Draft model – platform mergers

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1 General demand and basic results

This section does a first pass on differences between platform merger and standard horizontal merger analysis using general functional forms for demand on both sides of the market. The context is differentiated social media platforms potentially owned by the same parent that make a profit by selling ads.

The setup is inspired by Anderson and Coate (Restud 2005), but uses more general functional forms and allows consumers to multihome.

Setup

- Thera are two social media platforms $i \in \{1,2\}$ and an outside option $i = 0$. The platforms make money by attracting consumer time and attention and selling ad space facing marginal cost normalized to zero. In particular, platforms choose ad load a_i.
- Consumers allocate total time T between the two platforms so that $T = T_0 + T_1 + T_2$. Consumers like spending time on social media but dislike ads. Write the time spent on each platform as a function of ad load:

$$T_i(a_1, a_2) \quad \text{for } i = 1,2$$
$$T_0(a_1, a_2) \equiv T - T_1(a_1, a_2) - T_2(a_1, a_2)$$

- Advertisers pay a per-view price p_i for one person on platform i to see one ad. At a given price vector, advertisers choose whether to place an ad on each platform. Assume that p_i is set through some process to ensure that advertiser demand achieves the platform's desired ad load a_i, and write $p_i(a_1, a_2)$ as the inverse demand curve for ads.
- Assume that each ad has a uniform probability κ of being seen during each unit of time of social media use. Then platform revenue from ad load a_i is:

$$R_i(a_1, a_2) = a_i T_i(a_1, a_2) p_i(a_1, a_2) \kappa$$

  Note that it is equivalent, and possibly more intuitive, to consider the platform as choosing for a fraction $a_i \cdot \kappa$ of time spent on the social media platform to be devoted to viewing ads.

Comments on setup The setup is intentionally very stripped down, but is still a generalization of Anderson-Coate. A few places where extensions would be particularly valuable:

- *Consumer network effects*. In social media markets, consumer demand features one-sided network effects since a platform is more valuable when more people use it. I omit for now because characterizing the equilibrium is more challenging.

- *Consumer heterogeneity*. Social media use and substitution patterns varies with demographics, advertisers place different value on advertising to those demographics, and social media platforms price targeted ads accordingly. A conceptually-straightforward extension could index everything in the model by demographic groups with complete or partial segmentation across markets.
- *Diminishing returns to advertisement*. Advertisers demnd only depends on price per view, not the number or composition of users across platforms. While this can be microfounded using a model with advertisement decisions only made on the extensive margin, it is probably not that realistic.
- *Mapping from ad load to ad views*. The specification of moving from "ad load" to the number of ad views is a little ad hoc; extensions would be conceptually straightforward and could make the mdoel more realistic.

Monopolist problem Suppose a single firm owns both platforms. The firm solves:

$$\max_{a_1,a_2} \sum_i R_i(a_1,a_2)$$

The FOC is:

$$T_1 p_1 + a_1 \left[p_1 \frac{\partial T_1}{\partial a_1} + T_1 \frac{\partial p_1}{\partial a_1} \right] + a_2 \left[p_2 \frac{\partial T_2}{\partial a_1} + T_2 \frac{\partial p_2}{\partial a_1} \right] = 0 \tag{1}$$

Or written more compactly:

$$a_1 = \frac{T_1 p_1}{-\frac{\partial T_1 p_1}{\partial a_1}} + a_2 \left(\frac{\frac{\partial T_2 p_2}{\partial a_1}}{-\frac{\partial T_1 p_1}{\partial a_1}} \right) \tag{2}$$

FOCs for a_2 are similar. These expressions give some helpful intuition:

- Equation (1) illustrates the tradeoff from marginally raising ad load. Revenue increases directly based on the prevailing rate of revenue per ad $T_1 p_1$. In the second term, revenue decreases on inframarginal ad load on platform 1 because more ads both (i) reduces consumer use; and (ii) reduces the ad price that clears the ad market. The change in revenue on inframarginal ad load on platform 2 (third term) is ambiguous – if platforms are substitutes, then use goes up, but higher platform 1 ad load might also reduce platform 2 ad prices to the extent ads on different platforms are substitutes.
- Equation (2) shows two useful points. First, this FOC is exactly the same as for a differentiated good monopolist who chooses quantity with prices given by $P(q_1,q_2) = T(q_1,q_2)p(q_1,q_2)$. Second, the second term shows that the relevant diversion ratio is not just consumer substitution between the platforms, but the *revenue-per-ad* diversion ratio between the platofrms.
- Equation (2) also motivates a special case where the consumer diversion ratio is more directly relevant. Suppose that ad markets are completely segmented, so $\frac{\partial p_2}{\partial a_1} = 0$. Then $\frac{\partial T_2 p_2}{\partial a_1} = p_2 \frac{\partial T_2}{\partial a_1}$, and equation (2) becomes:

$$a_1 = \frac{T_1 p_1}{-\frac{\partial T_1 p_1}{\partial a_1}} + \frac{\frac{p_2}{p_1}\frac{\partial T_2}{\partial T_1}}{-\left(1 + \frac{T_1}{p_1}\frac{\partial p_1}{\partial T_1}\right)}$$

  The numerator contains direct consumer substitution between platforms, and the denominator has the elasticity of platform use with respect to price, more readily observable than platform use with respect to ad load.

