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TRƯỜNG ĐẠI HỌC BÁCH KHOA

Report Calculus Project

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221 Calculus CC17

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PREFACE

Calculus 1 is an important general subject for university students HCMUT in particular and students of science and technology - public sector technology in general. Therefore, devoting a certain amount of time to this subject and practice is indispensable to help students have a solid basis in subjects Science and technology and make a premise to study well in other subjects in the training program.

The development of computer science was born, which greatly supported the development of physics subjects. The application of informatics in the process of interpreting databases of physics, solving physics problems has shortened the time spent and brought higher efficiency.

During the process of writing the above essay, our team received much care and support, dedicated help from teachers, brothers and sisters and friends. In addition, the group would also like to express their most sincere gratitude to Mr. Nguyen Quoc Lan, is the instructor for this topic. Thanks to his wholeheartedly instructing, the group was able to complete the essay on time and solve problems well. His guidance has been the guideline for all actions of the group and maximized the supportive relationship between teachers and students in the educational environment. Also, on this occasion, we would like to thank Mr.Nguyen Tien Dung for his enthusiastic teaching of the theory very carefully so that we have a solid foundation to solve this math problem. Finally, once again, I would like to express my deep gratitude to the teachers and everyone who takes time to instruct the group. This is the belief, a great source of motivation for the group that this result can be achieved.

REQUIREMENTS OF THE TOPIC

1.1. Content:

Based on Section 6.1 & Section 7.7 (Stewart Textbook), approximate the area of one Vietnamese province.

+ 221 Semester. Topic 1b: Area of Vietnamese Province, start with letter L.

1.2. Requirements:

+ Students should use at least two different methods: One – numerical (for example Trapezoidal formula), Second – use quadratic function for three consecutive points.

+ Finalizing this project, students should compare calculated results to "real" value from other sources.

THEORETICAL BASIS

1.1. Introduction:

After researching the image of the province with the starting letter is "L", we finally chose Lào Cai because it's the least complicated shape at all.

Lào Cai is a province of the mountainous Northwest region of Vietnam. The province covers an area of 6,383.9 square kilometres and as of 2008 it had a population of 602,300 people.

1.2. Theory:

1.2.1 Area between Curves

Consider the region that lies between two curves and and between the vertical lines and $x = a$ and $x = b$, where $f$and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a,b]$. (See Figure 1.)

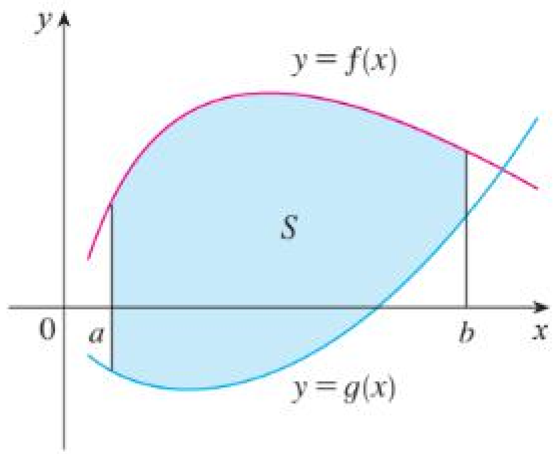


FIGURE 1
$S = \{(x, y) \mid a \leqslant x \leqslant b, g(x) \leqslant y \leqslant f(x)\}$

Figure 1.1: *Picture showing the area between 2 curves.*

Just as we did for areas under curves in Section 5.1, we divide S into n strips of equal width and then we approximate the ith strip by a rectangle with base Δx and height f(xi*) – g(xi)*. (See Figure 2. If we like, we could take all of the sample points to be right endpoints, in which case $f(x_i *) - g(x_i *)$.) The Riemann sum:

$$\sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\,\Delta x$$

is therefore an approximation to what we intuitively think of as the area of S.

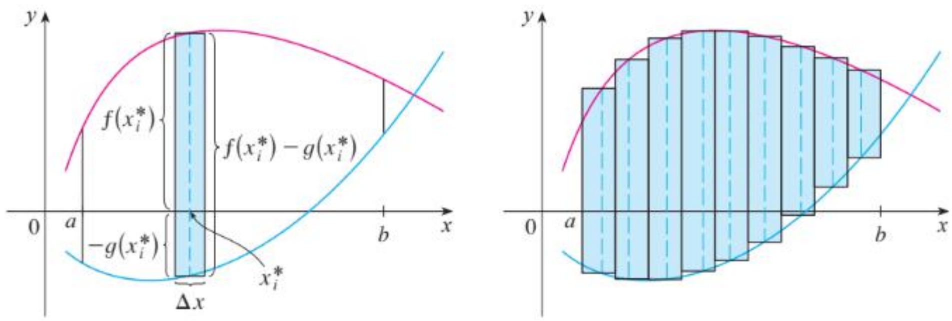


FIGURE 2 (a) Typical rectangle (b) Approximating rectangles

Figure 1.2: *Picture a and b are explaining the integral*

This approximation appears to become better and better as $n \to \infty$. Therefore we define the area A of the region S as the limiting value of the sum of the areas of these approximating rectangles.

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)] \Delta x$$

2 The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geqslant g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)]\, dx$$

Notice that in the special case where $g(x) = 0$, S is the region under the graph of f and our general definition of area reduces to our previous definition (Definition 2 in Section 5.1).

In the case where both and are positive, you can see from Figure 3 why is true:

$$A = [area\ under\ y = f(x)] - [area\ under\ y = g(x)]$$

$$= \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx$$

If we are asked to find the area between the curves $y = f(x)$ and $y = g(x)$ where $f(x) \geq g(x)$ for some values of x but $g(x) \geq f(x)$ for other values of x , then we split the given region into several regions S1, S2, . . . with areas A1, A2, . . . as shown in Figure 9.We then define the area of the region to be the sum of the areas of the smaller regions S1, S2, . . . , that is,A = A1 + A2 + ⋯ . Since

$$|f(x) - g(x)| = \begin{cases} f(x) - g(x) & \text{when } f(x) \geqslant g(x) \\ g(x) - f(x) & \text{when } g(x) \geqslant f(x) \end{cases}$$

we have the following expression for A.

3 The area between the curves $y = f(x)$ and $y = g(x)$ and between $x = a$ and $x = b$ is

$$A = \int_a^b |f(x) - g(x)|\, dx$$

2.2.2 Trapezoidal rule

$$\int_a^b f(x)\,dx \approx T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = (b-a)/n$ and $x_i = a + i\,\Delta x$.

The reason for the name Trapezoidal Rule can be seen from Figure 2, which illustrates the case with and . The area of the trapezoid that lies above the th sub-interval is.

$$\Delta x\left(\frac{f(x_{i-1}) + f(x_i)}{2}\right) = \frac{\Delta x}{2}\left[f(x_{i-1}) + f(x_i)\right]$$

and if we add the areas of all these trapezoids, we get the right side of the Trapezoidal Rule.

