Mr. Mina
12th Grade Computer Vision
9-12.CT.2, 9-12.CT.3, 9-12.CT.6

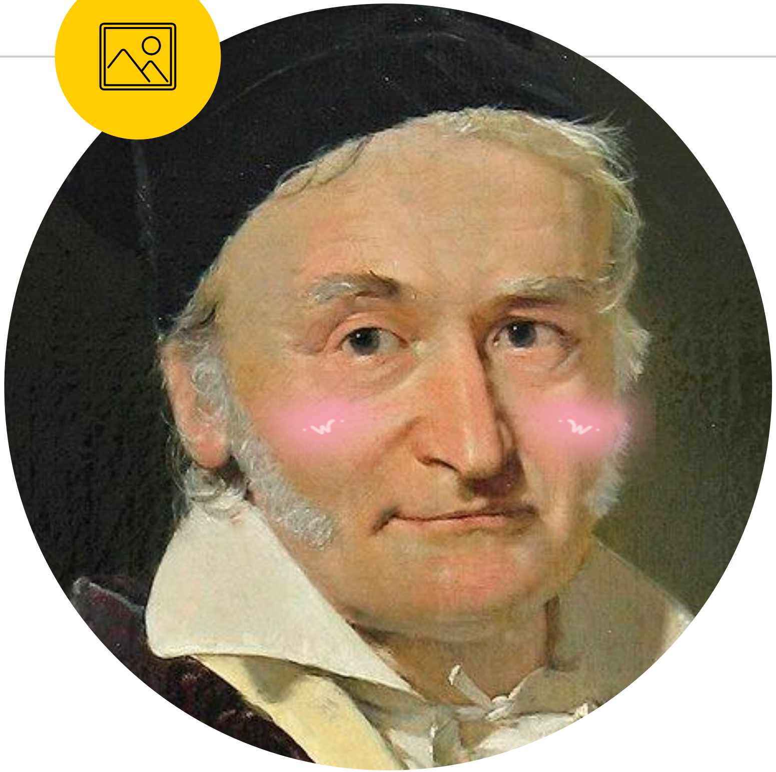
Lesson 05 - Gaussian Kernel

EQ: What is a Gaussian distribution and how do we apply it to images?

Do Now

Discuss with a partner:

- Have you ever heard of Gauss before?
- How did Homework 04 go? Any pain points?
- Do anything fun this weekend?

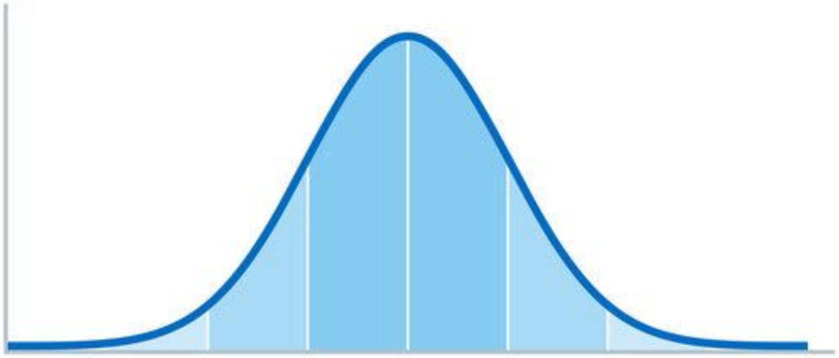


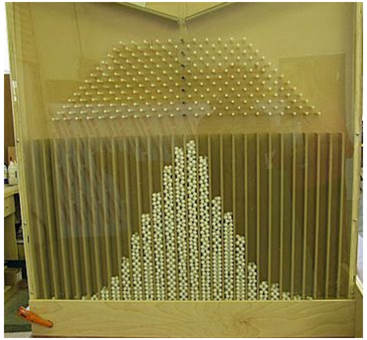
Carl Friedrich Gauss

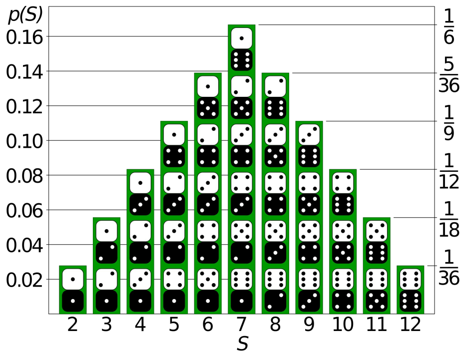
- Famous German mathematician for contributions in probability, geometry, in number theory
- Cool proofs
  - Sum of numbers 1–100 in one step (link)

