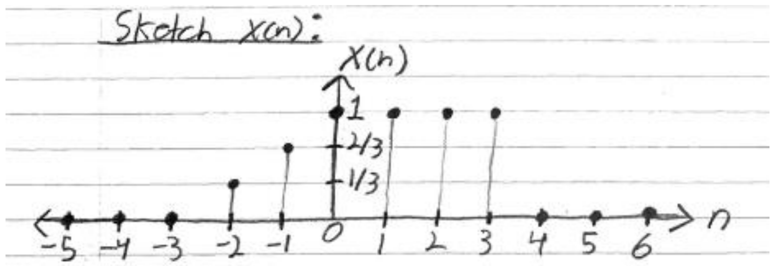
Module 2 Assignment: Discrete-Time LTI Systems

Digital Signal Processing (EN.525.627.8X)

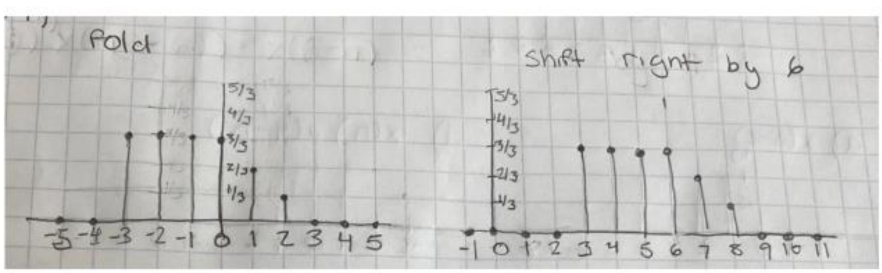
Problem 1:

a) Determine its values and sketch (by hand) the signal $x(n)$.

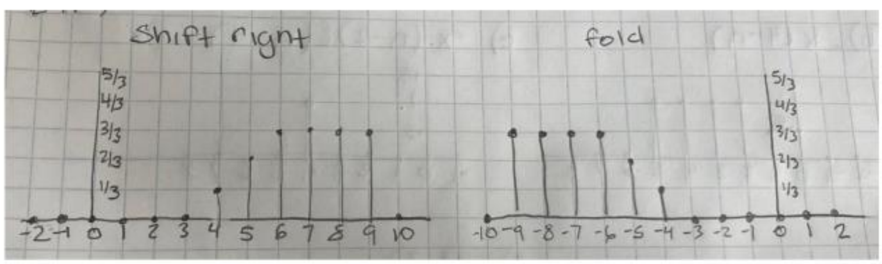


b) Sketch (by hand) the signals that result if we:

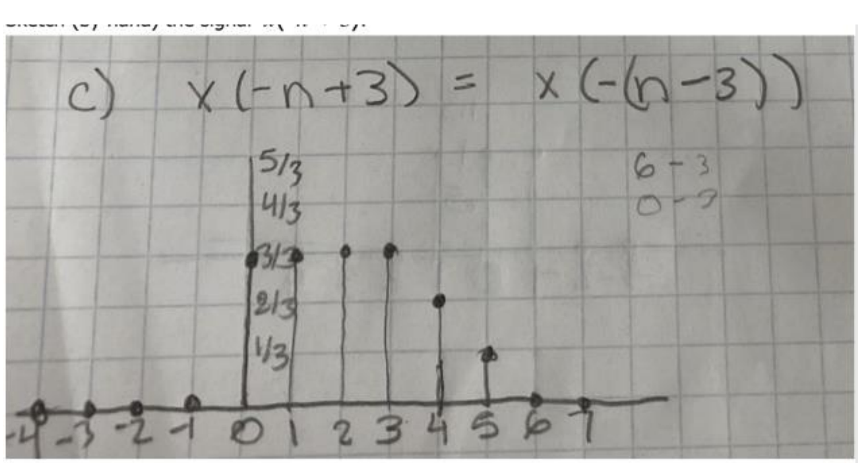
i. First fold $x(n)$ and then delay the resulting signal by six samples.



2 First delay $x(n)$ by six samples and then fold the resulting signal.



c) Sketch (by hand) the signal $x(-n+3)$.



d) Compare (a), (b) and (c) and determine a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

X(-n+k) is equivalent to x(-(n-k)): first flip the signal, then shift to the right for +k and left for -k.