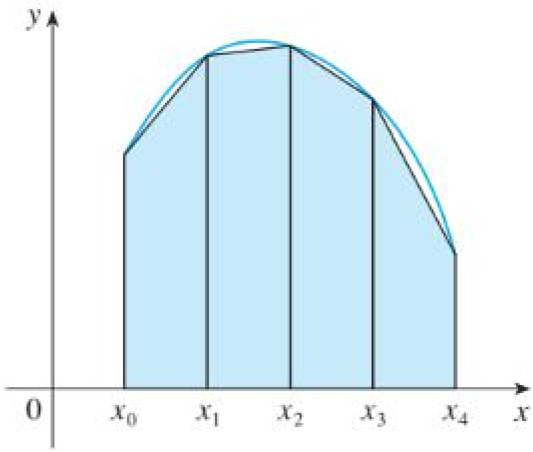


FIGURE 2
Trapezoidal approximation

Figure 1.3: *Picture showing trapezoidal approximation*

CALCULATION METHOD

To find the area of the province, we take the ratio 1:20 km on the side of the Google map. The image was taken into GeoGebra without changes in the ratio. Then we create the Ox and Oy axis, so the number of the image in GeoGebra is the right ratio as Google maps. In calculating progress, we just move the image, not zoom in or zoom out to ensure the ratio is maintained. The method and results below are calculated in the GeoGebra unit. In order to get the result (km^2) as the reality, we have to type it with 20.

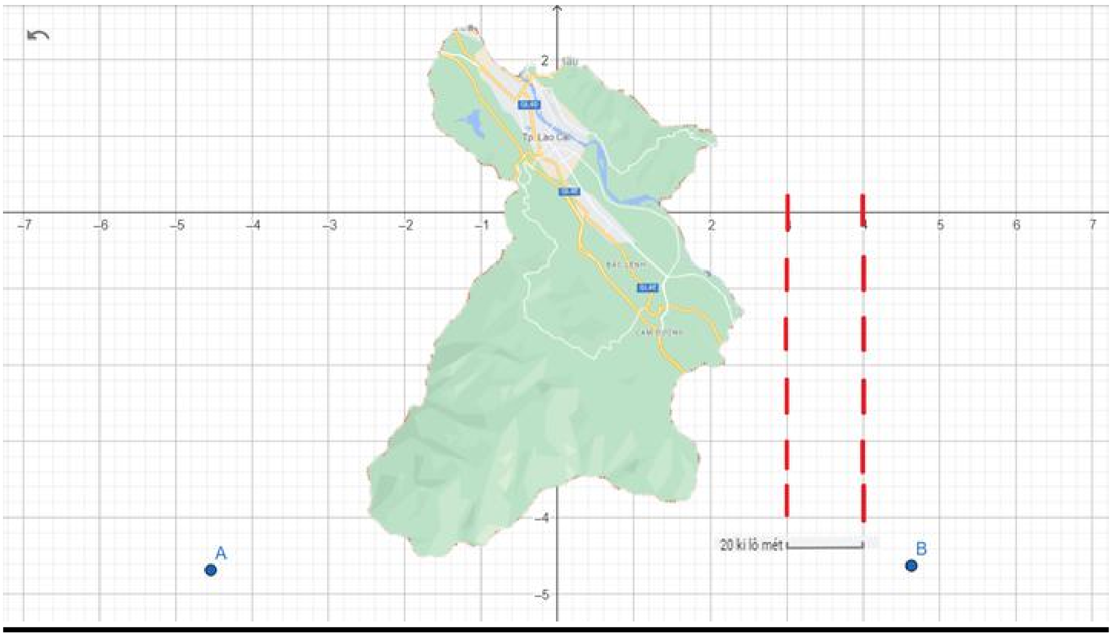


***Figure 2:** Picture show that 1 unit in Geogebra is exactly 20 km*

1.Trapezoid:

First we divide each corner of a map into many trapezoids to calculate the area more accurately. And the height of each trapezoid will be ⅖ unit from the coordinate of the app Geogebra.

