

HOMEWORK 10 – MATH 4341
DUE DATE: SUNDAY 11/19/2023

Problem 1. Let $S^n \subset \mathbb{R}^{n+1}$ be the standard unit n -sphere, i.e.

$$S^n = \{x \in \mathbb{R}^{n+1} : \|x\| = 1\}.$$

Suppose $\{A_k\}_{k=1}^\infty$ is a sequence of non-empty closed sets in S^n such that $A_1 \supset A_2 \supset \cdots \supset A_k \supset A_{k+1} \supset \cdots$. Show that $\bigcap_{k=1}^\infty A_k$ is non-empty.

Problem 2. Show that every compact subspace of a metric space is bounded and closed.

Problem 3. Show that a bounded and closed subset of a metric space is not always compact.

Problem 4. Suppose A is a compact subspace of the Hausdorff space X and $x \in X \setminus A$. Show that there exist disjoint open sets U and V of X containing x and A respectively.

Problem 5. Let A and B be disjoint compact subspaces of a Hausdorff space X . Show that there exist disjoint open sets U and V containing A and B respectively.