Draft model – ad competition

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This document presents a very simple model in the spirit of GSYY22 describing price setting in social media ad markets. The first section puts down the most stripped-down verison I could think of, and the second section loosely discusses extensions. Eventually this document will have synergies with my notes on institutional details of the social media ad market (link: work in progress).

1 Simple model

Setup Consider a highly targeted market for advertising to a single consumer i in a context with two platforms $j = 1,2$.

- The consumer exogenously splits her time on social media T between platforms so that $T = T_1 + T_2$;
- The number of ads that the platform shows i is a function of time spent on platform, where the number of ads seen is $a_j = \kappa T_j$. For now, we take κ as exogenous, although it is a choice variable for platforms;
  - Ad quantity as a function of time spent on platform is an ex-ante reasonable way for a platform to set up their sites, since the total amount of screen displayed is a function of how long someone spends scrolling and such as setup would ensure a consistent user experience;
  - Industry participants discuss time on platform and associated impact on ad "real estate" as a driver of high frequency price variation (link, see cost per click by day of the week section), suggesting that platforms do not substantially adjust ad density in real time. Unfortunately I have not been able to find anything on how frequently Facebook / Instagram dispaly advertisements, but I will add info on this if I find it.
- Homogeneous advertisers places value on a advertisements to consumer i $G_i(a)$, where $G_i(0) = 0, G_i' > 0, G_i'' < 0$.
  - $G_i'' < 0$ represents the industry concensus that additional impressions of an ad have diminishing value to advertisers, and is a special case of submodularity
- Social media companies own either one or both platforms. Companies set total price P to sell ad space a on their platforms.

Looking ahead, the objective is basically to measure or approximate the $G_i(\cdot)$ function, which controls incentives for platforms.

Comments on setup

- *No outside option*. It would be more realistic to add a non-strategic outside option to give the monopolist someone to compete against. However, we will see that adding an outside option does not change the main comparative statics that we will use for estimation, assuming that the total time spent on *any* social media platform doesn't change. So I omit an outside option to keep things a bit clearer.

- *Ad quantity as continuous rather than discrete*. In GSYY, networks sell a discrete number of ad slots to advertisers. In the social media context, consumer use – and hence ex-post advertising levels – are stochastic. So even if advertisers target discrete slots, they make decisions based on expected realizations, which will take continuous values. (It's also more tractable as a first passs).

- *Shape of $G_i(\cdot)$ function*. As discussed in detial on the companion document on the social media ad market, market participants think that the $G_i(\cdot)$ function has two additional features that I do not model here. First, instead of just being concave, it is actually S-shaped – $G_i'' > 0$ up to a point before switching to $G_i'' < 0$. This is because seeing an ad more than once initially "reinforces" its message, before consumers become desensitized and exhibit diminishing sensitivity. Second, there is a sense that viewing an ad in different contexts (here, on multiple platforms) "refreshes" the ad, meaning $G_i(a_1) + G_i(a_2) > G_i(a_1 + a_2)$ (where a_j represents the number of times an ad is shown on platform j). I omit these features for simplicity.

- *Quantities vs. prices*. Social media companies do not post prices, but instead make design choices which determine κ and then run auctions that dictate prices paid. There's a question as to whether quantities or prices are the strategic control (Bertrand vs. Cournot), but even if firms compete in a strategic sense Bertrand and just implement via quantities, the auction structure might matter. In this simple model, it probably does no harm to model as price setting rather than quantity setting. But it is worth flagging that this modeling choice may be inappropriate with markets for multiple consumers i. In a price setting model, the platforms could relatively easily price discriminate across advertisers who want access to different bundles of consumers. But if prices are set via auction and the platform can only choose κ (possibly κ_i) to implement desired prices, then the platform would institutionally not have this ability.

Monopoly equilibrium Suppose a single social media company owns both platforms. Then:

- $a_m = \kappa T$;
- The value to advertisers of advertising on all subsidiary platforms is $G_i(\kappa T)$;
- Therefore, the monopolist charges $P_m = G_i(\kappa T)$. Note that this leaves the advertiser with no surplus.
- Let cost-per-impression be given by $p_m \equiv \frac{G_i(\kappa T)}{\kappa T}$.

This is interesting because it presents two opportunities for measurement. First, suppose we observe some measure of advertiser spending P_m per campaign – I believe estimates of this are in Pathmatics data. Then:

$$\frac{\partial P_m}{\partial T} = \kappa G_i'(\kappa T)$$

With a source of variation in time spent on platform, we can estimate G_i' up to κ. Potential sources of such variation:

- Cross-sectionally across groups of users i, to the extent that we are willing to let G_i be homogeneous across users.
- Using higher-frequency variation for time spent on platform – e.g. weekends vs. weekdays, news events, weather (?). Problem with this is advertisers react to these patterns too and target differently based on them.
- Consider variation in residual ad load (elections instrument, Black Friday for B2B, etc) as soaking up a fraction of κT and hence informative about this elasticity.

For κ, we could either (i) treat this as a scale parameter and calibrate it depending on context; (ii) try to estimate directly; or (iii) put additional structure on G.

- For (ii), we see a measure of p_m with price data, P_m estimated from Pathmatics, and T from e.g. SensorTower. Then $\kappa = \frac{P_m}{p_m \cdot T}$.
- For (iii), if we assume $G_i(\cdot)$ is homogeneous of degree one, then $p_m = G_i(T)/T \implies Tp_m = G_i(T)$. Then with an estimate of $T \cdot p_m$ – both in principle observable – we can estimate $\frac{\partial T p_m}{\partial T} = G_i'(T)$.

