DSP HW #6

1) Determine all signals associated w/

$$X(z) = \frac{5z^{-1}}{(1-\frac{1}{2}z^{-1})(3-z^{-1})} = \frac{5z}{(z-1/2)(3z-1)}$$

$$\frac{5z}{(z-1/2)(3z-1)} = \frac{A}{z-1/2} + \frac{B}{3z-1} \Rightarrow 5z = A(3z-1) + B(z-1/2)$$

$$A = \frac{5(1/2)}{3(1/2)-1} = \frac{5}{2} \cdot \frac{2}{1} = 5$$

ROC

$z >$

$z <$

$< z <$

$$B = \frac{5(1/3)}{1/3-1/2} = -10$$

$$X(z) = \frac{5}{z-1/2} + \frac{-10}{3z-1}$$

$$\boxed{\begin{aligned} x_1[n] &= [5(\tfrac{1}{2})^n - 10(\tfrac{1}{3})^n]\,u[n] \\ x_2[n] &= [5(\tfrac{1}{2})^n - 10(\tfrac{1}{3})^n]\,u[-n-1] \\ x_3[n] &= 5(\tfrac{1}{2})^n u[n] - 10(\tfrac{1}{3})^n u[-n-1] \end{aligned}}$$

→ you can flip this around as well

DSP Ch6 HW

2a) $x[n] = \{3, 0, 0, 0, 0, \underset{\uparrow}{6}, 1, -4\}$ $n=0$ @ $x[n]=6$

$$X(z) = \sum_n x[n] z^{-n}$$

$$\boxed{X(z) = 3z^5 + 6 + z^{-1} - 4z^{-2} \quad \text{ROC entire z-plane but } z=0}$$

b) $x[n] = \begin{cases} (1/2)^n & n \geq 5 \\ 0 & \text{else} \end{cases}$

$x[n] = (1/2)^n u[n-5]$

$Z\{(\tfrac{1}{2})^n u[n]\} = \dfrac{1}{1 - 1/2 z^{-1}}$ TS by 5

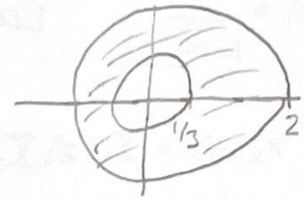
$$\boxed{X(z) = \frac{z^{-5}}{1 - 1/2 z^{-1}}}$$

DSP Ch6 HW

3a) $x[n] = \begin{cases} 1/3^{n} & n \geq 0 \\ \frac{1}{2}^{-n} & \text{other} \end{cases} = 1/3^{n} u[n] + \frac{1}{2}^{-n} u[-n-1]$

$\mathcal{Z}\{a^{n} u[n]\} = \frac{1}{1-az^{-1}}$ ROC: $|z| > |a|$

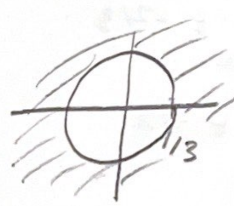
$\mathcal{Z}\{-a^{n} u[-n-1]\} = \frac{1}{1-az^{-1}}$ ROC: $|z| < a$



$$\boxed{X_1(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} \quad \text{ROC: } 1/3 < |z| < 2}$$

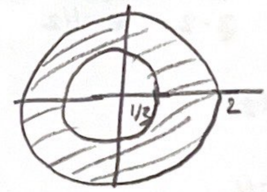
b) $x[n] = (\frac{1}{3}^{n} - 2^{n}) u[n]$

$$\boxed{X_2(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{1}{1-2z^{-1}} \quad \text{ROC: } |z| > 1/3 \quad z \neq \infty}$$



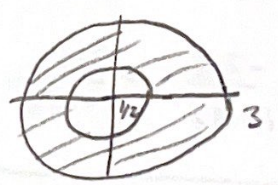
c) $x[n] = x_1[n+4]$

$$\boxed{X_3(z) = z^{4} X_1(z) \quad \text{ROC: } 1/3 < |z| < 2}$$



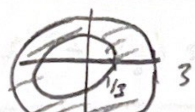
d) $x_4[n] = x_1[-n]$

$$\boxed{X_4(z) = \frac{1}{1-1/3z} - \frac{1}{1-2z} \quad \text{ROC: } \frac{1}{2} < |z| < 3}$$



e) $x_5[n] = x_1[n] + x_2[n]$

$$\boxed{X_5(z) = X_4(z) + X_2(z) \quad \text{ROC: } 1/3 < |z| < 3}$$

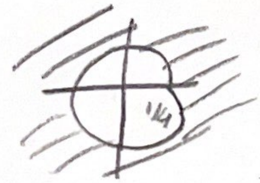


d) $x_6[n] = x_2[-2n]$

$x_2[2n] = (\frac{1}{3}^{2n} - 2^{2n}) u[2n] = (\frac{1}{9}^{n} - 4^{n}) u[n]$

$X_{2(2n)}(z) = \frac{1}{1-1/9z^{-1}} - \frac{1}{1-4z^{-1}}$ ROC: $|z| > 1/9$

$$\boxed{X_6(z) = \frac{1}{1-1/4z} - \frac{1}{1-4z} \quad \text{ROC: } |z| > 1/4}$$



