EQ: What is convolution and how can it be used?

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12th Grade Computer Vision
9-12.CT.2

Lesson 00 - Introduction to Convolution

Do Now

For each list below calculate the average between adjacent numbers and create a new list with the average calculated. Round the average to the nearest whole number. For example:

[11, 66, 43, 15] → [(11 + 66) / 2, (66 + 43) / 2, (43 + 15) / 2] → [39, 55, 29]

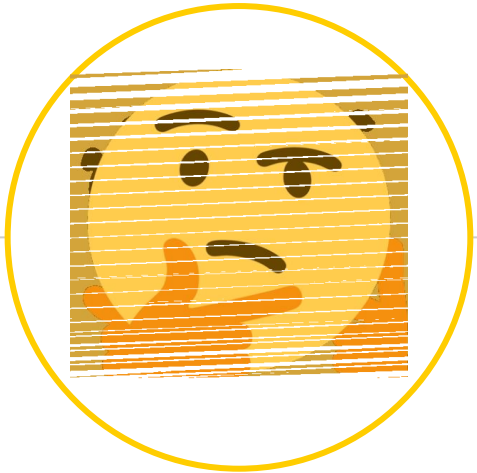
1. [78, 15, 50, 16, 72] → ??
2. [33, 52, 22, 80, 71] → ??

Mathematical Operations

- Addition and subtraction
  - two numbers → 2+3, 17-4
  - one step
- Multiplication, division, exponentiation, and modulus:
  - two numbers → 7*3, 7/3, 7^3, 7%3
  - multiple, repetitive steps -> add 7, three times
- Convolution: a ∗ b
  - two **lists** of numbers → [1, 2, 3] ∗ [4, 5, 6]
  - multiple, different steps

Convolution Operands

- First operand: a list of numbers
  - Input data that will be modified
- Second operand: a list of numbers
  - Modifier applied to the data
  - Dimensions are determined by how many or which elements we want to look at
  - Each element determines an input value's weight toward the final sum
  - Flipped *then* applied
  - Sometimes called a **kernel** or **filter**
- **NOTES:** for now, only compute for values that fit within the second operand



Why is convolution useful?

With convolution, we can:

- extract data (rolling average)
- derive information about an image (feature/object identification)
- modify images (apply filters)

Roll for confidence!

Think of convolution as a sliding window over a list.

– A wise man

Do Now Walkthrough - Question 1, Step 1

$$[78, 15, 50, 16, 72] \times [\tfrac{1}{2}, \tfrac{1}{2}] = [47, ?, ?, ?]$$

Do Now Walkthrough - Question 1, Step 2

$$[\,78,\ 15,\ 50,\ 16,\ 72\,]$$
$$\times\ [\,\tfrac{1}{2},\ \tfrac{1}{2}\,]$$
$$+$$
$$=$$
$$[\,47,\ 33,\ ?,\ ?\,]$$

Do Now Walkthrough - Question 1, Step 3

$$[\,78,\quad 15,\quad 50,\ \times\ [\,1/2,\quad 16,\ \times\ 1/2\,]\quad 72\,]$$

$$=$$

$$[\,47,\quad 50,\quad 33,\quad ?\,]$$

Do Now Walkthrough - Question 1, Step 4

$$[\,78,\ 15,\ 50,\ 16,\ 72\,] \times [\,\tfrac{1}{2},\ \tfrac{1}{2}\,]$$

$$= [\,47,\ 50,\ 33,\ 44\,]$$

Do Now Walkthrough - Question 1

[78, 15, 50, 16, 72]

$*$ (convolved with)

[½, ½]

=

[47, 50, 33, 44]

Do Now: Convolution Operands

- Question 1
  - Input: [78, 15, 50, 16, 72]
  - Kernel: [½, ½]
  - Convolution result: [47, 33, 33, 44]
- Question 2
  - Input: [33, 52, 22, 80, 71]
  - Kernel: [½, ½]
  - Convolution result: [43, 37, 51, 76]

Roll for confidence!

Convolution Practice

Questions

Work with a partner to solve the following convolutions. Pay special attention to the weights and size of the lists. For now, **only** calculate convolutions where all elements are covered (i.e. *don't* compute partial convolutions). Round to nearest integer.

1. [22, 91, 4, 5, 50] * [½, ½] ⟶ ??
2. [43, 30, 21, 34, 18] * [⅓, ⅓, ⅓] ⟶ ??
3. [37, 50, 34, 49, 6] * [1/10, 4/10, 4/10, 1/10] ⟶ ??
4. [28, 8, 48, 1, 24] * [7/10, 3/10] ⟶ ??

Convolution Practice - Answers

Don't forget to **flip** the second operand!!

1. [22, 91, 4, 5, 50] * [½, ½] → [57, 48, 5, 28]
2. [43, 30, 21, 34, 18] * [⅓, ⅓, ⅓] → [31, 28, 24]
3. [37, 50, 34, 49, 6] * [1/10, 4/10, 4/10, 1/10] → [42, 39]
4. [28, 8, 48, 1, 24] * [7/10, 3/10] → [14, 36, 15, 17]

Homework

00_homework on Google Classroom

Solve two 1D convolution problems similar to the ones we did in class. Don't forget to flip the kernel.

Due tomorrow