Final Project Math452 Due 05/01/20

- The methods, implementation and results must be well organized and presented with a typed report.
- It is recommended to use Matlab to perform all the computations, and submit command lines and supporting files.

1. Solve the boundary value problem using Finite Difference method

$$\begin{cases} \nabla^2 u + 2u = g \ , & \text{inside } \Omega = [0,1]\times[0,1] \\ u = 0 \ , & \text{on the boundary of } \Omega, \end{cases} \tag{1}$$

where $g(x,y) = (xy+1)(xy-x-y) + x^2 + y^2$. The exact solution is known as $u = \frac{1}{2}xy(x-1)(y-1)$. Use Gauss-Seidel procedure to solve the obtained linear equation starting with $u_0(x_i, y_j) = x_i y_j$.

2. Code and run the Crank-Nicolson method with different choices of h and k for the following parabolic equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial^2 x}, \qquad 0 < x < 2, \quad t > 0, \tag{2}$$

with

$$u(0,t) = u(2,t) = 0, \tag{3}$$

and

$$u(x,0) = \sin\left(\pi x (x - \frac{1}{2})\right). \tag{4}$$

Illustrate how the scheme converges with decreasing (h,k). Try $k = h$, for example.