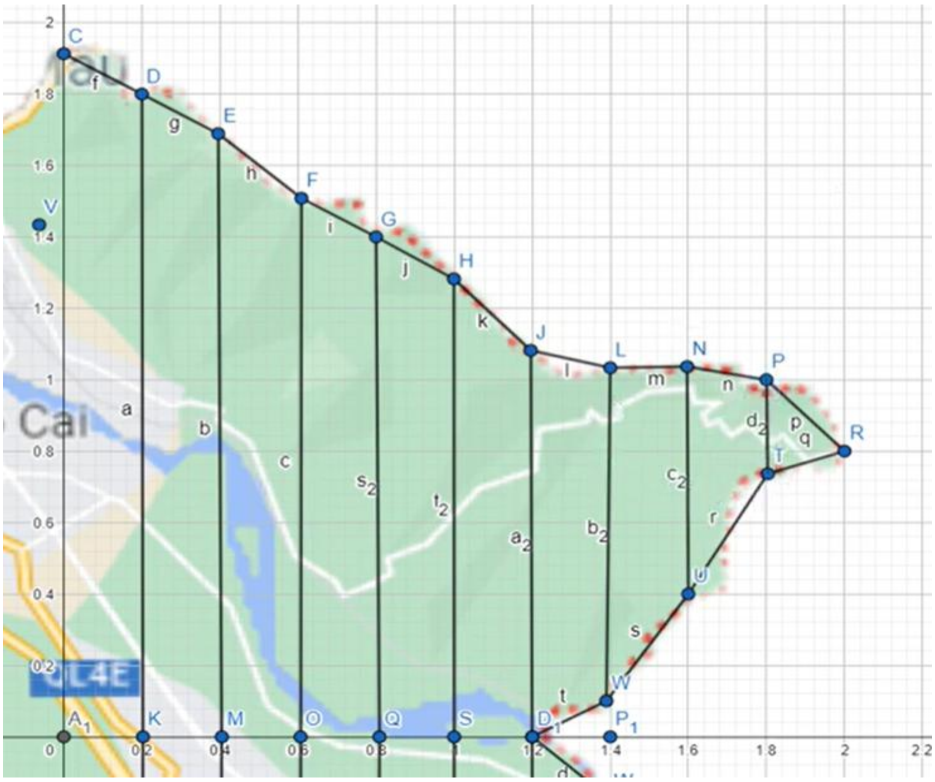


***Figure 2.1.1:** Divide trapezoids in first corner map*

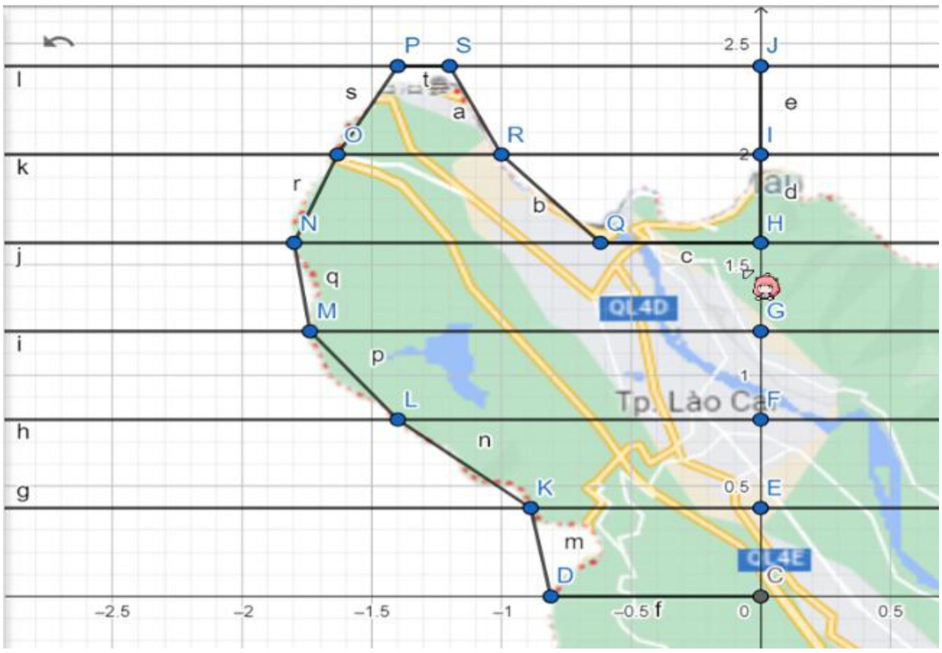


***Figure 2.1.2**: Divide trapezoids in second corner map*

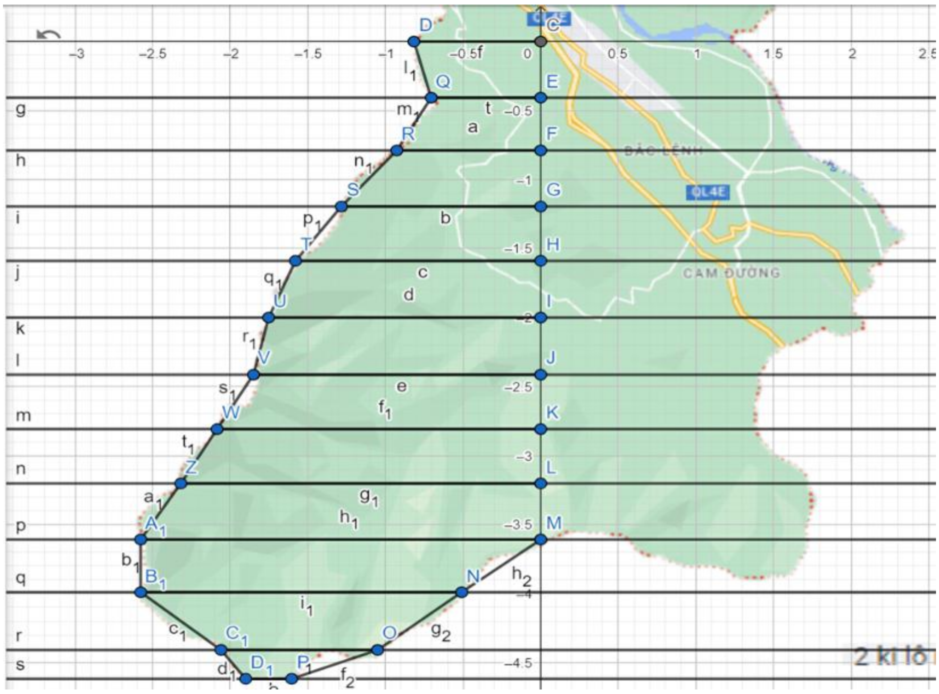


Figure 2.1.3: *Divide trapezoids in third corner map*

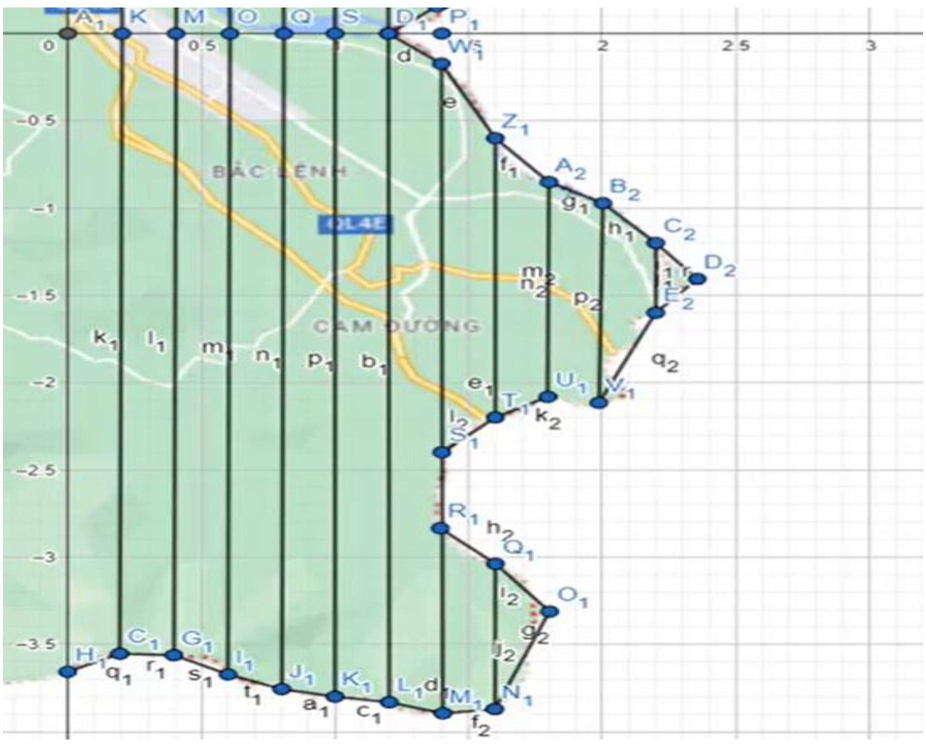


Figure 2.1.4: *Divide trapezoids in fourth corner map*

Then we use this tool in Geogebra to measure the width at the top and the bottom of each trapezoid exactly.