Gaussian Distribution

- Goes by many names
  - Bell curve
  - Normal distribution
  - Gaussian distribution
- Models probability of independent events over time
  - Plinko machine results
  - Sum of rolling two dice







Basic Gaussian formula

$$f(x) = e^{-x\hat{}2}$$

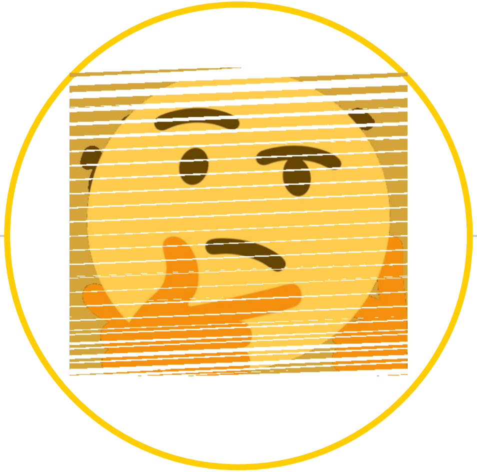
Extended Gaussian formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Gaussian formula: Explained

(Thank you 3Blue1Brown!)

Roll for confidence!



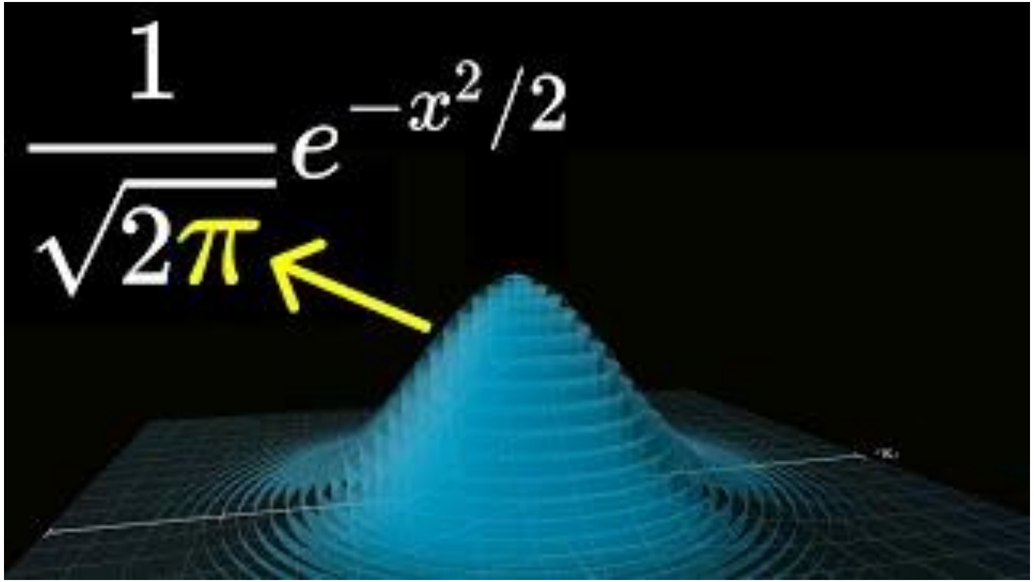
Why is a Gaussian distribution important?

Why Gaussian distribution?

- Symmetrical at the center
- Smooth falloff, values get smoothly smaller as you get to the edges
- Area under the curve = 1
- Great way for:
  - blurring without “blockiness” of block blur kernel
  - removing noise from an image

🙋🏾‍♂️ Mr. Mina, if the Gaussian distribution is one-dimensional, how do I apply it to images where there are two-dimensions???

2D Gaussian



2D Gaussian

- f(x, y) = height

Image

- f(col, row) = weight

REMEMBER: for images the x-coord is the column and the y-coord is the row

(Thank you again 3Blue1Brown!)

