Math 4301
Quiz 1

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Question 1

Let $A_n = \{x \in \mathbb{Q} : -3 + \frac{1}{4n^2+n-1} < x < \pi + \frac{(-1)^n}{3n-1}\}, n \in \mathbb{N}$ and $A = \bigcup_{n=1}^{\infty} A_n$. Find inf(A)

Solution: The answer is -3. 😄

Question 2

Let $A_n = \{x \in \mathbb{R} : 1 - \frac{1}{n^2} < x < 6 + \frac{\sqrt{2}}{n^2+n+5}\}$ and $A = \bigcap_{n=1}^{\infty} A_n$. Find sup(A)

Solution: The answer is 6 😄

Question 3

Choose all correct answers (there might be more than one correct answer)
(a) An ordered Field $\mathbb{F}$ has archimedean property if and only if for all $x \in \mathbb{F}, x > 0$ there is $n \in \mathbb{N}$ such that $0 < \frac{1}{n} < x$

(b) The set $\mathbb{R}$ of all real numbers is a complete ordered field.

(c) The set $s = \{x \in \mathbb{Q} : 0 < x < \sqrt{2}\} \subseteq \mathbb{R}$ has both the greatest and the lest upper bounds in $\mathbb{R}$.

(d) The set of all real numbers $\mathbb{R}$ does not satisfy archimedean property.

(e) The set $S = \{\frac{n^2+(-1)^n}{n} : n \in \mathbb{N}\} \subseteq \mathbb{R}$ is bounded above.

Solution: The answers are A,B, and C. 😄

Question 4

Choose all correct answers (there might be more than one correct answer)
(a) Well-ordering property and the Principle of Mathematical induction are equivalent statements for the set of all natural numbers.

(b) The set $\mathbb{Z}$ of all integers has the well-ordering property.

(c) The set $A - \{x \in \mathbb{Q} : 0 < x < 1\}$ has the greatest and the least upper bounds in $\mathbb{Q}$.

(d) The set of $\mathbb{Q}$ of all rational numbers is a complete ordered field.

(e) If $s \subseteq \mathbb{Z}, 0 \in S$ and $(k-1), (k+1) \in S$ whenever $k \in S$, then $S = \mathbb{Z}$.

Solution: The answers are A,C, and E. 😄