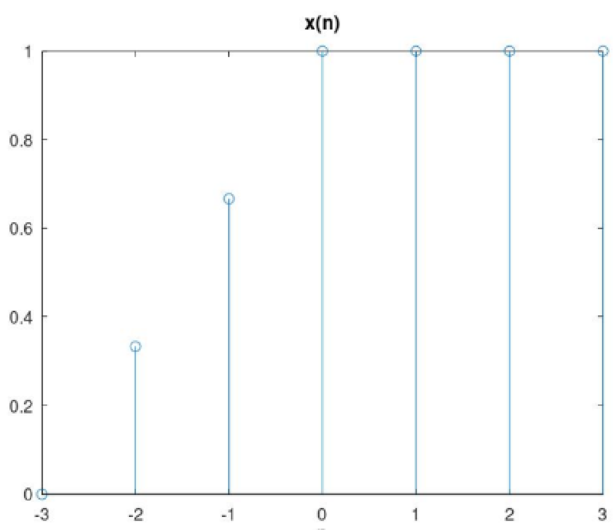
e) Express the signal $x(n)$ in terms of $\delta(n)$ and $u(n)$.

X(n) = 2/3δ(n+1) + 1/3 δ(n+2) + u(n) – u(n-4)

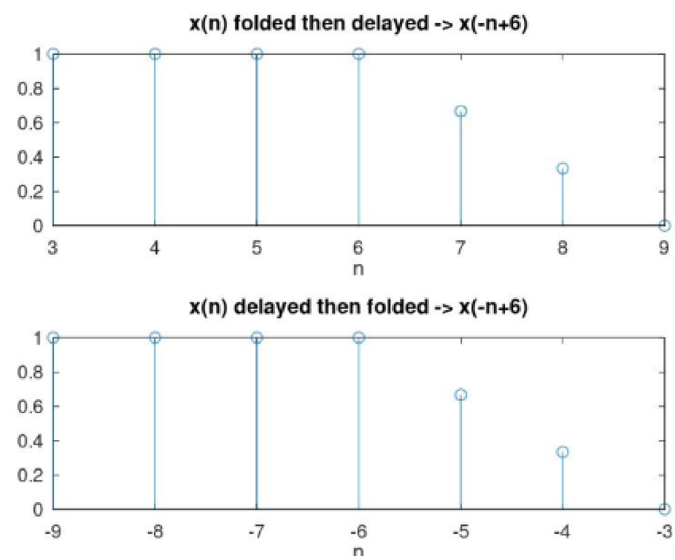
Problem 2

2. Repeat 1. Parts a), b), c) and e) using
   a. A MATLAB script file
   b. The "stem" plotting capability.
   c. Insert "comments" within your script file(s)
   d. Label the axes of your figures, plots.
   e. Insert a title to each plot or figure using the appropriate MATLAB commands within your script file.
   f. Include your script file within your single PDF file which is your for submission for this entire module assignment set.

