Prob.6.1

(a) Given that $X(\Omega)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

$$X(\Omega+2\pi)=\sum_{n=-\infty}^{\infty} x[n]e^{-j(\Omega+2\pi)n}=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}e^{-jn2\pi}$$

But $e^{-jn2\pi}=\cos(2n\pi)-j\sin(2n\pi)=1-j0=1$

Hence,

$$X(\Omega+2\pi)=\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}=X(\Omega)$$

(b) $X(\Omega+2\pi)=\dfrac{1}{1-ae^{-j(\Omega+2\pi)}}=\dfrac{1}{1-a^{-j\Omega}e^{-j2\pi}}$

But $e^{-j2\pi}=\cos(2\pi)-j\sin(2\pi)=1$

$$X(\Omega+2\pi)=\frac{1}{1-a^{-j\Omega}}=X(\Omega)$$

Prob. 6.10

(a) $y[n] = x[-n] \quad \Leftrightarrow \quad X(-\Omega)$

$$Y(\Omega) = \frac{2}{3 + e^{j\Omega}}$$

(b) $z[n] = nx[n] \quad \leftrightarrow \quad jX'(\Omega)$

$$Z(\Omega) = j\frac{d}{d\Omega}\left(\frac{2}{3 + e^{-j\Omega}}\right) = 2j(-1)(-je^{-j\Omega})(3 + e^{-j\Omega})^{-2}$$

$$= \frac{-2e^{-j\Omega}}{(3 + e^{-j\Omega})^2}$$

(c) $W(\Omega) = \frac{2}{3 + e^{-j\Omega}} + \frac{2}{3 + e^{-j\Omega}}e^{-j\Omega} = \frac{2(1 + e^{-j\Omega})}{3 + e^{-j\Omega}}$

(d) $v[n] = x[n]\cos(n\pi) = \frac{1}{2}x[n]\left[e^{j\pi n} + e^{-jn\pi}\right]$

$$V(\Omega) = \frac{1}{2}X(\Omega + \pi) + \frac{1}{2}X(\Omega - \pi) = \frac{1}{3 + e^{-j(\Omega-\pi)}} + \frac{1}{3 + e^{-j(\Omega+\pi)}}$$

$$= \frac{1}{3 + e^{-j\Omega}e^{j\pi}} + \frac{1}{3 + e^{-j\Omega}e^{-j\pi}}$$

But $e^{j\pi} = \cos\pi + j\sin\pi = -1, \qquad e^{-j\pi} = \cos(-\pi) + j\sin(-\pi) = -1$

$$V(\Omega = \frac{2}{3 - e^{-j\Omega}}$$

Prob. 6.17

Using the time-shifting property,

$$x[n+k] - x[n-k] \Leftrightarrow e^{j\Omega k}\mathrm{X}(\Omega) - e^{-j\Omega k}\mathrm{X}(\Omega)$$
$$= \mathrm{X}(\Omega)\left(e^{j\Omega k} - e^{-j\Omega k}\right)$$
$$= \mathrm{X}(\Omega)(\cos(k\Omega) + j\sin(k\Omega) - \cos(-k\Omega) - j\sin(-k\Omega))$$
$$= 2j\mathrm{X}(\Omega)\sin(k\Omega)$$

Prob. 6.19

$$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi nk/N}=\sum_{n=0}^{N-1}a^n e^{-j2\pi nk/N}=\sum_{n=0}^{N-1}\left(ae^{-j2\pi k/N}\right)^n$$

But $\displaystyle\sum_{n=0}^{\infty}q^n=\frac{1-q^N}{1-q}$

$$X[k]=\frac{1-(ae^{-j2\pi k/N})^N}{1-ae^{-j2\pi k/N}}=\frac{1-a^N e^{-j2\pi k}}{1-ae^{-j2\pi k/N}}$$

But $e^{-j2\pi k}=\cos(2\pi k)-j\sin(2\pi k)=1$

$$X[k]=\frac{1-a^N}{1-ae^{-j2\pi k/N}}$$

Prob. 6.20

(a) $\displaystyle X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi nk/N}$

