This problem has 2 conditions:

1) vowels == consonants
2) (vowels * consonants) % k == 0

To check the first condition for any sub-array i-j, we can compute it by O(1) with prefix sum.

let presum[i] = # of vowels − # of consonants in S[1 → i]
(S : 1-indexed style)

so: sub-array [i, j] has (1) ⟺ presum[j] == presum[i-1]

⇒ let create group[k] contain all i index that have presum[i] = k.

⇒ any pairs in each group always satisfy (1).

Now, we want to find how many pairs in each group satisfy (2)

Because vowels == consonants in valid sub-array

⇒ (2) ⟺ vowel² % k = 0

let consider sub arry [i+1, j] satisfy both condition.

we have: $\left(\frac{j-i}{2}\right)^2 \% k = 0$ (vowel = consonant, vowel + consonant = j − i)

⟺ $\frac{j^2 - 2ij + i^2}{2} \% k = 0$

The part i * j make the complexity $O(n^2)$ to check all i, j.

But we need some other direction to reduce the complexity

If the condition $\left(\frac{j-i}{2}\right)^2 \% k$ be equivalent by $\frac{j-i}{2} \% k_{new}$ it will reduce the complexity.
To be simply, let make this conditions just related to i, j.

$$\left(\frac{j-i}{2}\right)^2 \% k = 0$$
$$\Leftrightarrow \frac{(j-i)^2}{4} \% k = 0$$
$$\Leftrightarrow (j-i)^2 \% 4k = 0.$$

Let $k \leftarrow 4k$

So now now, make condition (2) equal new condition relate to $j-i$ only.

Let: $j - i = n$. (Find and for n to $n^2 \% k = 0$)

We need convert $n^2 \% k = 0$ to $n \% k_{new} = 0$

Let: $k = \prod_{i=1}^{m} p_i^{q_i}$ when p_i is prime number.

so $n^2 : k \Leftrightarrow n^2 = \prod_{i=1}^{m} p_i^{t_i} \times \ldots / t_i \geq q_i$

$$n = \prod_{i=1}^{m} p_i^{\frac{t_i}{2}} \times \ldots \quad / \quad \frac{t_i}{2} \geq \frac{q_i}{2}$$

so $\forall n \; / \; \frac{t_i}{2} \geq \frac{q_i}{2} \Rightarrow n^2 : k$.

$\Leftrightarrow \forall n : \boxed{k_{new} = \prod_{i=1}^{m} p_i^{\frac{q_i}{2}}}$ (*)

To compute k_{new} we have 2 way.

Factor k to find q_i, $i = \overline{1,m}$

or brute for k:

From (1) $\begin{cases} k_{new} \geq \sqrt{k} \\ k_{new}^2 \vdots k \end{cases}$

$\Rightarrow$ Check all $k_{new} = [\sqrt{k}, k]$, Find min $k_{new}^2 \vdots k$.

So $(j-i)^2 \vdots k \Leftrightarrow j - i \vdots k_{new}$

$\Leftrightarrow j \% k_{new} = i \% k_{new}$.

So, For each groups:

any pair i, j / $j \% k_{new} == i \% k_{new}$

will create beautiful string.