MATH 4334.001/CS 4334.001 Name : _____
Fall 2023
Paper Homework 6
Due 10/06/2023

Show ALL work to receive full credit.

1. Consider the four data points $(-2, 8),\ (0, 4),\ (1, 2),\ (3, -2)$.
   (a) Find a polynomial $p(x)$ of degree 3 or less passing through these points, using a divided-difference table.
   (b) Describe any other polynomials of degree 4 or less passing through the same points.

2. Let $p(x)$ be the degree 10 polynomial that is zero at $x = 1, \ldots, 10$ and satisfies $p(12) = 44$. Evaluate $p(0)$.

3. Consider four different points $P_1,\ P_2,\ P_3$, and P_4 lying on a parabola $y = ax^2 + bx + c$. How many degree 3 polynomials interpolate these four points? Explain your answer.

4. Consider the following census data. The approximate population of the United States was 150.7 million in 1950, 179.3 million in 1960, 203.3 million in 1970, 226.5 million in 1980, and 249.6 million in 1990.
   (a) Using Newton's interpolation polynomial for these data, find an approximate value for the population in the year 2000.
   (b) Use the same polynomial to estimate the population in 1920 based on these data. What conclusion should be drawn?

5. Consider the degree nine polynomial $p_9(x)$ that interpolates the function $f(x) = e^{-2x}$ at the 10 evenly spaced points $x = 0,\ 1/9,\ 2/9,\ 3/9, \ldots, 8/9,\ 1$.
   (a) Find an upper bound for the error $|f(1/2) - p_9(1/2)|$.
   (b) How many decimal places can you guarantee to be correct if $p_9(1/2)$ is used to approximate e^{-1}?

6. Suppose $\cos x$ is to be approximated by an interpolating polynomial of degree n, using $n + 1$ equally spaced nodes in the interval $[0, 1]$.
   (a) Find an upper bound on the error of the approximation in terms of n.
   (b) Find an upper bound for $n = 9$.
   (c) For what values of n is the error less than 10^{-7}?