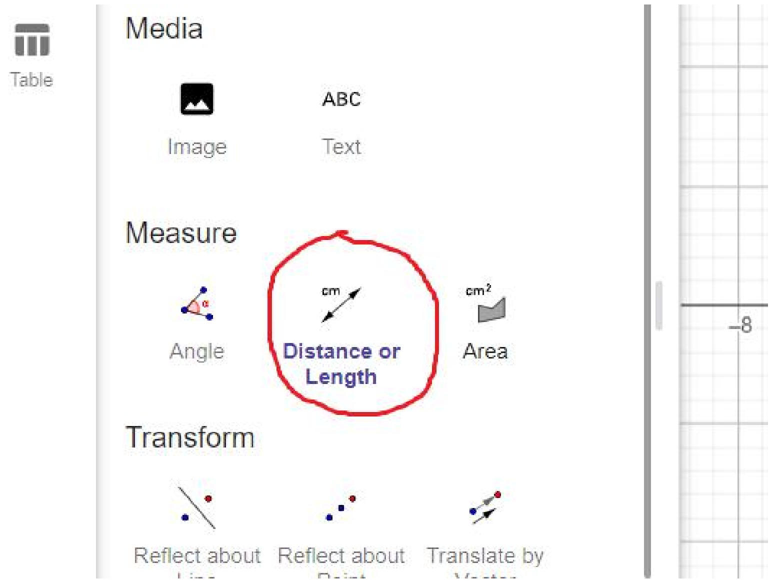


Figure 2.2.1: *Using the tool in Geogebra*

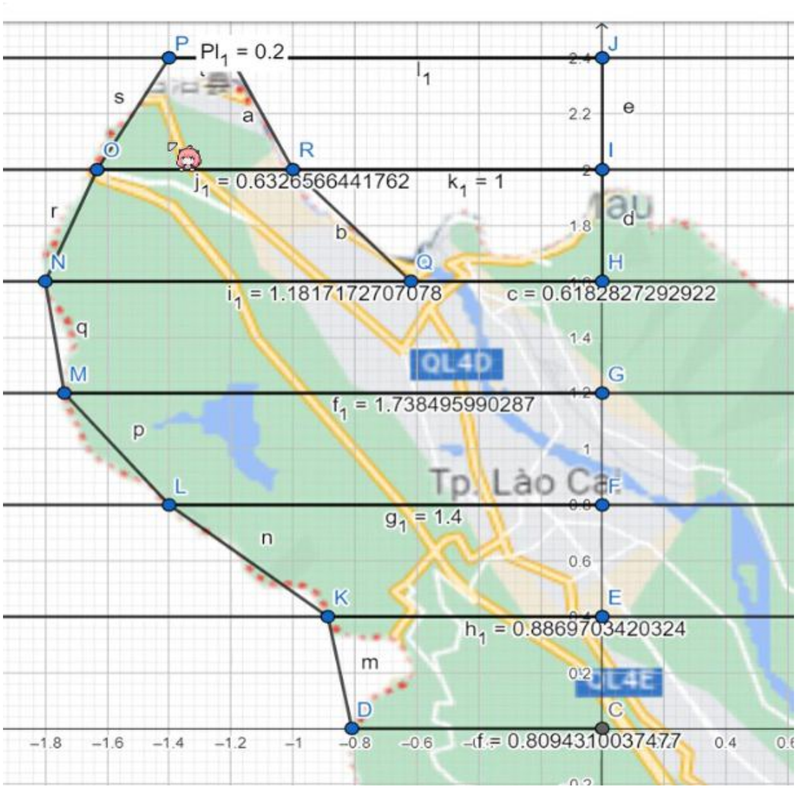


Figure 2.2.2: *The number displayed in each width at the top and bottom of trapezoids*

Some places are rough so we decide to divide the map more exactly by moving the trapezoids to the left.

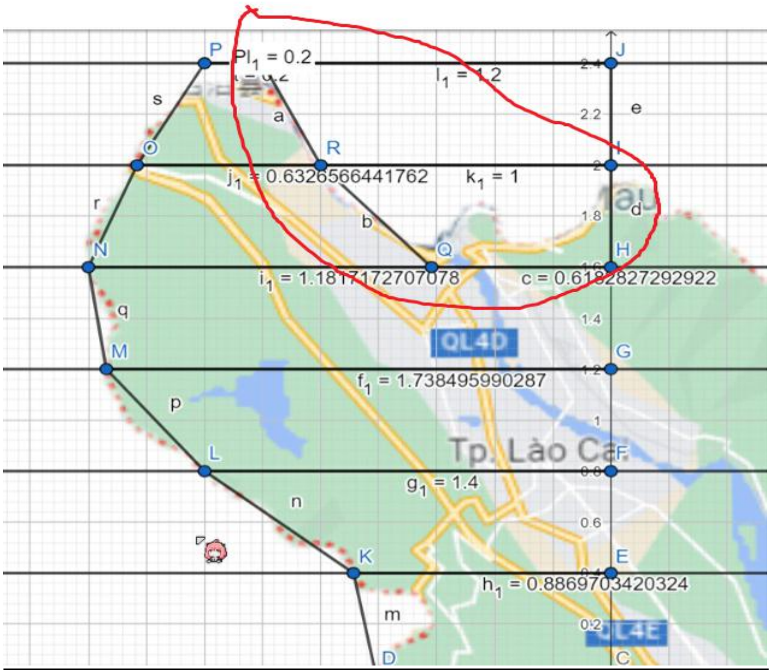
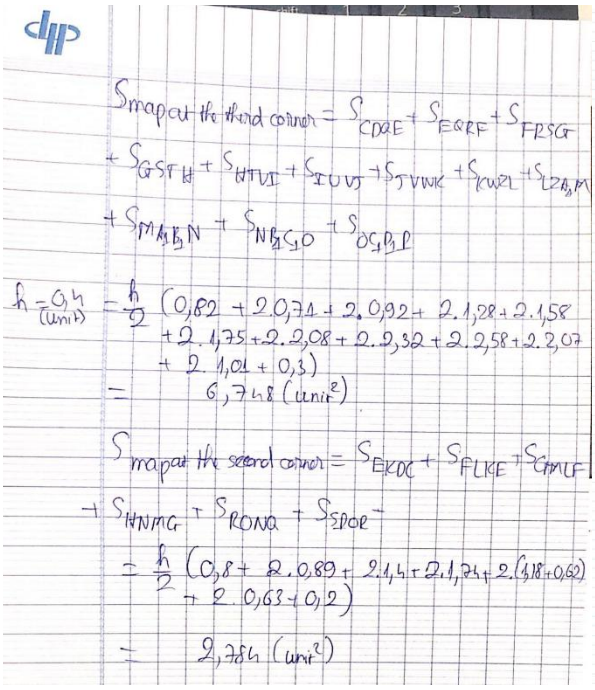


***Figure 2.2.3:** Some place is rough*

After that we start to calculate the area using all the numbers we got on the Geogebra with all the height of the trapezoids equal to 0.4 unit.



