STAT 716 - Class 9 - 2018-11-05

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Polynomial regression

global function - this can be an issue - especially at the outer edges

Step Functions

Step Functions localize everything, cut points categorical dummy variables

Basis functions

general funcionts

Regression Splines

piecewise polinomial regression

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piecwise cubic regression where to put knots?

spline - continuous on second derivative

Natural splines

the function is required to be linear at the boundary (in the region where X is smaller than the smallest knot, or larger than the largest knot).

Generalized Additive Models

We have discussed regular linear regression. We can standarize into GAM with the following equation:

$$y_i = \sum_{j=1}^{p} f_j(x_{ij}) + \epsilon_i$$

Lab

```r
library(ISLR)
first_fit <- lm(wage ~ poly(age, 4), data = Wage)

coef(summary(first_fit))
```

```
##                  Estimate Std. Error    t value     Pr(>|t|)
## (Intercept)    111.70361  0.7287409 153.283015 0.000000e+00
## poly(age, 4)1  447.06785 39.9147851  11.200558 1.484604e-28
## poly(age, 4)2 -478.31581 39.9147851 -11.983424 2.355831e-32
## poly(age, 4)3  125.52169 39.9147851   3.144742 1.678622e-03
## poly(age, 4)4  -77.91118 39.9147851  -1.951938 5.103865e-02
```

```r
coef(summary(lm(wage ~age + I(age^2) + I(age^3) + I(age^4),
                data = Wage)))
```

```
##                  Estimate   Std. Error  t value     Pr(>|t|)
## (Intercept) -1.841542e+02 6.004038e+01 -3.067172 0.0021802539
## age          2.124552e+01 5.886748e+00  3.609042 0.0003123618
## I(age^2)    -5.638593e-01 2.061083e-01 -2.735743 0.0062606446
## I(age^3)     6.810688e-03 3.065931e-03  2.221409 0.0263977518
## I(age^4)    -3.203830e-05 1.641359e-05 -1.951938 0.0510386498
```

Why don't the match? See here for an example: https://stackoverflow.com/questions/29999900/poly-in-lm-difference-between-raw-vs-orthogonal

```r
coef(summary(lm(wage ~poly(age, 4, raw = T),
                data = Wage)))
```

```
##                              Estimate   Std. Error  t value     Pr(>|t|)
## (Intercept)             -1.841542e+02 6.004038e+01 -3.067172 0.0021802539
## poly(age, 4, raw = T)1  2.124552e+01 5.886748e+00  3.609042 0.0003123618
## poly(age, 4, raw = T)2 -5.638593e-01 2.061083e-01 -2.735743 0.0062606446
## poly(age, 4, raw = T)3  6.810688e-03 3.065931e-03  2.221409 0.0263977518
## poly(age, 4, raw = T)4 -3.203830e-05 1.641359e-05 -1.951938 0.0510386498
```

Cuts

```r
coef(summary(lm(wage ~cut(age, 4),
                data = Wage)))
```

```
##                         Estimate Std. Error   t value     Pr(>|t|)
## (Intercept)            94.158392   1.476069 63.789970 0.000000e+00
## cut(age, 4)(33.5,49]   24.053491   1.829431 13.148074 1.982315e-38
## cut(age, 4)(49,64.5]   23.664559   2.067958 11.443444 1.040750e-29
## cut(age, 4)(64.5,80.1]  7.640592   4.987424  1.531972 1.256350e-01
```

Splines

```r
library(splines)
library(dplyr)

# cubic splines

lm(wage~bs(age,knots=c(25,40,60)),data=Wage) %>%
  coef
```

```
##                      (Intercept) bs(age, knots = c(25, 40, 60))1
##                         60.49371                         3.98050
## bs(age, knots = c(25, 40, 60))2 bs(age, knots = c(25, 40, 60))3
```

```
##                          44.63098                        62.83879
## bs(age, knots = c(25, 40, 60))4 bs(age, knots = c(25, 40, 60))5
##                          55.99083                        50.68810
## bs(age, knots = c(25, 40, 60))6
##                          16.60614
```

We have 3 knots - how many degrees of freedom?

