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AMATH 503

HOMEWORK 3

Exercises come from *Introduction to Partial Differential Equations by Peter J. Olver* as well as supplemented by instructor provided exercises.

1: Olver: 3.2.6 (a,c,e)

Solution:
TODO:

2: Olver: 3.3.2 and 3.3.3

Solution:
TODO:

3: Olver: 3.2.55

Solution:
TODO:

4: Olver: 3.4.6

Solution:
TODO:

5: Olver: 3.5.29

Solution:
TODO:

6: Olver: 3.5.43

Solution:
TODO:

7: We consider the complex orthonormal basis

$$\varphi_n = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{inx}$$

where $n = 0, 1, -1, 2, -2, \ldots.$ Consider the function $f_a(x) = \mathrm{e}^{ax}$ with real number $a \neq 0$ and compute the Fourier coefficient

$$\langle f_a, \varphi_n \rangle = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} f_a(x)\,\mathrm{e}^{-inx}\,dx.$$

Then prove the formula

$$\sum_{n=1}^{\infty} \frac{1}{a^2+n^2} = \frac{\pi}{2a}\coth(\pi a) - \frac{1}{2a^2}$$

(Hint: Plancherel's formula: the relation between L^2 norm of coefficients and $\langle f_a, f_a \rangle$.)

Solution:
TODO: