

Module 01: Assignment 01

Digital Signal Processing

Please complete the following problems and questions. Show all of your work in as much detail as possible.

1. Determine which of the following sinusoids are periodic and compute their fundamental period.

Problem 1 Solution:

“A discrete-time sinusoid is periodic only if its frequency f_0 is a rational number.”

Furthermore, the fundamental period of a sinusoid when it exists is equal to N where

$f_0 = \frac{k}{N}$ and k, N are relatively prime.

(a) $\cos(0.01\pi \cdot n)$

$$f = \frac{\omega}{2\pi} = \frac{0.01\pi}{2\pi} = \frac{1}{200} \text{ is rational and } (1, 200) \text{ are relatively prime.}$$

→ $\cos(0.01\pi \cdot n)$ is periodic with fundamental period $N = 200$.

(b) $\cos\left(\pi \cdot \frac{30}{105} n\right)$

$$f = \frac{\omega}{2\pi} = \frac{30\pi}{105} \cdot \frac{1}{2\pi} = \frac{1}{7} \text{ is rational and } (1, 7) \text{ are relatively prime.}$$

→ $\cos\left(\pi \cdot \frac{30}{105} n\right)$ is periodic with fundamental period $N = 7$.

(c) $\cos(3\pi \cdot n)$

$$f = \frac{\omega}{2\pi} = \frac{3\pi}{2\pi} = \frac{3}{2} \text{ is rational and } (3, 2) \text{ are relatively prime.}$$

→ $\cos(3\pi \cdot n)$ is periodic with fundamental period $N = 2$.

(d) $\sin(3 \cdot n)$

$$f = \frac{\omega}{2\pi} = \frac{3}{2\pi} = \frac{3}{2\pi} \text{ is not rational.}$$

→ $\sin(3 \cdot n)$ is non-periodic.

(e)

$$\sin\left(\pi \cdot \frac{62}{10}n\right)$$

$$f = \frac{\omega}{2\pi} = \frac{62\pi}{10} \cdot \frac{1}{2\pi} = \frac{31}{10} \text{ is rational and } (31,10) \text{ are relatively prime.}$$

$$\rightarrow \sin\left(\pi \cdot \frac{62}{10}n\right) \text{ is periodic with fundamental period } N = 10.$$

Problem 2 Solution:

2. Determine whether or not each of the following signals are periodic. If the signal is periodic, specify the fundamental period.

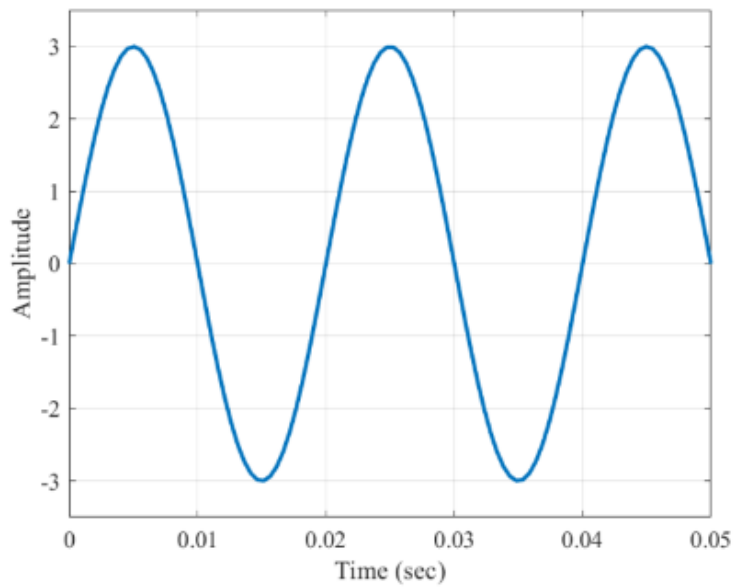
- a) **Periodic**, continuous time signal, no need to be rational.
- b) **Not Periodic**, $2\pi/5$ is not rational
- c) **Not Periodic**, $2\pi/(1/6)$ is not rational
- d) **Not Periodic**, $2\pi/8$ is not rational, $2\pi/8\pi$ is; however, product is non-periodic
- e) **Periodic**, all parts are rational and periodic: $2\pi/(\pi/2) \dots 2\pi/(\pi/8) \dots$

