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AMATH 503

HOMEWORK 1

Exercises come from *Introduction to Partial Differential Equations by Peter J. Olver* as well as supplemented by instructor provided exercises.

1: Olver 1.1
Solution:
TODO

2: Olver 1.17
Solution:
TODO

3: Olver 1.22
Solution:
TODO

4: We derive the advection-diffusion equation from the microscopic view. Define $u(x,t)$ as the density of the particles at location x and time t. Define the probability of jumping from the left as $p(x-\Delta x \to x, t) \approx \frac{1}{2} + \Delta x$ when Δx is small, and the probability of jumping from the right as $q(x+\Delta x \to x, t) \approx \frac{1}{2} - \Delta x)$ with small Δx. Assume $D := \lim_{\Delta x, \Delta t \to 0} \frac{\left(\Delta x\right)^2}{\Delta t}$. Establish the equation of $u(x,t)$ in the continuum limit.
Solution:
TODO

5: Consider the following boundary value problem (BVP).

$$\begin{cases} X'\prime(x) + \lambda X = 0 \\ X(0) = X(L) = 0 \end{cases}$$

where $L > 0$ is a constant. Solve the eigenpair:

$$(X_k, Y_k) = \left\{ \sin\left(\frac{k\pi x}{L}, \frac{k\pi}{L}\right)^2 \right\}_{k=0}^{\infty}$$

Solution:
TODO

6: Finish writing up information for question in class tomorrow.
Solution:
TODO