Euler theorem: if a and n are coprime: $a^{\varphi(n)} \equiv 1 \pmod{n}$

$\varphi(n) = n-1$ if n is prime

$n = a * b$; a, b coprime: $\varphi(n) = \varphi(a) * \varphi(b)$

Problem: $a^b \% 1337$

if: $a \% 1337 = 0 \Rightarrow 0$

if $a \not\vdots 7$ and $a \not\vdots 9 \Rightarrow$ a and 1337 are co-prime.

$\Rightarrow a^{\varphi(1337)} \equiv 1 \pmod{n}$

let $b = k\varphi(1337) + q$

$\Rightarrow a^b \% 1337 = a^{k\varphi(1337)+q} \% n = a^q \% n$

if $a \vdots 7$ or $a \vdots 9$. $\Rightarrow$ similar $a \vdots 9$

let assume: $a \vdots 7$

$\Rightarrow a = 7^k x$

$\Rightarrow a^b = 7^{bk} \times x^b$

$\Rightarrow a^b \% n = (7^{bk} x^b) \% n = ((7^{bk} \% 1337) \times (x^b \% 1337)) \% 1337$

$= ((7^{1140pk+qk} \% 1337) \times (x^{1140p+q} \% 1337)) \% 1337$

$= (7(7^{1140pk+qk-1})) \% 1337 \times (x^{1140p+q} \% 1337) \% 1337$

$= (7^{1140pk+qk-1} \% 191) \times (x^{1140p+q} \% 1337) \% 1337$

$= (7^{qk-1} \% 191) \times (x^{1140p+q} \% 1337) \% 1337$

$= ((7^{qk} \% 1337) \times (x^{1140p+q} \% 1337)) \% 1337$

$= (7^{qk} \% 1337) \times (x^q \% 1337) = (7^k x)^q \% 1337$