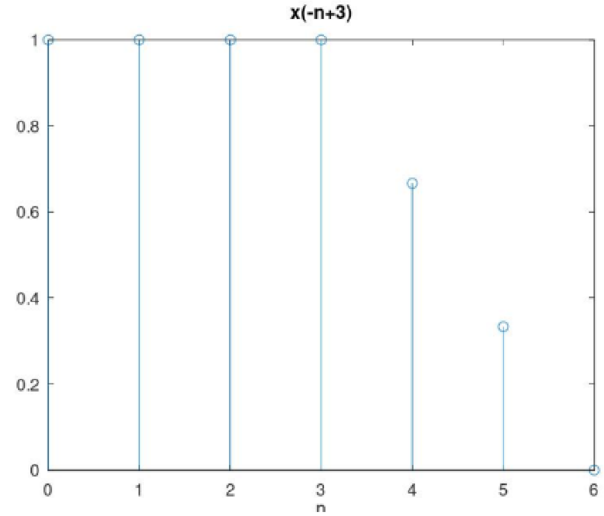
Problem 2a



Problem 2b



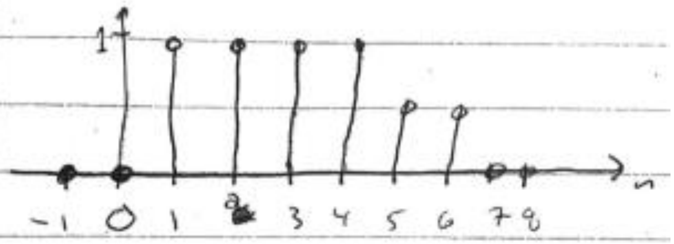
Problem 2c



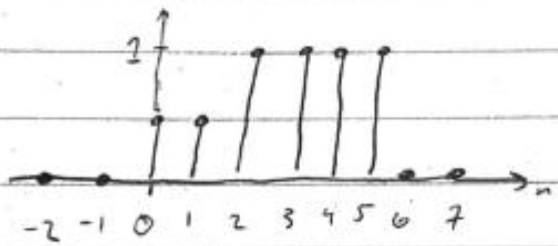

```
clear;
close;
clc;

n = -3:3;
x_n = [0 1/3 2/3 1 1 1 1];

#2a
stem(n, x_n);
title("x(n)");
xlabel("n");
saveas(gcf,'C:\Users\semmi\Desktop\Matlab\plots\DSP_hw2_prob2a.jpg');
close;



#2b
subplot(2,1,1);
stem(-n+6, x_n);  #Note we can delay a signal by adding the delay to -n.
title("x(n) folded then delayed -> x(-n+6)");
xlabel("n");

subplot(2,1,2);
stem(-n-6, x_n);  #Note we can delay a signal by adding the delay to -n.
title("x(n) delayed then folded -> x(-n+6)");
xlabel("n");
saveas(gcf,'C:\Users\semmi\Desktop\Matlab\plots\DSP_hw2_prob2b.jpg');
close;



#2c
stem(-n+3, x_n);  #Note we can delay a signal by adding the delay to -n.
title("x(-n+3)");
xlabel("n");
```