If it's piecewise cubic how many degrees of freedom? $4 + 4 + 4 + 4 = 16$

What do splines add? 1. Continuity 2. Continuity on 1st derivative 3. Continuiuty on 2nd derivative

Each constraint is a degree of freedom

remove $3 + 3 + 3 = 9$

Remaining $= 7$ which is what we see

```r
lm(wage~bs(age, df = 6),data=Wage) %>%
  coef
```

```
##      (Intercept) bs(age, df = 6)1 bs(age, df = 6)2 bs(age, df = 6)3
##         56.31384         27.82400         54.06255         65.82839
## bs(age, df = 6)4 bs(age, df = 6)5 bs(age, df = 6)6
##         55.81273         72.13147         14.75088
```

```r
attributes(bs(Wage$age, df = 6))
```

```
## $dim
## [1] 3000    6
##
## $dimnames
## $dimnames[[1]]
## NULL
##
## $dimnames[[2]]
## [1] "1" "2" "3" "4" "5" "6"
##
##
## $degree
## [1] 3
##
## $knots
##   25%   50%   75%
## 33.75 42.00 51.00
##
## $Boundary.knots
## [1] 18 80
##
## $intercept
## [1] FALSE
##
## $class
## [1] "bs"     "basis"  "matrix"
```

```r
lm(wage~bs(age),data=Wage) %>%
  coef
```

```
## (Intercept)    bs(age)1    bs(age)2    bs(age)3
```

```
##     58.68868    102.64368     48.76204     40.80330
```

```r
lm(wage~ns(age, knots=c(25,40,60)),data=Wage) %>% coef() %>% length()
```

```
## [1] 5
```

```r
lm(wage~bs(age, knots=c(25,40,60)),data=Wage) %>% coef() %>% length()
```

```
## [1] 7
```

```r
lm(wage~ns(age, knots=c(25,40,60)),data=Wage) %>% coef() %>% length()
```

```
## [1] 5
```

How many degrees of freedom do we get from add the natural spline contraint

only 2 degrees of freedom on the ends

- 4 degrees of freedom

2 + 4 + 4 + 2

- 2 - 3 - 2

```r
2 + 4 + 4 + 2 +
  - 2 - 3 - 2
```

```
## [1] 5
```

```r
ns_mod <- lm(wage~ns(age, df = 6),data=Wage)
bs_mod <- lm(wage~bs(age, df = 6),data=Wage)
poly_mod <- lm(wage~poly(age,6),data=Wage)

length(coef(ns_mod))
```

```
## [1] 7
```

```r
length(coef(bs_mod))
```