4) $x_1[n] = \frac{1}{3}^n u[n] + \frac{1}{2}^{-n} u[-n-1]$

$$X_1(z) = \frac{1}{1-1/3 z^{-1}} - \frac{1}{1-2z^{-1}} \quad ROC: 1/3 < |z| < 2$$

$$X_2(z) = \frac{1}{1-1/2 z^{-1}} \quad ROC: |z| > 1/2$$

$$x_1[n] * x_2[n] = X_1(z) X_2(z) = \frac{1}{(1-1/3 z^{-1})(1-1/2 z^{-1})} - \frac{1}{(1-2z^{-1})(1-1/2 z^{-1})}$$

$$\frac{z^2}{(z-1/3)(z-1/2)} = \frac{A_1}{z-1/3} + \frac{B}{z-1/2} \quad z^2 = A_1(z-1/2) + B(z-1/3)$$

$$A = \frac{1/9}{1/3-1/2} = -2/3$$

$$B = \frac{1/4}{1/2-1/3} = 3/2$$

$$\frac{z^2}{(z-2)(z-1/2)} = \frac{A}{z-2} + \frac{B}{z-1/2} \Rightarrow z^2 = A(z-1/2) + B(z-2)$$

$$A = \frac{4}{1.5} = 8/3$$

$$B = \frac{1/4}{-3/2} = \frac{-2}{12} = -1/6$$

$$X_1(z) X_2(z) = \frac{-2/3}{1-1/3 z^{-1}} + \frac{3/2}{1-1/2 z^{-1}} - \frac{8/3}{1-2z^{-1}} + \frac{1/6}{1-1/2 z^{-1}}$$

$$\boxed{x_1[n] * x_2[n] = \left[-2/3 \left(\tfrac{1}{3}\right)^n + 3/2 \left(\tfrac{1}{2}\right)^n\right] u[n] + \left[-8/3\, 2^n + 1/6 \left(\tfrac{1}{2}\right)^n\right] u[-n-1]}$$

DSP HW #6

5) $$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

$$
\begin{array}{r l}
 & 1 + 4z^{-1} + 7z^{-2} + 18z^{-3} \\
\hline
1 - 2z^{-1} + z^{-2} \,\big) & 1 + 2z^{-1} \\
 & -(1 - 2z^{-1} + z^{2}) \\
\hline
 & 0 + 4z^{-1} - z^{-2} \\
 & 4z^{-1} - 8z^{-2} + 4z^{-3} \\
\hline
 & 0 + 7z^{-2} + 4z^{-3} \\
 & 7z^{-2} - 14z^{-3} + 7z^{-4} \\
\hline
 & 0 + 18z^{-3} + 7z^{-4}
\end{array}
$$

$$\boxed{\begin{array}{l} X(0) = 1 \\ X(1) = 4 \\ X(2) = 7 \\ X(3) = 18 \end{array}}$$

This cannot get a closed form solution of the inverse Z-Transform but the first couple terms will match.

DSP Ch6 HW6

(G) $X(z) = \dfrac{1}{(1-1/2 z^{-1})(1-z^{-1})^2} = \dfrac{z^3}{(z-1/2)(z-1)^2}$

$$\frac{z^3}{(z-1/2)(z-1)^2} = \frac{A}{z-1/2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z^3 = A(z-1)^2 + B(z-1/2)(z-1) + C(z-1/2)$$

$$A = \frac{1/2^3}{(1/2-1)^2} = \frac{1/8}{1/4} = 1/2$$

$$C = \frac{1}{1-.5} = 2$$

$$B = \frac{d}{dz}\left[(z-1)^2 X(z)\right] = \frac{d}{dz}\left[\frac{z^3}{(z-1/2)}\right]\Bigg|_{z=1}$$

$$B = \frac{\cancel{(z-1)}^{0} 3z^2 - x^3}{(z-1/2)^2}\Bigg|_{z=1} = \frac{-1}{.25} = -4$$

$$\boxed{X[n] = \left[\frac{1}{2}\left(\frac{1}{2}\right)^n - 4(1)^n\right]u[n] + 2[n+1]u[n+1]}$$