More interestingly, we can approximately estimate G_i''. Consider:

$$\begin{aligned}\frac{\partial p_m}{\partial T} &= \frac{G_i'(\kappa T)}{T} - \frac{G_i(\kappa T)}{\kappa T^2} \\ &= \frac{1}{T}\left(G_i'(\kappa T) - \frac{G_i(\kappa T)}{\kappa T}\right)\end{aligned}$$

The Taylor approximation of G, G' around zero are (ignoring terms of order G_i''' and higher):

$$G_i(a) \approx G_i(0) + G_i'(0)a + \frac{1}{2}G_i''(0)a; \quad G_i'(a) \approx G_i'(0) + G_i''(0)a$$

Hence:

$$\frac{G_i(\kappa T)}{\kappa T} \approx G_i'(0) + \frac{1}{2}G_i''(0)\kappa T \implies \frac{\partial p_m}{\partial T} \approx \frac{\kappa}{2}G_i''(0)$$

which again gives an approximation for G_i'' up to κ. Intuitively, the change in price-per-impression p_m as time spent on platform increases reflects diminishing sensitivity because the rate at which the marginal impression becomes less valuable controls the rate at which average value declines. Price-per-impression and time on platform are both in principle observable, and the elasticity can be measured with an appropriate instrument.

A few comments:

- With an estimate of $G_i'(\kappa T)$ and $G_i''(0)$, if we assume that $G''' = 0$ then we can trace out the full G_i function. This is what we would need for counterfactuals – see below.
- I am not sure whether it matters for social media company incentives if we interpret curvature in G as diminishing sensitivity for additional impressions for an advertiser or just representing how the highest valuation for the marginal ad slot varies in a context with (i) heterogeneous advertisers and (ii) an efficient ad auction format.

Separated equilibrium Suppose each social media company owns only one platform. Each platform simultaneously chooses a pair (a_j, P_j) to sell ad space on its platform. Then the representative advertiser decides which (of any) of the two platforms to buy ads on. Using results from GSYY, the following is a Nash equilibrium:

$$\begin{aligned}P_1 &= G_i(\kappa T) - G_i(\kappa T_2) \\ P_2 &= G_i(\kappa T) - G_i(\kappa T_1)\end{aligned}$$

with $a_j = \kappa T_j \forall j$. This isn't quite a special case of GSYY, so to verify this is an equilibrium:

- First, the advertiser has no incentive to deviate from equilibrium. The advertiser gets positive surplus from advertising on both platforms, since:

  $$G_i(\kappa T) - \sum_k P_k = \sum_k G_i(\kappa T_k) - G_i(\kappa T) \geq 0$$

  Hence there's no profitable deviation to advertise on neither platform. Furthermore, by construction, there's no profitable deviation to drop advertising on either platform, since the incremental benefit to the advertiser from doing so equals the price.

- Second, neither platform has an incentive to offer a different bundle conditional on the bundle offered by the other. For bundle a_j, the maximum price a platform can charge is $P_j(a_j) = G_i(\sum_k a_k) - G_i(\kappa a_{-j})$:
  - Charge a higher price and advertisers reject because the cost exceeds the incremental benefit relative to advertising on $-j$;
  - Charge a lower price and there's a profitable deviation to increase the price with advertisers eventually accepting.

  Since $P_j' > 0$ because $G_i' > 0$, the platform chooses a corner solution $a_j = \kappa T_j$.

Some results:

1. If we know G_i approximately measured in the monopolist case, then we can estimate counterfactual P_j in the separated equilibrium using the above expressions.
2. Total prices paid by advertisers will fall, since:

$$(P_1 + P_2) - P_m = G_i(\kappa T) - \sum_k G_i(\kappa T_k) \leq 0$$

   We can also write an expression for the monopolist markup on cost-per-view on platform j:

$$\mu_j \equiv \frac{P_m - P_j}{\kappa T_j} = \frac{G_i(\kappa(T - T_j))}{\kappa T_j}$$

3. As a function of the time spent on platform for group i, the monopolist cost-per-view markup on platform j is:

$$\frac{\partial \mu_j}{\partial T_j} = -\frac{G_i'}{T_j} - \frac{G_i}{\kappa T_j^2} < 0$$

   meaning the change in cost-per-view after separation will be highest for platforms where users spend a lot of their time.

4. As a minor point, this section shows why so long as T is fixed, adding an outside option doesn't do much. For example:

$$\frac{\partial P_j}{\partial T_j} = \kappa G_i'(\kappa(T - T_j))$$

   which is similar to the monopolist comparative static used for estimation (although measured at a different point), and is the same up to normalization on T.

2 Extensions

- Industry practitioner guidance says that a big driver of ad prices for a targeted demographic is the amount of advertiser demand for that demographic. For non-overlapping targeted populations, this can probably be represented in reduced form as a different G_i function (higher for certain groups). But in reality, demographic targeting will be overlapping. For example, some advertisers will target 18-24 year olds in the Northeast, and others will target 18-34 year old women. This impacts the price of advertising to 18-24 year old women in the Northeast relative to 18-24 year old women in the Southeast.
- One thing I can try to work out next: in this setup, or in a more sophisticated setup with multiple overlapping targeted groups, Matt's suggestion that the curvature of G_i can be identified based on ad demand / prices.

- I touched on this point above, but I am not sure how important it is to model diminishing value to advertisements separately from heterogeneity in advertiser valuations. I think in the monopolist case it does not matter. It is not clear whether it matters in the separated equilibrium; I'd have to try to work out something more explicit.
- It could be useful to try for a more explicit parameterization of G_i.