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AMATH 503

HOMEWORK 6

Exercises come from *Introduction to Partial Differential Equations by Peter J. Olver* as well as supplemented by instructor provided exercises.

1: Solve the following wave equations by using D'Alambert's formula:

$$u_{tt} - 4u_{xx} = 0, -\infty x < \infty, t > 0,$$

(a) $u(x,0) = \mathrm{e}^x, u_t(x,0) = \sin(x)$.

Solution:
TODO

(b) $u(x,0) = \sin(x), u_t(x,0) = \cos(2x)$.

Solution:
TODO

2: Olver: 2.4.11 (c)
Solve the forced IVP

$$\begin{cases} u_{tt} - 4u_{xx} = \cos 2t, \quad -\infty < x < \infty, t \geq 0 \\ u(0,x) = \sin x, \\ u_t(0,x) = \cos x, \end{cases}$$

Solution:
TODO

3: Separation of variables to solve

$$\begin{cases} u_{tt} = u_{xx} + \mathrm{e}^{-t}\sin(x), & 0 < x < \pi, t > 0 \\ u(x,0) = \sin(3x), u_t(x,0) = 0, & 0 < x < \pi, \\ u(0,t) = 1, u(\pi,t) = 0, & t > 0. \end{cases}$$

Solution:

TODO

4: (Bonus question) Solve the following wave equation

$$\begin{cases} u_{tt} - 4u_{xx} = 0, & 0 < x < \infty, 0 < t < \infty \\ u(0,t) = 1, & t > 0, \\ u(x,0) = x, u_t(x,0) = \mathrm{e}^x, & x \geq 0. \end{cases}$$

Solution:
TODO

5: Separation of variables to solve

$$\begin{cases} u_{xx} + u_{yy} = 0, & 0 < x < \pi, 0 < y < \pi \\ u(0, y) = u_x(\pi, y) = u(x, 0) = 0 \\ u(x, \pi) = \sin\left(\frac{x}{2}\right) - 2\sin\left(\frac{3x}{2}\right). \end{cases}$$

Solution:

TODO

6: Olver: 4.3.34 (b) Solve the following boundary value problems for the Laplace equation on the annulus $1 < r < 2$ with

$$\begin{cases} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \\ u(1,\theta) = 0, u(2,\theta) = \cos\theta, \\ 1 \leq r < 2, 0 \leq \theta < 2\pi \end{cases}$$

Is this right?

Solution:

TODO

7: (Bonus) Consider the following Laplace equation

$$\begin{cases} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, & 0 \le r < 1, 0 \le \theta < 2\pi \\ u_r(1,\theta) + u(1,\theta) = \cos(2\theta) \end{cases}$$

Use the method of separation of variables to find a solution.

Solution:
TODO