

MATH 3379.001 Complex Variables

HW 1

Due : Tuesday 08/29, 11:59pm

Term: Fall 2023

Textbook recommended problems (do not turn in): p5: 2, 4; p7: 1, 2; p13: 1, 4, 5; p16: 9, 11, 14.

1. For the complex numbers $z = 2 + 3i$ and $w = 4 - 5i$ find
 - (a) zw
 - (b) $\frac{z}{w}$
 - (c) $\overline{(zw)}$
 - (d) $\overline{z}w$
 - (e) $z\overline{z}$
 - (f) $|z|^2$
2. Ohm's law for electric circuit says, the voltage V (measured in volts) is the product of current I (measured in amps) and the impedance Z (ohms); i.e. $V = IZ$
 - (a) If the current $I = 24 - 5i$ amps and impedance $Z = 4 - 2i$ ohms find the voltage V
 - (b) If the voltage $V = 24 - 5i$ volts and impedance $Z = 4 - 2i$ ohms, find the current I .
3. The combined electrical complex impedance Z of two parallel complex impedance Z_1 and Z_2 is given by

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}.$$

If $Z_1 = 3 + 4i$ and $Z_2 = 7 - 5i$, find Z .

4. Sketch the following regions in complex plane.
 - (a) $|z - 1 + i| \leq 3$
 - (b) $z = x + iy : x \geq 1, y \leq 2$
5. Prove the following
 - (a) A complex number z is real if and only if $z = \overline{z}$
 - (b) A complex number z is pure imaginary if and only if $z = -\overline{z}$
6. Compute $(1 + \sqrt{3}i)^6$