DSP HW #3

1a) $x(t) = \begin{cases} Ae^{-at} & t \ge 0 \\ 0 & t < 0 \end{cases}$ $\quad a > 0$

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{0}^{\infty} A e^{-at} e^{-j\Omega t}\, dt$$

$$= A\int_{0}^{\infty} e^{-at - j\Omega t}\, dt = A\int_{0}^{\infty} e^{-t(a+j\Omega)}\, dt$$

$$X(\Omega) = \frac{-A e^{-t(a+j\Omega)}}{a+j\Omega}\Bigg|_{0}^{\infty} = \frac{-A \cancel{e^{-\infty(a+j\Omega)}}^{\,0}}{a+j\Omega} - \frac{-A \cancel{e^{0}}^{\,1}}{a+j\Omega}$$

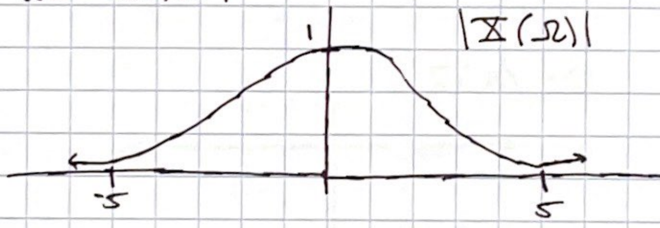
$$X(\Omega) = \frac{A}{a+j\Omega}$$

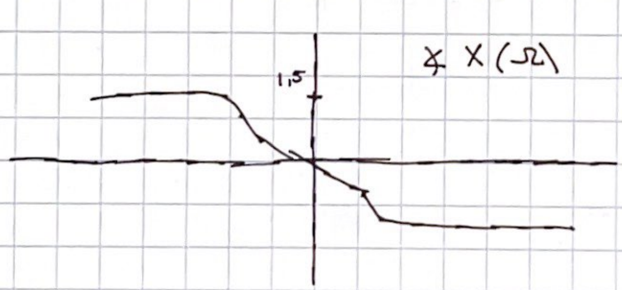
$$|X(\Omega)| = \frac{A(a-j\Omega)}{a^2+\Omega^2} \Rightarrow \left[\left[\frac{Aa}{(a^2+\Omega^2)}\right]^2 + \left[\frac{-Aj\Omega}{(a^2+\Omega^2)}\right]^2\right]$$

$$|X(\Omega)| = \frac{A^2(a^2+\Omega^2)}{(a^2+\Omega^2)^2}$$

$$\measuredangle X(\Omega) = \arctan\left[\frac{\cancel{Real}}{\cancel{Imag}}\right] = \arctan\left[\frac{Im}{Re}\right] = \arctan\left[\frac{-j\Omega}{a}\right]$$

Plot with $a = A = 1$





DSP HW #3

1b) $x(t) = Ae^{-a|t|}$

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{0} Ae^{at} e^{-j\Omega t}\, dt + \int_{0}^{\infty} Ae^{-at} e^{-j\Omega t}\, dt$$

$$= A\int_{-\infty}^{0} e^{t(a-j\Omega)}\, dt + \underbrace{A\int_{0}^{\infty} e^{-t(a+j\Omega)}\, dt}_{= \frac{A}{a+j\Omega}}$$

$$A\int_{-\infty}^{0} e^{t(a-j\Omega)}\, dt = \left.\frac{Ae^{t(a-j\Omega)}}{a-j\Omega}\right|_{-\infty}^{0} = \frac{A}{a-j\Omega} - \cancel{Ae^{-\infty}}$$

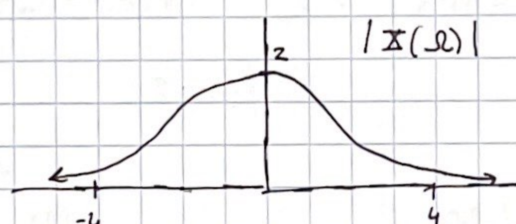
$$X(\Omega) = \frac{A}{a+j\Omega} + \frac{A}{a-j\Omega} = \frac{A(a-\cancel{j\Omega})}{a^2+\Omega^2} + \frac{A(a+\cancel{j\Omega})}{a^2+\Omega^2}$$

