

General Relativity (I)

homework for week 2

due: week 4

1. [invisible Lorentz contraction/ download and read paper] 30%

The idea of Lorentz contraction is about the *measurement* of length *simultaneously* in the inertial observer's frame. In comparison, when we "see" a moving object, we receive photons emitted from the object at the *different time* but arrive our eyes at the *same time*. Visual effects when "seeing" objects move at nearly the speed of light is demonstrated in the paper "[First-person visualizations of the special and general theory of relativity](#)" by U Kraus (*p.s. you can download the paper when using the NTNU internet*):

(a) explain the physics behind figure 1 of the paper.

(b) In (a), the dice is moving from *left to right*. If the dice is moving from *right to left*, how would the result change and why?

2. [superluminal motion/ find resources] 10%

When an object moving with a speed close to the speed of light, its transverse velocity on the sky may seem faster than the speed of light to a distant observer. Find related references (e.g. Box 4.3 of "Gravity: an introduction to Einstein's general relativity" by James B. Hartle, and/or google the key word) and explain such visual illusion with more details. (*p.s. remember to write down the reference(s) you find and read*)

3. [coordinate and metric construction] 30%

In spherical coordinate (r, θ, ϕ) the position (three-) vector is

$$\mathbf{r} = r \sin\theta \cos\phi \mathbf{i} + r \sin\theta \sin\phi \mathbf{j} + r \cos\theta \mathbf{k},$$

where $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ are orthonormal basis vectors associated with the Cartesian coordinate (x, y, z) :

(a) The three (natural) basis vectors can be constructed by $\mathbf{e}_r = \partial\mathbf{r}/\partial r$, $\mathbf{e}_\theta = \partial\mathbf{r}/\partial\theta$, $\mathbf{e}_\phi = \partial\mathbf{r}/\partial\phi$. Show that $\mathbf{e}_r \cdot \mathbf{e}_\theta = \mathbf{e}_\theta \cdot \mathbf{e}_\phi = \mathbf{e}_\phi \cdot \mathbf{e}_r = 0$

(b) use $g_{ij} \equiv \mathbf{e}_i \cdot \mathbf{e}_j$ to compute g_{rr} , and $g_{\theta\theta}$, $g_{\phi\phi}$.

(c) use $g^{ij}g_{jk} = \delta_k^i$ to compute g^{rr} , $g^{\theta\theta}$, and $g^{\phi\phi}$, where δ_k^i is the Kronecker delta defined by $\delta_k^i = 1$ (or 0) if $i = j$ (or $i \neq j$).

3. [four-vectors] 30%

In the inertial frame \mathcal{O} , given the four-vectors $\vec{\mathbf{A}} = (0, 2, -4, 1)$ and $\vec{\mathbf{B}} = (6, 4, 0, 3)$:

(a) what are the components: A^1 , A_1 , B^0 , B_0 ?

(b) Is $\vec{\mathbf{A}}$ time-like, space-like, or null? how about $\vec{\mathbf{B}}$?

(c) compute $A^\alpha B_\alpha$

(d) find the components of $\vec{\mathbf{A}}$ in another inertial frame \mathcal{O}' , which moves at a speed of $0.8c$ with respect to \mathcal{O} in the positive x direction.

- (e) find the components of $\vec{\mathbf{B}}$ in another inertial frame \mathcal{O}' .
- (f) compute $A^\alpha B_\alpha$ in \mathcal{O}' frame.