$S_{map\ at\ the\ third\ corner} = S_{CDQE} + S_{EQRF} + S_{FRSG} + S_{GSTH} + S_{HTUI} + S_{IUVJ} + S_{JVWK} + S_{KWZL} + S_{LZA_1M} + S_{MA_1B_1N} + S_{NB_1C_1O} + S_{OC_1B_1P}$

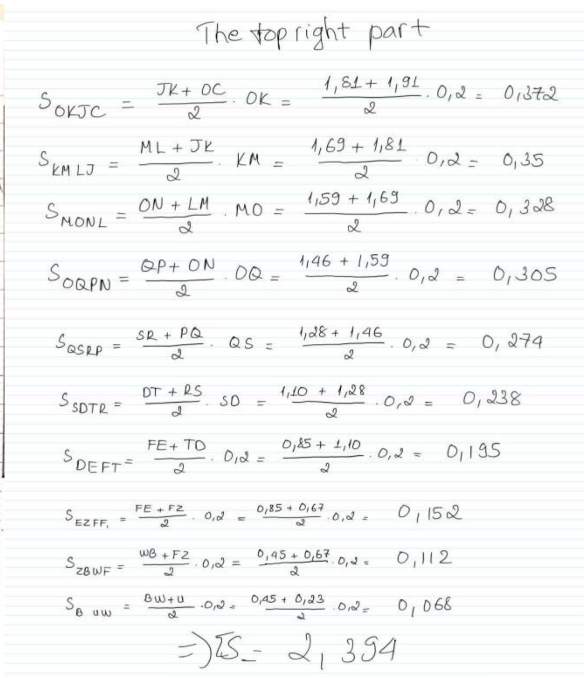
$h = 0,4$ (unit) $= \frac{h}{2}(0,82 + 2.0,71 + 2.0,92 + 2.1,28 + 2.1,58 + 2.1,75 + 2.2,08 + 2.2,32 + 2.2,58 + 2.2,07 + 2.1,01 + 0,3)$

$= 6,748$ (unit²)

$S_{map\ at\ the\ second\ corner} = S_{EKDC} + S_{FLKE} + S_{GMLF} + S_{HNMG} + S_{RONQ} + S_{SPOR}$

$= \frac{h}{2}(0,8 + 2.0,89 + 2.1,4 + 2.1,74 + 2.(1,18 + 0,62) + 2.0,63 + 0,2)$

$= 2,784$ (unit²)



The top right part

$S_{OKJC} = \frac{JK + OC}{2} . OK = \frac{1,81 + 1,91}{2} . 0,2 = 0,372$

$S_{KMLJ} = \frac{ML + JK}{2} . KM = \frac{1,69 + 1,81}{2} . 0,2 = 0,35$

$S_{MONL} = \frac{ON + LM}{2} . MO = \frac{1,59 + 1,69}{2} . 0,2 = 0,328$

$S_{OQPN} = \frac{QP + ON}{2} . OQ = \frac{1,46 + 1,59}{2} . 0,2 = 0,305$

$S_{QSRP} = \frac{SR + PQ}{2} . QS = \frac{1,28 + 1,46}{2} . 0,2 = 0,274$

$S_{SDTR} = \frac{DT + RS}{2} . SD = \frac{1,10 + 1,28}{2} . 0,2 = 0,238$

$S_{DEFT} = \frac{FE + TD}{2} . 0,2 = \frac{0,85 + 1,10}{2} . 0,2 = 0,195$

$S_{EZFF} = \frac{FE + FZ}{2} . 0,2 = \frac{0,85 + 0,67}{2} . 0,2 = 0,152$

$S_{ZBWF} = \frac{WB + FZ}{2} . 0,2 = \frac{0,45 + 0,67}{2} . 0,2 = 0,112$

$S_{BUW} = \frac{BW + U}{2} . 0,2 = \frac{0,45 + 0,23}{2} . 0,2 = 0,068$

$\Rightarrow \Sigma S = 2,394$

***Figure 2.3.1:** Calculation for the second, third corner and the first corner map*

The bottom right part

$$S_{A_1KG_1H_2} = \frac{3{,}66+3{,}55}{2} \cdot 0{,}2 = 0{,}721$$

$$S_{KG_1I_1M} = \frac{3{,}55+3{,}56}{2} \cdot 0{,}2 = 0{,}711$$

$$S_{MOJ_1I_1} = \frac{3{,}56+3{,}68}{2} \cdot 0{,}2 = 0{,}724$$

$$S_{OQK_1J_1} = \frac{3{,}68+3{,}76}{2} \cdot 0{,}2 = 0{,}744$$

$$S_{QSL_1K_1} = \frac{3{,}76+3{,}8}{2} \cdot 0{,}2 = 0{,}756$$

$$S_{SD_1M_1L_1} = \frac{3{,}8+3{,}83}{2} \cdot 0{,}2 = 0{,}763$$

$$S_{C_2D_2O_1} = 0{,}83 \cdot \frac{1}{2} \cdot 0{,}2 = 0{,}083$$

$$S_{D_1C_1N_1M_1} = \frac{3{,}83+3{,}68}{2} \cdot 0{,}2 = 0{,}751$$

$$S_{C_1D_1R_1S_1} = \frac{2{,}2+1{,}6}{2} \cdot 0{,}2 = 0{,}38$$

$$S_{Q_1T_1U_1R_1} = \frac{1{,}6+1{,}22}{2} \cdot 0{,}2 = 0{,}282$$

$$S_{T_1U_1W_1V_1} = \frac{1{,}22+1{,}16}{2} \cdot 0{,}2 = 0{,}238$$

$$S_{V_1Z_1A_2W_1} = \frac{1{,}16+0{,}4}{2} \cdot 0{,}2 = 0{,}156$$

$$S_{E_2C_2D_1N_1} = \frac{1{,}05+0{,}83}{2} \cdot 0{,}2 = 0{,}188$$

$$S_{A_2B_2Z_1} = 0{,}4 \cdot 0{,}14 \cdot \frac{1}{2} = 0{,}028$$

$$\Rightarrow \Sigma S = 6{,}525$$

Figure 2.3.2: *Calculation for the fourth corner map*

So the total area of the map by using the trapezoid method is :

Smap = 6.525 + 2.394 + 2.784 + 6.748 = 18.451 (unit^2).

