Math 464 Semester Project Report

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For my Math 464 semester project, I will be constructing and solving a linear optimization problem based on the decisions that go into planning public transit routes. I will be basing my information off of meetings with the Spokane Transit Authority. The purpose of this project is to gain insight into how the mathematical practices for this class can be applied to complicated problems in the industry. This document includes the planning and initial meeting summaries, as well as the finalized problem statement and solution.

1 Contact List

I have a handful of companies I am thinking about contacting for this project. They range from smaller companies to bigger companies, mainly based in Spokane, WA.

- Avista - Determine the best way to provide power to everyone?
- Red Lion Hotels Corporation - Optimizing hotel booking?
- Tidyman's - Optimize shift booking or advertising?
- Spokane Transit Authority - Determine the best way to route the buses?
- Gravity Jack - Optimize sales promotions?
- Northwest Farm Credit Services - Optimize profits for the company?
- Imprezzio - Optimize their data usage?

I chose these companies because I think that their businesses make plenty of strategic decisions everyday, which would provide excellent optimization problems.

2 Contact Plan

The companies that I am planning on contacting primarily are Spokane Transit Authority, Northwest Farm Credit Services, and Red Lions Hotels Corporation. I understand that these may be somewhat larger companies for this project, but I believe that they exhibit great opportunities for this project. They all have significant decisions being made every day that I am excited to begin optimizing. I have done some research into these three companies, and found people from each to contact. Each email is very similar; they only differ in names and the ways in which I consider helping them. Note that for some I could not find specific emails, but I most likely could through some phone calls or other types of online forms. Here are my rough drafts of emails to start:

Spokane Transit Authority

I am a student in a Mathematical Optimization course at Washington State University. As part of my course project, I am considering researching the best way to schedule and route buses in the transit system. I was hoping that you, as the Chief Operations Officer, would have some time to discuss the process and strategies behind making these types of business decisions in your company. A quick 20-30 minute meeting would give me all the information needed to get started on the project. I could meet with you any time Friday, or Monday/Wednesday after 3. Please let me know, and feel free to contact my professor at tasaki@wsu.edu if you have any questions.

Best,

Hunter Mitchell

Red Lions Hotels Corporation

I am a student in a Mathematical Optimization course at Washington State University. As part of my course project, I am considering researching the most profitable way to organize and schedule hotel reservations. I was hoping that you would have some time to discuss the process and strategies behind making these types of business decisions in your company. A quick 20-30 minute meeting would give me all the information needed to get started on the project. I could meet with you any time Friday, or Monday/Wednesday after 3. Please let me know, and feel free to contact my professor at tasaki@wsu.edu if you have any questions.

Best,

Hunter Mitchell

Northwest Farm Credit Services

I am a student in a Mathematical Optimization course at Washington State University. As part of my course project, I am considering researching the most profitable way for companies to organize loans and other various types of banking. I was hoping that you would have some time to discuss the process and strategies behind making these types of business decisions in your company. A quick 20-30 minute meeting would give me all the information needed to get started on the project. I could meet with you any time Friday, or Monday/Wednesday after 3. Please let me know, and feel free to contact my professor at tasaki@wsu.edu if you have any questions.

Best, Hunter Mitchell

3 Meeting Synopsis

After contacting Spokane Transit Authority, I arranged a quick 20 minute meeting with Kathleen Weinand, the Principal Transit Planner.

Our meeting began with her explaining what her occupation consists of. She then explained the process of how they plan routes. They typically begin with the destination that is needed, then work on the stops and planning for the drivers. This then gave me some further questions concerning the drivers, since there seem to be some constraints there. I got some great information from her that can be summarized as follows.

Spokane transit Authority have more than 40 bus routes. The bus routes are less busy on the weekends as one would expect, so they must take this into account during planning. Furthermore, they must plan the routes to be in 30 minute intervals: 30,60,90, and sometimes 120 minutes. Each route must have at least 15% of the run time dedicated for layover. In other words, this time is for the operators to be get snacks, use the restroom, etc. The company pays the operators by the hour, so it is in their best interest financially to minimize these layover times. They also must maintain the right amount of vehicles. They must have spares in case of maintenance issues, but cannot have too much due to federal regulations. They must also take into consideration run cutting. This is essentially the same thing as crew scheduling while minimizing cost.

All of this information makes great for building a linear program. The main optimization problem I can think of is scheduling the bus routes closest to the 30 minute intervals to save money. This would certainly have constraints ($\geq$15% layover time per route) and a cost objective function. I could also perhaps make an optimization problem revolved around the number of buses they own to minimize costs and keep their company running. I am planning on talking to Kathleen again with some more specific questions. She also provided me with the website they use to plan the routes may serve to be useful. We will see what sort of optimization problem we develop given all of this information.

3.1 Problem Statement

Spokane Transit Authority must plan bus routes to be as close to 30-minute periods as possible (i.e. $30, 60, 90, 120$ minutes). These routes can hit any number of n stops. The time taken to get from stop i to stop j is $T_{i,j}$ minutes where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$. What is the route that gets closest to a 30 minute interval while making sure to come back to the station afterward?

3.2 Optimization Model

Our decision variables will be a matrix representing the stops in a bus route. We will let

$$A_{i,j} = \begin{cases} 0 & \text{if the bus } \textbf{does not} \text{ go from stop i to stop j} \\ 1 & \text{if the bus } \textbf{does} \text{ go from stop i to stop j} \end{cases}$$

The objective function which we will minimize can then be written as

$$z = |(\sum A_{i,j}T_{i,j}) - 30m| \text{ for all } i = 1, 2, \ldots, n, j = 1, 2, \ldots, n$$

where m represents the 30-minute interval we are trying to achieve ($m = 1$ means 30 minutes, $m = 2$ means 60 minutes, etc.).

