HOMEWORK 8 – MATH 4341
DUE DATE: MONDAY 10/30/2023

Problem 1. Consider $\mathbb{R}$ with the topology $\mathcal{T} = \{\emptyset, \mathbb{Q}, \mathbb{I}, \mathbb{R}\}$, where $\mathbb{Q}$ is the set of all rational numbers and $\mathbb{I}$ is the set of all irrational numbers.

(a) Is $(\mathbb{R}, \mathcal{T})$ connected?

(b) Is $(\mathbb{R}, \mathcal{T})$ path connected?

Problem 2. Let X be a topological space. Define a relation $\sim_p$ on X by declaring that $x \sim_p y$ if and only if there is a path connected set $A \subset X$ such that $x, y \in A$.

(a) Show that $\sim_p$ is an equivalence relation.

(b) The equivalence classes of $\sim_p$ are called the path connected components of X. Show that any path connected component is a path connected subset of X.

Problem 3. Let $\mathbb{I}$ be the subspace of $\mathbb{R}$ consisting of all irrational numbers.

(a) Find all connected components of $\mathbb{I}$.

(b) Find all path connected components of $\mathbb{I}$.

Problem 4. Let $n \geq 2$ be an integer.

(a) Show that $\mathbb{R}^n \setminus \{\mathbf{0}\}$ is path connected, where $\mathbf{0}$ is the origin in $\mathbb{R}^n$.

(b) Show that $\mathbb{R}^n \not\cong \mathbb{R}$.