

Chapter 14.6: Linearization and Tangent Planes

D2: Tangent Planes and Linear Approximations. I can find equations for tangent planes to surfaces and linear approximations of functions at a given point and apply these to solve problems.

Mechanics

1. Find the linearization of $f(x, y) = e^{2y-x}$ at $(1, 2)$. Without doing any more calculations, find an equation of the tangent plane of the surface $f(x, y)$ at $(1, 2)$.
2. Find the linearization of $f(x, y, z) = \arctan(xyz)$ at $(1, 1, 0)$.
3. Use the linearization to approximate $f(2.95, 7.1)$ for the function $f(x, y) = \sqrt{x^2 + y}$, knowing that $f(3, 7) = 4$.
4. Find an equation of the tangent plane to the unit sphere $x^2 + y^2 + z^2 = 1$ at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}\right)$.

Applications

5. Suppose you are shining a flashlight on a smooth surface. The *angle of incidence*, θ_i is the angle at which a light beam hits the surface, measured with respect to the surface normal (i.e., the normal vector to the tangent plane at point of contact). The *angle of reflection* θ_r is the angle of the reflected light beam measured with respect to the surface normal. The *law of reflection* states that in a vacuum, we must have $\theta_i = \theta_r$. Draw a labeled picture to convince yourself that this is reasonable.

Now, consider the paraboloid $z = x^2 + y^2$, and a light ray traveling along the path $\mathbf{r}(t) = (-2, -3, 2)t + (3, 4, 0)$. Compute the angle of reflection at the point $(1, 1, 2)$ [*Hint: How can one find the angle between two vectors?*].

Extensions

6. Use software to graph the function $z = x^{1/3}y^{1/3}$. Examine the graph at $(0, 0)$ - does it look like the function has a tangent plane there? Use this to deduce a necessary condition for a function $f(x, y)$ to have a tangent plane.