MATH 3379.001 Complex Variables
Fall 2023

Due: 09/12, Tuesday, 11:59 pm

1. Verify the following limits [Use the theorem on sec 2.17]

   (a) $\displaystyle\lim_{z \to -i} \frac{iz + 3}{z^2 + 1} = \infty$

   (b) $\displaystyle\lim_{z \to \infty} \frac{3z + i}{6z + 1} = \frac{1}{2}$

   (c) $\displaystyle\lim_{z \to \infty} \frac{2z^3 - 4z}{z^2 + 1} = \infty$

2. Let $T(z) = \dfrac{az + b}{cz + d}$, $(ad - bc \neq 0)$, use the theorem on section 2.17 to show that

   (a) $\displaystyle\lim_{z \to \infty} T(z) = \infty$ if $c = 0$

   (b) $\displaystyle\lim_{z \to \infty} T(z) = \frac{a}{c}$ if $c \neq 0$

   (c) $\displaystyle\lim_{z \to -\frac{d}{c}} T(z) = \infty$ if $c \neq 0$.

3. Test the continuity of following function at $z = 2i$

$$f(z) = \begin{cases} \dfrac{z^2 + 4}{z - 2i} & \text{if } z \neq 2i \\ 4i & \text{if } z = 2i \end{cases}$$

4. Test the continuity of following function at $z = 0$

$$f(z) = \begin{cases} \dfrac{z}{\bar{z}} & \text{if } z \neq 0 \\ 1 & \text{if } z = 0 \end{cases}$$

5. Find all point(s) of discontinuity of the following functions

   (a) $f(z) = \dfrac{2z - 3}{z^2 + 2z + 2}$

   (b) $f(z) = \dfrac{3z^2 + 4}{z^4 - 16}$