# HOMEWORK 6 – MATH 4341
# DUE DATE: MONDAY 10/16/2023

**Problem 1**. (a) Show that all metric spaces (with the metric topology) are Hausdorff.
(b) Show that all metric spaces (with the metric topology) are first-countable.

**Problem 2**. (a) Show that $\mathcal{B} = \{(a, \infty) \mid a \in \mathbb{R}\}$ is a basis for some topology on $\mathbb{R}$.
(b) Is $\mathbb{R}$ Hausdorff in the topology generated by $\mathcal{B}$?

**Problem 3**. Let $\mathbb{R}^\omega$ be the countably infinite product of $\mathbb{R}$, i.e., $\mathbb{R}^\omega = \prod_{i=1}^{\infty} X_i$ where each $X_i = \mathbb{R}$. We equip $\mathbb{R}^\omega$ with the product topology. Let $A$ be the subset of $\mathbb{R}^\omega$ consisting of all points whose coordinates are positive, i.e. $A = \{(x_1, x_2, \cdots) \mid x_i > 0 \ \forall i = 1, 2, \cdots\}$.
(a) Show that the origin $\mathbf{0} = (0, 0, \cdots)$ is a limit point of $A$.
(b) Construct an explicit sequence in $A$ converging to $\mathbf{0}$ in $\mathbb{R}^\omega$.