Prob. 3.3

(a) 2t=2(t-4) + 8

x(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)

$$X(s)=\frac{2}{s^2}e^{-4s}+\frac{8}{s}e^{-4s}=\left(\frac{2}{s^2}+\frac{8}{s}\right)e^{-4s}$$

(b) $$X(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty 5\cos t\delta(t-2)e^{-st}dt=5\cos te^{-st}\Big|_{t=2}=5\cos 2e^{-2s}$$

(c) $$e^{-t}=e^{-(t-\tau)}e^{-\tau}$$

$$x(t)=e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$X(s)=e^{-\tau}e^{-\tau s}\frac{1}{s+1}=\frac{e^{-\tau(s+1)}}{s+1}$$

(d) $$\sin 2t=\sin[2(t-\tau)+2\tau]=\sin 2(t-\tau)\cos 2\tau+\cos 2(t-\tau)\sin 2\tau$$

$$x(t)=\cos 2\tau\sin 2(t-\tau)u(t-\tau)+\sin 2\tau\cos 2(t-\tau)u(t-\tau)$$

$$X(s)=\cos 2\tau e^{-\tau s}\frac{2}{s^2+4}+\sin 2\tau e^{-\tau s}\frac{s}{s^2+4}$$

Prob. 3.11

(a) $x(t)=u(t)+2u(t-1)$

(b) $$Y(s)=\frac{10}{(s-1)(s-4)}=\frac{A}{s-1}+\frac{B}{s-4}$$

$$A=Y(s)(s-1)\Big|_{s=1}=-\frac{10}{3}$$

$$B=Y(s)(s-4)\Big|_{s=4}=\frac{10}{3}$$

$$Y(s)=\frac{10}{3}\left[-\frac{1}{s-1}+\frac{1}{s-4}\right]$$

$$y(t)=\frac{10}{3}(-e^{t}+e^{4t})u(t)$$

(c) $$Z(s)=\frac{s-2}{(s+1)^2+9}=\frac{s+1}{(s+1)^2+3^2}-\frac{3}{(s+1)^2+3^2}$$

$$z(t)=\left(e^{-t}\cos 3t-e^{-t}\sin 3t\right)u(t)$$

Prob. 3.14

(a) $F(s)=\dfrac{20(s+2)}{s(s^2+6s+25)}=\dfrac{A}{s}+\dfrac{Bs+C}{s^2+6s+25}$

$20(s+2)=A(s^2+6s+25)+Bs^2+Cs$

Equating components,

s^2 : $0 = A + B$ or $B = -A$

s: $20 = 6A + C$

constant: $40 = 25$ A or $A = 8/5$, $B = -8/5$, $C = 20 - 6A = 52/5$

$$F(s)=\frac{8}{5s}+\frac{-\frac{8}{5}s+\frac{52}{5}}{(s+3)^2+4^2}=\frac{8}{5s}+\frac{-\frac{8}{5}(s+3)+\frac{24}{5}+\frac{52}{5}}{(s+3)^2+4^2}$$

$$f(t)=\frac{8}{5}u(t)-\frac{8}{5}e^{-3t}\cos 4t+\frac{19}{5}e^{-3t}\sin 4t$$

(b) $P(s)=\dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}+\dfrac{C}{s+3}$

$$A=\frac{6-36+20}{(-1+2)(-1+3)}=-5$$

$$B=\frac{24-72+20}{(-1)(1)}=28$$

$$C=\frac{54-108+20}{(-2)(-1)}=-17$$

$$P(s)=\frac{-5}{s+1}+\frac{28}{s+2}-\frac{17}{s+3}$$

$$p(t)=(-5e^{-t}+28e^{-2t}-17e^{-3t})u(t)$$

Prob. 3.18

(a) $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5(1+1/s)}{(1+2/s)(1+3/s)} = 5$

