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例 6.4

依 $E(x_i)=\mu$ 　 $V(x_i)=\sigma^2=E(x_i^2)-\mu^2$

則 $E(\bar{x})=\mu$ 　 $V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x}^2)-\mu^2$

$$E(\hat{\theta}_1)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})}{n}\right]=\frac{1}{n}\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$$

$$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$$

$$=\sigma^2+\mu^2-\frac{\sigma^2}{n}-\mu^2$$

$$=\sigma^2-\frac{\sigma^2}{n}$$

$=\frac{n-1}{n}\sigma^2 \Rightarrow E(\hat{\theta}_1)$未滿足之不偏估計量 $\Rightarrow$ 偏誤估計量

$$E(\hat{\theta}_2)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right]=\frac{1}{n}E\left[\sum_{i=1}^{n}(x_i-\bar{x})^2\right]$$

$$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$$

$$=\left(\frac{n\sigma^2}{n-1}+\frac{n\mu^2}{n-1}-\frac{\sigma^2}{n-1}-\frac{n\mu^2}{n-1}\right)$$

$$=\frac{n\sigma^2}{n-1}-\frac{\sigma^2}{n-1}$$

$=\sigma^2 \Rightarrow E(\hat{\theta}_2)$滿足 σ^2 之不偏估計量 $\Rightarrow$ 不偏估計量