HOMEWORK 2 – MATH 4341
DUE DATE: WEDNESDAY 09/06/2023

Problem 1. Suppose X is a finite set of cardinality n. Show that $\mathcal{P}(X)$, the power set of X, is a finite set of cardinality 2^n.

Problem 2. Describe all topologies on the set $X = \{a, b, c\}$. Justify your answer.

Problem 3. Let $\mathcal{I}$ be the set of all irrational numbers. We define $\mathcal{T}$ to be the collection of all subsets U of $\mathcal{I}$ such that either $U = \emptyset$ or $\mathcal{I} \setminus U$ is countable. Show that $\mathcal{T}$ is a topology on $\mathcal{I}$.

Problem 4. Let
$$\mathcal{B}_\ell = \{[a, b) \mid a, b \in \mathbb{R}\}.$$
Show that $\mathcal{B}_\ell$ is a basis for a topology on $\mathbb{R}$.

Problem 5. Let $K = \{1/n \mid n \in \mathbb{N}\} \subset \mathbb{R}$ and let
$$\mathcal{B}_K = \{(a, b) \mid a, b \in \mathbb{R}\} \bigcup \{(a, b) \setminus K \mid a, b \in \mathbb{R}\}.$$
Show that $\mathcal{B}_K$ is a basis for a topology on $\mathbb{R}$.