Communication Systems

Chapter 6 Principles of Digital Data Transmission

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March 25, 2024

1 Baseband Line Coding

Digital data can be transmitted with multiple different line codes (such as on/off, polar, bipolar). They each have their own advantages but the properties a line code should have is

- **Low Bandwidth** - Transmission bandwidth should be as low as possible.
- **Power Efficiency** - For a given bandwidth and specified detection error rate, the power should be as low as possible.
- **Error Detection and Correction** - Ability to detect and correct errors (CH13).
- **Favorable Spectral Density** - It is desireable to have 0 PSD at DC.
- **Adequate Timing Content** - It should be able to extract timing/clock information.
- **Transparency** - It should be possible to correctly receive any pattern of 0, 1's.

1.1 PSD of Various Baseband Line Codes

In this section $p(t)$ is a generic pulse with Fourier Transform of $P(f)$. The line code symbol at time k is a_k and $T_b = 1/R_b$ (bit rate).

$$y(t) = \sum a_k p(t - kT_b) \tag{1}$$

A way to describe the PSD of $y(t)$, not depending on $p(t)$ is to select a PAM signal $x(t)$ that uses unit impulses for the basic then pass it through a filter with impulse response $h(t) = p(t)$. The PSD will be

$$S_y(f) = |P(f)|^2 S_x(f) \tag{2}$$

A very good derivation on page 372 of the autocorrelation $\mathcal{R}(\tau)$ where

$$\mathcal{R}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t-\tau)dt \tag{3}$$

$$\mathcal{R}(\tau) = \frac{1}{T_b} \sum_n R_n \delta(\tau - nT_b) \tag{4}$$

where ($|\overline{x}|$ is the time average)

$$R_n = \lim_{N\to\infty} \frac{T_b}{T} \sum_k a_k a_{k+n} = |\overline{a_k a_{k+n}}| \tag{5}$$

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2 = |\overline{a_k^2}| \tag{6}$$

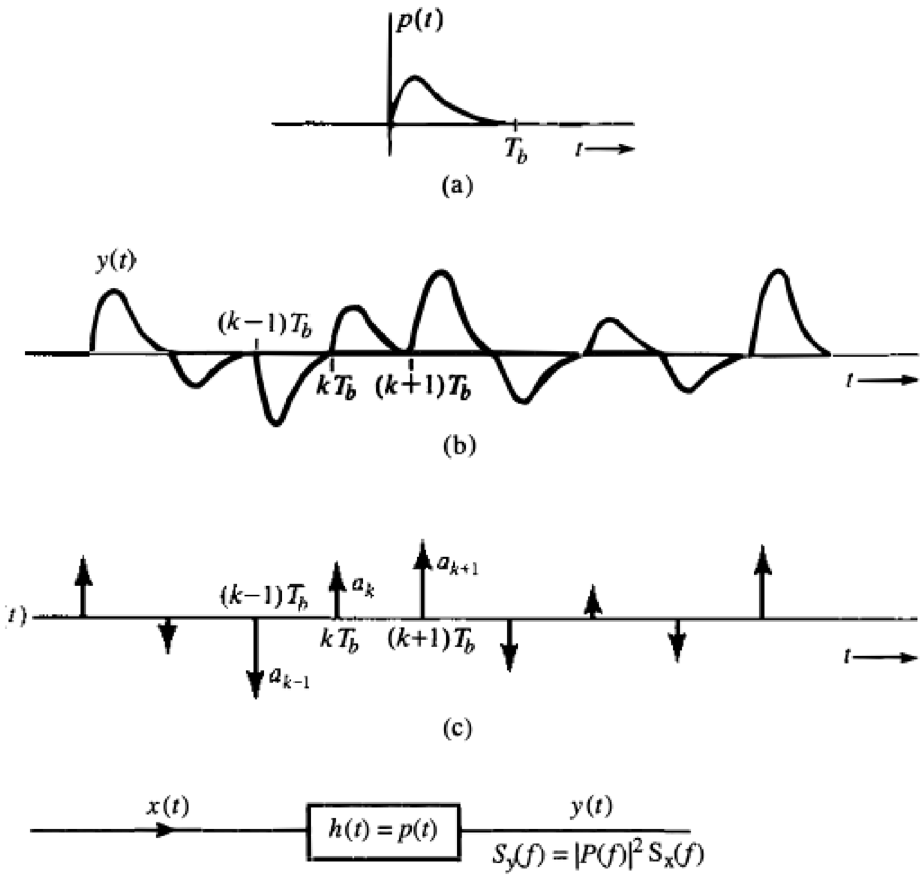

Figure 1: PSD of $p(t)$

The PSD $S_x(f)$ is the fourier transform of $\mathcal{R}(\tau)$,

$$S_x(f) = \frac{1}{T_b}\sum_n R_n e^{-jn2\pi f T_b} \tag{7}$$

Since $\mathcal{R}(\tau)$ is an even function of τ

$$S_x(f) = \frac{1}{T_b}[R_o + 2\sum_{n=1}^{\infty} R_n \cos n2\pi f T_b] \tag{8}$$

