General Relativity (I)

solutions for week 15-16

Week 15:
1(a): hint:$R_{00} = -3\frac{\ddot{a}}{a}$

1(b): There are many ways to do this.

hint(1): note that $T^{00} = \rho$, $T^{11} = \frac{p}{a^2}$, $T^{22} = \frac{p}{a^2 r^2}$, $T^{00} = \frac{p}{a^2 r^2 \sin^2\theta}$.

hint(2): $g_{0\mu} T^{\mu\nu}_{;\nu} = 0$ gives the continuity equation: $(\rho u^\mu)_{;\mu} + p u^\mu_{;\mu} = 0$; see also problem 1(a) of week 6 homework.

2(g): $\rho_{rad} \propto T^4 \propto t^{-2}$.

Week 16: bonus homework:
1(a): compare with $T^{\alpha\beta}$ of the ideal fluid: $T^{\alpha\beta} = (\rho + p) u^\alpha u^\beta + p g^{\alpha\beta}$, with $u^\alpha = (1, 0, 0, 0)$

1(c): hint: show $\rho + 3p < 0$, then $a = 0$ must take place in the past in a finite time

3(c): fainter