**HOMEWORK 9 – MATH 4341**
**DUE DATE: SUNDAY 11/12/2023**

**Problem 1**. Let $X$ be a topological space. Suppose $\{x_n\}_n$ is a sequence in $X$ converging to $x \in X$. Show that the set $A = \{x_n\}_n \cup \{x\}$ is a compact subspace of $X$.

**Problem 2**. Show that any finite union of compact subspaces of a topological space $X$ is a compact subspace of $X$.

**Problem 3**. Show that any intersection of compact subspaces of a Hausdorff space $X$ is a compact subspace of $X$.

**Problem 4**. Suppose $\mathbb{R}$ has the topology consisting of all subsets $A$ such that $\mathbb{R} \setminus A$ is either finite or all of $\mathbb{R}$. Show that every subspace of $\mathbb{R}$ is compact.

**Problem 5**. Show that a compact subspace of a topological space is not always closed.