and finally

$$S_y(f) = |P(f)|^2 S_x(f) \tag{9}$$

$$S_y(f) = \frac{|P(f)|^2}{T_b}[\sum_n R_n e^{-jn2\pi f T_b}] \tag{10}$$

$$S_y(f) = \frac{|P(f)|^2}{T_b}[R_0 + 2\sum_{n=1}^{\infty} R_n \cos n2\pi f T_b] \tag{11}$$

1.2 Polar Signaling

In polar signaling, a binary 1 is sent as $p(t)$ and a binary 0 is sent as $-p(t)$. In this case a_k is equally likely to be 1 or -1 and $a_k^2 = 1$.

$$R_0 = \lim_{N\to\infty}\frac{1}{N}\sum_k a_k^2 = \lim_{N\to\infty}\frac{1}{N}(N) = 1 \tag{12}$$

Equation 5 shows that R_n is the time average of $a_k a_{k+n}$ and since a_k is equally likely to be 1 or -1,

$$R_1 = \lim_{N\to\infty} \frac{1}{N}[\frac{N}{2}(1) + \frac{N}{2}(-1)] = 0 \tag{13}$$

and therefore

$$R_n = 0 \quad n > 0 \tag{14}$$

$$S_y(f) = \frac{|P(f)|^2}{T_b} R_0 = \frac{|P(f)|^2}{T_b} \tag{15}$$

1.3 Constructing a DC Null in PSD by Pulse Shaping

Since $S_y(f)$ has a multiplicative factor of $|P(f)|^2$ we can force the PSD to have a DC null such that $p(t)$ doenst have a DC offset.

$$P(0) = \int_{-\infty}^{\infty} p(t)dt \tag{16}$$

If the area under $p(t)$ is zero then there is a DC null.

2 Digital Carrier Systems

2.1 Basic Binary Carrier Modulations

2.1.1 ASK

ASK is known as **Amplitude Shift Keying**. The on-off baseband signal $m(t)$ is $(a_k = 0, 1)$

$$m(t) = \sum_k a_k p(t - kTb) \quad \text{where} \quad p(t) = \Pi(\frac{t - T_b/2}{T_b}) \tag{17}$$

The ASK signal is

$$\varphi_{ASK}(t) = m(t)\cos\omega_c t \tag{18}$$

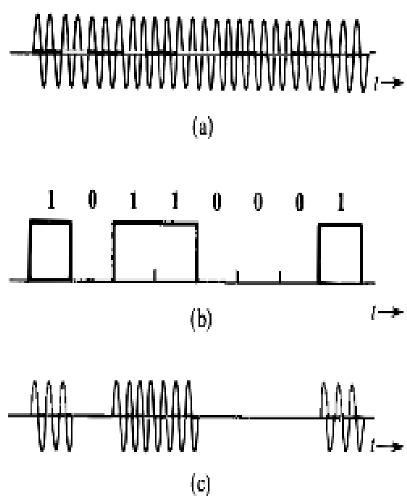

Figure 2: On-off keyed $m(t)$ modulated by ASK

2.1.2 PSK

PSK is known as **Phase Shift Keying**. If a baseband signal $m(t)$ is polar, transmitting a 1 would produce a pulse $p(t)\cos\omega_c t$ and transmitting a 0 would produce $-p(t)\cos\omega_c t$. This can be rewritten as

$$-p(t)\cos\omega_c t = p(t)\cos(\omega_c t + \pi) \tag{19}$$

which is a signal that is π radians (180deg) apart in phase. So when the line code is polar ($a_k = -1, 1$), then

$$\varphi_{PSK}(t) - m(t)\cos\omega_c t \qquad m(t) = \sum_k a_k p(t - kT_b) \tag{20}$$

2.1.3 FSK

FSK is known as **Frequency shift keying**. This is done by varying the instantaneous frequency, where a 0 is transmitted with ω_{c_0} and a 1 is transmitted with ω_{c_1}. FSK can be viewed as a sum of two alternating ASK signals, one with carrier ω_{c_0} and the other with carrier ω_{c_1}. The binary ASK expression to write a FSK signal is

$$\varphi_{FSK}(t) = \sum_k a_k p(t - kT_b)\cos\omega_{c_1} t + \sum_k (1 - a_k) p(t - kT_b)\cos\omega_{c_0} t \tag{21}$$

where $a_k = 0, 1$ is on off keying. The FSK is the superposition of two ASK signals with different frequencies but complementary amplitudes.

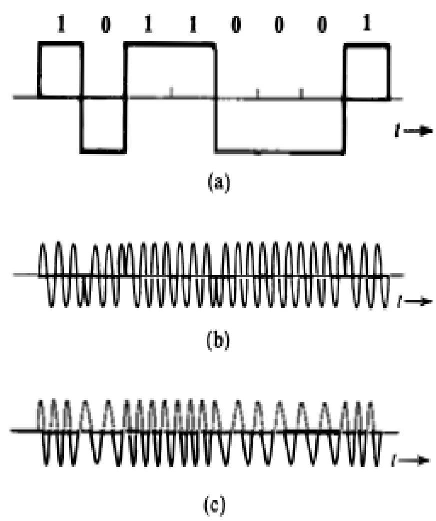

Figure 3: Polar $m(t)$ modulated with PSK then FSK

2.2 PSD of Digital Carrier Modulation

In the current section it was shown that ASK, PSK and FSK can be written in the format of $m(t)\cos\omega_c t$.

$$\varphi(t) = m(t)\cos\omega_c t \tag{22}$$

The PSD of $\varphi(t)$ is

$$S_\varphi(f) = \lim_{T\to\infty} \frac{|\Psi_T(f)|^2}{T} \tag{23}$$

where $\Psi_T(f)$ if the Fourier transform of the truncated signal

$$\varphi_T(t) = \varphi(t)[u(t + T/2) - u(t - T/2)] \tag{24}$$

$$\varphi_T(t) = m(t)[u(t + T/2) - u(t - T/2)]\cos\omega_c t \tag{25}$$

$$\varphi_T(t) = m_T(t)cos\omega_c t \tag{26}$$

The PSD of $m_T(t)$ is

$$S_m(f) = \lim_{T\to\infty} \frac{|M_T(f)|^2}{T} \tag{27}$$

and then applying the frequency shift from the cos term,

$$\Psi_T(f) = \frac{1}{2}[M_T(f - f_c) + M_T(f + f_c)] \tag{28}$$

so finally the PSD of the modulated signal is

$$S_\varphi(f) = \lim_{T\to\infty} \frac{1}{4T}|M_T(f + f_c) + M_T(f - f_c)|^2 \tag{29}$$

$M(f + f_c)$ and $M(f - f_c)$ have zero overlap as $T \to \infty$ if f_c is larger than the bandwidth of $M(f)$, so

$$S_\varphi(f) = \frac{1}{4}S_M(f + f_c) + \frac{1}{4}S_M(f - f_c) \tag{30}$$

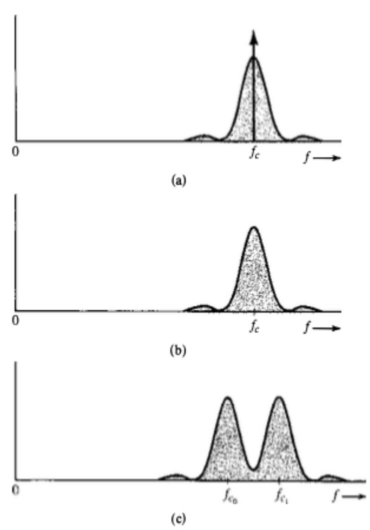

Figure 4: PSD of ASK (on-off), PSK and FSK (polar)

For FSK,

$$\varphi_{FSK}(t) = \sum_k a_k p(t - kT_b)\cos\omega_{c_1} t + \sum_k (1 - a_k)p(t - kT_b)\cos\omega_{c_0} t \tag{31}$$

where baseband signal $m_1 = \sum_k a_k p(t - kT_b)$ and $m_0 = \sum_k (1 - a_k)p(t - kT_b)$ the PSD would be

$$S_{FSK}(f) = \frac{1}{4}S_{M_0}(f + f_c) + \frac{1}{4}S_{M_0}(f - f_c) + \frac{1}{4}S_{M_1}(f + f_c) + \frac{1}{4}S_{M_1}(f - f_c) \tag{32}$$

2.3 Demodulation

2.3.1 ASK Detection

ASK can be demodulated both coherently (using synchronous detection) or non-coherently with an envelope detector. The coherent method takes more processing but has better performance, especially when the SNR is low. When the SNR is high, envelope detection works just as well and is simpler, so coherent demodulation is not used often.