2D Square Gaussian Formula

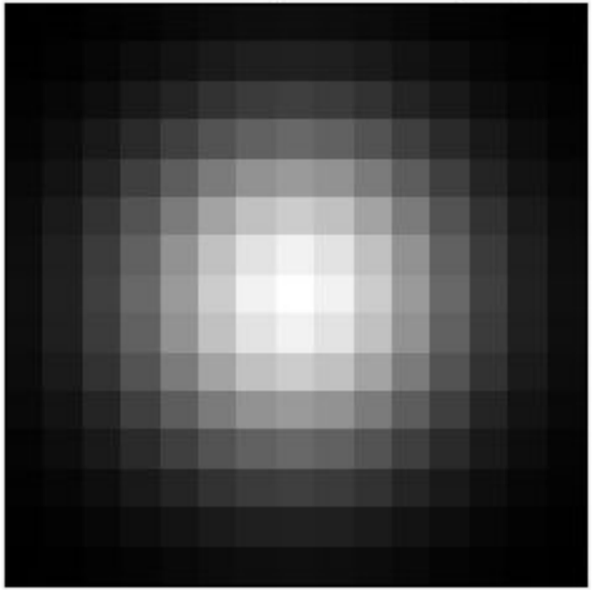
- `µ` is the center coordinate (only one since it's a square)
- `d` is the squared distance from the center (think Pythagorean Theorem)
- `exp(num)` is shorthand for `e^num`
- The first coefficient is a normalization factor
  - we can normalize **way** easier with numpy

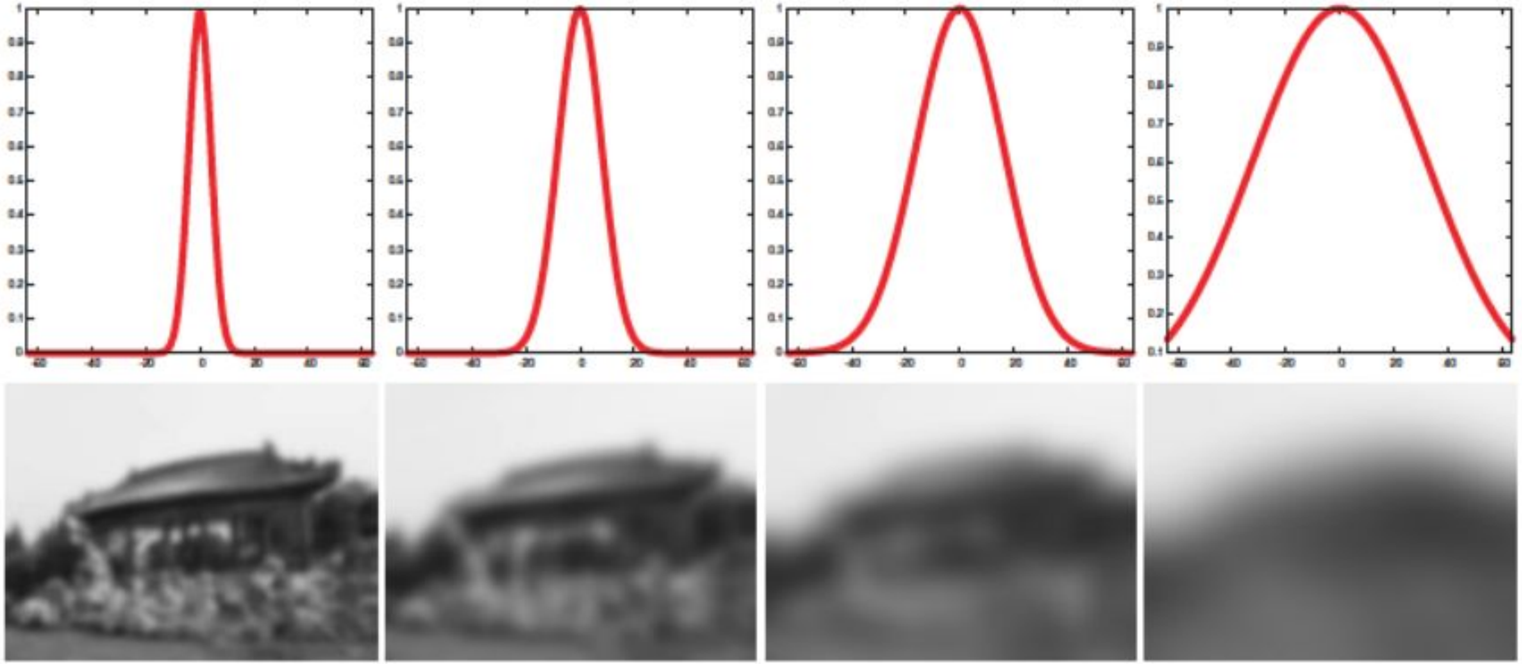
$$d = (x - \mu)^2 + (y - \mu)^2$$

$$G(x, y) = \frac{1}{\sigma\sqrt{2\pi}} * \exp\left(-\frac{1}{2\sigma^2} * d\right)$$

