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AMATH 502

HOMework 1

Exercises come from *Nonlinear Dynamics and Chaos* by Steven H. Strogatz

1: 2.2.3

Solution:

TODO

2: 2.2.7

Solution:

TODO

3: 2.2.8

Solution:

TODO

4: 2.2.10

Solution:

TODO

5: 2.2.13 (a,b,c,d)

(a) $m\dot{v} = mg - kv^2$ with initial condition $v(0) = 0$

Solution:

Let's begin by dividing by m and factoring out a g term.

$$\begin{aligned} m\dot{v} &= mg - kv^2 \\ \dot{v} &= g - \frac{kv^2}{m} \\ \dot{v} &= g \left(1 - \frac{kv^2}{gm} \right). \end{aligned}$$

Now rewriting \dot{v} as the derivative of v with respect to t we have

$$\begin{aligned} \frac{dv}{dt} &= g \left(1 - \frac{kv^2}{gm} \right) \\ \frac{1}{\left(1 - \frac{kv^2}{gm} \right)} dv &= g dt \\ \int \frac{1}{\left(1 - \sqrt{\frac{k}{gm}} v \right) \left(1 + \sqrt{\frac{k}{gm}} v \right)} dv &= \int g dt. \end{aligned}$$

Now, we can do partial fractions on the left and integrate both sides

$$\begin{aligned}
& \int \frac{1}{\left(1 - \sqrt{\frac{k}{gm}} v\right) \left(1 + \sqrt{\frac{k}{gm}} v\right)} dv = \int g dt \\
& \int \frac{1/2}{\left(1 - \sqrt{\frac{k}{gm}} v\right)} dv + \int \frac{1/2}{\left(1 + \sqrt{\frac{k}{gm}} v\right)} dv = gt + C \\
& -\frac{1}{2} \sqrt{\frac{gm}{k}} \log \left(1 - \sqrt{\frac{k}{gm}} v\right) + \frac{1}{2} \sqrt{\frac{gm}{k}} \log \left(1 + \sqrt{\frac{k}{gm}} v\right) = gt + C \\
& \frac{1}{2} \sqrt{\frac{gm}{k}} \left(\log \left(1 + \sqrt{k/(gm)} v\right) - \log \left(1 - \sqrt{k/(gm)} v\right) \right) = gt + C
\end{aligned}$$

Interesting ... now I need to input the initial condition

$$\begin{aligned}
\frac{1}{2} \sqrt{\frac{gm}{k}} \left(\log \left(1 + \sqrt{k/(gm)} 0\right) - \log \left(1 - \sqrt{k/(gm)} 0\right) \right) &= g0 + C \\
\frac{1}{2} \sqrt{\frac{gm}{k}} \left(\log(1) - \log(1) \right) &= C \\
0 &= C.
\end{aligned}$$

Plugging this in and simplifying we have

$$\begin{aligned}
\frac{1}{2} \sqrt{\frac{gm}{k}} \left(\log \left(1 + \sqrt{k/(gm)} v\right) - \log \left(1 - \sqrt{k/(gm)} v\right) \right) &= gt \\
\log \left(\frac{1 + \sqrt{k/(gm)} v}{1 - \sqrt{k/(gm)} v} \right) &= 2gt \sqrt{k/(gm)}.
\end{aligned}$$

Now we can exponentiate and solve for v

$$\begin{aligned}
\frac{1 + \sqrt{k/(gm)} v}{1 - \sqrt{k/(gm)} v} &= e^{2gt \sqrt{k/(gm)}} \\
1 + \sqrt{k/(gm)} v &= e^{2gt \sqrt{k/(gm)}} (1 - \sqrt{k/(gm)} v) \\
1 + \sqrt{k/(gm)} v &= e^{2gt \sqrt{k/(gm)}} - \sqrt{k/(gm)} v e^{2gt \sqrt{k/(gm)}} \\
\sqrt{k/(gm)} v + \sqrt{k/(gm)} v e^{2gt \sqrt{k/(gm)}} &= e^{2gt \sqrt{k/(gm)}} - 1 \\
v &= \frac{e^{2gt \sqrt{k/(gm)}} - 1}{\sqrt{k/(gm)} + \sqrt{k/(gm)} e^{2gt \sqrt{k/(gm)}}}.
\end{aligned}$$

Some final simplifications gives us

$$\begin{aligned}
v &= \frac{1}{\sqrt{k/(gm)}} \frac{e^{2gt \sqrt{k/(gm)}} - 1}{1 + e^{2gt \sqrt{k/(gm)}}} \\
v &= \sqrt{\frac{gm}{k}} \left(\frac{e^{2gt \sqrt{k/(gm)}} - 1}{e^{2gt \sqrt{k/(gm)}} + 1} \right).
\end{aligned}$$

Therefore our final analytical solution is

$$v = \sqrt{\frac{gm}{k}} \left(\frac{e^{2gt\sqrt{k/(gm)}} - 1}{e^{2gt\sqrt{k/(gm)}} + 1} \right).$$

□

(b)

(c)

(d)

6: 2.3.2

Solution:

TODO

7: 2.3.6

Solution:

TODO

8: 2.4.7

Solution:

TODO