Separated platforms Suppose each platform is separately owned and firms compete static Nash-in-ad load. Firms solve:

$$\max_{a_i} a_i T_i(a_i, a_{-i}) p_i(a_i, a_{-i})$$

The FOC is:

$$0 = T_1 p_1 + a_1 p_1 \frac{\partial T_1}{\partial a_1} + a_1 T_1 \frac{\partial p_1}{\partial a_1}$$

Rearranging gives a standard single-product oligopoly quantity setting rule:

$$a_1 = \frac{T_1 p_1}{-\frac{\partial T_1 p_1}{\partial a_1}} \tag{3}$$

Would ad load rise or fall due to a platform separation? Consider an equilibrium (a_1^m, a_2^m) that satisfies (2) and the equivalent FOC for a_2. At the initial equilibrium, the first term on the RHS of (3) would be the same as the first term on the RHS of (2). The LHS would be higher or lower depending on the sign of the second term in (2) – if this second term is negative, then equilibrium ad load would be higher under separation; if the second term is positive, then equilibrium ad load would be lower under separation.

The denominator is unambiguously positive so the sign depends on the numerator:

$$\frac{\partial T_2 p_2}{\partial a_1} = p_2 \frac{\partial T_2}{\partial a_1} + T_2 \frac{\partial p_2}{\partial a_1}$$

The first term captures consumer substitution from platform 1 to platform 2 as a result of the ad load change and is positive. This reflects how relative to monopoly, separated platforms may have lower ad load to compete against each other for users. The second term captures advertiser substitution and is negative. This reflects how relative to monopoly, separated platforms have less of an incentive to hold back ad volumes to raise prices.

The sign is ambiguous. This means the impact of platform separation is also ambiguous. This is an important insight relative to (i) what we initially thought and (ii) relative to Anderson-Coate, which makes functional form assumptions so that $\frac{\partial p_2}{\partial a_1} = 0$. A few more comments:

- This result is nice because it implies any quantification exercises are not *just* for magnitudes, but directions too;
- On the other hand, since there are forces working in both direction I'm assuming that once we put numbers on this we would get a small impact on the change in ad volume. This means the "behavioral" upshot that generates surprising results for consumer surplus might have less of a quantitative punchline.
- This result conforms to the intuition of some of the industry people that Hunt spoke with who seemed to think the

2 Functional forms / parameterization

This section puts down some ideas I've brainstormed about possible functional forms to the extent we want to calibrate a full model. Benefits of this are that (i) we can compute exact rather than approximate equilibria; and (ii) welfare impacts in my view become more transparent. These are early-stage thoughts rather than anything very fleshed out.

Advertiser demand A good starting place is Anderson and Coate (2005):

- M advertisers are monopoly producers of a single good each produced at $mc = 0$.
- Advertisers have quality $\sigma \in (0,1), \sigma \sim F$, where σ gives the probability that on viewing an ad, a consumer has WTP $\omega > 0$ for the product.
- Since advertisers are monopolists they will charge $p = \omega$.
- The expected profit from advertising on a platform with T_i time spent on platform is:

$$T_i \kappa \sigma \omega$$

  so advertisers buy an ad slot so long as total cost $T_i \kappa p_i < T_i \kappa \sigma \omega \implies \sigma \geq p_i/\omega$. This implies the demand curve:

$$a_i = 1 - F\left(\frac{p_i}{\omega}\right)$$

  which also implicitly defines the inverse demand curve.

A few comments:

- In this setup, $\frac{\partial p_{-i}}{\partial a_i} = 0$, so platform separation will unambiguoulsy reduce ad volume.
- It might make more sense to move towards a specification along the lines of GSYY22 where there's diminishing value to ads shown mulitple times to the same person.

Consumer demand The goal is to specify a simple way to get multihoming. As before, consumers split their time across platforms and the outside option. They do this because incrmenetal time spent doing the same thing has diminishing value.

- The baseline value of the first increment of time on platform i is:

$$v_i - \gamma a_i$$

  where γ captures disutility from ads and $a_0 = 0$.
- Total utility is:

$$U(T_0, T_1, T_2) = \frac{1}{1-\eta} \sum_{m=0}^{2} \left[T_m \left(v_m - \gamma a_m\right)\right]^{1-\eta}$$

- The consumer problem is:

$$\max_{T_0, T_1, T_2} \frac{1}{1-\eta} \sum_{m=0}^{2} \left[T_m \left(v_m - \gamma a_m\right)\right]^{1-\eta} \text{ s.t. } T_0 + T_1 + T_2 = T$$

  The solution is characterized by:

$$\frac{T_1}{T_2} = \left[\frac{v_1 - \gamma a_1}{v_2 - \gamma a_2}\right)^{\frac{1-\eta}{\eta}}$$

A few remarks:

- We could add heterogeneity on the baseline values $v_i^j \sim G$ or $\eta_j \sim H$ to capture heterogeneous substitution patterns across different consumers.

- It might make more sense to use log-linear rather than linear baseline values, so that the first incrmeent of time is worth $v_i/(\gamma a_i)$. Then the solution looks cleaner to work with:

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\frac{\gamma_1 a_1}{\gamma_2 a_2}\right)^{\frac{1-\eta}{\eta}}$$

  This highlights how relative time use depends on relative initial value, relative ad load, and substitution parameters. It is noteworthy that if $\gamma_1 = \gamma_2$ then γ would not impact the ratio of time spent on the platforms.

- It would make sense to have different parameters η_0 and η_1 that control substitution between $(T_1 + T_2)$ and T_0 and T_1 and T_2 – this is straightforward but adds notation.