When $k=0$,

$$X[0]=\sum_{n=0}^{N-1}x[n]e^{-j0}=\sum_{n=0}^{N-1}x[n]$$

(b) When $k=N/2$,

$$X[N/2]=\sum_{n=0}^{N-1}x[n]e^{-jn\pi}$$

But $e^{-jn\pi}=\cos n\pi-j\sin n\pi=(-1)^n$

Hence,

$$X[N/2]=\sum_{n=0}^{N-1}(-1)^n x[n]$$

Prob. 6.22

(a) $$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi nk/N}$$

$$X[0]=\sum_{n=0}^{3}x[n]=-1+1+0+2=2$$

$$X[1]=\sum_{n=0}^{3}x[n]e^{-j\pi n/2}=-1+(1)e^{-j\pi/2}+0+2e^{-j3\pi/2}=-1-j+2j=-1+j$$

$$X[2]=\sum_{n=0}^{3}x[n]e^{-j\pi n}=-1+(1)e^{-j\pi}+0+2e^{-j3\pi}=-1-1-2=-4$$

$$X[3]=\sum_{n=0}^{3}x[n]e^{-j3\pi n/2}=-1+(1)e^{-j3\pi/2}+0+2e^{-j9\pi/2}=-1+j-j2=-1-j$$

(b) $$X[k]=\sum_{n=0}^{3}x[n]e^{-j2\pi nk/N}$$

$$X[0]=\sum_{n=0}^{3}x[n]=1+2+3-1=5$$

$$X[1]=\sum_{n=0}^{3}x[n]e^{-j\pi n/2}=1+2e^{-j\pi/2}+3e^{-j\pi}-1e^{-j3\pi/2}=1-j2-3-j=-2-3j$$

$$X[2]=\sum_{n=0}^{3}x[n]e^{-j\pi n}=1+2e^{-j\pi}+3e^{-j2\pi}-1e^{-j3\pi}=1-2+3+1=3$$

$$X[3]=\sum_{n=0}^{3}x[n]e^{-j3\pi n/2}=1+2e^{-j3\pi/2}+3e^{-j3\pi}-1e^{-j9\pi/2}=1+2j-3+j=-2+j3$$

Prob. 6.25

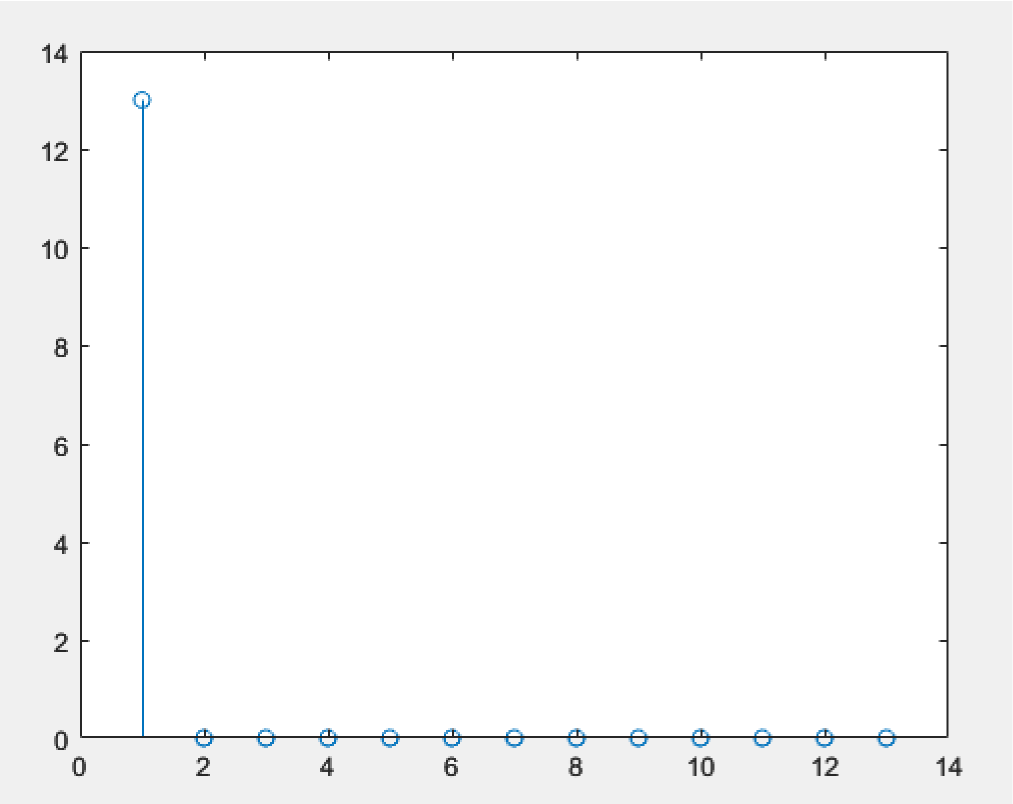
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi nk/N} = \frac{1}{5}\sum_{k=0}^{4} X[k]e^{j2\pi nk/5}$$

