

Chapter 15.2: General Double Integrals

I1: Double & Triple Integrals. I can set up double and triple integrals as iterated integrals over any region. I can sketch regions based on a given iterated integral.

I2: Iterated Integrals. I can compute iterated integrals of two and three variable functions, including applying Fubini's Theorem to change the order of integration of an iterated integral.

Mechanics

1. Write iterated integrals for $\iint_R 1 \, dA$ over the regions R in both orders $dx dy$ and $dy dx$.
 - (a) R bounded by $y = e^{-x}$, $y = 1$, and $x = \ln 3$.
 - (b) R bounded by $y = x^2$ and $y = x + 2$
2. Evaluate each of the following by sketching the region of integration and deciding whether or not to swap the order of integration. Why did you choose to swap the ones you did?

(a)

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) \, dx \, dy$$

(b)

$$\int_0^{\pi} \int_0^x x \sin(y) \, dy \, dx$$

(c)

$$\iint_R xy^2 \, dA$$

where R is the region bounded by $x = 0$ and $x = \sqrt{1 - y^2}$

Applications

3. Find the mass of a triangular plate with vertices $(0, 0)$, $(4, 0)$ and $(0, 1)$, given that its density at any point is $\rho(x, y) = x^2 + y^2$

Extensions

4. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant.