Table 1: Test results of the four methods for problem 3.1

| | اح | | | | | | | | | | | | | | | | | | | | | |
|----------------|------|------------|------------|------------|------------|------------|------------|---------|------------|------------|------------|------------|------------|------------|---------|------------|------------|------------|------------|------------|------------|----------|
| | Norm | I | I | 0 | I | I | 0 | 0 | 0 | I | I | I | I | 0 | 0 | 0 | I | I | I | I | 0 | 0 |
| NMPD | PT | I | I | 0.03588 | I | I | 0.00315 | 0.00654 | 0.06145 | I | I | I | I | 0.01259 | 0.01192 | 0.09145 | I | 1 | 1 | 1 | 0.01598 | 0.0178 |
| | FE | I | 1 | 09 | I | I | 13 | 13 | 61 | I | I | I | I | 13 | 13 | 52 | 1 | I | I | I | 13 | 13 |
| | LIZ | I | I | ∞ | I | I | П | П | 9 | I | I | I | I | П | П | 9 | 1 | 1 | 1 | 1 | П | П |
| | Norm | 3.1760E-09 | 4.4039E-09 | 7.1159E-09 | 3.1583E-09 | 4.5665E-09 | 1.6478E-09 | 0 | 7.1018E-09 | 9.8475E-09 | 6.3521E-09 | 9.5227E-09 | 7.2094E-09 | 3.2391E-09 | 0 | 3.0959E-09 | 1.8124E-09 | 1.7436E-09 | 7.9916E-09 | 2.2103E-09 | 4.5808E-09 | 0 |
| DLP | PT | 0.0391 | 0.0121 | 0.0177 | 0.0197 | 0.0252 | 0.0091 | 0.0058 | 0.0619 | 0.0656 | 0.0648 | 0.0659 | 0.0484 | 0.0775 | 0.0133 | 0.1067 | 0.1005 | 0.1348 | 0.1027 | 0.0819 | 0.1044 | 0.0200 |
| | Æ | 30 | 31 | 34 | 40 | 30 | 25 | 13 | 30 | 31 | 20 | 46 | 30 | 47 | 13 | 35 | 37 | 26 | 32 | 31 | 47 | 13 |
| | LIZ | 16 | 15 | 14 | 19 | 16 | 11 | 1 | 16 | 15 | 17 | 21 | 16 | 14 | 1 | 16 | 18 | 21 | 18 | 18 | 14 | 1 |
| | Norm | ı | 0 | 0 | 4.3072E-09 | 4.3749E-09 | 0 | 0 | 0 | I | I | 2.6925E-09 | 2.7014E-09 | 0 | 0 | ı | 9.6707E-09 | 3.5804E-09 | 3.9058E-09 | 3.9122E-09 | 0 | 0 |
| ACGD | PT | ı | 0.0287 | 0.0493 | 0.0419 | 0.0362 | 0.0087 | 0.0072 | 0.0797 | I | I | 0.0856 | 0.0970 | 0.0338 | 0.0181 | I | 0.2027 | 0.2039 | 0.1775 | 0.1654 | 0.0343 | 0.0244 |
| | 표 | ı | 25 | 47 | 37 | 37 | 13 | 13 | 36 | I | I | 36 | 36 | 13 | 13 | I | 22 | 54 | 36 | 36 | 13 | 13 |
| | LIZ | ı | 17 | 15 | 16 | 16 | 1 | 1 | 15 | I | I | 18 | 18 | 1 | 1 | I | 19 | 19 | 18 | 18 | 1 | 1 |
| | Norm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SHZM | PT | 0.0200 | 0.0130 | 0.0091 | 0.0162 | 0.0144 | 0.0095 | 0.0186 | 0.0356 | 0.0221 | 0.0221 | 0.0409 | 0.0358 | 0.0288 | 0.0376 | 0.0631 | 0.0581 | 0.0418 | 0.0600 | 0.0551 | 0.0566 | 0.0402 |
| 6, | FE | 22 | 13 | 13 | 25 | 22 | 13 | 13 | 22 | 13 | 13 | 25 | 25 | 13 | 13 | 25 | 13 | 13 | 22 | 22 | 13 | 13 |
| | LIN | 2 | _ | 1 | 7 | 7 | 1 | 1 | 7 | 1 | 1 | 7 | 7 | 1 | 1 | 7 | _ | 1 | 7 | 7 | _ | 1 |
| Ipt | | x | ž | х х | x 4 | x5 | 9x | ×, | x | χ | х 3 | x 4 | x5 | 9x | × | x1 | ž | х х | x 4 | х 5 | 9x | X |
| Pnum Nvars Ipt | | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
| Pnum | | 3.1 | | | | | | | | | | | | | | | | | | | | |

Table 2: Test results of the four methods for problem 3.2

| | Norm | ı | I | I | I | I | I | I | I | I | I | I | ı | I | I | I | I | I | I | I | I | I |
|----------|---------------|------------|------------|------------|------------|------------|------------|--------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|--------|
