HOMEWORK 4 – MATH 4341
DUE DATE: MONDAY 09/25/2023

Problem 1. (a) Suppose $\mathcal{T}_1$ and $\mathcal{T}_2$ are two different topologies on a set X. When is the identity map $id : X \to X$ given by $id(x) = x$ a continuous map from $(X, \mathcal{T}_1)$ to $(X, \mathcal{T}_2)$?

(b) Show that the subspace topology $\mathcal{T}_Y$ is the smallest topology on $Y \subset X$ for which the inclusion $\iota : Y \to X$ is a continuous map.

Problem 2. (a) Let $Y \subset X$ be an open subset of a topological space X. Show that a set $U \subset Y$ is open in the subspace topology on Y if and only if U is open in X.

(b) Let $Y \subset X$ be a closed subset of a topological space X. Show that a set $U \subset Y$ is closed in the subspace topology on Y if and only if U is closed in X.

Problem 3. Let (X_1, d_1) and (X_2, d_2) be metric spaces. Define a function on $X_1 \times X_2$ by

$$d((x_1, x_2), (y_1, y_2)) = \max(d_1(x_1, y_1), d_2(x_2, y_2)).$$

(a) Show that d is a metric on $X_1 \times X_2$.

(b) Show that the metric topology on $X_1 \times X_2$ induced by d is the product topology, where X_1 and X_2 have the metric topologies from d_1 and d_2 respectively.