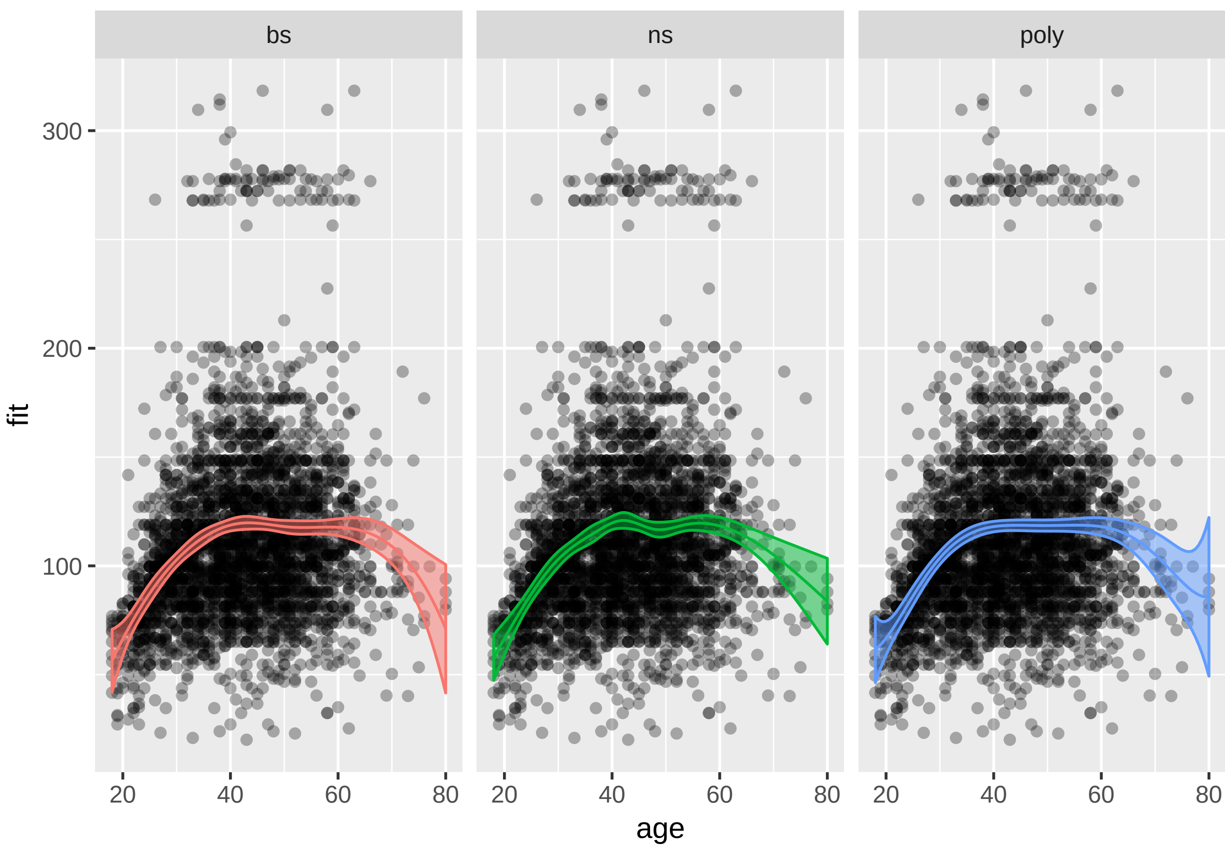
```
## [1] 7
```

```r
length(coef(poly_mod))
```

```
## [1] 7
```

```r
age_df  <- data.frame(age = seq(min(Wage$age), max(Wage$age), length.out =  100))

predict_se_fit <- function(model, newdata,...){

  predictions <- predict(model, newdata, se.fit = T)
  data.frame(
    fit   = predictions$fit,
    se.fit = predictions$se.fit
  )
}


all_models  <- bind_rows(
  age_df %>%
    nest() %>%
    mutate(mod = map(data, ~predict_se_fit(ns_mod, .)),
           model = "ns") %>%
```

```
    unnest(),
  age_df %>%
    nest() %>%
    mutate(mod = map(data, ~predict_se_fit(bs_mod, .)),
           model = "bs") %>%
    unnest(),
  age_df %>%
    nest() %>%
    mutate(mod = map(data, ~predict_se_fit(poly_mod, .)),
           model = "poly") %>%
    unnest()

) %>%
  mutate(fit_low = fit - 2*se.fit,
         fit_high = fit + 2*se.fit)

all_models %>%
  ggplot(aes(x = age, y = fit, fill = model, color = model)) +
  geom_point(aes(x = age, y = wage), data = Wage, inherit.aes = F, alpha = 0.3) +
  geom_ribbon(aes(ymax = fit_high, ymin = fit_low), alpha = 0.5) +
  geom_line() +
  facet_wrap(~model) +
  theme(legend.position = "none")
```



Smoothing Splines

$$\sum_{i=1}^{n}(y_i - g(x_i))^2 + \lambda \int g''(t)^2 dt$$

Loss + penalty function

2nd dervative is a measure of roughness

The function g(x) that minimizes (7.11) can be shown to have some special properties: it is a piecewise cubic polynomial with knots at the unique values of x1,. . . ,xn, and continuous first and second derivatives at each knot. Furthermore, it is linear in the region outside of the extreme knots. In other words, the function g(x) that minimizes (7.11) is a natural cubic spline with knots at x1,. . . ,xn!

effective parameters

n to 2 as lamda goes to ininity

you can use loocv to calc

$$RSS_{cv} = \sum_{i=1}^{n}(\frac{y_i - \hat{g}_\lambda(x_i)}{1 - \{S_\lambda\}_{ii}})^2$$

$\hat{g}$ is the fitted values for the full spline

```
cv_smooth_spline <-with(Wage, smooth.spline(age,wage,cv=TRUE))
```

```
## Warning in smooth.spline(age, wage, cv = TRUE): cross-validation with non-
## unique 'x' values seems doubtful
```

```
gcv_smooth_spline <- with(Wage, smooth.spline(age,wage))
```

```
library(gam)
```