| PD | PT | ı | 1 | 1 | 1 | ı | I | I | 1 | 1 | ı | I | ı | ı | I | 1 | 1 | 1 | 1 | I | 1 | 1 |
| NMPE | · EE | ı | 1 | 1 | 1 | ı | ı | ı | 1 | ı | ı | ı | ı | ı | ı | ı | 1 | 1 | ı | ı | 1 | ı |
| | | | | | | | | | | | | | | | | | | | | | | |
| | LIN | I | ı | ı | ı | ı | ı | ı | ı | ı | ı | ı | ı | ı | I | ı | ı | ı | ı | I | ı | I |
| | Norm | 8.9876E-09 | 4.5086E-09 | 3.8555E-09 | 3.8110E-09 | 3.7539E-09 | 7.1860E-09 | 0 | 7.7872E-09 | 4.8854E-09 | 5.2409E-09 | 8.4393E-09 | 8.4134E-09 | 4.5030E-09 | 0 | 3.8789E-09 | 6.9055E-09 | 5.8589E-09 | 4.1959E-09 | 4.1894E-09 | 7.1498E-09 | 0 |
| DLP | \mathbf{PT} | 0.0202 | 0.0209 | 0.0211 | 0.0219 | 0.0198 | 0.0290 | 0.0069 | 0.1136 | 0.1309 | 0.1062 | 0.1081 | 0.1239 | 0.1644 | 0.0233 | 0.1784 | 0.1836 | 0.1919 | 0.1726 | 0.1755 | 0.2541 | 0.0479 |
| | FE | 69 | 73 | 74 | 89 | 89 | 84 | 25 | 72 | 7 | 83 | 89 | 89 | 66 | 22 | 22 | 1 | 83 | 7 | 7 | 106 | 22 |
| | LIN | 21 | 22 | 22 | 21 | 21 | 24 | 7 | 22 | 23 | 24 | 22 | 22 | 25 | 7 | 23 | 23 | 24 | 23 | 23 | 26 | 2 |
| | Norm | 3.5068E-09 | 2.6660E-09 | 2.8568E-09 | 4.5104E-09 | 4.4844E-09 | 0 | 0 | 7.8423E-09 | 6.8232E-09 | 6.4767E-09 | 1.9759E-09 | 1.9736E-09 | 0 | 0 | 2.1024E-09 | 9.7809E-09 | 9.1846E-09 | 2.8269E-09 | 2.8253E-09 | 0 | 0 |
| ACGD | ${ m Ld}$ | 0.0500 | 0.0298 | 0.0562 | 0.0330 | 0.0462 | 0.0193 | 0.0201 | 0.1436 | 0.1395 | 0.1563 | 0.1230 | 0.1429 | 0.0803 | 0.0621 | 0.2316 | 0.2470 | 0.2250 | 0.2121 | 0.2327 | 0.1752 | 0.0844 |
| | FE | 65 | 99 | 74 | 28 | 28 | 37 | 25 | 65 | 99 | 74 | 62 | 62 | 44 | 25 | 69 | 99 | 74 | 62 | 62 | 29 | 22 |
| | LIN | 15 | 14 | 15 | 14 | 14 | Ŋ | 7 | 14 | 14 | 16 | 15 | 15 | ^ | 7 | 15 | 15 | 15 | 15 | 15 | 6 | 2 |
| | Norm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SHZM | ${ m LI}$ | 0.0293 | 0.0246 | 0.0102 | 0.0211 | 0.0176 | 0.0146 | 0.0424 | 0.0423 | 0.0807 | 0.0513 | 0.0430 | 0.0881 | 0.0354 | 0.0907 | 0.1084 | 0.0894 | 0.0759 | 0.1005 | 0.1568 | 0.0855 | 0.1540 |
| G | FE | 25 | 25 | 25 | 25 | 37 | 25 | 61 | 25 | 25 | 25 | 25 | 37 | 25 | 49 | 25 | 25 | 25 | 25 | 37 | 25 | 49 |
| | IIN | 7 | 7 | 7 | 7 | ϵ | 7 | rV | 7 | 7 | 7 | 7 | ϵ | 7 | 4 | 7 | 7 | 7 | 7 | ϵ | 7 | 4 |
| Ipt | | x1 | x2 | x3 | x 4 | x 5 | 9x | x7 | x1 | x2 | x3 | x 4 | x 5 | 9x | x 7 | x1 | x2 | x3 | x 4 | x5 | 9x | x7 |
| Nvars | | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
| Pnum | | 3.2 | | | | | | | | | | | | | | | | | | | | |

Table 3: Test results of the four methods for problem 3.3

| SHZM | SHZM | SHZM | SHZM | 1 | | | AC | ACGD | | | D | DLP | | | Z | NMPD | 1 |
|--------------------------------|-------------------------|---------------|---------------|----------|------|-----|----|---------------------|------|-----|---|---------------|------|-----|---|---------------------|------|
| NIT FE PT Norm | . FE PT | . FE PT | \mathbf{PT} | | Norm | LIZ | H | PT | Norm | LIZ | Æ | \mathbf{PT} | Norm | LIZ | H | PT | Norm |
| 1000 x1 2 25 0.0491 0 | 2 25 | | | 0.0491 0 | 0 | I | I | I | I | I | I | I | I | I | ı | I | I |