Problem 3 Solution:

3. Consider the following analog sinusoidal signal: $x_a(t) = 3\sin(100\pi t)$

- a. Sketch the signal, $x_a(t)$, for $0 \leq t \leq 50$ milliseconds (ms)
- b. The signal, $x_a(t)$, is sampled with a sampling frequency $F_s = 300$ samples/second. Determine the frequency of the discrete-time signal, $x(n) = x_a(nT_s)$, where is the sampling period and show that $x(n)$ is periodic.

a)

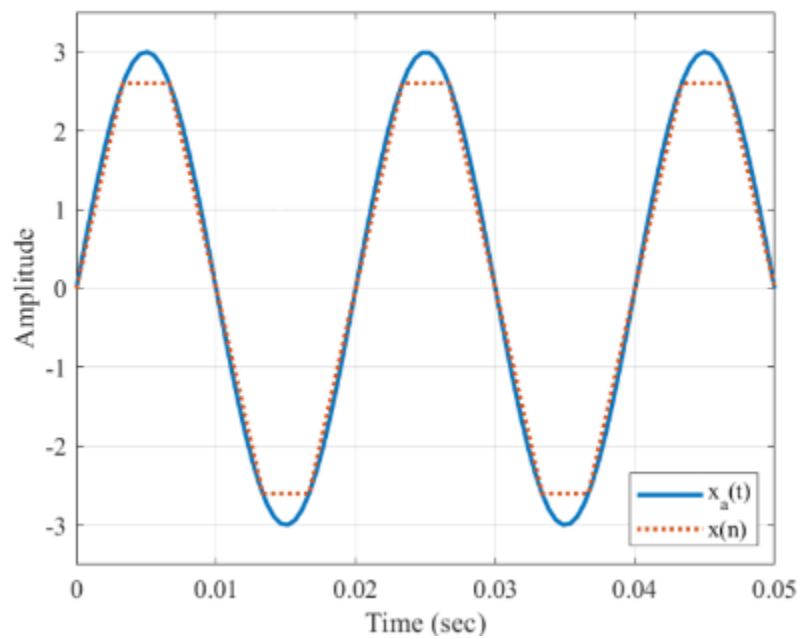


b) $3\sin(100\pi n/300) = 3\sin(\pi n/3)$

*use same method as Problem 1 to solve for $f=1/6$, periodic with $N=6$.

c. Sketch the signals $x_a(t)$ and $x(n)$, as defined in 2(a) and 2(b) on the same plot.

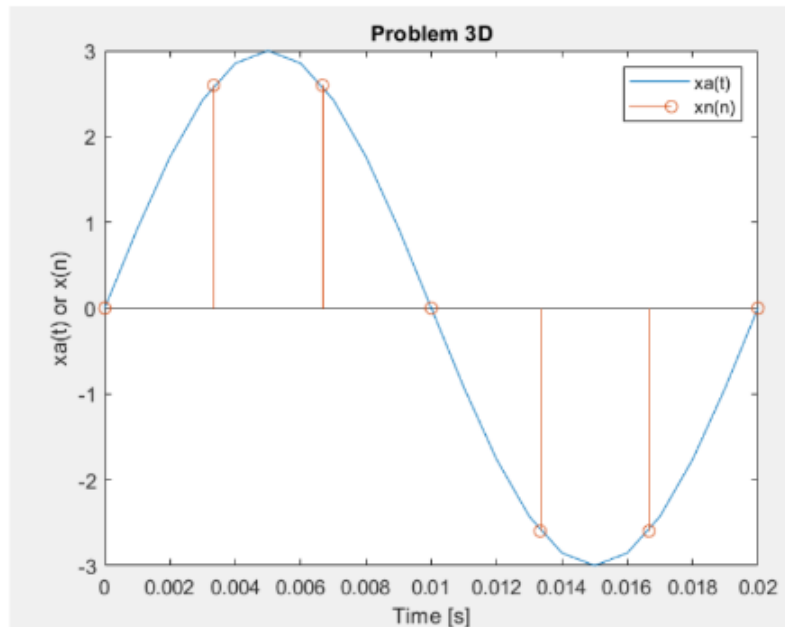
c)



- d. Compute the sample values in one period of $x(n)$. Using MATLAB, plot $x(n)$ using the stem function and also plot $x_a(t)$ showing it to be a continuous function. What is the period of the discrete-time signal, $x(n)$, in milliseconds? Using MATLAB, label each axis and each curve.

d)

$$T = 6 * 1/300 = 0.02 \text{ seconds}$$



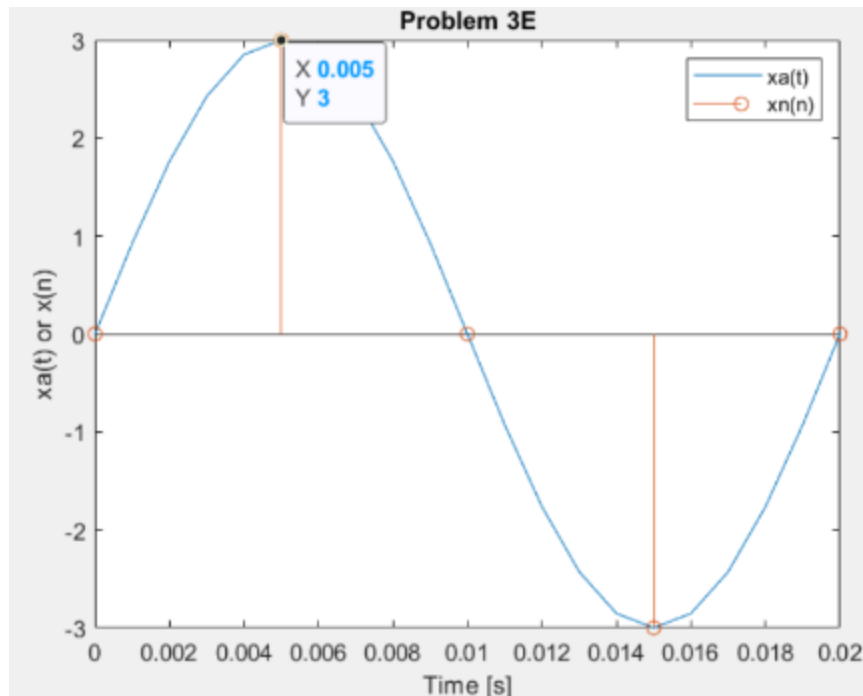
- e. Find a sampling frequency, F_s , such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?

e)

Solve for where $\omega = \pi/2$:

$$2\pi * 50 / F_s = \pi / 2$$

$$F_s = 200 \text{ Hz}$$



Problem 4-8 Solution:

4. According to the information and video available on the course website, what must a student do in order to perform well in this course?
 5. Should you use your work version of MATLAB for this class?
 6. Should you use your laptop which still runs Windows 98 for this course?
 7. What consists of a proper response to a (homework) Assignment problem?
 8. What are the four different frequency definitions discussed in Module 1? What are their units?
- 4: Stay dedicated and on top of your work**
- 5: No – please use Student Edition**
- 6: No – please use up to date machine**

7: Repeat the problem, include thorough steps, commentary, and justifications. Be sure to address all parts of the problem statement. MATLAB code isn't necessary unless specifically asked for, but can be included as an appendix if you wish.

8:

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|---------------------------------------|---------------------------|
| 1. Continuous Time Frequency: | F (Hz or cycles/second) |
| 2. Continuous Time Angular Frequency: | Ω (rad/second) |
| 3. Discrete Time Frequency: | f (cycles/sample) |
| 4. Discrete Time Angular Frequency: | ω (rad/sample) |