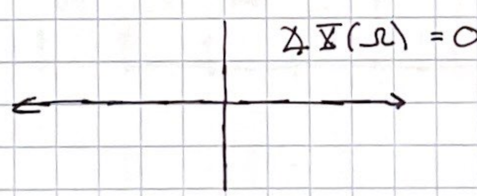
$$X(\Omega) = \frac{2Aa}{a^2+\Omega^2}$$

Since $x(t)$ is real + even, $X(\Omega)$ is Real

$\angle X(\Omega) = 0$

Plot for $A = a = 1$





DSP HW #3

2. $x[n] = \{\ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots\}$

Ir $c_k = \frac{1}{9}\sum_{n=0}^{N-1} e^{-j\pi 2kn/N} \cdot x[n]$

$c_0 = \frac{1}{9}[1 + 0 + 1 + 2 + 3 + 2 + 1 + 0 + 1] = 11/9$

$c_1 = \frac{1}{9}[1 \cdot 1 + 0 \cdot e^{-2\pi/9} + 1e^{j2\pi 2/N} + 2e^{-j\frac{2\pi 3}{9}} + 3e^{-\frac{2\pi 4}{9}} + 2e^{-\frac{2\pi 5}{9}} + e^{-\frac{2\pi 6}{9}} + e^{-\frac{2\pi 7}{9}}$

$c_1 = \frac{1}{9}[1 + e^{-j\pi\frac{4}{9}} + 2e^{-j6\pi/9} + 3e^{-j8\pi/9} + 2e^{-j\frac{10}{9}\pi} + e^{-j\frac{12}{9}\pi} + e^{-j16/9}]$

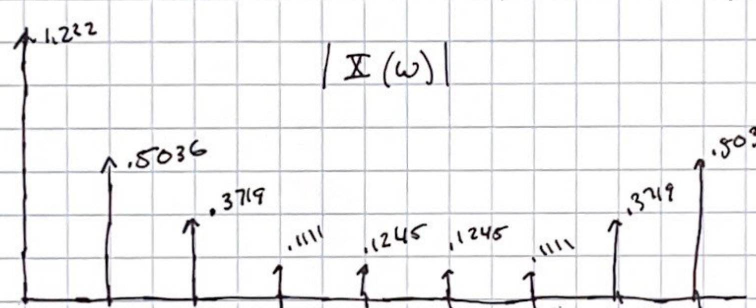
$c_1 = -.584 - j.172$

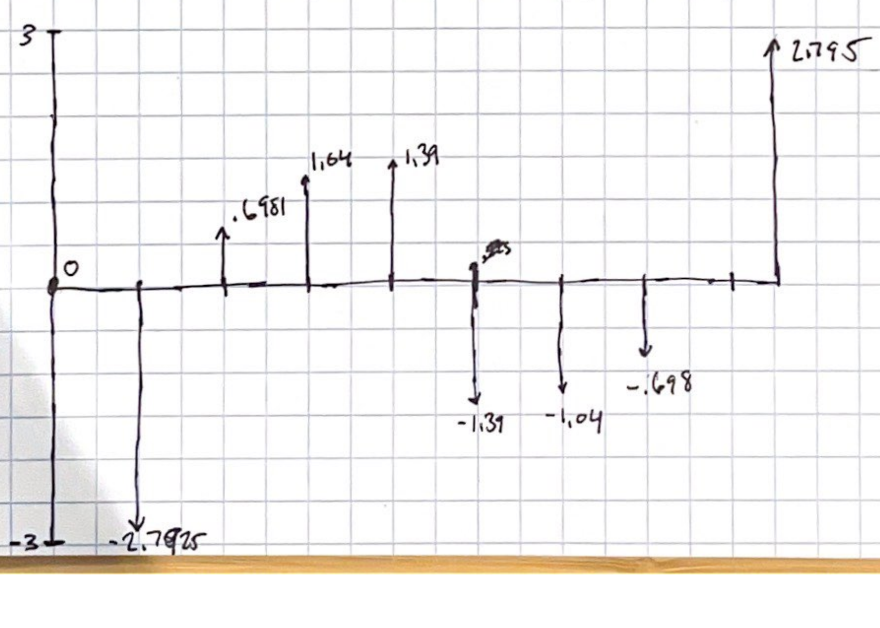
$\cancel{c_2 = \frac{1}{9}[1 + 1e^{-j6/9\pi} + 2e^{-j\frac{12\pi}{9}} + 3e^{-j24\pi/9} + 2e^{-j48/9}}$

$c_2 = \frac{1}{9}[1 + e^{-j\pi 8/9} + 2e^{-j\pi 12/9} + 3e^{-j\pi 16/9} + 2^{-j\pi 20/9} +$

The rest calculatd using matlab.

