SOLUTION SET

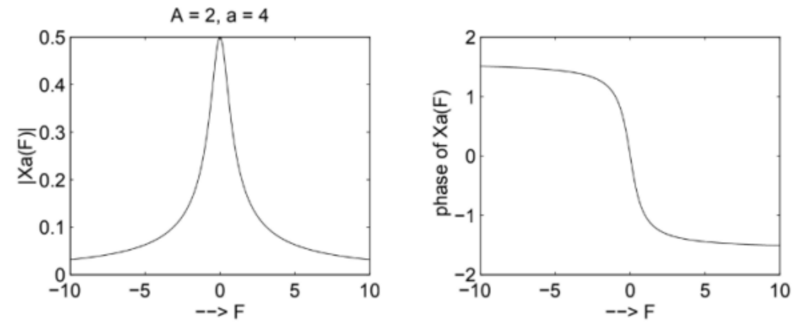
Module 3 Assignment: Frequency Analysis of Signals

Digital Signal Processing (EN.525.627.8X)

1.

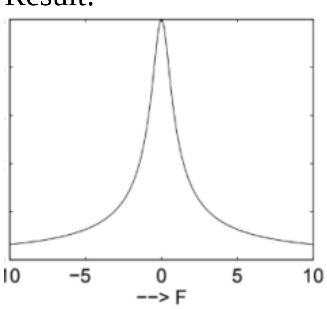
a)

$$
\begin{aligned}
x_a(t) &= Ae^{-at}u(t), \qquad a > 0 \\
X_a(F) &= \int_0^{\infty} Ae^{-at}e^{-j2\pi Ft}dt \\
&= \frac{A}{-a - j2\pi F}e^{-(a+j2\pi F)t}\Big]_0^{\infty} \\
&= \frac{A}{a + j2\pi F} \\
|X_a(F)| &= \frac{A}{\sqrt{a^2 + (2\pi F)^2}} \\
\angle X_a(F) &= -tan^{-1}(\frac{2\pi F}{a})
\end{aligned}
$$



b) Same steps as part (a):

Result:

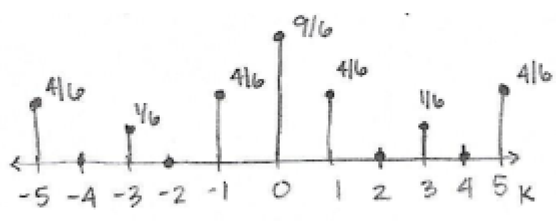


, Phase = 0

2.

a)

$$
\begin{aligned}
x(n) &= \left\{\ldots,1,0,1,2,\underset{\uparrow}{3},2,1,0,1,\ldots\right\} \\
N &= 6 \\
c_k &= \frac{1}{6}\sum_{n=0}^{5} x(n)e^{-j2\pi kn/6} \\
&= \left[3+2e^{\frac{-j2\pi k}{6}}+e^{\frac{-j2\pi k}{3}}+e^{\frac{-j4\pi k}{3}}+2e^{\frac{-j10\pi k}{6}}\right] \\
&= \frac{1}{6}\left[3+4cos\frac{\pi k}{3}+2cos\frac{2\pi k}{3}\right] \\
\text{Hence, } c_0 &= \frac{9}{6}, c_1=\frac{4}{6}, c_2=0, c_3=\frac{1}{6}, c_4=0, c_5=\frac{4}{6}
\end{aligned}
$$



, phase is zero

b)

$$
\begin{aligned}
P_t &= \frac{1}{6}\sum_{n=0}^{5}|x(n)|^2 \\
&= \frac{1}{6}(3^2+2^2+1^2+0^2+1^2+2^2)
\end{aligned}
$$

, Pt = 19/6

$$
\begin{aligned}
P_f &= \sum_{n=0}^{5}|c(n)|^2 \\
&= \left[(\frac{9}{6})^2+(\frac{4}{6})^2+0^2+(\frac{1}{6})^2+0^2+(\frac{4}{6})^2)\right]
\end{aligned}
$$

