HOMEWORK 5 – MATH 4341
DUE DATE: MONDAY 10/09/2023

Problem 1. Let X be a topological space and $Y \subset X$. Show that $\mathrm{Int}Y$ is equal to the set of all points $x \in X$ such that some neighborhood of x is a subset of Y.

Problem 2. Let X be a topological space and $Y \subset X$. Show that:

(1) $\mathrm{Int}Y = X \setminus \overline{(X \setminus Y)}$.
(2) $\overline{Y} = X \setminus \mathrm{Int}(X \setminus Y)$.

Problem 3. Let X be a topological space and $Y, Z \subset X$. Show that:

(1) $\overline{Y \cup Z} = \overline{Y} \cup \overline{Z}$.
(2) $\overline{Y \cap Z} \subset \overline{Y} \cap \overline{Z}$. Find an example where $\overline{Y \cap Z} \neq \overline{Y} \cap \overline{Z}$.
(3) $\mathrm{Int}Y \cup \mathrm{Int}Z \subset \mathrm{Int}(Y \cup Z)$. Find an example where $\mathrm{Int}Y \cup \mathrm{Int}Z \neq \mathrm{Int}(Y \cup Z)$.
(4) $\mathrm{Int}Y \cap \mathrm{Int}Z = \mathrm{Int}(Y \cap Z)$.

Problem 4. Let $\mathbb{R}_\ell$ be the set of all real numbers $\mathbb{R}$ with the lower limit topology, and $\mathcal{I}$ be the set of all irrational real numbers. Prove that

(1) $\partial\mathcal{I} = \mathbb{R}_\ell$.
(2) $\mathcal{I}$ is dense in $\mathbb{R}_\ell$.