Chapter 14.8: Lagrange Multipliers

D3: Optimization. I can locate and classify critical points of functions of two variables. I can find absolute maxima and minima on closed bounded sets. I can use the method of Lagrange multipliers to maximize and minimize functions of two or three variables subject to constraints. I can interpret the results of my calculations to solve problems.

Mechanics

1. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

2. Find the extreme values of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.

Applications

3. A rectangular box without a lid is to be made from 12 m^2 of cardboard. Find the maximum volume of such a box.

4. A niche restaurant in midtown Atlanta serves only garlic bread (denoted by g) and bunches of kale (denoted by k). The cost of producing these goods is given by the function $C(g, k) = 5g^2 + 2gk + 3k^2 + 10$. Assuming that the total amount of items to be produced is 40, compute the minimal production cost.

5. The height of a mountain is given by $h(x, y) = 300 - (3x^2 + 4xy + 3y^2)$. Compute the height of the lowest point on the mountain within $\sqrt{200}$ units of the origin. Where might this point occur? Are there multiple points where this occurs? *[Hint: First, justify why none of x, y, λ can be zero. Then solve for λ.]*

Extensions

6. The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin. *[Hint: It may be helpful algebraically to work with the square of the distance to the origin.]*

7. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r.