| $1000 x^2 2 25 0.0363 0$ | x2 2 25 0.0363 0 | 2 25 0.0363 0 | 25 0.0363 0 | 0.0363 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| 1000 x3 2 25 0.0268 0 | x3 2 25 0.0268 0 | 2 25 0.0268 0 | 25 0.0268 0 | 0.0268 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x4 2 25 | 2 25 | 2 25 0.0265 0 | 25 0.0265 0 | 0.0265 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x5 2 25 | 2 25 | 2 25 0.0273 0 | 25 0.0273 0 | 0.0273 0 | 0 | I | I | I | I | I | I | 1 | I | I | I | I | I |
| x6 2 25 | 2 25 | 2 25 0.0463 0 | 25 0.0463 0 | 0.0463 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x7 2 25 | | | | 0.0289 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x1 2 25 | | | | 0.1266 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x_2 2 25 | 2 25 | | | 0.1027 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x3 2 25 | 2 25 | | | 0.0992 0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| x4 2 25 | 2 25 | | | 0 9960.0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| x5 2 | 2 25 0.1125 | 0.1125 | 0.1125 | | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| x6 2 25 | 2 25 0.1044 | 0.1044 | 0.1044 | | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| | | | | 0.1053 0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| 10000 x1 2 25 0.1871 0 | | | | 0.1871 0 | 0 | I | 1 | I | I | I | I | 1 | I | I | I | 1 | I |
| x^2 2 25 (| 2 25 (| | | 0.1709 0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| x3 2 25 (| • | • | • | 0.1886 0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| x4 2 25 (| 2 25 0.1842 | | | | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| 10000 x5 2 25 0.1504 0 | x5 2 25 0.1504 0 | 2 25 0.1504 0 | 25 0.1504 0 | 0.1504 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| 10000 x6 2 25 0.1869 0 | 2 25 0 | 2 25 0.1869 0 | 25 0.1869 0 | 0.1869 0 | 0 | I | I | I | I | I | I | I | I | I | 1 | I | I |
| 10000 x 7 2 25 0.1920 0 | 2 25 (| _ | _ | 0.1920 0 | 0 | I | I | I | I | I | I | I | I | I | I | I | I |
| | | | | | | | | | | | | | | | | | |

Table 4: Test results of the four methods for problem 3.4

| Pnum | Pnum Nvars | Ipt | | S | SHZM | | | A | ACGD | | | | DLP | | | Z | NMPD | |
|------|------------|------------|-----|-----|--------|------|-----|----|--------|------|-----|----|--------|------------|-----|-----|--------|------|
| | | | LIZ | FE | PT | Norm | LIZ | ŁΕ | PT | Norm | LIN | FE | PT | Norm | LIZ | ŁΕ | PT | Norm |
| 3.4 | 1000 | x1 | 4 | 38 | 0.0183 | 0 | 11 | 18 | 0.0174 | 0 | 14 | 28 | 0.0191 | 6.3387E-09 | ı | I | 1 | ı |
| | 1000 | ζ | ^ | 73 | 0.0315 | 0 | 24 | 95 | 0.0746 | 0 | 13 | 17 | 0.0153 | 3.5880E-09 | 10 | 102 | 0.0300 | 0 |
| | 1000 | 8 3 | 18 | 203 | 0.0786 | 0 | 22 | 78 | 0.0400 | 0 | 16 | 34 | 0.0235 | 3.2473E-09 | I | I | I | I |
| | 1000 | x 4 | 9 | 62 | 0.0334 | 0 | 12 | 32 | 0.0496 | 0 | 16 | 29 | 0.0148 | 1.6676E-09 | 2 | 14 | 0.0061 | 0 |
| | 1000 | x 5 | 9 | 62 | 0.0281 | 0 | 16 | 49 | 0.0553 | 0 | 17 | 32 | 0.0174 | 4.7949E-09 | 2 | 14 | 0.0061 | 0 |
| | 1000 | 9x | 8 | 15 | 0.0103 | 0 | ^ | 6 | 0.0288 | 0 | 13 | 15 | 0.0116 | 3.9725E-09 | 3 | 21 | 0.0087 | 0 |
| | 1000 | χ' | 4 | 16 | 0.0317 | 0 | 10 | 12 | 0.0352 | 0 | 16 | 18 | 0.0198 | 1.2776E-09 | വ | 46 | 0.0173 | 0 |
| | 2000 | x1 | 4 | 38 | 0.0850 | 0 | 19 | 46 | 0.1589 | 0 | 18 | 33 | 0.0784 | 1.2034E-09 | I | I | ı | ı |
