Math 3379
Homework 1

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Question 1

For the complex numbers $z = 2 + 3i$ and $w = 4 - 5i$ find
(a) zw
(b) $\frac{z}{w}$
(c) $(\overline{zw})$
(d) $\overline{zw}$
(e) $z\bar{z}$
(f) $|z|^2$

Solution: **(a)** We have that

$$zw = (2 + 3i) \cdot (4 - 5i) = 23 + 2i$$

(b) We have that

$$\frac{z}{w} = \frac{z}{w} \cdot \frac{\bar{w}}{\bar{w}} = \frac{2 + 3i}{4 - 5i} \cdot \frac{4 + 5i}{4 + 5i} = (-\frac{7}{41}) + (\frac{22}{41})i$$

(c) We have that

$$(\overline{zw}) = \bar{z} \cdot \bar{w} = (2 - 3i \cdot 4 + 5i) = 23 - 2i$$

(d) We have that

$$\overline{zw} = \bar{z} \cdot \bar{w} = (2 - 3i) \cdot (4 + 5i) = 23 - 2i$$

(e) We have that

$$z\bar{z} = (2 + 3i) \cdot (2 - 3i) = 13$$

(f) We have that

$$|z|^2 = |2 + 3i|^2 = 13$$

😄

Question 2

Ohm's law for electric circuit says, the voltage V (measured in volts) is the product of current I (measured in amps) and the impedance Z (ohms); i.e. $V = IZ$
(a) If the current $I = 24 - 5i$ amps and impedance $Z = 4 - 2i$ ohms find the voltage V.

(b) If the voltage $V = 24 - 5i$ volts and impedance $Z = 4 - 2i$ ohms, find the current I.

Solution: **(a)** Finding the voltage, we have

$$V = (24 - 5i) \cdot (4 - 2i) = 86 - 68i$$

(b) Finding the current, we have

$$24 - 5i = (x + yi) \cdot 4 - 2i = \frac{53}{10} + \frac{7}{5}i$$

😄

Question 3

The combined electrical complex impedance Z of two parallel complex impedance Z_1 and Z_2 is given by

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{z_2}$$

if $Z_1 = 3 + 4i$ and $Z_2 = 7 - 5i$, find Z.

Solution: To find Z, we have that

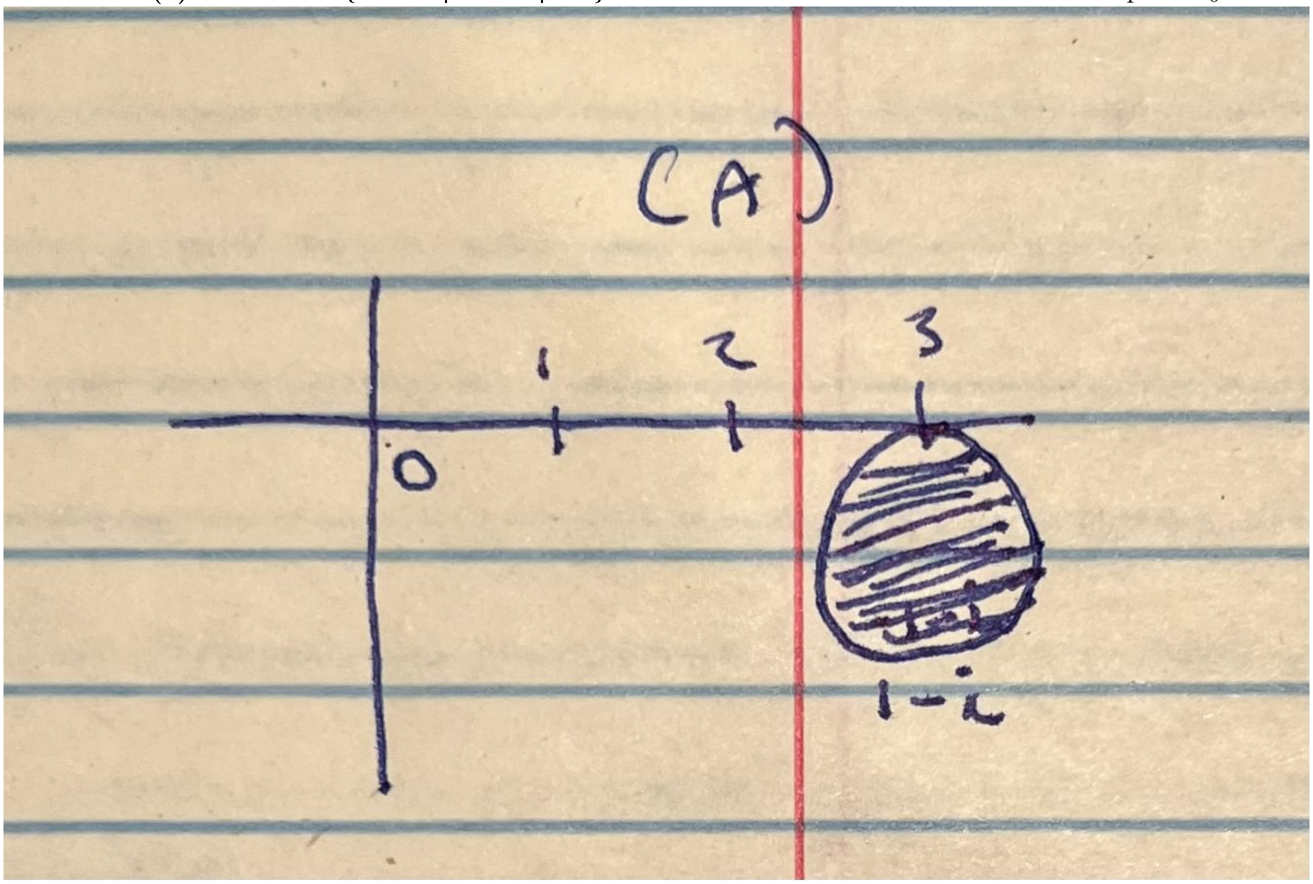
$$\begin{aligned} \frac{1}{Z} &= \frac{1}{Z_1} + \frac{1}{Z_2} \\ &= \frac{1}{3+4i} + \frac{1}{7-5i} \\ &= \frac{397}{101} + \frac{171}{101}i \end{aligned}$$