Problem 3

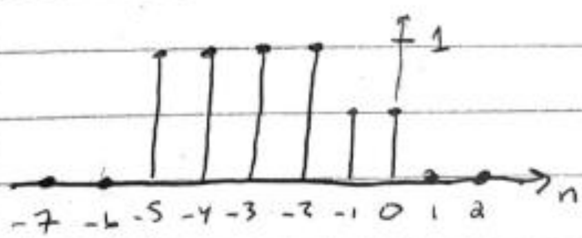
Problem 3a: $x(n-2)$



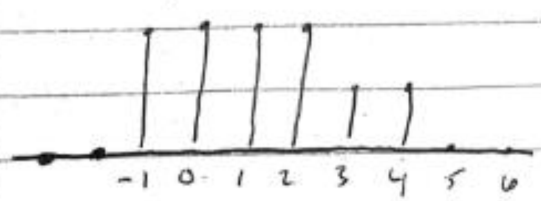
Problem 3b: $x(4-n) = x(-(n+4))$

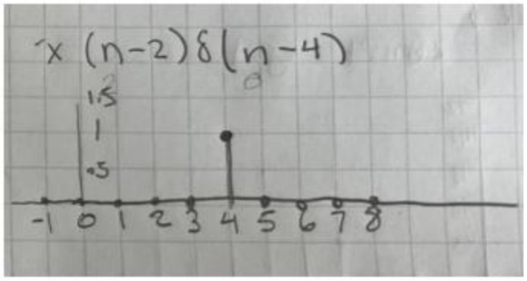


Problem 3c: $x(n+4)$

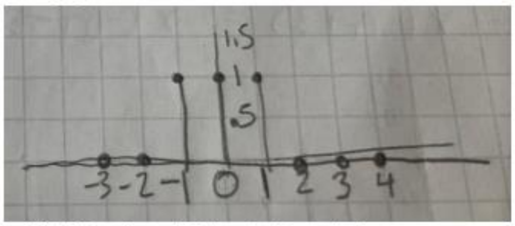


Problem 3d: $x(n)\,u(4-n)$

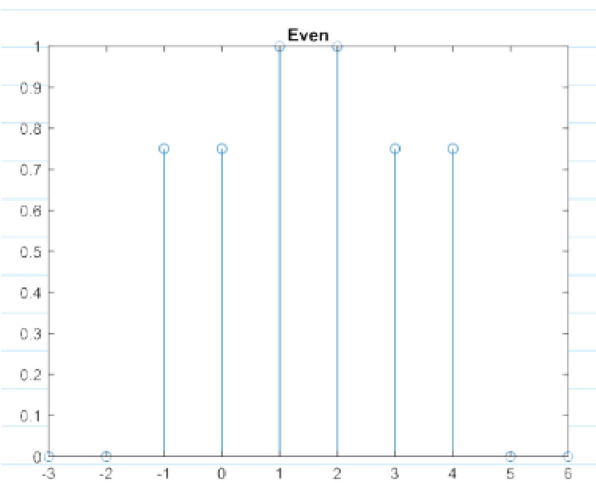




f) $x(n^3)$



e) $x(n-2)\delta(n-4)$



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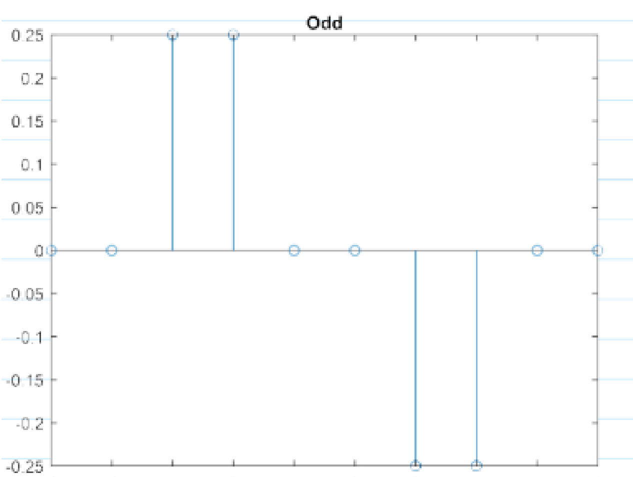

H

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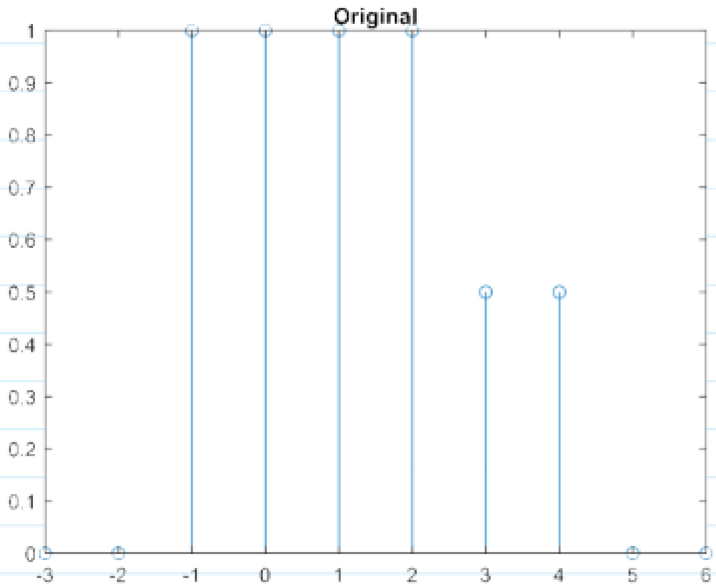
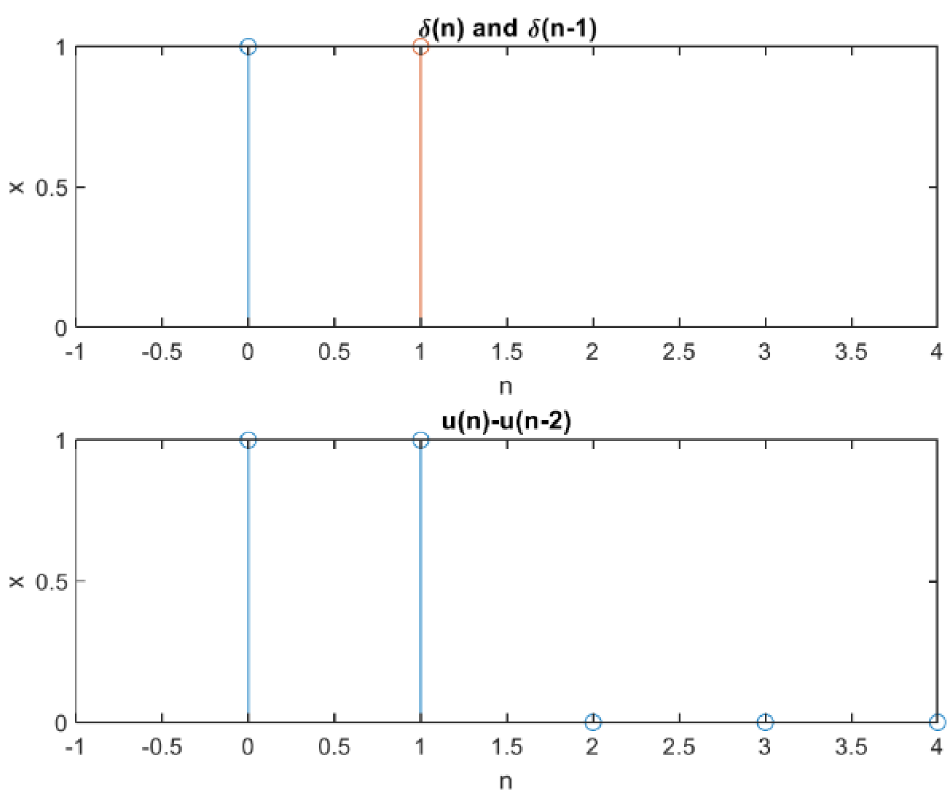
i)

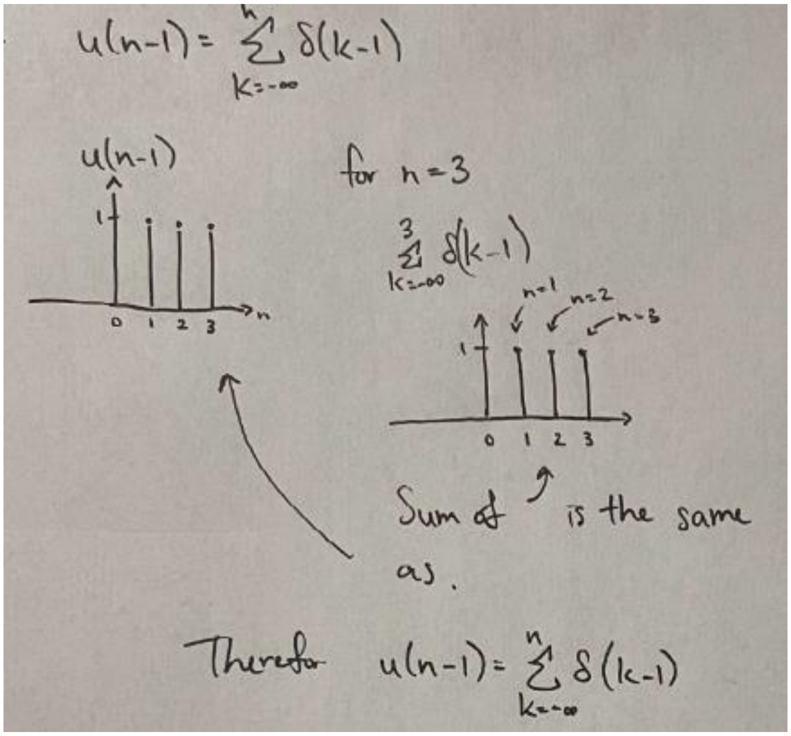


Problem 4

a) Using MATLAB, show that $\delta(n)+\delta(n-1)=u(n)-u(n-2)$



b) By hand and with sketch(es), show that $u(n-1)=\sum_{k=-\infty}^{n}\delta(k-1)$



Problem 5

5. Show that a "necessary and sufficient condition" for a causal LTI system to be BIBO stable is:

$$\sum_{n=-\infty}^{\infty}|h(n)|\leq M<\infty$$

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ |y(n)| &\leq \sum_k |h(k)||x(n-k)| \\ &\leq M_x \sum_k |h(k)| \end{aligned}$$

where $|x(n-k)|\leq M_x$. Therefore, $|y(n)|<\infty$ for all n, if and only if

$$\sum_k |h(k)|<\infty.$$

Problem 6

6. Compute (using MATLAB's conv.m function) and plot (using the stem.m capability) the convolutions $x(n)*h(n)$ and $h(n)*x(n)$ for the pairs of signals shown below:

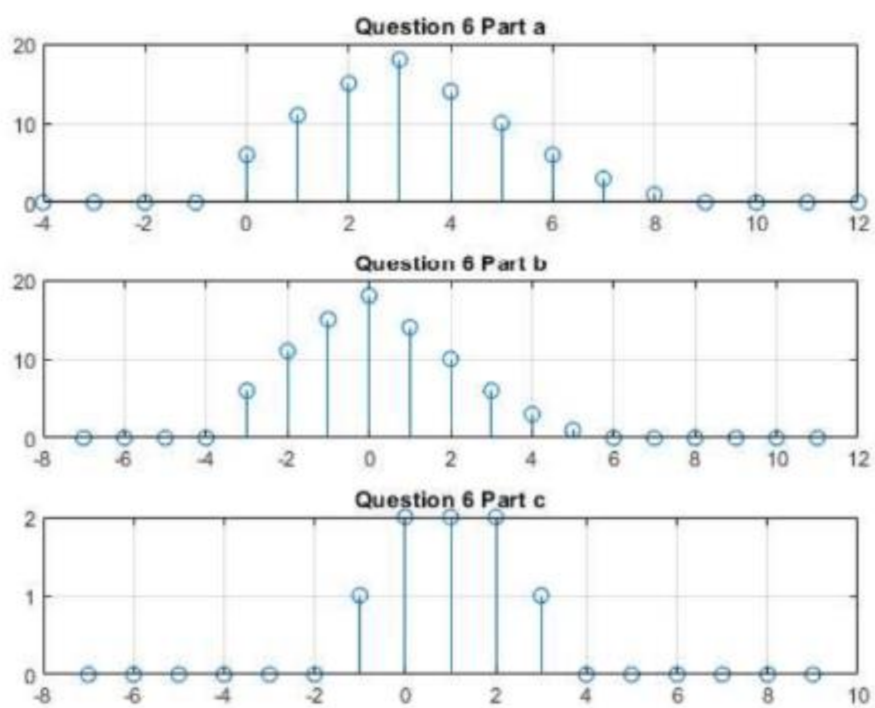
```
clear all;
clear graph;

x = [0 0 1 1 1 1 0]; nx = -2:4;
h = [0 0 6 5 4 3 2 1 0 0 0]; nh = -2:8;
subplot(3,1,1)
[y,ny] = conv_m(x, nx, h, nh)
[y1,ny1] = conv_m(h, nh, x, nx)
stem(ny,y)
grid on
title('Question 6 Part a')
x = [0 0 1 1 1 1 0 0]; nx = -2:5;
h = [0 0 6 5 4 3 2 1 0 0 0 0]; nh = -5:6;
[y,ny] = conv_m(x, nx, h, nh)
[y1,ny1] = conv_m(h, nh, x, nx)
subplot(3,1,2)
stem(ny,y)
grid on
title('Question 6 Part b')
x = [0 0 0 0 0 1 1 1 1 0 0]; nx = -2:8;
h = [0 1 1 0 0 0 0]; nh = -5:1;
[y,ny] = conv_m(x, nx, h, nh)
[y1,ny1] = conv_m(h, nh, x, nx)
subplot(3,1,3)
stem(ny,y)
grid on
title('Question 6 Part c')

nyb = nx(1)+nh(1); nye = nx(length(x)) + nh(length(h));

ny = [nyb:nye];

y = conv(x,h);
```



Problem 7

7. Consider a system with the impulse response:

$$h(n)=\begin{cases}\left(\frac{1}{2}\right)^n, & 0\le n\le 4\\ 0, & \text{elsewhere}\end{cases}$$

Determine the input $x(n)$ for $0\le n\le 8$ that will generate the following output:

$$y=[1,\ 2,\ 2.5,\ 3,\ 3,\ 3,\ 2,\ 1,\ 0]$$

And

$$n=[0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9]$$

$$\begin{aligned}
h(n) &= \left\{\underset{\uparrow}{1},\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}\right\}\\
y(n) &= \left\{\underset{\uparrow}{1},2,2.5,3,3,3,2,1,0\right\}\\
x(0)h(0) &= y(0)\Rightarrow x(0)=1\\
\frac{1}{2}x(0)+x(1) &= y(1)\Rightarrow x(1)=\frac{3}{2}
\end{aligned}$$

By continuing this process, we obtain

$$x(n)=\left\{1,\frac{3}{2},\frac{3}{2},\frac{7}{4},\frac{3}{2},\ldots\right\}$$
