General Relativity (I)

homework for week 2

due: Oct. 5th, 2020

1. [invisible Lorentz contraction/ download and read paper] 30%
The idea of Lorentz contraction is about the *measurement* of length *simultaneously* in the inertial observer's frame. In comparison, when we "see" a moving object, we are receive photons emit from the object at the *different time* object but arrive our eyes at the *same time*. Visual effects when "seeing" objects move at nearly the speed of light is demonstrated in the paper "First-person visualizations of the special and general theory of relativity" by U Kraus (*p.s. you can download the paper when using the NTNU internet*):

(a) explain the physics behind figure 1 of the paper.

(b) In (a), the dice is moving from *left to right*. If the dice is moving from *right to left*, how would the result change and why?

2. [superluminal motion/ find resources] 10%
When a object moving with a speed close to the speed of light, its transverse velocity on the sky may seem faster than the speed of light to a distant observer. Find related references (e.g. Box 4.3 of "Gravity: an introduction to Einstein's general relativity" by James B. Hartle, and/or google the key word) and explain such visual illusion with more details. (*p.s. remember to write down the reference(s) you find and read*)

3. [coordinate and metric construction] 30%
In spherical coordinate (r, θ, ϕ) the position (three-) vector is

$$\mathbf{r} = r\sin\theta\cos\phi\,\mathbf{i} + r\sin\theta\sin\phi\,\mathbf{j} + r\,\cos\theta\,\mathbf{k}\,,$$

where $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ are orthonormal basis vectors associated with the Cartesian coordinate (x, y, z):

(a) The three basis vectors can be constructed by $\mathbf{e}_r = \partial\mathbf{r}/\partial r$, $\mathbf{e}_\theta = \partial\mathbf{r}/\partial\theta$ $\mathbf{e}_\phi = \partial\mathbf{r}/\partial\phi$. Show that $\mathbf{e}_r \cdot \mathbf{e}_\theta = \mathbf{e}_\theta \cdot \mathbf{e}_\phi = \mathbf{e}_\phi \cdot \mathbf{e}_r = 0$

(b) use $g_{ij} \equiv \mathbf{e}_i \cdot \mathbf{e}_j$ to compute g_{rr}, and $g_{\theta\theta}$, $g_{\phi\phi}$.

(c) use $g^{ij}g_{jk} = \delta^i_k$ to compute g^{rr}, $g^{\theta\theta}$, and $g^{\phi\phi}$, where δ^i_k is the Kronecker delta defined by $\delta^i_k = 1$ (or 0) if $i = j$ (or $i \neq j$).

3. [four-vectors] 30%
In the inertial frame $\mathcal{O}$, given the four-vectors $\mathbf{A} = (0, 2, -4, 1)$ and $\mathbf{B} = (6, 4, 0, 3)$:

(a) what are the components: A^1, A^1, B^0, B_0?

(b) Is $\mathbf{A}$ time-like, space-like, or null? how about $\mathbf{B}$?

(c) compute $A^\alpha B_\alpha$

(d) find the components of $\mathbf{A}$ in another inertial frame $\mathcal{O}'$, which moves at a speed of $0.8c$ with respect to $\mathcal{O}$ in the positive x direction.

(e) find the components of **B** in another inertial frame $\mathcal{O}'$.

(f) compute $A^{\alpha} B_{\alpha}$ in $\mathcal{O}'$ frame.