

Unit Connection	
<b>11/22 Completing the Square</b>	We recently finished operations with radicals and complex numbers. The previous lesson was on solving equations by taking square roots. This unit will eventually lead to the full completing the square technique to solve quadratics including radicals, complex numbers and the quadratic formula.
Desired Results	
<b>Learning Target(s): I can set up equations so one side is a binomial squared (leading to completing the square).</b>	<b>Standards:.</b> Solve equations and inequalities in one variable. 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
Learning Activities	
<p>Warm-up</p> <ul style="list-style-type: none"> <li>- New song recommendations. What is a song/artist your parents/grandparents might recommend?</li> </ul> <p>Structure of Lesson:</p> <p>There are 4 sections that build on each other and gradually get more complex, leading towards setting up a binomial square. For each section there will be one example for the whole class, time to work on that section in groups, and then a whole class discussion about patterns, shortcuts, and explanations.</p> <p>A) Review of squaring a binomial.</p> <ul style="list-style-type: none"> <li>- What do you notice about the middle and last term of the expression that you expanded?</li> <li>- How does the + or - change the expression?</li> </ul> <p>B) Starting with the first 2 terms of an expression, find a constant to add to create a binomial squared</p> <ul style="list-style-type: none"> <li>- How does the pattern/shortcut for part 1 help us to figure these out?</li> <li>- How can you check?</li> <li>- What patterns/shortcuts did you see?</li> </ul> <p>C) Adding the constant to both sides of the equation to complete the square</p> <ul style="list-style-type: none"> <li>- Whatever you do to one side of the equation you need to do to the other.</li> <li>- How are you picking the constant to add?</li> <li>- Why is the constant you add always positive?</li> </ul> <p>D) Starting with an equation, manipulate the constant until you can make a binomial square.</p> <ul style="list-style-type: none"> <li>- Why do I subtract a constant only to add one back immediately after? Do you have to ?</li> <li>- How does this problem look like what we did the previous day?</li> </ul> <p>*) For groups that finish early, preview of the next days lesson</p> <ul style="list-style-type: none"> <li>- Use what you learned yesterday to solve for BOTH answers to the last 6 problems.</li> </ul>	
Evidence of Student Learning/Engagement	
<p>Check for understanding as students work on problems for each section (Students that are not working effectively with the people around them should be moved/regrouped).</p> <p>Ask them to explain their method as well as any shortcuts or strategies they may have. Are you sure they always work?</p> <p>Check that they are able to explain the process they used with their peers as well as correct each other's errors.</p>	

Name: \_\_\_\_\_

Organizing for completing the square

Expand each expression, look for patterns as you go.

Example: $(x + 3)^2$	$(x + 5)^2$
$(x - 7)^2$	$(x + 11)^2$
$(x - 2)^2$	$(x - 8)^2$
$(x + 4)^2$	$(x + 9)^2$

What patterns do you notice? Can you explain a shortcut to expanding this?

Write the number you would need to add to create a squared binomial.

Example: $x^2 + 10x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$	$x^2 - 18x + \underline{\hspace{1cm}} = (x - 9)^2$
$x^2 + 12x + \underline{\hspace{1cm}} = (x + 6)^2$	$x^2 - 6x + \underline{\hspace{1cm}} = (x - 3)^2$
$x^2 - 14x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$	$x^2 + 2x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$
$x^2 + 22x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$	$x^2 - 8x + \underline{\hspace{1cm}} = (\underline{\hspace{1cm}})^2$

What patterns do you notice? Can you explain a shortcut to expanding this?

Fill in the blanks by adding the same number to each side so that you can complete a square binomial.

<p>Example</p> $x^2 - 20x + \underline{\hspace{2cm}} = 21 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$	$x^2 + 14x + \underline{\hspace{2cm}} = 11 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$
$x^2 - 8x + \underline{\hspace{2cm}} = 30 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$	$x^2 + 6x + \underline{\hspace{2cm}} = -10 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$
$x^2 + 18x + \underline{\hspace{2cm}} = 21 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$	$x^2 - 12x + \underline{\hspace{2cm}} = 0 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$
$x^2 - 4x + \underline{\hspace{2cm}} = 21 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$	$x^2 - 22x + \underline{\hspace{2cm}} = -50 + \underline{\hspace{2cm}}$ $(\underline{\hspace{2cm}})^2 = \underline{\hspace{2cm}}$

What patterns or shortcuts do you notice?

Rewrite each equation as a squared binomial (you do not need to solve it today).

<p>Example</p> $x^2 + 8x + 7 = 9$ <p>1) Add/subtract from both sides to make space to complete the square  2) Select the number need to complete the square and add it to BOTH sides  3) Write it as a square binomial equal to a number.  *) Tomorrow we will solve after this.</p>	
$x^2 + 6x + 2 = 42$	$x^2 - 18x + 67 = 6$
$x^2 + 14x + 1 = 2$	$x^2 - 4x + 14 = 28$
$x^2 - 12x + 20 = 9$	$x^2 + 10x + 50 = 125$

Name: \_\_\_\_\_

### Solving Quadratics by completing the square (a=1)

Solve for **ALL** values of the variable in simplest radical form.

$$x^2 + 16x + 39 = 0$$

1) Add/Subtract terms from both sides until you get it in the form  $x^2 + \#x + \underline{\hspace{1cm}} = \# \underline{\hspace{1cm}}$  (include the spaces).

2) Add the same term to both sides of the equation so you can make a squared binomial (take half of the x term and square it).

3) Take the square root of both sides, remember to include + and - roots.

4) Solve both equations and simplify

$$x^2 - 10x + 35 = 1$$

$$x^2 + 8x + 10 = 6$$

Name: \_\_\_\_\_

Organizing for completing the square

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What patterns do you notice? Can you explain a shortcut to expanding this?

Fill in the blanks by adding the same number to each side so that you can complete a square binomial.

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### Completing the Square (Pro Level)

Completing the square is easiest when:

- The  $x^2$  has a coefficient of 1
- The  $x$  has a coefficient that is an even number.

It is possible to solve quadratics using completing the square but there are some extra steps and possibly some fractions.

When  $x^2$  has a coefficient greater than 1.

- Divide everything by the coefficient of  $x^2$ , this might introduce fractions.

$$3x^2 + 18x = 30x$$