2.Integral:

Firstly, we cover the areas in each corner of the map by the lines made of points whose coordinates are displayed on the toolbar of GeoGebra Classic

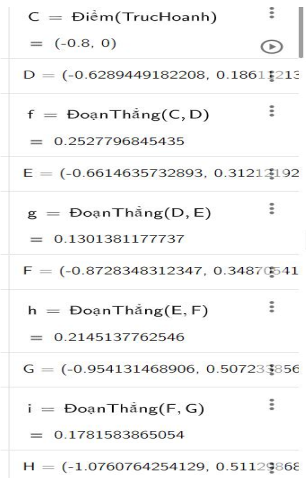


Figure 3.1.1: *The position* ***Figure 3.1.2:*** *Setting points on the map (first corner)*

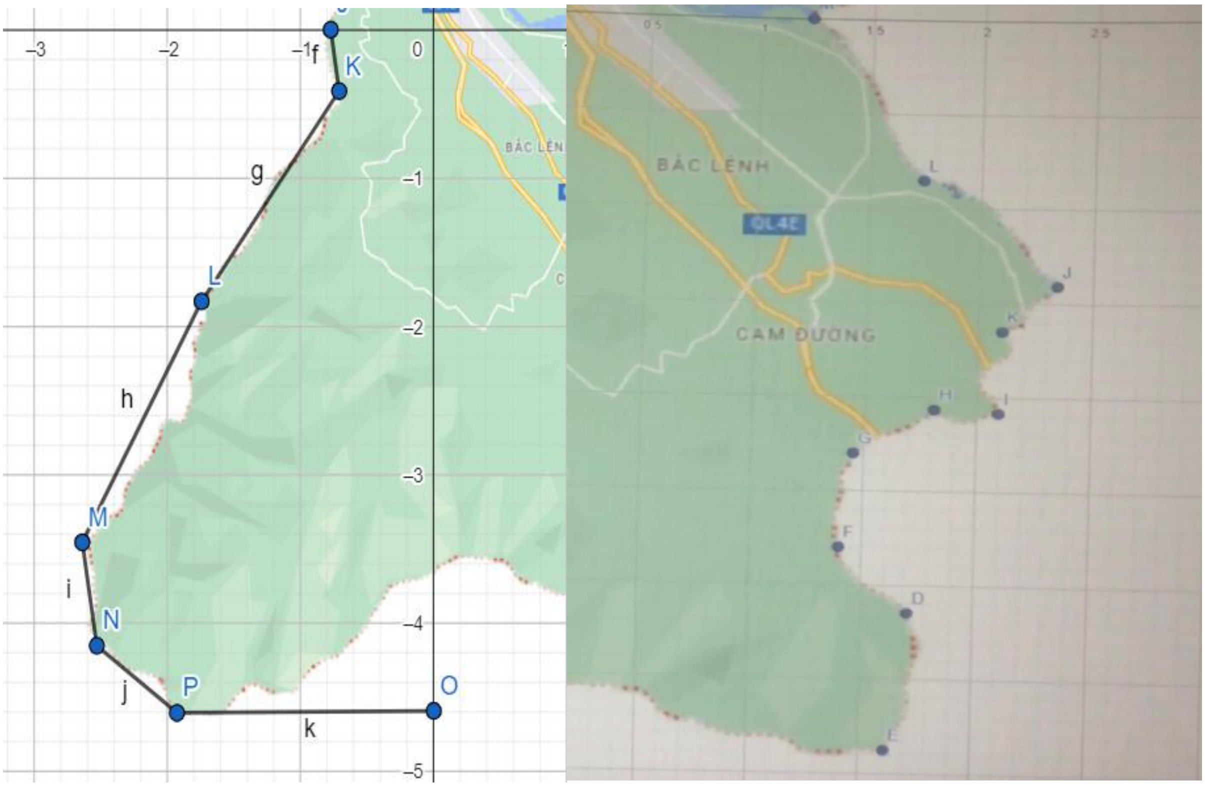


Figure 3.1.3 & 3.1.4 *The third and fourth corner map*

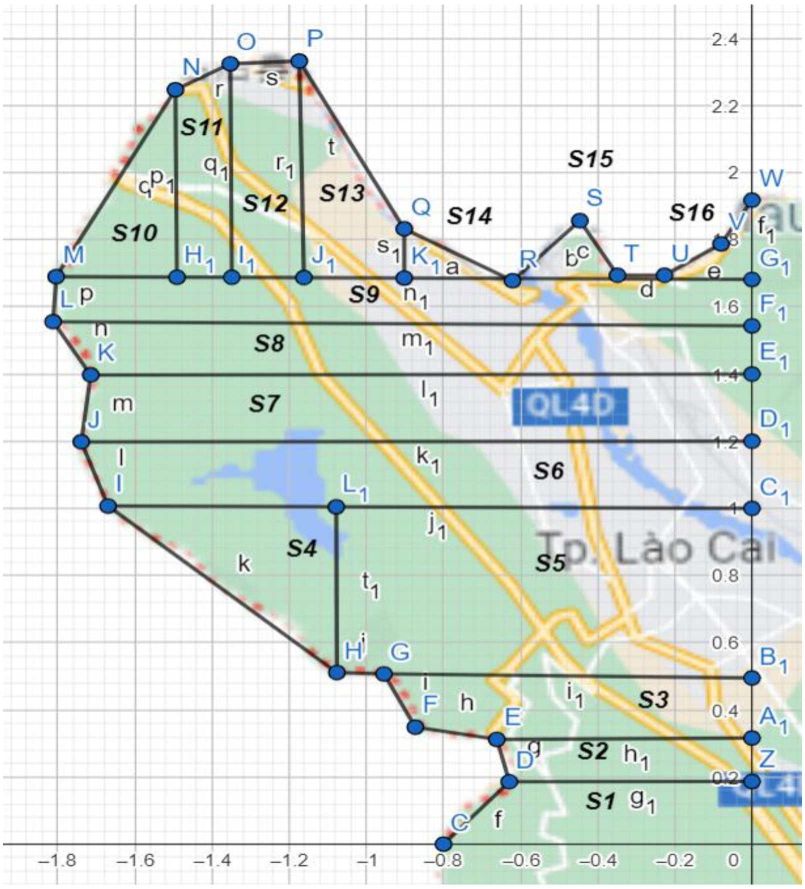


***Figure 3.2.1:** Divide the map into some function to calculate the integral (second corner of the map)*

Then we divide the overall area into small-scale parts(S1,S2,S3,.....) to improve the accuracy of calculations

