AMATH 582: HOMEWORK 1

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Abstract. The following is a detailed explanation and analysis of an application of a method of signal processing to track a signal being emitted from a moving object. We discuss the process of using the Fourier transform and applying a gaussian filter on the transformed signal in the frequency space. We also address the challenges of applying such a method to a high dimensional problem. Various shapes of gaussian filters are considered and analyzed. Potential methods to be applied in the future for further analysis of the object tracking problem are proposed.

1. Misc Notes While Analyzing the data

TODO: Remove later I am currently trying to visualize the data more on my own with the packages provided by matplotlib and plotly. It is confusing to me a bit since we are really visualizing something that has 4 dimensions. There are signals or values which are occurring in a 3 dimensional space.

2. Introduction and Overview

Signal processing is a vast field with an array of methodologies as well as applications. Signal processing is not only relevant to analyzing easily recognizable signals like audio or EKG's but can also be applied to any number of problems where you have some means of gathering data from some signal emitting source. The problem presented to us in this assignment was to track an unknown submarine moving about in the Puget Sound. We have been given a broad spectrum recording of acoustics pressure data obtained over 24 hours in half-hour increments. The data is noisy and requires careful analysis. In order to track the submarine using the data we aimed to complete the following 3 tasks:

(1) Through **averaging of the Fourier transform** determine the dominant frequency (center frequency) generated by the submarine. Verify your results through visualization.

(2) Design and implement a **Filter** to extract this center frequency in order to denoise the data and determine a more robust path of the submarine. Visualize the denoised measurement the 3D path of the submarine and inspect the validity and effectiveness of the denoising.

(3) Determine and **plot the** x-y coordinates of the submarine path during the 24 hour period. This information can be used to deploy a sub-tracking aircraft to keep an eye on your submarine in the
future.

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In the endevor to track the unknown submarine and complete the requisite tasks I made extensive use of several important Python packages. Namely I used matplotlib to create all plots and animations [**?**].

Make sure you use the correct citation commands (i.e., `\cite`) to keys from your bib file like this [**?**]. If you want to cite more than one reference simply use [**?**, **?**]. You can grab latex citations from Google Scholar. Just keep in mind that they often need to be cleaned up.

3. Theoretical Background

TODO: Actually edit this later You dedicate this section to the theoretical background of the methods and frameworks that you used in your homework. This is not meant to reproduce material from the lectures or references you used but rather to demonstrate your understanding of the mathematical foundations of the methods and algorithms. You can create equations like this

$$f(x) = \int_A \sin(\pi x) dx.$$

You do not need to label your equations unless they are referenced in the text. In that case simply use

$$-\frac{\partial^2 u}{\partial x^2} = \sin(\pi x). \tag{1}$$

Also look up the `align` or `aligned` environments if you want multi-line equations. You can then reference your equations in text using the `\eqref` command as such (1).

4. Algorithm Implementation and Development

TODO: Actually edit this later Here you discuss the algorithms and software packages that you used. Not much to it. Just make sure you cite the packages properly and avoid including code. You are welcome to use LaTeXpackages that are specifically designed to show algorithms such as this, but it is not always worth the effort and real estate.

5. Computational Results

TODO: Actually edit this later In our work to locate and track the suspected submarine in the puget sound, it was of utmost importance to be able to think through the dimensionality of this problem. We are tracking a signal from a location in 3 dimensional space, which inherently becomes a 4 dimensional problem. I had to come up with methods of thinking about what it meant to average something that is 3 dimensional across its fourth dimension (time). I had to visualize things in lower dimensions in my mind to make sense of the actions that we need to take.
One thing that helped was taking slices of the average frequencies across time to confirm what we found by calculation. As seen in Figure 1 by looking at the slices corresponding to the location of the max frequency in our cube of frequencies.

This helped confirm that I had identified the correct dominant frequency at which to center the gaussian filter to help us eliminate the noise. Furthermore we visualized the path of the of the submarine in Figure 2

Moreover, I was able to generate several other important visualizations in gif form which are viewable on my Github. This is the animation of the 3 dimensional path and this is the animation of the path in just the x-y plane ($z = 0$).

You will most definitely need tables and figures. So here is an example.

Make sure your table is labeled and referenced withing the text using `\ref` as such Table 1. In fact, you can use `\ref` to cite anything else in the document such as sections (ex. Section 2). This will create hyperlinks in your pdf after compilation and automatically update the numbers and tags whenever you change anything.

FIGURE 1. **TODO: increase font size and give a detailed caption.**

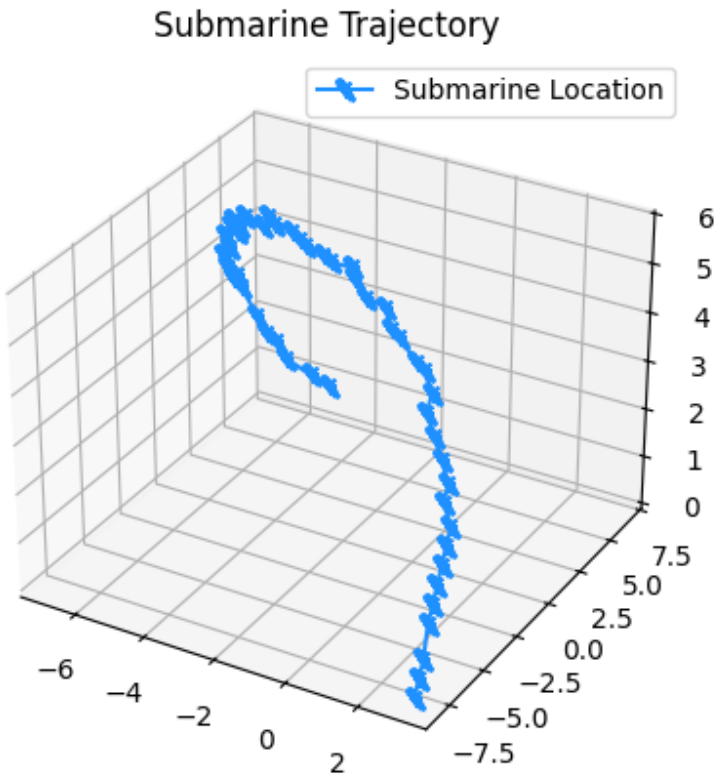

FIGURE 2. **TODO: give a detailed caption.**

Figures are very similar to tables. Here's an example:

You may also need to include multiple figures:

Once again, make sure all your figures are referenced like Figure **??** or Figure **??** in the text body of the report and discussed in detail. This is where you will make observations about your results and we will look at these very closely.

row 1	column 1	column 2
row 2	column 1	column 2
row 3	column 1	column 2

TABLE 1. Don't forget to include a caption for your table. Say a few words about what is being shown.

Also note, I am using PDF figures. These give you the best looking graphs but PNG works well too. I advise staying away from JPG as it always looks weird and low quality.] Both Python and MATLAB can output figures in PDF or PNG.

6. Summary and Conclusions

Wrap up your report with a brief summary of what you did and what you discovered. Finish with some conclusions and possibly future directions if any.

Acknowledgements

The author is thankful to Jaxon Tuggle for useful discussions about the process to find the dominant frequency in task 1. Additionally Hailey Sparks was thoroughly helpful when comparing methods by which we identified the correct location of our gaussian filter for task 2. I would also like to thank Professor Eli Shlizerman for carefully instructing us in class and answering questions in Office Hours.