Chapter 15.3: More Double Integrals

I1: Double & Triple Integrals. I can set up double and triple integrals as iterated integrals over any region. I can sketch regions based on a given iterated integral.

I2: Iterated Integrals. I can compute iterated integrals of two and three variable functions, including applying Fubini's Theorem to change the order of integration of an iterated integral.

Mechanics

1. Consider the function $f(x, y) = xy$. Without performing any computations, do you think the average value of f is larger over the square $0 \leq x \leq 1, 0 \leq y \leq 1$, or over the quarter circle $x^2 + y^2 \leq 1$ *in the first quadrant*? Verify your guess by integrating

2. A metal triangular plate with vertices $(0, 0)$, $(2, 0)$ and $(2, 4)$ has temperature equal to $C(x, y) = xe^{xy}$ degrees Celsius. Compute the average temperature of the plate. *[Hint: Choose a favourable order of integration.]*

Applications

3. If $f(x, y) = 100(y + 1)$ represents the population density in people per square mile of a planar region on Earth, where x and y are measured in miles, find the number of people in the region bounded by the curves $x = y^2$ and $x = 2y - y^2$.

4. A rectangular can of Pringles chips may be modelled by the prism $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 5$. Assuming that the Pringles container is filled up with chips until the surface $z = x^2 - y^2 + 3$, are there more chips or air in the can? *[Note: The Pringles enthusiast may complain that their containers are supposed to be cylinders, not prisms. This nuance will be addressed when we work with polar coordinates.]*

Extensions

5. An organism can be initially described as the solid with base $[0, 1] \times [0, 1]$ and height $z = e^{x+y}$. Suppose that the base of this organism grows at a rate of t units per second in both the positive x and positive y directions. Compute the rate of change of the volume of the organism at $t = 4$ seconds. *[Hint: Set up an integral expression for the volume in terms of t, evaluate the integral, then differentiate with respect to t.]*