2.3.2 FSK Detection

Since binary FSK can be viewed as two ASK signals (a_k, b_k) and carriers, f_{c_0} and f_{c_1}, it can be demodulated coherently or non-coherently. In non-coherent detection, the incoming signal is applied to a pair of narrowband filters, $H_0(f)$ and $H_1(f)$, tuned to f_{c_0} and f_{c_1}. Each filter is followed by an envelope detector. The two methods are shown in the next figure.

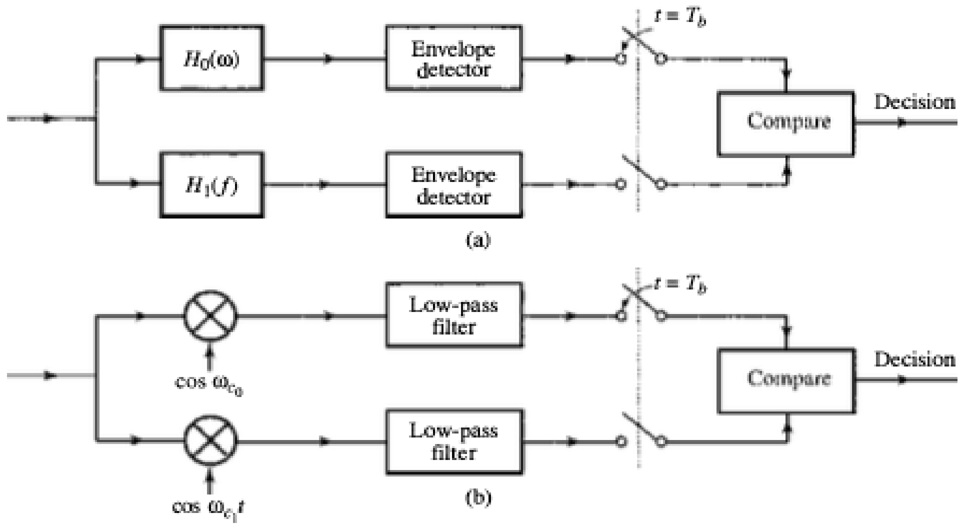

Figure 5: Non-coherent and Coherent Demodulation for FSK

2.3.3 PSK Detection

In binary PSK, a 1 is transmitted by a pulse $A\cos\omega_c t$ and a 0 is transmitted with a pulse $A\cos(\omega_c t + \pi) = -A\cos\omega_c t$. The information resides in the phase, as such these signals cannot be demodulated with an envelope detector. The demodulator is shown in the next figure.

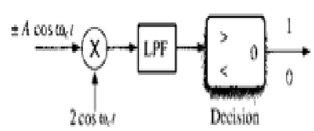

Figure 6: Coherent Demodulation for PSK

2.3.4 Differential PSK

DPSK, differential PSK uses differential detection, where the receiver detects the relative phase change between successively modulated phases: θ_k and θ_{k-1}. Since the phase in binary PSK is 0 or π, the transmitter can encode the information into the phase difference $\theta_k - \theta_{k-1}$. For example a phase difference of 0 would represent 0b and a phase difference of π would represent 1b. This is known as differential encoding.

In the demodulator, we avoid generation of an LO since the signal is the carrier with a sign ambiguity. We can delay the signal by T_b. If the pulse is identical to the previous one, the product

$$y(t) = A^2\cos^2\omega_c t = (A^2/2)(1 + \cos(2\omega_c t)) \tag{33}$$

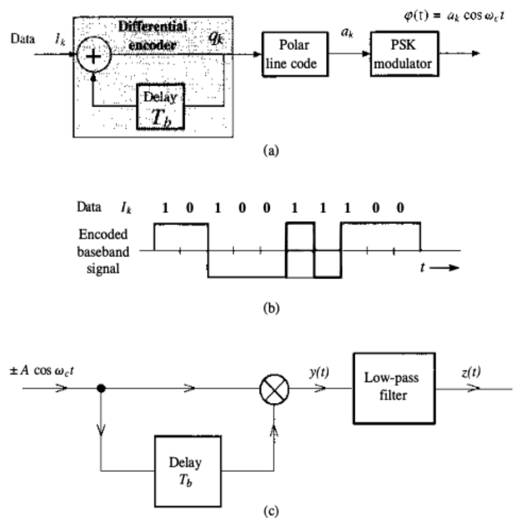

Figure 7: DPSK

and low pass filtered to

$$z(t) = A^2/2 \tag{34}$$

and the bit is decided to be 0. On the other hand if the pulse and the previous pulse are opposite polarity,

$$y(t) = -A^2 \cos^2 \omega_c t = -(A^2/2)(1 + \cos(2\omega_c t)) \tag{35}$$

and low pass filtered to

$$z(t) = -A^2/2 \tag{36}$$

and the bit is decided to be 1.