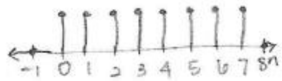
, Pf = 19/6

3. Compute the Fourier transform of:

a) $x(n)=u(n)-u(n-8)$

a) $x(n) = u(n) - u(n-8)$

$x(n)$



aka $x(n) = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & \text{otherwise} \end{cases}$

$$X(\omega) = \sum_{n=0}^{7} \overset{(1)}{x(n)}\, e^{-j\omega n}$$

$$\sum_{n=0}^{7} e^{-j\omega n}$$

$$X(\omega) = 1 + e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega} + e^{-6j\omega} + e^{-7j\omega}$$

$$\boxed{X(\omega) = \frac{1-e^{-8j\omega}}{1-e^{-j\omega}}} \leftarrow \frac{1-r^N}{1-r}$$

b) $x(n)=2^n u(-n)$

b) $x(n) = 2^n u(-n)$

aka $\begin{cases} 2^n & -\infty \le n \le 0 \\ 0 & \text{elsewhere} \end{cases}$

$$X(\omega) = \sum_{n=-\infty}^{0} (2^n)\, e^{-j\omega n}$$

$$X(\omega) = 1 + \frac{1}{2e^{-j\omega}} + \frac{1}{2e^{-2j\omega}} + \frac{1}{2e^{-3j\omega}} + \ldots$$

$$\frac{1}{1-\frac{e^{j\omega}}{2}} \quad \frac{(2)}{(2)} \leftarrow \frac{1}{1-\frac{1}{2e^{-j\omega}}} \quad \Leftarrow \frac{a}{1-r}$$

$$\frac{2}{2-\frac{\cancel{2}e^{j\omega}}{\cancel{2}}} \qquad \boxed{X(\omega) = \frac{2}{2-e^{j\omega}}}$$

c) $x(n)=\left(\frac{1}{3}\right)^n u(n+2)$

c) $x(n) = \left(\frac{1}{3}\right)^n u(n+2)$ aka $\begin{cases} \left(\frac{1}{3}\right)^n & -2 \le n \le \infty \\ 0 & \text{elsewhere} \end{cases}$

$$\frac{a}{1-r} \qquad X(\omega) = \sum_{n=-2}^{\infty} \left(\frac{1}{3}\right)^n e^{-j\omega n}$$

$$\sum_{n=-2}^{\infty} \left(\frac{1}{3e^{j\omega}}\right)^n$$

$$\frac{\left(\frac{1}{3e^{j\omega}}\right)^{-2}}{1-\left(\frac{1}{3e^{j\omega}}\right)} = \frac{(3e^{j\omega})^2}{1-\frac{1}{3}e^{-j\omega}}$$

$$\left(\frac{1}{3e^{j\omega}}\right)^{-2} + \left(\frac{1}{3e^{j\omega}}\right)^{-1} + 1 + \frac{1}{3e^{j\omega}} + \left(\frac{1}{3e^{j\omega}}\right)^{2} + \ldots$$

$$X(\omega) = \frac{(3e^{j\omega})^2}{1-\frac{1}{3}e^{-j\omega}}$$

d) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$ $X(\omega) = \sum_{n=-2}^{2} x(n) e^{-j\omega n}$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$2j\sin\theta = e^{j\theta} - e^{-j\theta}$

$= -2e^{2j\omega} - 1e^{j\omega} + e^{-j\omega} + 2e^{-2j\omega}$

$-2(e^{2j\omega} - e^{-2j\omega}) + -1(e^{j\omega} - e^{-j\omega})$

$-2(2j\sin 2\omega) - 2j\sin\omega$

$-4j\sin 2\omega - 2j\sin\omega$

$\boxed{X(\omega) = -2j(2\sin 2\omega - \sin\omega)}$

4.

a) $X(0)$ = -1

$$X(0) = \sum_{n=-2}^{2} x(n) * e^{0}$$

$X(0) = \sum x(n) * e^{0}$ = -1

b) $\angle X(\omega)$ = π

$$X(w) = \sum_{n=-2}^{2} x(n) * e^{-jwn}$$

solve to get π

c) $\int_{-\pi}^{\pi} X(\omega) d\omega$ = -6π

$$x(n) = 1/2\pi \int_{-\pi}^{\pi} X(w) e^{jwn}$$

n=0, solve to get -6 π

d) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$ = 38π

$$\sum_{n=-2}^{2} |x(n)|^2 = 1/2\pi \int_{-\pi}^{\pi} |X(w)|^2$$

solve to get 38 π