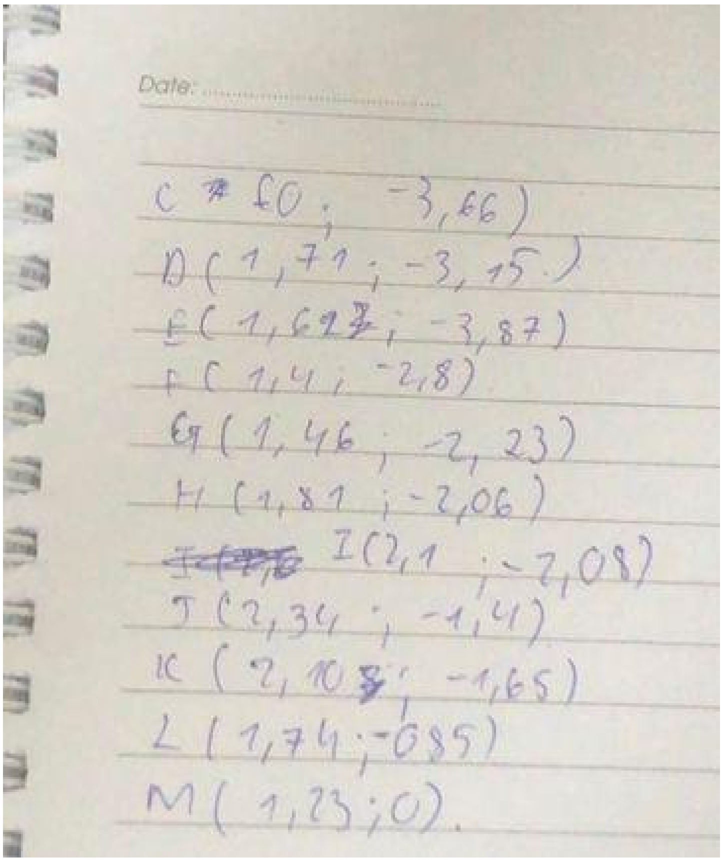


***Figure 3.2.2:** The coordinate of each point to use for the integral*

Next, we determine each point coordinates that limit the area of each part

C(-0.8,0) D(-0.63,0.19) Z(0,0.19) → $S1$

D(-0.63,0.19) E(-0.66,0.31) Z(0,0.19) A1(0,0.32) → $S2$

H(-1.08,0.51) I(-1.67,1.01) L1(-1.07,1) → $S4$

Then we solve for the functions of the straight lines and curves which covering the areas:

+ Determine the kind of function(Linear, quadratic,cubic,...)

+ Build the equations from the positions (f(x); g(x))

+ Calculate the parameters from the equations and build the functions

Use the integration with the functions f(x) and g(x) on intervals (a,b) to calculate the areas

$$A = \int_c^d f(y) - g(y)\, dy \qquad f(y) \geq g(y)$$

N(-1.5,2.25) M(-1.8,1.7) H1(-1.49,1.7) → Linear function y=ax+b

$$\rightarrow \begin{cases} -1.5a + b = 2.25 \\ -1.8a + b = 1.7 \end{cases}$$

$$\rightarrow \begin{cases} a = \frac{11}{6} \\ b = 5 \end{cases}$$

$$\rightarrow y = \frac{11}{6}x + 5$$

$$\rightarrow \text{S10} = \int_{-1.8}^{-1.49} \frac{11}{6}x + 5 - 1.7 = 0.088$$

Finally, we calculate all the areas of all parts in 4 corners of the map and sum them

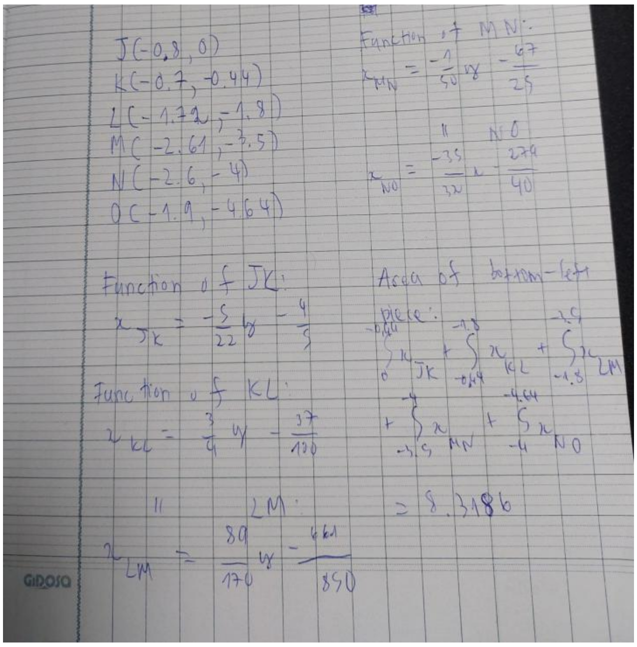


***Figure 3.2.3:** The calculation for the first and third corner of the map on paper*

To calculate the area of the 4th corner of the province, we take points on the border of the province. Then create line equations from existing points along the border.

The next step is to calculate the integral bounded by the coordinates, the created line equations, the line that runs through the two points that make up that line. Finally, adding those integrals together will calculate the area of the province located in the 4th
quadrant.

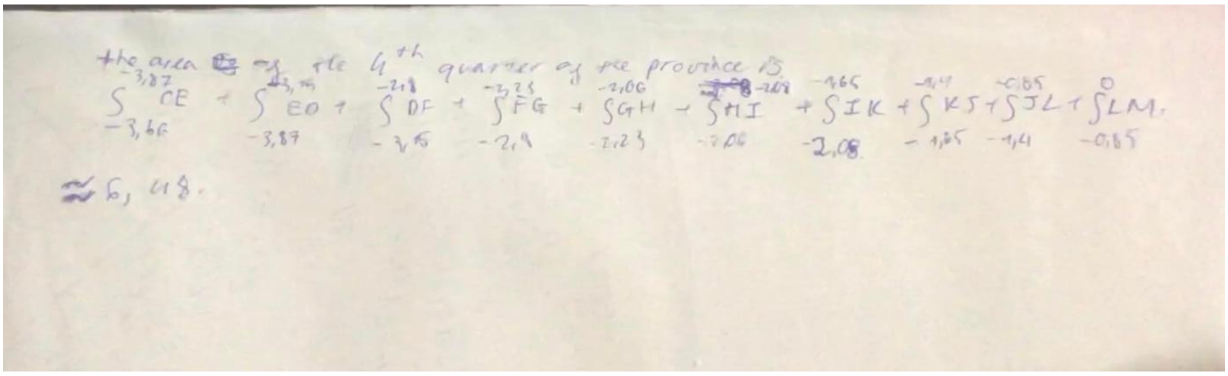


Figure 3.2.4: *The calculation using the point to solve for the area(fourth corner map)*

Linear function: y=ax+b

C(-0.8,0) D(-0.63,0.19) Z(0,0.19) → $S1 = \int_{0}^{0.19} \left(\frac{-17}{19}x + \frac{4}{5}\right) = 0.136$

D(-0.63,0.19) E(-0.66,0.31) Z(0,0.19) A1(0,0.32) → $S2 = \int_{0.19}^{0.32} \left(\frac{1}{4}x + \frac{233}{400}\right) = 0.084$

H(-1.08,0.51) I(-1.67,1.01) L1(-1.07,1) → $S4 = \int_{0.51}^{1} \left(\left(\frac{59}{50}x + \frac{2391}{5000}\right) - 1.07\right) = 0.147$

L1(-1.07,1) H(-1.08,0.51) B1(0,0.51) C1(0,1) → $S5 = \int_{0.51}^{1} (1.08) = 0.53$

