Show ALL work to receive full credit.

1. Consider the equation
$$x^4 = x^3 + 10$$

   (a) Find an interval $[a, b]$ of length one inside which the equation has a solution.

   (b) How many steps of the Bisection Method are required to calculate the solution, starting with $[a, b]$, with an error of at most 10^{-10}? Answer with an integer.

2. The equation $x - Rx^{-1} = 0$ has $x = \pm R^{1/2}$ for its solution. Establish Newton's iterative scheme, in simplified form, for this situation. Carry out three steps for $R = 25$ and $x_0 = 1$.

3. The iteration formula
$$x_{n+1} = x_n - (\cos x_n)(\sin x_n) + R \cos^2 x_n$$
for some positive constant R, was obtained by applying Newton's method to some function $f(x)$. Determine $f(x)$.

4. Consider the function
$$f(x) = x^4 - 7x^3 + 18x^2 - 20x + 8$$
Does Newton's Method converge quadratically to the zero $r = 2$? Explain your answer. Then determine $\lim_{n \to \infty} e_{n+1}/e_n$, where e_n denotes the error at the nth step.

5. If we use the secant method on
$$f(x) = x^3 - 2x + 2$$
starting with with $x_0 = 0$ and $x_1 = 1$, what is x_2?