Hence, we have found that $Z = \frac{397}{101} + \frac{171}{101}i$. ☺

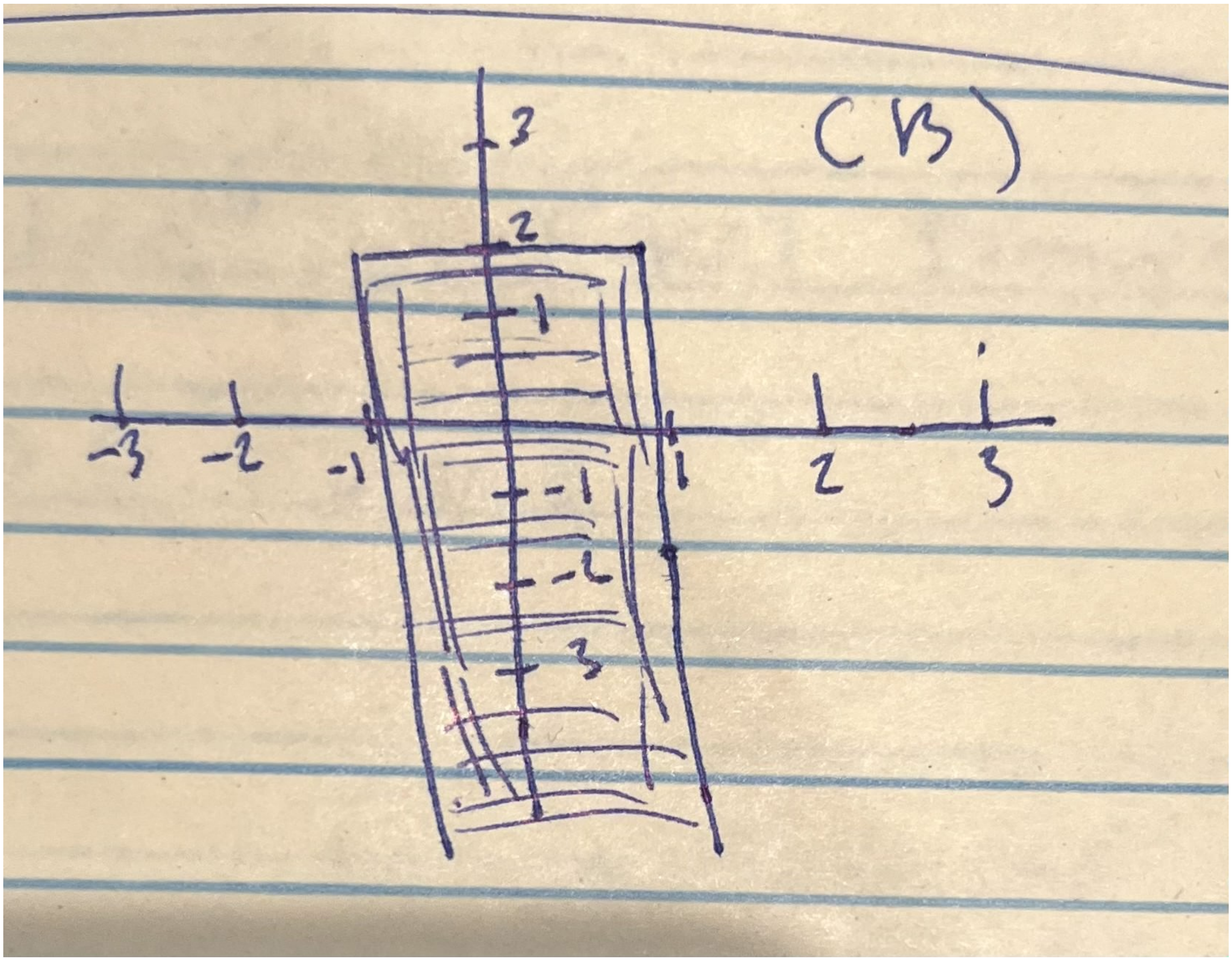
Question 4

Sketch the following regions in complex plane.
(a) $|z - 1 + i| \leqslant 3$
(b) $z = x + iy : x \geqslant 1, y \leqslant 2$

Solution: **(a)** The set $A = \{z \in \mathbb{C} : |z - 1 + i| \leqslant 3\}$ is the closed disk of radius 3 centered at the point $z_0 = 1 - i$.



(b) We have that the region defined by $x \geqslant 1$ and $y \leqslant 2$ in the complex plane corresponds to the set of complex numbers where the real part $x \geqslant 1$, and the imaginary part $y \leqslant 2$.


(B)
3
2
1
-3
-2
-1
1
2
3
-1
-2
3

😄

Question 5

Prove the following
(a) A complex number z is real if and only if $z = \bar{z}$
(b) A complex number z is pure imaginary if and only if $z = -\bar{z}$

Proof: **(a)** Let $z = x + iy$ such that $x, y \in \mathbb{R}$. Then, we have that

$$z = \bar{z} \iff x + iy = x - iy \iff \begin{cases} x = x \\ y = -y \end{cases} \iff y = 0 \iff z = x$$