$$= \frac{1}{5}\left[1 + (1+j2)e^{j2\pi n/5} + (1-j)e^{j4\pi n/5} + (1+j)e^{j6\pi n/5} + (1-2j)e^{j8\pi n/5}\right]$$

$$x[0] = 1, x[1] = -0.5257, x[2] = -0.8507, x[3] = 0.8507, x[4] = 0.5257$$

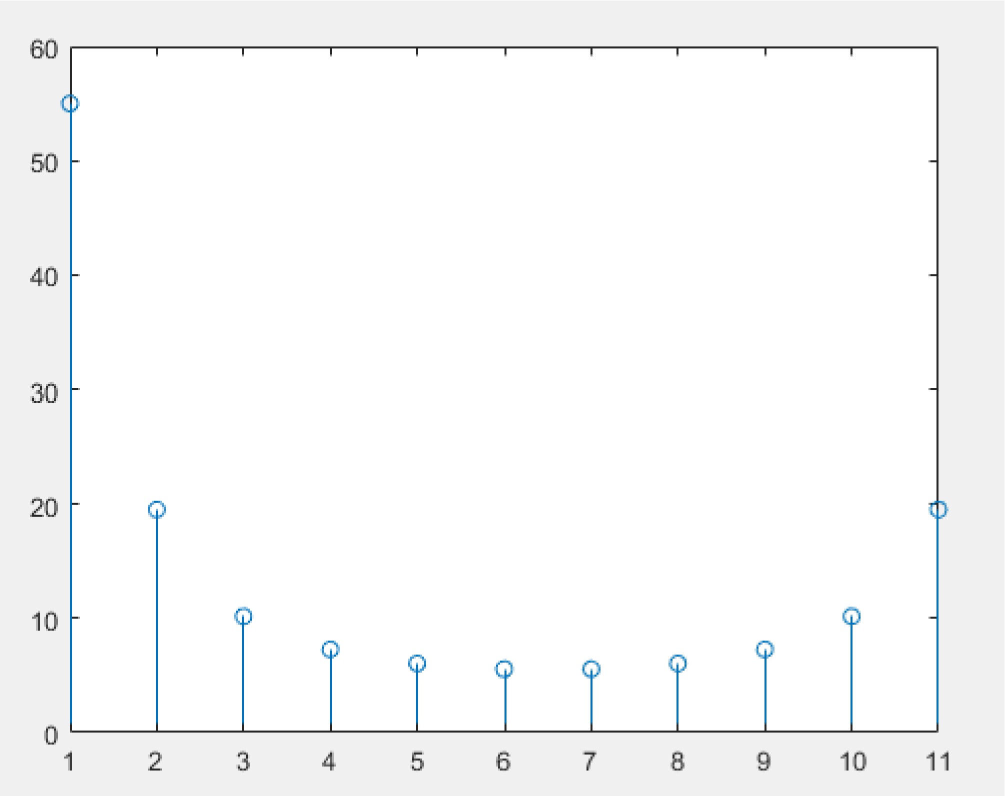
Prob. 6.28 (a)

```
x = [1 1 1 1 1 1 1 1 1 1 1 1 1];
X = fft(x);
stem(abs(X));
```