$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

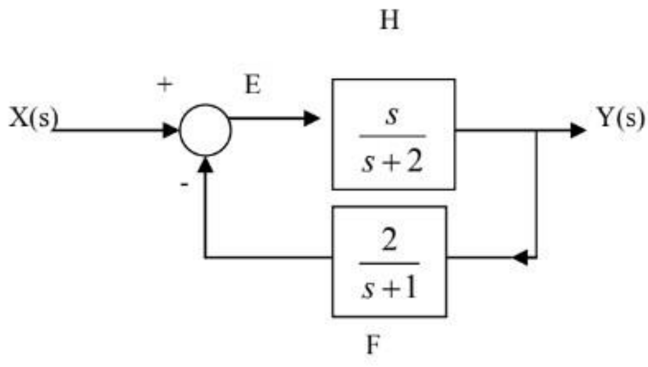
(b) $F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$$A = \frac{5(-1)}{1} = -5, \qquad B = \frac{5(-2)}{-1} = 10$$

$$F(s) = \frac{-5}{s+2} + \frac{10}{s+3} \quad \longrightarrow \quad f(t) = -5e^{-2t} + 10e^{-3t}$$

f(0) = -5 + 10 = 5
f(∞)= -0 + 0 = 0.

Prob. 3.22



$E = X - FY$

$Y = EH = XH - FHY$

$Y(1+FH) = XH$

$\frac{Y}{X} = H_o = \frac{H}{1+FG}$

$$H_o(s) = \frac{\frac{s}{s+2}}{1+\left(\frac{2}{s+1}\right)\left(\frac{s}{s+2}\right)} = \frac{s(s+1)}{(s+1)(s+2)+2s} = \frac{s(s+1)}{s^2+5s+2}$$

Prob. 3.29

$$\left[s^2Y(s)-sy(0)-y'(0)\right]+7\left[sY(s)-y(0)\right]+12Y(s)=\frac{1}{s+1}$$

$$\left(s^2+7s+12\right)Y(s)=\frac{1}{s+1}-s+2-7=\frac{-s^2-6s-4}{s+1}$$

$$Y(s)=\frac{-s^2-6s-4}{(s+1)\left(s^2+7s+12\right)}=\frac{-s^2-6s-4}{(s+1)(s+3)(s+4)}$$

$$Y(s)=\frac{A}{s+1}+\frac{B}{s+3}+\frac{C}{s+4}$$

$$A=(s+1)Y(s)\Big|_{s=-1}=1/6$$

$$B=(s+3)Y(s)\Big|_{s=-3}=-5/2$$

$$C=(s+4)Y(s)\Big|_{s=-4}=4/3$$

$$Y(s)=\frac{1/6}{s+1}-\frac{5/2}{s+3}+\frac{4/3}{s+4}$$

$$y(t)=\left(\frac{1}{6}e^{-t}-\frac{5}{2}e^{-3t}+\frac{4}{3}e^{-4t}\right)u(t)$$

Prob. 3.46

```
num = [ 1 6  10];
den = [ 1  7  11  5];
[r,p,k] = residue(num,den)
```

r =

    0.3125
    0.6875
    1.2500

p =

   -5.0000
   -1.0000
   -1.0000

k =

    []

$$H(s)=\frac{0.3125}{s+5}+\frac{0.6875}{s+1}+\frac{1.25}{(s+1)^2}$$

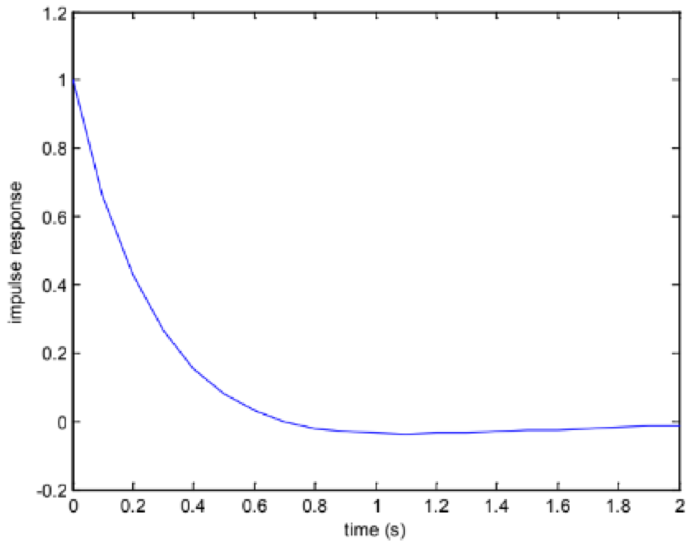
The inverse Laplace transform of this is:

$$x(t)=0.3125e^{-5t}+0.6875e^{-t}+1.25te^{-t}, t>0$$

Prob. 3.50

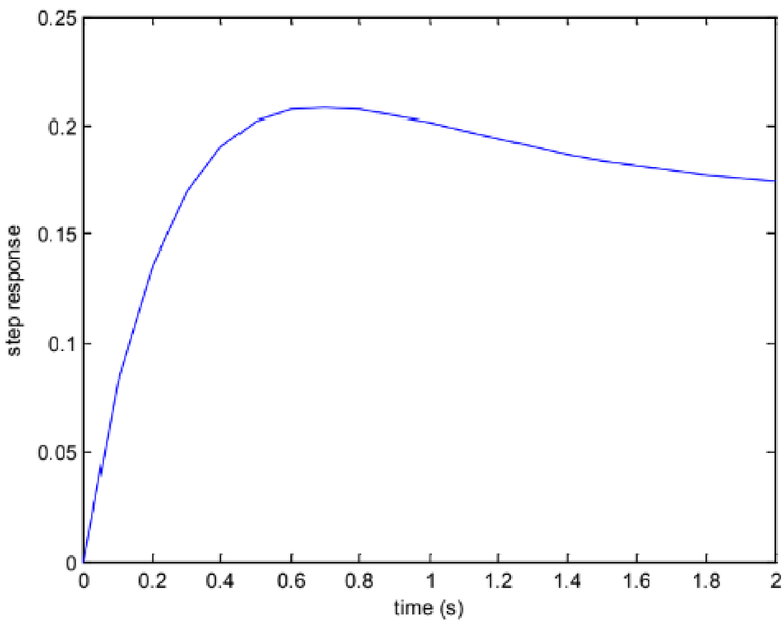
(a) The MATLAB code for the impulse response is shown below.

```
num = [ 1  1 ];
den = [1  5   6];
t = 0:0.1:2;
y=impulse(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('impulse response')
```



The following MATLAB code is for the step response.

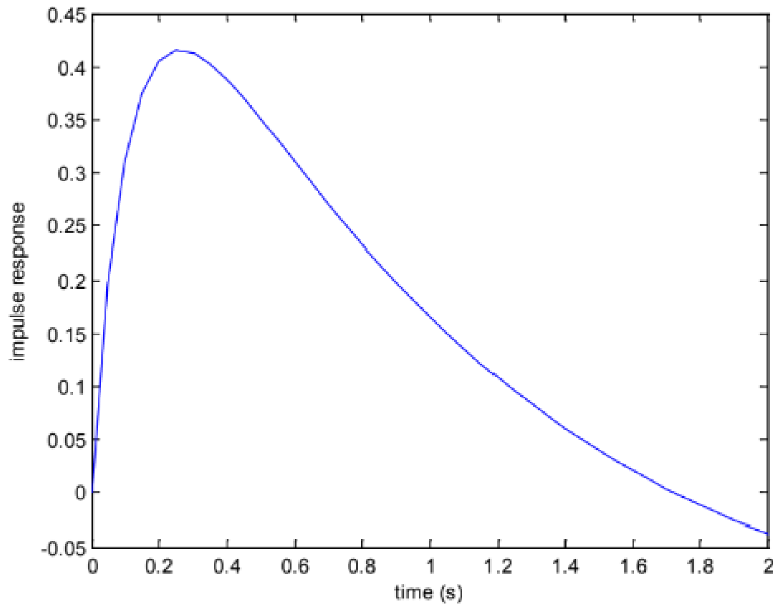
```
num = [ 1  1 ];
den = [1  5   6];
t = 0:0.1:2;
y=step(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('step response')
```