| | 2000 | 3 | ^ | 73 | 0.1455 | 0 | 12 | 21 | 0.0918 | 0 | 13 | 17 | 0.0508 | 6.0894E-09 | 7 | 73 | 0.0718 | 0 |
| | 2000 | 8 3 | ^ | 7 | 0.1365 | 0 | 13 | 22 | 0.0972 | 0 | 17 | 32 | 0.0639 | 4.9163E-09 | I | I | I | I |
| | 2000 | x 4 | 9 | 62 | 0.1210 | 0 | 13 | 25 | 0.1353 | 0 | 18 | 31 | 0.0644 | 4.5253E-09 | 2 | 14 | 0.0148 | 0 |
| | 2000 | x 5 | 9 | 62 | 0.1229 | 0 | 12 | 24 | 0.1150 | 0 | 18 | 40 | 0.0819 | 2.0320E-09 | 2 | 14 | 0.0205 | 0 |
| | 2000 | 9x | 8 | 15 | 0.0553 | 0 | ^ | 6 | 0.0453 | 0 | 14 | 18 | 0.0517 | 6.4696E-09 | 4 | 39 | 0.2106 | 0 |
| | 2000 | χ' | 4 | 16 | 0.0607 | 0 | 6 | 11 | 0.0475 | 0 | 19 | 28 | 0.0828 | 0 | 9 | 29 | 0.1376 | 0 |
| | 10000 | x1 | 4 | 38 | 0.1392 | 0 | 14 | 37 | 0.2229 | 0 | 17 | 27 | 0.1120 | 4.3170E-09 | I | I | I | I |
| | 10000 | ζ | ^ | 73 | 0.2309 | 0 | 12 | 21 | 0.1685 | 0 | 14 | 18 | 0.0900 | 2.5432E-09 | I | I | I | I |
| | 10000 | х х | Ŋ | 47 | 0.1819 | 0 | 15 | 26 | 0.1868 | 0 | 16 | 56 | 0.1232 | 2.7320E-09 | I | I | 1 | ı |
| | 10000 | x 4 | 9 | 62 | 0.2114 | 0 | 16 | 45 | 0.2225 | 0 | 19 | 33 | 0.1289 | 3.1972E-09 | 2 | 14 | 0.0389 | 0 |
| | 10000 | x 5 | 9 | 62 | 0.2226 | 0 | 17 | 46 | 0.2462 | 0 | 19 | 32 | 0.1348 | 5.3086E-09 | 2 | 14 | 0.0508 | 0 |
| | 10000 | 9x | 8 | 15 | 0.0657 | 0 | ^ | 6 | 0.0852 | 0 | 16 | 23 | 0.1291 | 1.2138E-09 | rC | 26 | 0.2005 | 0 |
| | 10000 | x, | 4 | 16 | 0.0677 | 0 | 6 | 11 | 0.1381 | 0 | 22 | 42 | 0.1529 | 8.1259E-09 | 9 | 70 | 0.2327 | 0 |

Table 5: Test results of the four methods for problem 3.5

| | l | ı | | | | | | | | | | | | | | | | | | | | |
|------------|---------------|--------|----------|--------|------------|------------|--------|--------|--------|----------|--------|------------|------------|--------|------------|--------|----------|--------|------------|------------|--------|--------|
| | Norm | 0 | I | I | I | I | 0 | 0 | 0 | I | I | I | I | 0 | 0 | I | I | I | I | I | 0 | 0 |
| NMPD | \mathbf{PT} | 0.0434 | I | I | I | I | 0.0106 | 0.0325 | 0.2095 | I | I | I | I | 0.0467 | 0.0816 | I | I | I | I | I | 0.0717 | 0.1844 |
| Z | FE | 28 | I | I | I | ı | 35 | 31 | 123 | I | I | I | I | 35 | 41 | I | I | I | I | I | 34 | 52 |
| | NIT | 7 | I | I | I | I | 4 | 4 | 11 | I | I | I | I | 4 | ιυ | I | I | I | I | I | 4 | 9 |
| | Norm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| DLP | PT | 0.0138 | 0.0229 | 0.0135 | 0.0175 | 0.0170 | 0.0091 | 0.0175 | 0.0627 | 0.1146 | 0.0846 | 0.0603 | 0.0780 | 0.0596 | 0.0968 | 0.1119 | 0.1667 | 0.1665 | 0.1194 | 0.1371 | 0.1133 | 0.1862 |
| | FE | 38 | 38 | 38 | 26 | 45 | 16 | 25 | 30 | 39 | 48 | 26 | 43 | 31 | 28 | 30 | 51 | 51 | 41 | 51 | 37 | 82 |
| | LIN | 18 | 18 | 17 | 15 | 18 | 10 | 12 | 16 | 19 | 19 | 15 | 20 | 14 | 18 | 16 | 22 | 22 | 18 | 23 | 15 | 22 |
| | Norm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ACGD | PT | 0.0446 | 0.0273 | 0.0465 | 0.0347 | 0.0194 | 0.0206 | 0.0218 | 0.1249 | 0.1125 | 0.1266 | 0.0861 | 0.1020 | 0.1033 | 0.0885 | 0.1468 | 0.1922 | 0.2051 | 0.1792 | 0.1543 | 0.1413 | 0.1651 |
| ⋖ | FE | 31 | 34 | 32 | 32 | 33 | 30 | 33 | 31 | 34 | 30 | 28 | 28 | 36 | 31 | 33 | 34 | 36 | 41 | 28 | 28 | 41 |
| | LIN | 14 | 15 | 14 | 15 | 15 | 13 | 14 | 14 | 15 | 13 | 13 | 13 | 17 | 13 | 15 | 15 | 16 | 15 | 13 | 12 | 18 |