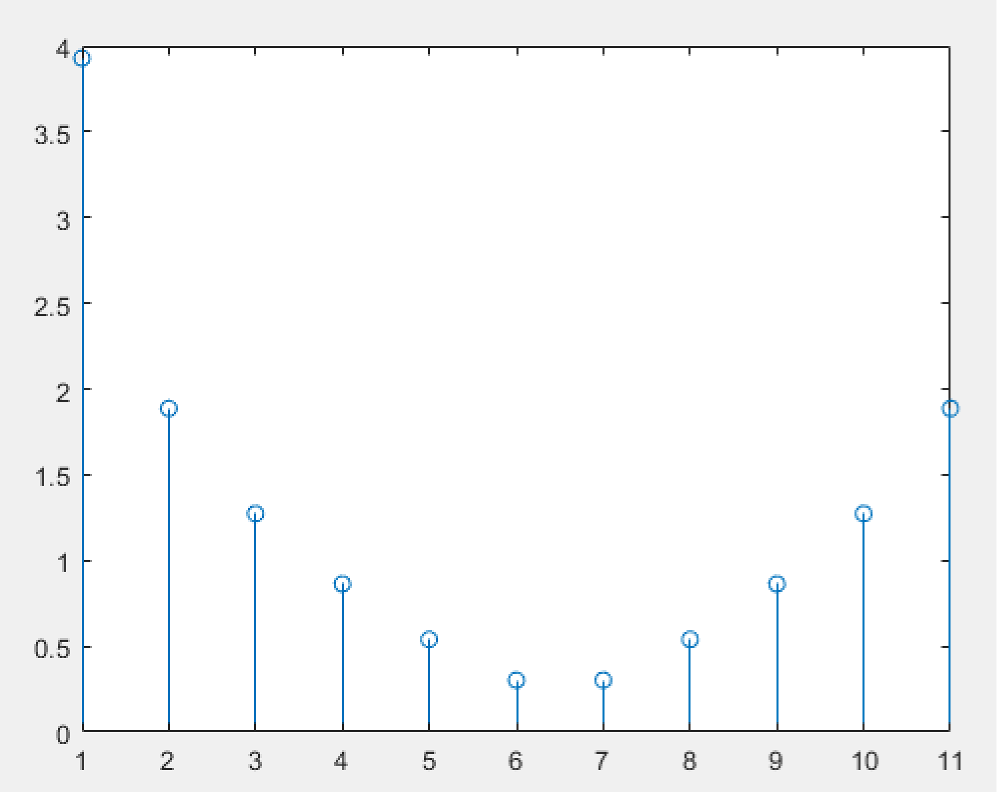
Prob. 6.28 (b)

```
x = [ 0 1 2 3 4 5 6 7 8 9 10];
X = fft(x);
stem(abs(X));
```



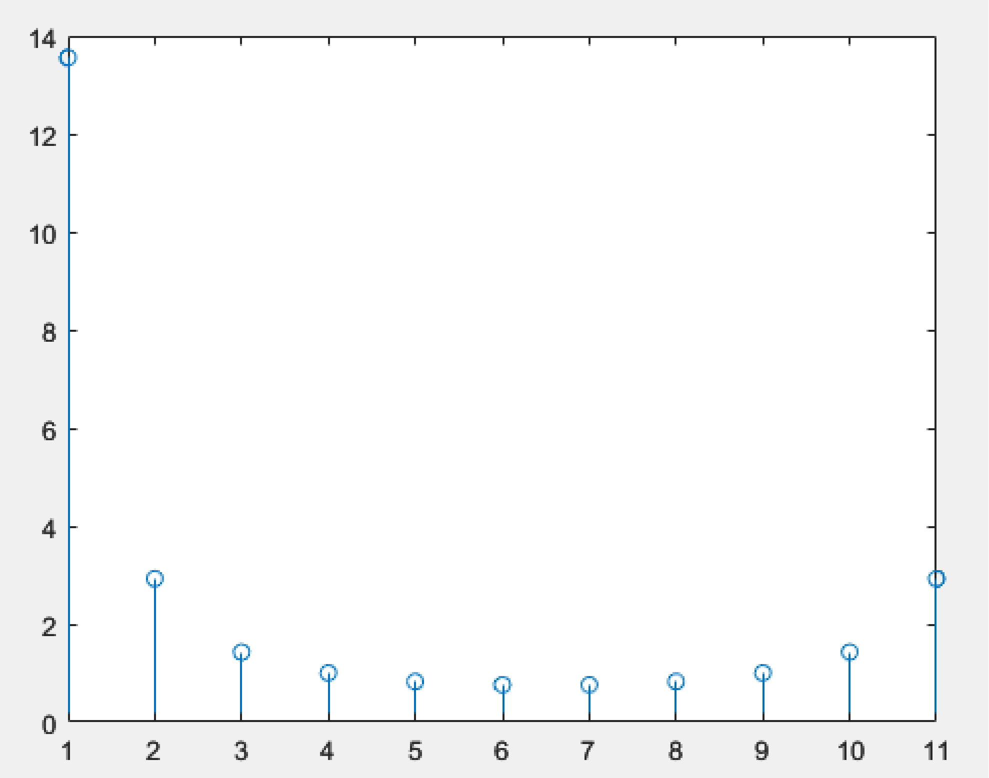
Prob. 6.28 (c)

```
x = [1 1 1/2 1/3 1/4 1/5 1/6 1/7 1/8 1/9 1/10];
X = fft(x);
stem(abs(X));
```