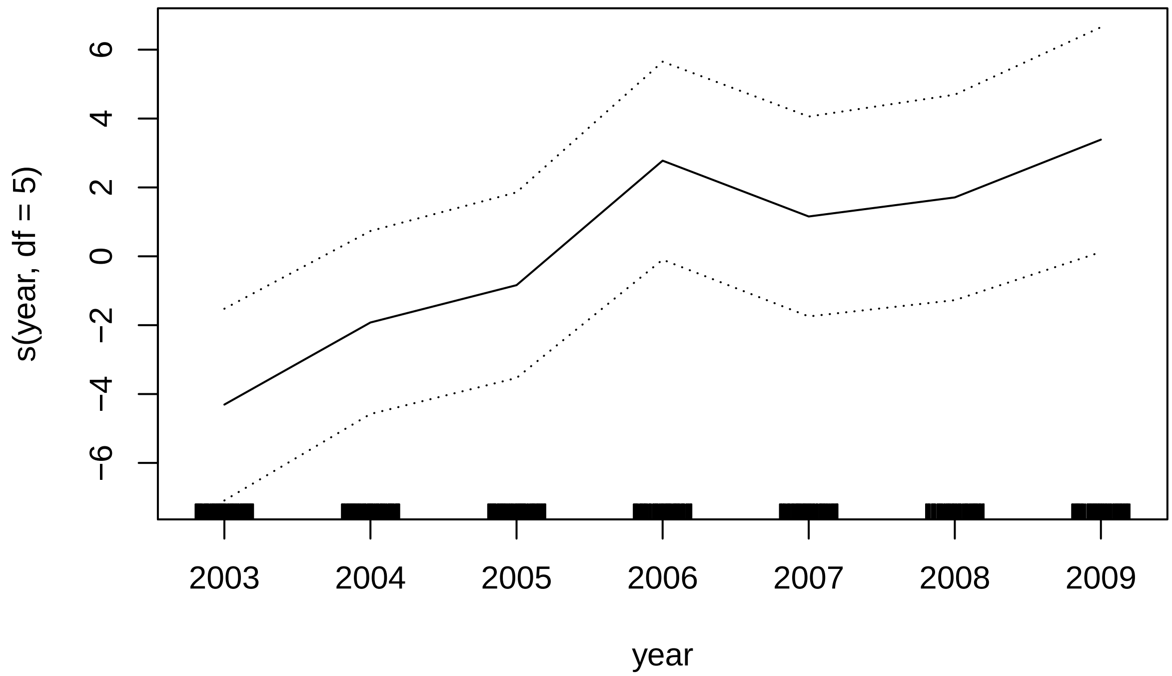
```
## Loading required package: foreach
```

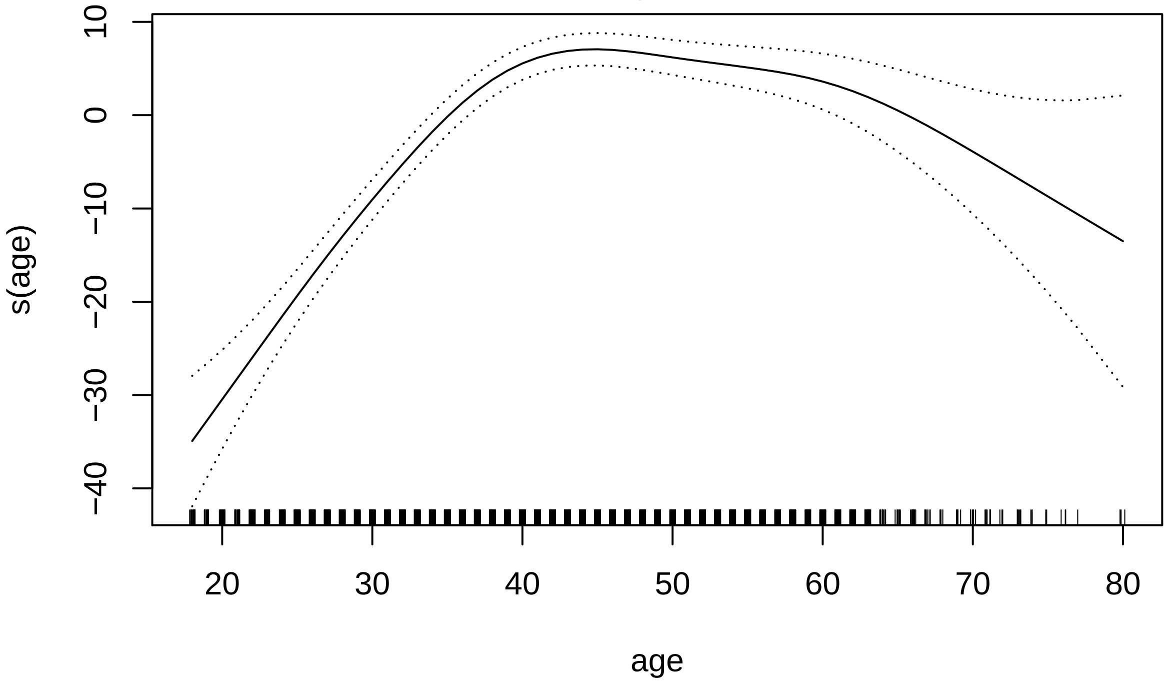
```
##
## Attaching package: 'foreach'
```