Prob. 3.50

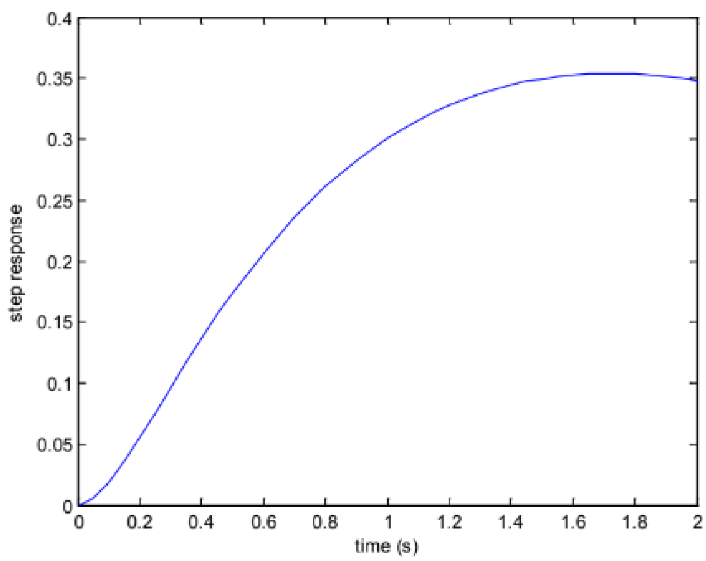
(b) The MATLAB code for the impulse response is presented below.

```
num = [ 5  0 ];
den = [1  10  10   4];
t = 0:0.05:2;
y=impulse(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('impulse response')
```



The MTLAB code for the step response is shown below.

```
num = [ 5  0 ];
den = [1  10  10   4];
t = 0:0.05:2;
y=step(num,den,t)
plot(t,y)
xlabel('time (s)')
ylabel('step response')
```



Prob. 3.51

(a) We first expand the denominator to get

$$H(s) = \frac{s-2}{s^2+2s+10}$$

```
num = [ 1  -2 ];
den = [1  2  10];
z = roots(num)
p = roots(den)
```

z =

2

p =

-1.0000 + 3.0000i
-1.0000 - 3.0000i

(b)

```
num = [ 1  2  5 ];
den = [1  4  13  0];
z = roots(num)
p = roots(den)
```

z =

-1.0000 + 2.0000i
-1.0000 - 2.0000i

p =

0.0000 + 0.0000i
-2.0000 + 3.0000i
-2.0000 - 3.0000i

(c)

```
num = [1  10  5 ];
den = [1  4  10  6];
z = roots(num)
p = roots(den)
```

z =

-9.4721
-0.5279

p =

-1.5956 + 2.2075i
-1.5956 - 2.2075i
-0.8087 + 0.0000i

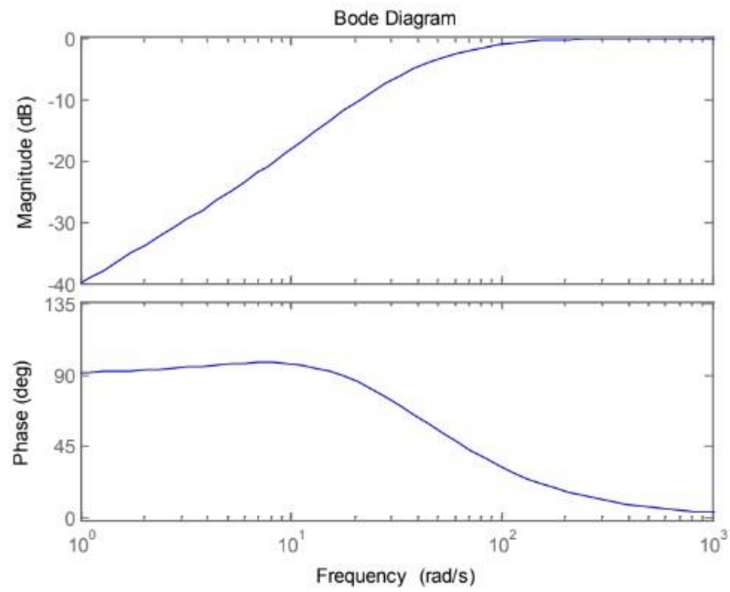
Prob. 3.53

(a) We expand H(s) as

$$H(s) = \frac{s^2 + 10s}{s^2 + 70s + 1000}$$

The MATLAB code with the Bode plot is presented below.

```
num = [ 1  10  0];
den = [ 1 70 1000];
bode(num,den)
```



(b) We first expand the denominator of H(s) as follows.

$$H(s) = \frac{s+1}{s^3 + 24.5s^2 + 61s + 32}$$

The MATLAB code with the result is shown below.

```
num = [ 1  1 ];
den = [ 1  24.5  61  32 ];
bode(num,den)
```