HOMEWORK 1 – MATH 4341
DUE DATE: WEDNESDAY 08/30/2023

Problem 1. Let A, B, C be three sets. Use definition to show that

$$\begin{aligned} A \setminus (B \cup C) &= (A \setminus B) \cap (A \setminus C), \\ A \setminus (B \cap C) &= (A \setminus B) \cup (A \setminus C). \end{aligned}$$

Problem 2. Let $\{A_i\}_{i\in I}$ and $\{B_j\}_{j\in J}$ be two collections of sets. Show that

$$\begin{aligned} \left(\bigcup_{i\in I} A_i\right) \cap \left(\bigcup_{j\in J} B_j\right) &= \bigcup_{i\in I,\, j\in J} (A_i \cap B_j), \\ \left(\bigcap_{i\in I} A_i\right) \cup \left(\bigcap_{j\in J} B_j\right) &= \bigcap_{i\in I,\, j\in J} (A_i \cup B_j). \end{aligned}$$

Problem 3. (a) Suppose C is a subset of $\mathbb{R} \times \mathbb{R}$ such that C is equal to the Cartesian product of two subsets of $\mathbb{R}$. Show that if two points (a_1, b_1) and (a_2, b_2) are elements in C then two points (a_1, b_2) and (a_2, b_1) are also elements in C.

(b) Determine whether the subset $C = \{(x, y) \mid x^2 + y^3 > 7\}$ of $\mathbb{R} \times \mathbb{R}$ is equal to the Cartesian product of two subsets of $\mathbb{R}$.

Problem 4. Let $f : A \to B$ be a function. We define a relation C on A by setting xCy if $f(x) = f(y)$. Show that C is an equivalence relation.

Problem 5. Define a relation on $\mathbb{Q}$ by

$$C = \{(x, y) \mid x - y \text{ is an even integer}\}.$$

(a) Show that C is an equivalence relation.
(b) Describe the set of equivalence classes of C.