Instructions: Name your file `hw2.py` and submit on CCLE. Add comments to each function.

- **Problem 1**:
  Write a function `longestpath(d)` that finds the length of the longest path, $(a : b) \rightarrow$ $(b : c) \rightarrow \cdots$, in a dictionary `d`. It counts each pointer from a key to a value as one step. For example, the path $(a : b) \rightarrow (b : c)$ has length 2. To avoid cycles, we do not allow any key to appear more than once in a path (as a key).

  Test cases:
  ```
  d1 = {"a":"b","b":"c"}
  d2 = {"a":"b","b":"c","c":"d","e":"a","f":"a","d":"b"}
  longestpath(d1) should return 2.
  longestpath(d2) should return 5.
  ```

- **Problem 2**:
  Implement Newton's method (also known as the Newton-Raphson method) to find a root (zero) of a function. No prior knowledge of this algorithm is needed. Just follow the steps.

  Given a function $f(x)$, the function's derivative $f'(x)$, and a desired tolerance ϵ (usually a very small positive number), your goal is to find a desired value x^* which is close enough to a root of $f(x)$ such that $|f(x^*)| \leq \epsilon$. The algorithm is as follows:

  Algorithm:

  1. Starting from an initial guess x_0, calculate the error of your guess $f(x_0)$.

  2. If $|f(x_0)| \leq \epsilon$, then you are done because x_0 is close enough to the root. Otherwise, a better approximation than x_0 is given by $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$.

  3. Keep updating your guess x_n using the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ until you have $|f(x_n)| \leq \epsilon$.

  Instructions:

  - Write your algorithm in a `solve` function that takes as input a function $f(x)$, its derivative $f'(x)$, an initial guess x_0 and the tolerance ϵ. This function can be called like this:

    ```
    solve(lambda x: [x**2-1, 2*x], 3, 0.0001)
    ```

  - Test your `solve` function using the following functions $f(x)$, their derivatives $f'(x)$, and initial guesses x_0:

    $$f(x) = x^2 - 1, \ f'(x) = 2x, \ x_0 = 3$$
    $$f(x) = x^2 - 1, \ f'(x) = 2x, \ x_0 = -1$$
    $$f(x) = \exp(x) - 1, \ f'(x) = \exp(x), \ x_0 = 1$$
    $$f(x) = \sin(x), \ f'(x) = \cos(x), \ x_0 = 0.5.$$

    Use a calculator to test if the solutions provided by your code are correct, and put results in comment in your script.

Suggestions:

– You can start by hard-coding the function, its derivative, the initial guess, and the tolerance in your script without getting user input. This will help you better understand the algorithm.

– If you are still confused, watch this video for an example of Newton's method with two iterations: `https://www.youtube.com/watch?v=xdLgTDlFwrc`.