| | Norm | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SHZM | PT | 0.0230 | 0.0304 | 0.0106 | 0.0259 | 0.0127 | 0.0163 | 0.0170 | 0.0427 | 0.0426 | 0.0391 | 0.0328 | 0.0329 | 0.0318 | 0.0496 | 0.0886 | 0.0774 | 0.0835 | 0.0509 | 0.0707 | 0.0797 | 0.0389 |
| U J | FE | 25 | 25 | 25 | 17 | 17 | 13 | 13 | 25 | 25 | 25 | 17 | 17 | 13 | 13 | 25 | 25 | 25 | 17 | 17 | 13 | 13 |
| | LIN | 2 | 7 | 7 | 7 | 7 | ┰ | ┰ | 7 | 7 | 7 | 7 | 7 | ┰ | \vdash | 7 | 7 | 7 | 7 | 7 | П | Π |
| Ipt | • | x1 | x | x3 | x 4 | x 5 | 9x | ×, | x1 | x | x3 | x 4 | x 5 | 9x | x 7 | x1 | x | x3 | x 4 | x 5 | 9x | ×, |
| Nvars | | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 1000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 2000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 | 10000 |
| Pnum | | 3.5 | | | | | | | | | | | | | | | | | | | | |

Table 6: Test results of the four methods for problem 3.6

| MIT FE PT Norm NIT FE PT Norm Norm <th> </th> <th>Pnum Nvars Ipt</th> <th>Ipt</th> <th></th> <th>S</th> <th>SHZM</th> <th></th> <th></th> <th>V</th> <th>ACGD</th> <th></th> <th></th> <th></th> <th>DLP</th> <th></th> <th></th> <th>_</th> <th>NMPD</th> <th></th> | | Pnum Nvars Ipt | Ipt | | S | SHZM | | | V | ACGD | | | | DLP | | | _ | NMPD | |
|--|---------------|----------------|----------------|----------|----|--------|------|-----|----|--------|------|----------|----|--------|------|-----|----|--------|------|
| x1 1 13 0.0155 0 1 13 0.0136 0 1 13 0.0136 0 1 13 0.0136 0 1 13 0.0136 0 1 13 0.0136 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 0.0049 0 1 13 | | | | LIZ | FE | | Norm | LIN | 표 | PT | Norm | LIN | FE | PT | Norm | LIN | FE | PT | Norm |
| x3 1 13 0.0094 0 1 13 0.0139 0 1 13 0.0049 0 1 x4 1 13 0.0102 0 1 13 0.0049 0 1 13 0.0049 0 1 x5 1 13 0.0104 0 1 13 0.0049 0 1 13 0.0049 0 1 x6 1 13 0.0145 0 1 13 0.0049 0 1 13 0.0049 0 1 x7 1 13 0.0145 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 <th>1</th> <th>1000</th> <th>x1</th> <td></td> <td>13</td> <td>0.0155</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0091</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0136</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0077</td> <td>0</td> | 1 | 1000 | x1 | | 13 | 0.0155 | 0 | П | 13 | 0.0091 | 0 | П | 13 | 0.0136 | 0 | 1 | 13 | 0.0077 | 0 |
| x3 1 13 0.0102 0 1 13 0.0067 0 1 13 0.0049 0 1 x4 1 13 0.0154 0 1 13 0.0151 0 1 13 0.0069 0 1 x6 1 13 0.0145 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.0069 0 1 13 0.00136 0 | | 1000 | 8 | ₩ | 13 | 0.0094 | 0 | П | 13 | 0.0139 | 0 | П | 13 | 0.0070 | 0 | 1 | 13 | 0.0095 | 0 |
| x4 1 13 0.0194 0 1 13 0.0151 0 1 13 0.0069 0 1 x5 1 13 0.0186 0 1 13 0.0200 0 1 13 0.0054 0 1 x7 1 13 0.0145 0 1 13 0.0048 0 1 13 0.0048 0 1 x1 13 0.0142 0 1 13 0.0048 0 1 13 0.0048 0 1 x2 1 13 0.0176 0 1 13 0.0049 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0134 0 1 13 0.0134 0 1 13 0.0134 0 <th></th> <th>1000</th> <th>x3</th> <td>1</td> <td>13</td> <td>0.0102</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0067</td> <td>0</td> <td>_</td> <td>13</td> <td>0.0049</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0200</td> <td>0</td> | | 1000 | x3 | 1 | 13 | 0.0102 | 0 | 1 | 13 | 0.0067 | 0 | _ | 13 | 0.0049 | 0 | 1 | 13 | 0.0200 | 0 |