$c_0 = 1.22$ $c_1 = -.4732 - j.1722$ $c_2 = .2849 + j.239$
$c_3 = .0556 + j.0962$ $c_4 = .0216 + j.1226$ $c_5 = .0216 - j.1226$
$c_6 = .0556 - j.0962$ $c_7 = .2849 - j.2391$ $c_8 = -.4732 + j.1772$





PSP HW3 2b

2b) $P_x = \frac{1}{N} \sum_n |x(n)|^2 = \sum_k |c_k|^2$

$\frac{1}{9}[1^2 + 1^2 + 2^2 + 3^2 + 2^2 + 1^2 + 1^2] = 2.33\ W$

$1.22^2 + .5036^2 + .3719^2 + (1/9)^2 + 2(.1245)^2 + (1/9)^2 + .3719^2 + .5036^2 = 2.33 W$

$\therefore$ It proves parsevals relation since $\frac{1}{N} \sum_n |x(n)|^2 = \sum_k |c_k|^2$

DSP HW #3

3) Find the Fourier Transforms

a) $x[n] = u[n] - u[n-8]$ $\rightarrow$ Time Shifted square pulse

$y[n] = u[n + \frac{N}{2}] - u[n - \frac{N}{2}]$ $\quad \mathcal{F}\{y[n]\} = Y(\omega) = \dfrac{\sin(\frac{\omega N}{2})}{\sin(\frac{\omega}{2})}$

Time shift property $\rightarrow$ $x[n-k] = e^{-j\omega k} X(\omega)$

$$\boxed{X(\omega) = e^{-j\omega 4} \frac{\sin(2\omega)}{\sin(\omega/2)}}$$

b) $x[n] = 2^n u[-n]$ $\rightarrow$ Time reversal of decaying exponential

$x[n] = (\frac{1}{2})^{-n} u[-n]$ $\quad \mathcal{F}\{\frac{1}{2}^n u[n]\} = \dfrac{1}{1-.5e^{-j\omega}}$

Time reversal property

$x(-n) = X(-\omega)$

$$\boxed{X(\omega) = \frac{1}{1-.5e^{j\omega}}}$$

c) $x[n] = (\frac{1}{3})^n u[n+2]$

$X(\omega) = \sum_n x[n] e^{-j\omega n} = \sum_{n=-2}^{\infty} (\frac{1}{3})^n e^{-j\omega n} = \sum_{n=-2}^{\infty} (\frac{e^{-j\omega}}{3})^n$

$= \sum_{n=-1}^{\infty} \left[(\frac{e^{-j\omega}}{3})^n \right] + (\frac{e^{-j\omega}}{3})^{-2}$

$$\boxed{X(\omega) = \frac{1}{1-\frac{1}{3}e^{-j\omega}} + \left(\frac{e^{-j\omega}}{3}\right)^{-2} + \left(\frac{e^{-j\omega}}{3}\right)^{-1}}$$

$\sum_n (ae)^n = \dfrac{1}{1-ae^n}$

d) $x[n] = \{-2, -1, 0, 1, 2\}$

$X(\omega) = \sum_n x[n] e^{-j\omega n}$ $= \boxed{-2e^{2j\omega} - e^{j\omega} + e^{-j\omega} + e^{-2j\omega} = X(\omega)}$

DSP HW #3

4) $x(n) = \{-1, 2, -3, 2, -1\}$ → Real + even

a) $X(0) = \sum_n x(n) e^{-j\omega n}\Big|_{\omega=0} = \sum_n x(n) = \boxed{-1 = X(0)}$

b) Since $x(n)$ is real + even $X_I(\omega) = 0$

$$\boxed{\measuredangle X(\omega) = \arctan\left(\frac{X_I(\omega)}{X_R(\omega)}\right) = 0}$$

c) $x(n) = \cancel{\sum} \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$

$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$ when $n = 0$

$x(0) = \cancel{\quad} = -3$

$$\boxed{\int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi(-3) = -6\pi}$$

d) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_n |x(n)|^2 = 2\pi[1 + 4 + 9 + 4 + 1]$

$$\boxed{\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi(19)}$$