This problem has 2 conditions:
1) vowels == consonants
2) (vowels * consonants) % k == 0

To check the first condition for any sub-array $i-j$, we can compute it by $O(1)$ with prefix sum.

let presum[i] = # of vowels − # of consonants in $S[1 \to i]$
(s : 1-indexed style)

so : sub-array $[i, j]$ has (1) $\Leftrightarrow$ presum[j] == presum[i−1]

$\Rightarrow$ let create group[k] contain all i ndexet that have presum[i] = k.

$\Rightarrow$ any pairs in each group alway satisfy (1).

Now, we want to find how many pairs in each group satisfy (2)

Becaus vowels == consonants in valid sub-array

$\Rightarrow$ (2) $\Leftrightarrow$ $vowel^2 \% k = 0$

let consider subarray $[i+1, j]$ satisfy both conditi.

we hv: $\left(\frac{j-i}{2}\right)^2 \% k = 0$ (vowel = consonant, vowel + consonant $= j - i$)

$\Leftrightarrow \frac{j^2 - 2ij + i^2}{2} \% k = 0.$

The part $i * j$ make the computation $O(n^2)$ to check all i, j.

But we need some other direction to reduce the complexity

If the condition $\left(\frac{j-i}{2}\right)^2 \% k$ be equival to $\frac{j-i}{2} \% k_{new}$

it will reduce the complexity.

To be simply, let make this conditions just related to i, j,

$$\left(\frac{j-i}{2}\right)^2 \% k = 0$$

$$\Leftrightarrow \frac{(j-i)^2}{4} \% k = 0$$

$$\Leftrightarrow (j-i)^2 \% 4k = 0.$$

Let $k \leftarrow 4k$

so now, make condition (2) equal new condition relate to $j-i$ only.

Let : $j-i = n$. (Find cond for n to $n^2 \% k == 0$)

we need convert $n^2 \% k = 0$ to $n \% k_{new} = 0$

Let: $k = \prod_{i=1}^{m} p_i^{q_i}$ with p_i is prime number.

so $n^2 \vdots k \Leftrightarrow n^2 = \prod_{i=1}^{m} p_i^{t_i} \times \dots / t_i \geq q_i$

$$n = \prod_{i=1}^{m} p_i^{\frac{t_i}{2}} \times \dots \quad / \quad \frac{t_i}{2} \geq \frac{q_i}{2}$$

so $\forall n \ / \ \frac{t_i}{2} \geq \frac{q_i}{2} \Rightarrow n^2 \vdots k.$

$$\Leftarrow \forall n \vdots \boxed{k_{new} = \prod_{i=1}^{m} p_i^{\frac{q_i}{2}}} \quad (*)$$

To compute k_{new} we have 2 way.

Factor k to find q_i, $i = \overline{1, m}$

or brute for k:

From (*) $\begin{cases} k_{new} \geq \sqrt{k} \\ k_{new}^2 \,\vdots\, k. \end{cases}$

$\Rightarrow$ Check all $k_{new} = [\sqrt{k}, k]$, Find $\min k_{new}^2 \,\vdots\, k$.