \quad x[2] = 0, \quad x[3] = 2$

(b) $x[0] = 1, \quad x[1] = 2, \quad x[2] = 3, \quad x[3] = -1$

6.25 The DFT of a signal $x[n]$ is

$$X(0) = 1, \quad X(1) = 1 + j2, \quad X(2) = 1 - j, \quad X(3) = 1 + j, \quad X(4) = 1 - j2$$

Compute $x[n]$.

Simulation part

6.28 Use MATLAB to compute the FFT of the following signals. For each signal, plot $|X(k)|$.

(a) $x[n] = 1,\ 0 \le n \le 12$

(b) $x[n] = n,\ 0 \le n \le 10$

(c) $x[n] = \begin{cases} 1, & n = 0 \\ 1/n, & n = 1, 2, \ldots, 10 \\ 0, & \text{otherwise} \end{cases}$

(d) $x[n] = n(0.8)^n, \quad 0 \le n \le 10$

6.29 In Example 6.3, the DTFT of the signal $x[n] = a^{|n|}$ is

$$X(\Omega) = \frac{1-a^2}{1-2a\cos\Omega + a^2}$$

For $a = 0.75$ and $0 < \Omega < 2\pi$, plot $|X(\Omega)|$.

6.30 Use MATLAB to find the DFT of the discrete signal

$$x[n] = \{1, 2, 0, -1, -2, 1, 5, 4\}$$