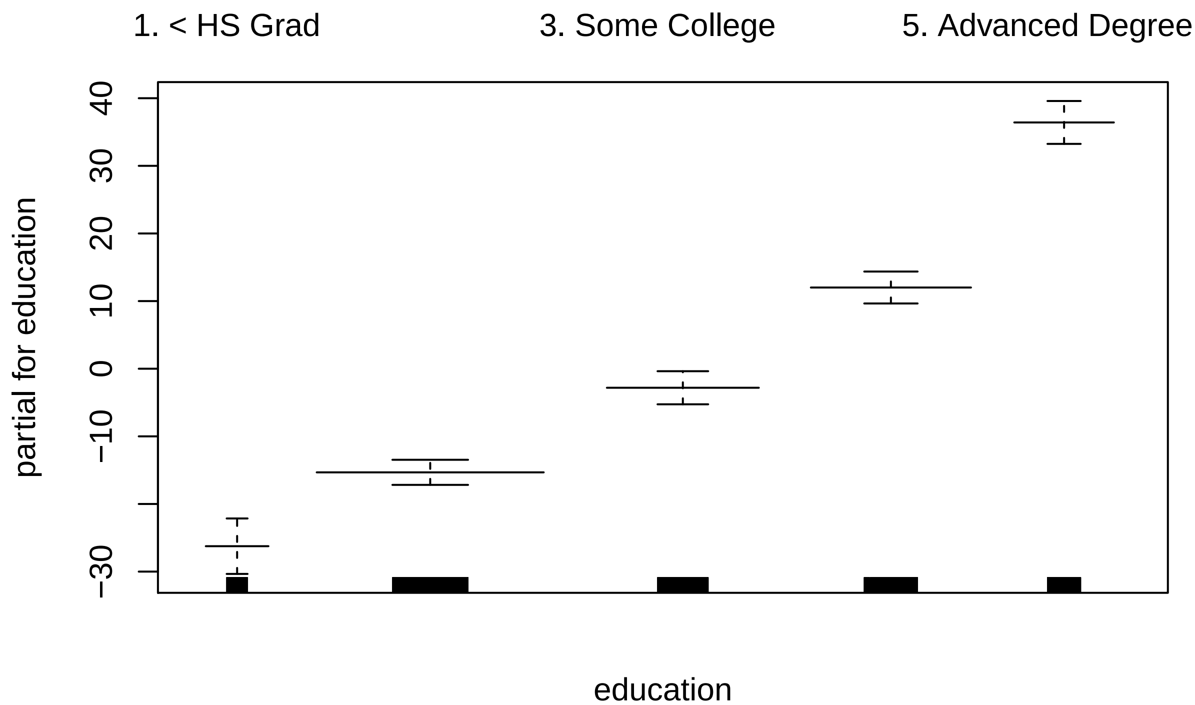
```
## The following objects are masked from 'package:purrr':
##
##     accumulate, when
```

```
## Loaded gam 1.16
```

```
gam_mod <- gam(wage ~ s(year) + s(age) + education, data = Wage)
gam_mod <- gam(wage ~ s(year, df = 5) + s(age) + education, data = Wage)
plot(gam_mod, se = T)
```


6
4
2
0
−2
−4
−6
s(year, df = 5)
2003
2004
2005
2006
2007
2008
2009
year


10
0
−10
−20
−30
−40
s(age)
20
30
40
50
60
70
80
age



Local Regression

weighted regresion,

choose points within x_0. can be linear or w/e