| x5 1 13 0.0186 0 1 13 0.0200 0 1 13 0.0084 0 1 x6 1 13 0.0067 0 1 13 0.0048 0 1 x1 13 0.0145 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0048 0 1 13 0.0149 0 1 13 0.0134 0 1 1 | | 1000 | x 4 | П | 13 | 0.0194 | 0 | П | 13 | 0.0151 | 0 | П | 13 | 0.0069 | 0 | П | 13 | 0.0218 | 0 |
| x6 1 13 0.0045 0 1 13 0.0067 0 1 13 0.0048 0 1 x7 1 13 0.0142 0 1 13 0.0149 0 1 13 0.0048 0 1 x2 1 13 0.0205 0 1 13 0.0208 0 1 13 0.0139 0 1 x4 1 13 0.0208 0 1 13 0.0208 0 1 13 0.0134 0 1 x4 1 13 0.0208 0 1 13 0.0208 0 1 13 0.0134 0 1 x5 1 13 0.0289 0 1 13 0.0135 0 1 13 0.0135 0 1 x6 1 13 0.0289 0 1 13 0.0135 0 1 <th></th> <th>1000</th> <th>x5</th> <td>₩</td> <td>13</td> <td>0.0186</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0200</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0054</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0087</td> <td>0</td> | | 1000 | x 5 | ₩ | 13 | 0.0186 | 0 | П | 13 | 0.0200 | 0 | П | 13 | 0.0054 | 0 | 1 | 13 | 0.0087 | 0 |
| x7 1 13 0.0142 0 1 13 0.0117 0 1 13 0.0148 0 1 13 0.0148 0 1 13 0.0156 0 1 13 0.0156 0 1 13 0.0158 0 1 13 0.0158 0 1 13 0.0158 0 1 13 0.0158 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0289 0 1 13 0.0289 0 1 13 0.0289 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 0.0138 0 1 13 | | 1000 | 9x | 1 | 13 | 0.0145 | 0 | 1 | 13 | 0.0067 | 0 | _ | 13 | 0.0048 | 0 | 1 | 13 | 0.0093 | 0 |
| x1 1 13 0.0421 0 1 13 0.0356 0 1 13 0.0137 0 1 x2 1 13 0.0206 0 1 13 0.0208 0 1 13 0.0134 0 1 x3 1 13 0.0215 0 1 13 0.0208 0 1 13 0.0134 0 1 x4 1 13 0.0289 0 1 13 0.0135 0 1 13 0.0135 0 1 x4 1 13 0.0289 0 1 13 0.0135 0 1 13 0.0135 0 1 x4 1 13 0.0368 0 1 13 0.0327 0 1 x4 1 13 0.0348 0 1 13 0.0343 0 1 13 0.0343 0 1 | | 1000 | x ₇ | \vdash | 13 | 0.0142 | 0 | 1 | 13 | 0.0117 | 0 | П | 13 | 0.0048 | 0 | 1 | 13 | 0.0093 | 0 |
| x2 1 13 0.0156 0 1 13 0.0206 0 1 13 0.0134 0 1 x3 1 13 0.0215 0 1 13 0.0234 0 1 13 0.0134 0 1 x4 1 13 0.0289 0 1 13 0.0134 0 1 13 0.0134 0 1 x6 1 13 0.0289 0 1 13 0.0134 0 1 13 0.0139 0 1 x7 1 13 0.0269 0 1 13 0.0368 0 1 13 0.0139 0 1 x4 1 13 0.0368 0 1 13 0.0329 0 1 13 0.0139 0 1 x4 1 13 0.0325 0 1 13 0.0243 0 1 <th></th> <th>2000</th> <th>x1</th> <td>П</td> <td>13</td> <td>0.0421</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0356</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0137</td> <td>0</td> <td>П</td> <td>13</td> <td>0.0222</td> <td>0</td> | | 2000 | x1 | П | 13 | 0.0421 | 0 | П | 13 | 0.0356 | 0 | П | 13 | 0.0137 | 0 | П | 13 | 0.0222 | 0 |
| x3 1 13 0.0215 0 1 13 0.0208 0 1 13 0.0281 0 1 13 0.0135 0 1 x4 1 13 0.0281 0 1 13 0.0285 0 1 13 0.0135 0 1 x6 1 13 0.0289 0 1 13 0.0135 0 1 13 0.0135 0 1 13 0.0135 0 1 13 0.0139 0 1 13 0.0139 0 1 13 0.0350 0 1 13 0.0257 0 1 x4 1 13 0.0322 0 1 13 0.0323 0 1 13 0.0243 0 1 x4 1 13 0.0323 0 1 13 0.0232 0 1 1 x4 1 13 0.0243 | | 5000 | ζ | 1 | 13 | 0.0150 | 0 | 1 | 13 | 0.0206 | 0 | П | 13 | 0.0138 | 0 | 1 | 13 | 0.0438 | 0 |
