# Chapter 15.1: Double Integrals on Rectangles

**I1: Double & Triple Integrals.** I can set up double and triple integrals as iterated integrals over any region. I can sketch regions based on a given iterated integral.

**I2: Iterated Integrals.** I can compute iterated integrals of two and three variable functions, including applying Fubini's Theorem to change the order of integration of an iterated integral.

## Mechanics

1. Compute $\iint_R (xy - 3xy^2)\ dA$, where $R$ is the square $0 \le x \le 2, 1 \le y \le 2$.
2. Use Fubini's Theorem to evaluate the integral
$$\int_0^1 \int_0^3 xe^{xy}\ dx\ dy$$
Why was it a good idea to exchange the order of integration?
3. Find the volume of the region bounded above by the paraboloid $z = 16 - x^2 - y^2$ and below by the square $R : 0 \le x \le 2, 0 \le y \le 2$.

## Extensions

4. Evaluate the double integral $\iint_R (4-2y)\ dA$, where $R = [0,1]\times[0,1]$ **without integrating** by identifying it as the volume of a solid *[Hint: It is a prism cut by some plane.]*
5. The integral $\iint_R \sqrt{9-y^2}$, where $R = [0,4] \times [0,2]$, represents the volume of a solid. Sketch the solid.
6. This problem explores a failure of Fubini's theorem. Consider the two iterated integrals, which differ by swapping the order of integration:
$$\int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\ dy\ dx \quad \text{and} \quad \int_0^1 \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\ dx\ dy$$
Use the fact that
$$\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right) = \frac{\partial}{\partial x}\left(\frac{-x}{x^2+y^2}\right) = \frac{x^2-y^2}{(x^2+y^2)^2}$$
and that $\frac{d}{dt}\arctan(t) = 1/(1+t^2)$ to show that the iterated integrals are different. Why does Fubini's theorem fail?