

Assignment No. 4: Merge k Ordered Lists Efficiently

Allocated time: 2 hours

Implementation

You are required to implement **correctly** and **efficiently** an $O(n \log k)$ method for **merging k sorted sequences**, where n is the total number of elements. (Hint: use a heap, see seminar no. 2 notes).

Implementation requirements:

- Use linked lists to represent the k sorted sequences and the output sequence

Input: k lists of numbers $\langle a_1^i, a_2^i, \dots, a_{m_i}^i \rangle$, $\sum_{i=1}^k m_i = n$

Output: a permutation of the union of the input sequences: $\langle a'_1 \leq a'_2 \leq \dots \leq a'_n \rangle$

$a'_1 \leq a'_2 \leq \dots \leq a'_n$

Thresholds

Threshold	Requirements
5	Generate k random sorted lists, having n elements in total (n and k given as parameters); merge 2 lists
7	Adapt heap operations to work on new structure (list_index, key); use min-HEAP
9	Correct and complete algorithm implementation, with demo on a small-sized input
10	Evaluation, interpretations, discussion

Evaluation

! Before you start to work on the algorithms evaluation code, make sure you have a correct algorithm! You will have to show your algorithm works on a small-sized input (e.g. $k=4$, $n=20$).

We will make the average case analysis of the algorithm. Remember that, in the average case, you have to repeat the measurements several times. Since both k and n may vary, we will make each analysis in turn:

1. Choose, in turn, 3 constant values for k ($k_1=5$, $k_2=10$, $k_3=100$); generate k **random** sorted lists for each value of k so that the combined number of elements in all the lists (n) varies between 100 and 10000, with a maximum increment of 400 (we suggest 100); run the algorithm for all values of n (for each value of k); generate a chart that represents the sum of assignments and comparisons done by the merging algorithm for each value of k as a curve (total 3 curves).
2. Set $n = 10.000$; the value of k must vary between 10 and 500 with an increment of 10; generate k **random** sorted lists for each value of k so that the combined number of elements in all the lists is 10000; test the merging algorithm for each value of k and generate a chart that represents the sum of assignments and comparisons as a curve.
3. Interpret your charts.