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AMATH 502

**HOMEWORK 8**

Exercises come from the assignment sheet provided by the professor on canvas.

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**1:** See assignment document for background on the *Sierpinski Carpet*.

(a) Show that the (Lebesgue) measure of the resulting fractal is 0. Justify your work.

*Solution:*
**TODO**

(b) Find the similarity dimension of the limiting fractal. Show and explain your work.

*Solution:*
**TODO**

(c) Show that the box-counting dimension of this fractal is the same as the similarity dimension. (You may not be able to complete this until box-counting dimension is defined in class).

*Solution:*
**TODO**

(d) Show that there are uncountably many points on the interior of the limiting fractal (i.e. infinitely many points without x = 0, x = 1, y = 0, or y = 1).
**Hint:** Try showing this in just one dimension, e.g., show that there are uncountably many points (x, y) with a fixed value of y, perhaps y = 0.5 is a good choice.

*Solution:*
**TODO**

**2:** In this problem we construct what is called a middle-halves Cantor set. Consider the following Cantor set construction. Start with the interval [0, 1], then remove the middle half. Continue this process for each sub-interval.

(a) Draw $S_1$ and $S_2$.

*Solution:*
**TODO**

(b) Find the similarity dimension of the set.

*Solution:*
**TODO**

(c) Find the measure of the set.

*Solution:*
**TODO**

**3:** See assignment sheet for details of the *fat fractal*.

(a) Find the (Lebesgue) measure of this cantor set. Show your work.

*Solution:*
**TODO**

(b) Is the fractal self similar? Justify your answer.
**Hint:** Can you find the similarity dimension of this set? What happens when you try?)
**Note:** You may find this part to be difficult. If you are struggling with it, you may want to skip it for now and come back to it later.

**4:** The tent map on the interval [0, 1] is defined by $x_{n+1} = f(x_n)$, where

$$f(x) = \begin{cases} rx, & 0 \le x \le \frac{1}{2} \\ r(1-x), & \frac{1}{2} < x \le 1 \end{cases}.$$

Assume that $r > 2$. Then some points get mapped outside of the interval [0, 1]. If $f(x_0) > 1$ then we say that $x_0$ has "escaped" after one iteration. Similarly, if $f^n(x_0) > 1$ for some finite $n$ and $n$ is the smallest integer for which this is true, then we say $x_0$ has escaped after $n$ iterations.

**Hint:** You should plot the function $f(x)$ and think about orbits of different initial conditions when you are solving this problem, it will make your work much easier. Use these plots to guide your analysis, the plots alone are not enough.

(a) Find the set of initial conditions $x_0$ that escape after one iteration.

*Solution:*
**TODO**

(b) Find the set of initial conditions $x_0$ that escape after two iterations.

*Solution:*
**TODO**

(c) Describe the set of $x_0$ that never escape. This is called the invariant set.
**Hint:** First look at what happens for $r = 3$. Does this look like a set you recognize?

*Solution:*
**TODO**

(d) Find the box dimension of the invariant set (for general $r$, not $r = 3$).

*Solution:*
**TODO**