| x4 1 13 0.0433 0 1 13 0.0281 0 1 13 0.0135 0 1 x5 1 13 0.0289 0 1 13 0.0196 0 1 13 0.0137 0 1 x6 1 13 0.0189 0 1 13 0.0137 0 1 x1 13 0.0203 0 1 13 0.0350 0 1 13 0.0227 0 1 x3 1 13 0.0350 0 1 13 0.0352 0 1 13 0.0243 0 1 x4 1 13 0.0242 0 1 13 0.0542 0 1 13 0.0232 0 1 x4 1 13 0.0243 0 1 13 0.0132 0 1 x6 1 13 0.0243 | | 2000 | x3 | П | 13 | 0.0215 | 0 | 1 | 13 | 0.0208 | 0 | П | 13 | 0.0134 | 0 | 1 | 13 | 0.0214 | 0 |
| x5 1 13 0.0289 0 1 13 0.0196 0 1 13 0.0137 0 1 x6 1 13 0.0189 0 1 13 0.0131 0 1 x1 1 13 0.0203 0 1 13 0.0139 0 1 13 0.0139 0 1 x2 1 13 0.0348 0 1 13 0.0227 0 1 13 0.0243 0 1 x3 1 13 0.0348 0 1 13 0.0542 0 1 13 0.0243 0 1 x4 1 13 0.0323 0 1 13 0.0493 0 1 13 0.0232 0 1 x6 1 13 0.0493 0 1 13 0.0197 0 1 x6 1 13 | | 2000 | x 4 | \vdash | 13 | 0.0433 | 0 | 1 | 13 | 0.0281 | 0 | \vdash | 13 | 0.0135 | 0 | 1 | 13 | 0.0314 | 0 |
| x6 1 13 0.0353 0 1 13 0.0189 0 1 13 0.0368 0 1 13 0.0368 0 1 13 0.0368 0 1 13 0.0368 0 1 13 0 1 </td <th></th> <th>2000</th> <th>x5</th> <td>\vdash</td> <td>13</td> <td>0.0289</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0196</td> <td>0</td> <td>⊣</td> <td>13</td> <td>0.0137</td> <td>0</td> <td>1</td> <td>13</td> <td>0.0355</td> <td>0</td> | | 2000 | x 5 | \vdash | 13 | 0.0289 | 0 | 1 | 13 | 0.0196 | 0 | ⊣ | 13 | 0.0137 | 0 | 1 | 13 | 0.0355 | 0 |
| x7 1 13 0.0203 0 1 13 0.0368 0 1 13 0.0350 0 1 13 0.0350 0 1 13 0 x4 1 13 0.0270 0 1 13 0.0497 0 1 13 0 x5 1 13 0.0270 0 1 13 0 1 13 0 1 <t< td=""><th></th><th>2000</th><th>9x</th><td>1</td><td>13</td><td>0.0353</td><td>0</td><td>1</td><td>13</td><td>0.0189</td><td>0</td><td>\vdash</td><td>13</td><td>0.0131</td><td>0</td><td>1</td><td>13</td><td>0.0198</td><td>0</td></t<> | | 2000 | 9x | 1 | 13 | 0.0353 | 0 | 1 | 13 | 0.0189 | 0 | \vdash | 13 | 0.0131 | 0 | 1 | 13 | 0.0198 | 0 |
| x1 1 13 0.0404 0 1 13 0.0350 0 1 13 0.0322 0 1 13 0.0322 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 x6 1 13 0.0234 0 1 13 0.0493 0 1 13 0 x7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | | 2000 | χ' | П | 13 | 0.0203 | 0 | 1 | 13 | 0.0368 | 0 | \vdash | 13 | 0.0139 | 0 | 1 | 13 | 0.0296 | 0 |
| x2 1 13 0.0348 0 1 13 0.0322 0 1 13 0.0542 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 1 13 0 x5 1 13 0.0226 0 1 13 0.0493 0 1 13 0 x6 1 13 0.0234 0 1 13 0.0543 0 1 13 0 x7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | $\overline{}$ | 00001 | x1 | П | 13 | 0.0404 | 0 | T | 13 | 0.0350 | 0 | \vdash | 13 | 0.0227 | 0 | 1 | 13 | 0.0327 | 0 |
| x3 1 13 0.0318 0 1 13 0.0542 0 1 13 0.0323 0 1 13 0 x4 1 13 0.0226 0 1 13 0.0493 0 1 13 0 x6 1 13 0.0234 0 1 13 0.0543 0 1 13 0 x7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | | 00001 | x | П | 13 | 0.0348 | 0 | 1 | 13 | 0.0322 | 0 | П | 13 | 0.0243 | 0 | 1 | 13 | 0.0314 | 0 |
| x4 1 13 0.0226 0 1 13 0.0323 0 1 13 0.0493 0 1 13 0 x5 1 13 0.0226 0 1 13 0.0493 0 1 13 0 x6 1 13 0.0234 0 1 13 0.0497 0 1 13 0 x7 1 13 0.0270 0 1 13 0 1 13 0 | | 00001 | x3 | П | 13 | 0.0318 | 0 | П | 13 | 0.0542 | 0 | П | 13 | 0.0188 | 0 | П | 13 | 0.0478 | 0 |
| x5 1 13 0.0226 0 1 13 0.0493 0 1 13 0 x6 1 13 0.0234 0 1 13 0.0543 0 1 13 0 x7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | | 00001 | x 4 | ₩ | 13 | 0.0226 | 0 | Π | 13 | 0.0323 | 0 | П | 13 | 0.0232 | 0 | 1 | 13 | 0.0543 | 0 |
| x6 1 13 0.0234 0 1 13 0.0543 0 1 13 0 x7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | ٠, | 00001 | x 5 | П | 13 | 0.0226 | 0 | 1 | 13 | 0.0493 | 0 | П | 13 | 0.0237 | 0 | 1 | 13 | 0.0427 | 0 |
| x 7 1 13 0.0270 0 1 13 0.0497 0 1 13 0 | ١, ١ | 00001 | 9x | 1 | 13 | 0.0234 | 0 | 1 | 13 | 0.0543 | 0 | ⊣ | 13 | 0.0197 | 0 | 1 | 13 | 0.0281 | 0 |
| | | 00001 | X | П | 13 | 0.0270 | 0 | T | 13 | 0.0497 | 0 | \vdash | 13 | 0.0153 | 0 | 1 | 13 | 0.0425 | 0 |

Table 7: Test results of the four methods for problem 3.7

| Pnum | Pnum Nvars | Ipt | | S | SHZM | | | | ACGD | | | | DLP | | | _ | NMPD | |
|------|------------|------------|-----|----|--------|------|-----|-----|--------|------------|-----|------|--------|------------|-----|-----|--------|------|
| | | | LIZ | Æ | PT | Norm | LIN | FE | PT | Norm | NIT | FE | PT | Norm | LIN | FE | PT | Norm |
| 3.7 | 1000 | x1 | 4 | 49 | 0.0216 | 0 | 34 | 246 | 0.1088 | 8.8730E-09 | 12 | 82 | 0.0265 | 0 | ı | I | ı | ı |
| | 1000 | x | R | 61 | 0.0408 | 0 | 52 | 376 | 0.2128 | 9.7685E-09 | 75 | 479 | 0.1388 | 3.2131E-09 | I | I | I | ı |
| | 1000 | x3 | 9 | 62 | 0.0659 | 0 | 54 | 391 | 0.2067 | 6.8230E-09 | 62 | 354 | 0.1434 | 6.6984E-09 | I | I | I | I |
| | 1000 | x 4 | 3 | 37 | 0.0327 | 0 | Ŋ | 25 | 0.0301 | 0 | 33 | 257 | 0.1108 | 0 | Ŋ | 34 | 0.0166 | 0 |
| | 1000 | x 5 | 3 | 37 | 0.0324 | 0 | 26 | 466 | 0.2640 | 7.2649E-09 | 46 | 283 | 0.1100 | 5.5437E-09 | I | I | I | I |
| | 1000 | 9x | 7 | 25 | 0.0125 | 0 | 4 | 30 | 0.0570 | 0 | 4 | 36 | 0.0146 | 0 | 20 | 101 | 0.0424 | 0 |
| | 1000 | x | 7 | 14 | 0.0190 | 0 | 7 | 25 | 0.0244 | 0 | 2 | 22 | 0.0083 | 0 | 7 | 14 | 0.0089 | 0 |
| | 5000 | x1 | 4 | 49 | 0.1643 | 0 | 51 | 368 | 0.7555 | 8.4208E-09 | Ŋ | 56 | 0.0665 | 0 | 14 | 119 | 0.3060 | 0 |
| | 5000 | x | R | 61 | 0.1201 | 0 | 26 | 406 | 0.8067 | 0 | 48 | 283 | 0.4076 | 5.6757E-09 | I | I | I | I |
| | 2000 | x 3 | 9 | 62 | 0.1928 | 0 | 48 | 349 | 0.7303 | 7.0074E-09 | 52 | 316 | 0.4402 | 3.2294E-09 | 14 | 95 | 0.1776 | 0 |
| | 5000 | x 4 | 4 | 38 | 0.1027 | 0 | 4 | 33 | 0.0812 | 0 | ^ | 72 | 0.1308 | 0 | 51 | 8 | 0.3429 | 0 |
| | 2000 | x 5 | 3 | 37 | 0.0988 | 0 | I | I | I | I | I | I | ı | ı | 4 | 21 | 0.0915 | 0 |
| | 2000 | 9x | 7 | 14 | 0.0643 | 0 | 8 | 30 | 0.0563 | 0 | 3 | 26 | 0.0380 | 0 | 7 | 14 | 0.0564 | 0 |
| | 2000 | x 7 | 7 | 14 | 0.0688 | 0 | 7 | 14 | 0.0411 | 0 | 2 | 14 | 0.0309 | 0 | 7 | 14 | 0.0397 | 0 |
| | 10000 | x1 | 4 | 49 | 0.1625 | 0 | 75 | 538 | 1.7755 | 9.8675E-09 | Ŋ | 29 | 0.1097 | 0 | 14 | 121 | 0.4711 | 0 |
| | 10000 | x 2 | Ŋ | 61 | 0.2214 | 0 | 26 | 411 | 1.3987 | 0 | 52 | 301 | 0.7749 | 6.6958E-09 | I | I | I | ı |
| | 10000 | x3 | 9 | 62 | 0.2484 | 0 | 51 | 370 | 1.3060 | 7.8892E-09 | 26 | 497 | 1.2296 | 5.4829E-09 | 18 | 112 | 0.3648 | 0 |
| | 10000 | x 4 | 4 | 38 | 0.1997 | 0 | 8 | 80 | 0.3043 | 0 | 3 | 27 | | 0 | R | 4 | 0.1895 | 0 |
| | 10000 | x 5 | 8 | 37 | 0.1329 | 0 | I | I | I | ı | 190 | 1743 | | 7.5137E-09 | 4 | 22 | 0.1117 | 0 |
| | 10000 | 9x | 7 | 14 | 0.0985 | 0 | 8 | 30 | 0.1099 | 0 | 3 | 26 | 0.0912 | 0 | 7 | 14 | 0.0905 | 0 |
| | 10000 | x7 | 2 | 14 | 0.0580 | 0 | 2 | 14 | 0.0800 | 0 | 2 | 14 | 0.0484 | 0 | 2 | 14 | 0.0691 | 0 |
| | | | | | | | | | | | | | | | | | | |