1. Find the least residue of 7^{80} mod 33 by the same method we used in class for 3^{50} mod 100.

2.
(a) Show that $10^k \equiv (-1)^k$ mod 11 for every nonnegative integer k.

(b) Show that

$$a_0 + a_1 10 + a_2 10^2 + a_3 10^3 + \ldots a_k 10^k \equiv a_0 - a_1 + a_2 - a_3 + \ldots + (-1)^k a_k \text{ mod } 11$$

(c) Part (a) gives us a test for divisibility by 11. For example, we can verify that $132 \equiv 0$ mod 11 by observing that $132 = 2 + 3 \cdot 10 + 1 \cdot 10^2$ and verifying that $2 - 3 + 1 \equiv 0$ mod 11. Use part (a) to determine if the following integer is divisible by 11. Show your work.

$$27, 182, 818, 284, 590, 452.$$

3. Show that mod 13, every cube is congruent to 0, ± 1, or ± 8.

4. Solve each of the following linear congruence equations separately, or state (with explanation) that no solutions exist:

(a) $3x \equiv 1$ mod 17

(b) $4x \equiv 6$ mod 18

(c) $15x \equiv 14$ mod 20