2D Square Gaussian Kernel

- Smooth falloff from the center
- A good sigma is usually `size/5`
- Bigger sigma = more blur
- Smaller sigma = less blur
  - We're widening the Gaussian and getting more values to average





$\sigma = 4$ $\sigma = 8$ $\sigma = 16$ $\sigma = 32$

Fig. 2.1 Example of Gaussian linear filtering with different σ. Top row shows the profile of a 1D Gaussian kernel and bottom row the result obtained by the corresponding 2D Gaussian convolution filtering. Edges are lost with high values of σ because averaging is performed over a much larger area.

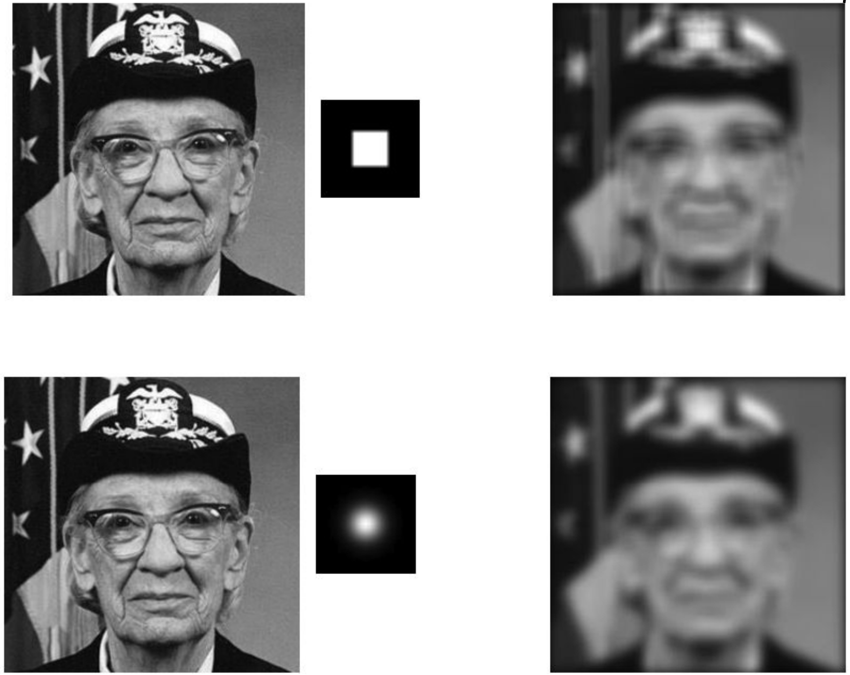
Credit: Fredo Durand @ MIT

Roll for confidence!

Block vs. Gaussian Blur

- Block kernel leaves rigid lines where the average wasn't taken
- Gaussian blur smoothes the image out

(Hi, Grace Hopper!)

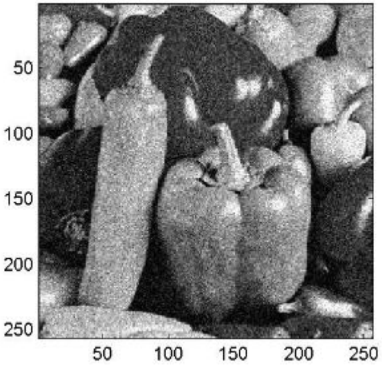


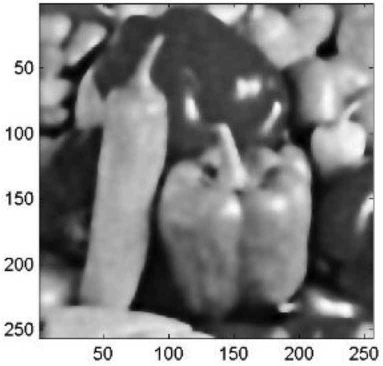
Credit: Ioannis Stamos @ Hunter College

Noise removal with Gaussian

- Sometimes, images can be low-quality and have noise
  - **noise:** random variation of brightness or color in images produced by sensors and cameras
- Gaussian blur smoothes the image out and makes noise less noisy (which will be helpful for edge detection!)

Original image





Smoothed with Gaussian

Homework

05_*homework* on Google Classroom (link)

Using Slides 11 and 12, create a function that creates 2D square Gaussian kernel with odd size. The function should take the size and sigma as parameters.