Hence, a complex number z is real if and only if $z = \bar{z}$.

(b) Let $z = x + iy$ such that $x, y \in \mathbb{R}$. Then, we have that

$$z = -\bar{z} \iff x + iy = -(x - iy) \iff \begin{cases} x = -x \\ y = y \end{cases} \iff x = 0 \iff z = y$$

Hence, a complex number z is pure imaginary if and only if $z = -\bar{z}$. ☺

Question 6

Compute $(1+\sqrt{3}i)^6$

Solution: We have that

$$(1+\sqrt{3}i)^6 = \sqrt{1^2+(\sqrt{3})^2} = 2$$

and now the argument is

$$\tan^{-1}(\frac{\sqrt{3}}{1}) = \frac{\pi}{3}$$

taken to polar form we have $2[\cos(\frac{\pi}{3}+i\sin(\frac{\pi}{3})]$ so $(1+\sqrt{3}i)^6 \longrightarrow 2[\cos(\frac{\pi}{3}+i\sin(\frac{\pi}{3})]^6$. Computing, we have

$$\begin{aligned} 2[\cos(\frac{\pi}{3}+i\sin(\frac{\pi}{3})]^6 &= 2^6(\cos(\frac{6\pi}{3})+i\sin(\frac{6\pi}{3})) \\ &= 64(\cos 2\pi + i\sin 2\pi) \\ &= 64(1+0) \\ &= 64 \end{aligned}$$

Hence, we have shown that $(1+\sqrt{3}i)^6 = 64$. 😃