1. What exactly is wrong with the following proof?

STATEMENT: In any set of n flowers, the flowers have the same color.

Proof.
Base case: If $n = 1$ then there is only one flower in the set so clearly all flowers in the set have the same color.

Inductive step:
Suppose the claim is true for a set of n flowers.
Now consider a set of $n + 1$ flowers. Remove one flower from the set. Then the remaining n flowers have the same color by the hypothesis above. It remains to show that the flower we removed is also of this color. Put that flower back in the set and remove a different flower. Applying the inductive hypothesis again shows that the remaining n flowers have the same color. Therefore all $n + 1$ flowers have the same color and the claim is true for $n + 1$.

By induction, the claim is true for all natural numbers n. $\square$

2. Use induction to prove that

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + 4 \cdot 2^4 + \ldots + n \cdot 2^n = 2 + (n-1)2^{n+1}.$$

3. The Fibonacci numbers F_n are defined as follows:

$$F_1 = 1,\ F_2 = 1,$$

and

$$F_k = F_{k-1} + F_{k-2}$$

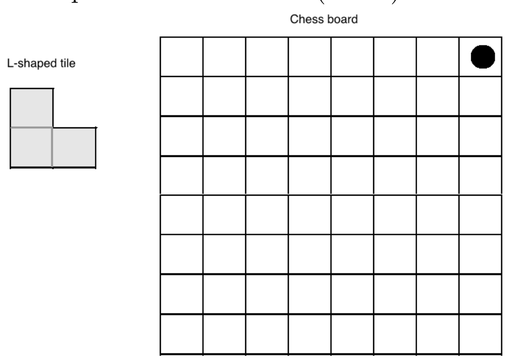
for $k > 2$. Use induction to prove that

$$F_1 + F_2 + F_3 + \ldots + F_n = F_{n+2} - 1$$

for all $n \in \mathbb{N}$.

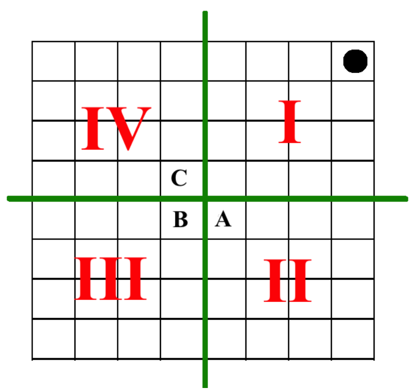
4. A $2^n \times 2^n$ chess board has a stone placed on one of its a corner squares. The rest of the board needs to be completely covered by non-overlapping L-shaped tiles (an L-shaped tile consists of 3 squares arranged in the shape of an L). Prove by induction that this can be done for any $n \in \mathbb{N}$.

The following picture depicts an 8×8 board ($n = 3$).



Try to do it before reading the following hint

Hint: For the inductive step, divide the board into quarters (as pictured below for the case $n = 3$). What are the dimensions of each quarter? The quarter labeled I can be covered with L-shaped tiles apart from the square with the stone – Why? The quarter labeled II can be covered with L-shaped tiles apart from the square labeled A – Why? The quarter labeled III can be covered with L-shaped tiles apart from the square labeled B – Why? The quarter labeled IV can be covered with L-shaped tiles apart from the square labeled C – Why? Finally it remains to cover the squares A, B, C – can you do it?



5. Use induction to prove that $3^{2n} - 1$ is divisible by 8 for every natural number n.