Prob. 6.28 (d)

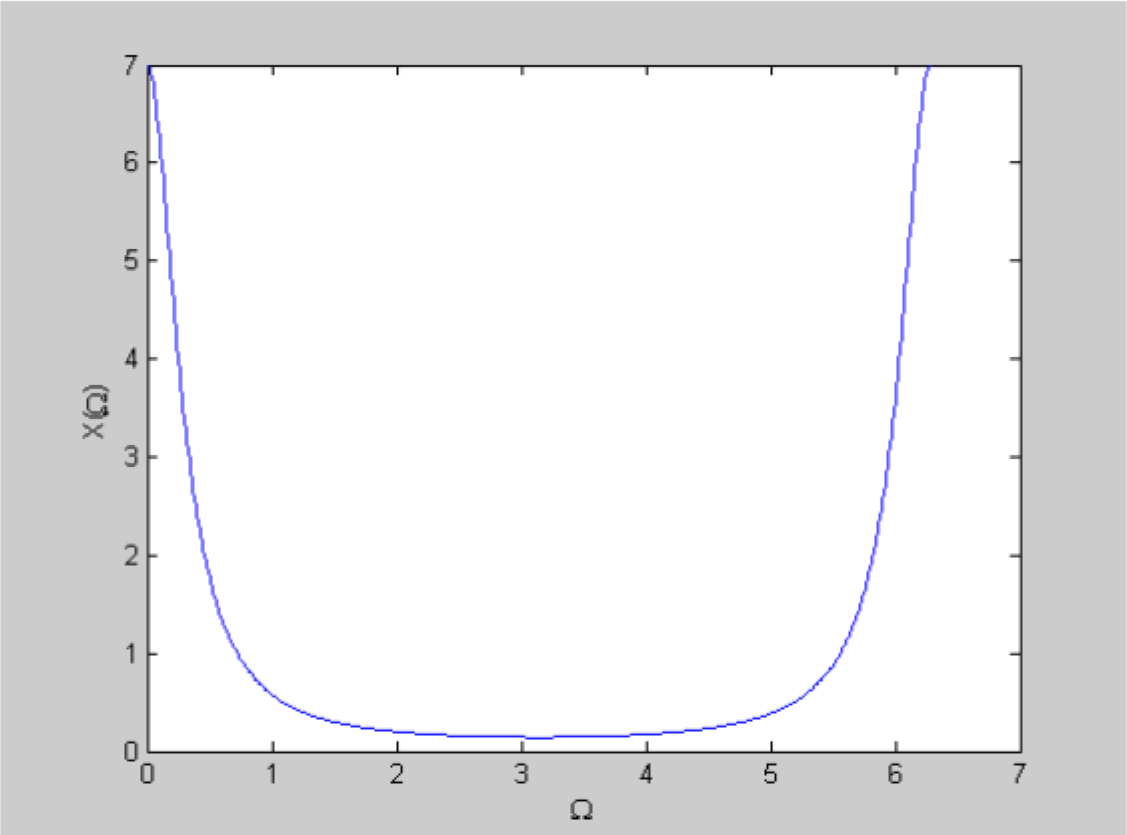
```
x = [ 0 0.8 2*(0.8^2) 3*(0.8^3) 4*(0.8^4) 5*(0.8^5) 6*(0.8^6) 7*(0.8^7) 8*(0.8^8) 9*(0.8^9) 10*(0.8^10)];
X = fft(x);
stem(abs(X));
```



Prob. 6.29

The MATLAB code and the plot are shown below.

```
a=0.75;
Omega= 0:0.01:2*pi;
X =(1-a^2)./(1- 2*a*cos(Omega)+ a^2);
plot(Omega,abs(X));
xlabel('\Omega')
ylabel('X(\Omega)')
```



Prob. 6.30

The MATLAB code with the result is shown below.

```
x = [1  2  0  -1  -2  1  5  4 ];
X = fft(x)
```

```
X =  10.0000
     7.2426 + 7.8284i
     -6.0000
     -1.2426 - 2.1716i
     -2.0000
     -1.2426 + 2.1716i
     -6.0000
     7.2426 - 7.8284i
```