

**HOMEWORK 11 – MATH 4341**  
**DUE DATE: SUNDAY 12/03/2023**

**Problem 1.** Given a path  $f : [0, 1] \rightarrow X$  with  $f(0) = p$  and  $f(1) = q$ . Let  $e_r$  be the constant path at  $r \in X$ , i.e.  $e_r(x) = r$  for  $x \in [0, 1]$ .

- (a) Find explicit formulas for  $f \star e_q$  and  $e_p \star f$ .
- (b) Find an explicit formula for a path homotopy from  $f$  to  $f \star e_q$ .
- (c) Find an explicit formula for a path homotopy from  $f$  to  $e_p \star f$ .

**Problem 2.** Given a path  $f : [0, 1] \rightarrow X$  with  $f(0) = p$  and  $f(1) = q$ . Let  $\bar{f} : [0, 1] \rightarrow X$  be the reverse path of  $f$ , i.e.  $\bar{f}(x) = f(1 - x)$  for  $x \in [0, 1]$ .

- (a) Find explicit formulas for  $f \star \bar{f}$  and  $\bar{f} \star f$ .
- (b) Find an explicit formula for a path homotopy from  $e_p$  to  $f \star \bar{f}$ .
- (c) Find an explicit formula for a path homotopy from  $e_q$  to  $\bar{f} \star f$ .

**Problem 3.** Given paths  $f, g, h : [0, 1] \rightarrow X$  with  $f(1) = g(0)$  and  $g(1) = h(0)$ .

- (a) Find explicit formulas for  $(f \star g) \star h$  and  $f \star (g \star h)$ .
- (b) Find an explicit formula for a path homotopy from  $(f \star g) \star h$  to  $f \star (g \star h)$ .

**Problem 4.** Let  $h : X \rightarrow Y$  be a continuous function between two topological spaces. Given paths  $f, g : [0, 1] \rightarrow X$  with  $f(1) = g(0)$ . Show that

$$h \circ (f \star g) = (h \circ f) \star (h \circ g).$$

**Problem 5.** Find an explicit formula for the map  $r(x)$  in the proof of Theorem 7.11 (Brouwer fixed point theorem for  $D^2$ ) in the lecture notes.