I(-1.67,1.01) J(-1.74,1.2) → $S6 = \int_{1.01}^{1.2} \left(\frac{7}{19}x + \frac{1233}{950}\right) = 0.32395$

K(-1.71,1.4) J(-1.74,1.2) → $S7 = \int_{1.2}^{1.4} \left(\frac{-3}{20}x + \frac{48}{25}\right) = 0.345$

L(-1.81,1.56) K(-1.71,1.4) → $S8 = \int_{1.4}^{1.56} \left(\frac{5}{8}x + \frac{167}{200}\right) = 0.2816$

M(-1.8,1.7) L(-1.81,1.56) → $S9 = \int_{1.56}^{1.7} \left(\frac{-1}{14}x + \frac{269}{140}\right) = 0.2527$

N(-1.5,2.25) M(-1.8,1.7) H1(-1.49,1.7) → $S10 = \int_{-1.8}^{-1.49} \left(\left(\frac{11}{6}x + 5\right) - 1.7\right) = 0.088$

O(-1.35,2.32) N(-1.5,2.25) H1(-1.49,1.7) I1(-1.35,1.7) → $S11 = \int_{-1.49}^{-1.35} \left(\left(\frac{7}{15}x + \frac{59}{20}\right) - 1.7\right) = 0.082$

P(-1.17,2.33) O(-1.35,2.32) I1(-1.35,1.7) J1(-1.16,1.7) → $S12 = \int_{-1.35}^{-1.16} \left(\left(\frac{1}{18}x + \frac{479}{200}\right) - 1.7\right) = 0.119$

P(-1.17,2.33) J1(-1.16,1.69) Q(-0.9,1.83) K1(-0.9,1.7) → $S13 = \int_{-1.16}^{-0.9} \left(\left(\frac{-50}{27}x + \frac{49}{300}\right) - 1.83\right) + \int_{-1.16}^{-0.9} (1.83 - 1.7) = 0.096$

Q(-0.9,1.83) K1(-0.9,1.7) R(-0.62,1.7) → $S14 = \int_{-0.9}^{-0.62} \left(\left(\frac{-15}{28}x + \frac{1887}{1400}\right) - 1.7\right) = 0.0154$

R(-0.62,1.7) S(-0.45,1.86) T(-0.35,1.7) → $S15 = \int_{-0.62}^{-0.45} \left(\left(\frac{16}{17}x + \frac{1941}{850}\right) - 1.7\right) + \int_{-0.45}^{-0.35} \left(\left(\frac{-8}{5}x + \frac{57}{50}\right) - 1.7\right) = 0.022$

Quadratic function: $y=ax^2+bx+c$

E(-0.66,0.31) F(-0.87,0.35) G(-0.95,0.5) $\rightarrow$ $S3 = \int_{0.31}^{0.5} \left(\frac{-95}{4}x^2 + \frac{837}{40}x - \frac{5671}{1600}\right) = 0.183$

U(-0.23,1.7) V(-0.08,1.79) W(0,1.92) $\rightarrow$ $S16 = \int_{-0.23}^{0} \left(\frac{25}{6}x^2 + \frac{47}{24}x + \frac{48}{25}\right) - 1.7 = 0.016$

Total Area S= 2.72165

- Total area in first corner: 8.3186
- Total area in second corner: 2.72165
- Total area in third corner: 2.4
- Total area in fourth corner: 6.48

Finally, we calculate the sum of all areas in the 4 corner of the map

Smap = 8.3186 + 2.72165 + 6.48 + 2.4 = 19.92(unit^2)

==> So the total area of LAO CAI city by using the Integral method is : 19.92 (unit^2).

Convert the result into real unit (km^2)

Because we have shown that 1 unit in Geogebra is exactly equal to 20km. So the area of Lao Cai Province according to two method:

- The integral method 19.92 * 20^2 = 7968 (km^2)
- The trapezoid method 18.451 * 20^2 = 7380.4 (km^2)

Evaluate and analysis

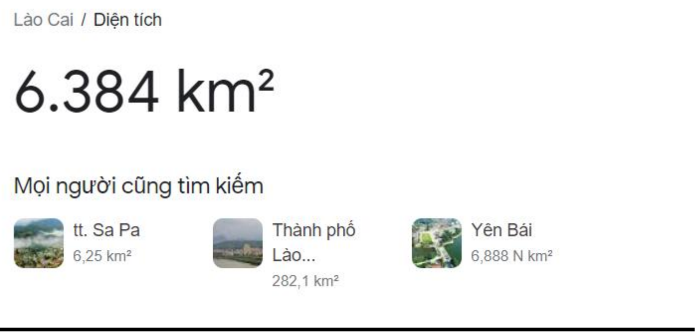


***Figure 4:** The real value from Google*

Compare two results from what we have done. We can see that the area we have calculated is much more than the real value. The trapezoid method is more accurate than the integral method. We think that because in the trapezoid method; we have divided the map more detailed than the integral one. And because the map has some places that are rough so it makes the process of calculating the area harder and less accurate.

*To improve:

- We think that maybe if we have divided the map into smaller trapezoid or have setted the point more specifically, the result would be more alike to the real value from Google.

- The app can also be improved for the user to edit the map into the coordinate easier.

Conclusion

Thus, we have gone from general problems to quite complex specific problems that require lots of computational work with problem solvers. However, with the support of the Geogebra tool, solving and surveying problems becomes easy, lively and direct. We can easily use Geogebra to simulate or calculate the area of each part of the map.

Advantages:

- Calculation is easy and convenient, giving accurate results like the common calculation method.

- Save operation and calculation time compared to common calculation methods.

Weakness:

- The difficulty of some commands in the input bar especially for students and teachers with no prior programming experience

EVALUATE THE WORK OF EACH MEMBER

Evaluation of group members' attitude			
Member	***Fair***	***Satisfactory***	***Poor***
Nguyễn Tiến Hưng	✓		
Nguyễn Vũ Hà	✓		
Huỳnh Mai Quốc Khang	✓		
Trần Cẩm Hòa	✓		
Đoàn Quốc Huy	✓		

- Calculate the first and fourth corner map by using trapezoid method: Nguyễn Vũ Hà.

- Calculate the second and third corner map by using trapezoid method: Nguyễn Tiến Hưng.

- Calculate the first and third corner map by using integral method: Đoàn Quốc Huy

- Calculate the second corner map by using integral method: Huỳnh Mai Quốc Khang

- Calculate the fourth corner map by using integral method: Trần Cẩm Hòa

- Record the work: All members

-Edit video: Đoàn Quốc Huy.

-Edit the report: Nguyễn Vũ Hà; Nguyễn Tiến Hưng; Huỳnh Mai Quốc Khang; Trần Cẩm Hòa; Đoàn Quốc Huy.

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