As for constraints, we must make sure that we go from each stop no more than once, and to each stop no more than once. Hence,

$$\sum_{j=1}^{n} A_{i,j} \leq 1 \text{ for all } i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} A_{i,j} \leq 1 \text{ for all } j = 1, 2, \ldots, n$$

Next, we must make sure that the route goes from the station, and to the station.

$$\sum_{j=1}^{n} A_{1,j} = 1$$

$$\sum_{i=1}^{n} A_{i,1} = 1$$

For this to work, the station **must** be represented as the first row and column of the matrices.

Lastly, we must eliminate all subtours. We can do this by taking a set of bus stops S, and making sure that the total number of connections between these stops is less than the amount of those stops. That is,

$$\sum_{i,j \in S} A_{i,j} \leq |S| - 1 \text{ for all } S \subset n, S \neq \emptyset$$

Altogether, our model is

$$
\begin{aligned}
\min_{A} \quad & z = |(\sum A_{i,j} T_{i,j}) - 30m| \\
\text{s.t.} \quad & \sum_{i=1}^{n} A_{i,j} \leq 1, \quad j = 1, 2, \ldots, n \\
& \sum_{j=1}^{n} A_{i,j} \leq 1, \quad i = 1, 2, \ldots, n \\
& \sum_{j=1}^{n} A_{1,j} = 1 \\
& \sum_{i=1}^{n} A_{i,1} = 1 \\
& \sum_{i,j \in S} A_{i,j} \leq |S| - 1 \text{ for all } S \subset n, S \neq \emptyset \\
& A_{i,j} \in \{0, 1\} \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n \\
& A \in \mathbb{Z}^{n \times n}
\end{aligned}
$$

4 Solution

I do not have specific data from the company that I can work with, so I will be creating "synthetic data" that will be used to test out the model in several different scenarios.

To begin, I will start with a scenario that only has one solution, to make sure that the model returns the correct output. For this, I am letting there be one other stop for the bus to take. To do this, we will let

$$T = \begin{bmatrix} k & 15 \\ 15 & k \end{bmatrix}$$

I defined $k = 300$ so that the model makes sure not to use those, since this would represent the time taken to go to and from the same station, which doesn't make sense. When I implement this into MATLAB, I get what I should be getting which is $A_{1,2} = 1, A_{2,1} = 1$ with a zero optimal value, since this route is exactly 30 minutes. I also did not implement the subtour constraints in this, since there is no way for there to be any subtours with one stop. I should add a quick assumption that I didn't think about when making the model. For these examples, we are assuming that the time it takes to get to stop i from stop j is the same amount of time to get to stop j from stop i. This makes it a bit more practical, with less options, and makes the matrix symmetric.

The next scenario we will try is the following:

$$T = \begin{bmatrix} k & 8 & 13 \\ 8 & k & 12 \\ 13 & 12 & k \end{bmatrix}$$

This scenario does not have as obvious of a solution. The route has four different options: stop 1 then stop 2, stop 2 then stop 1, just stop 1, or just stop 2. The output for this example is $A_{1,2} = 1, A_{2,3} = 1, A_{3,1} = 1, w* = 3$. This means that the bus travels from the station to stop 2, then to stop 3, then back to the station, for a total time of 33 minutes.

The last scenario we will try will be

$$T = \begin{bmatrix} k & 11 & 23 & 16 \\ 11 & k & 7 & 12 \\ 23 & 7 & k & 21 \\ 16 & 12 & 21 & k \end{bmatrix}$$

For this, we will see how close to 60 minutes we can get the route, so $m = 2$. This one takes a bit longer to code, since there are 17 decision variables in total. Surprisingly enough, we are able to get exactly 60 minutes from these random times! We get this by going from the station to stop 3 (23 minutes), then from stop 3 to stop 4 (21 minutes), and finally back to the station (16 minutes). I also tried this with 30 minutes to see how it performed, and the fastest way was to go to and from stop 4 for a total of 32 minutes. This seems accurate as well, since I can't find a better option.

To make sure that the subtour constraints work, we will try one more example with a 5×5 matrix:

$$T = \begin{bmatrix} k & 7 & 11 & 8 & 14 \\ 7 & k & 18 & 15 & 12 \\ 11 & 18 & k & 9 & 6 \\ 8 & 15 & 9 & k & 10 \\ 14 & 12 & 6 & 10 & k \end{bmatrix}$$

We'll try 60 minutes again to see how it performs. This one is definitely harder to implement in the code, because of all the subtours possible that we must constrain. After running this, we get a time of 61 minutes for the route. The route goes from stop 1 (station) to stop 3 (11 minutes), then stop 4 (9 minutes), then stop 2 (15 minutes), then stop 5 (12 minutes), then back to the station (14 minutes). I am very happy with how this worked out, since there are no subtours in play. I also went ahead and tried to see how close we could get to 30 minutes, and we can actually get a perfect timed route by going to stop 2 (7 minutes), then stop 4 (15 minutes), then back to the station (8 minutes). Yay!

5 Reflection

Overall, I am very pleased with how this project turned out. When I started, I really wanted to create a model that could benefit a real company, and I believe that this does. Due to certain problems and circumstances, it doesn't follow all of the constraints I originally had after my meeting with Kathleen. I had to pick and choose exactly what I wanted to minimize and what to include. Furthermore, halfway through refining my optimization model, I realized that I had a notably *really* hard problem on my hands. It turns out that I was dealing with a version of the traveling salesman problem. I have read about this problem prior to working on this project, so I knew how complicated it really was. After consulting with Tom, he told me just to implement the subtour constraints with one bus route trying to reach a specific time. Although this wasn't exactly what I was trying to do initially, it is very close, and most importantly could still be used by the Spokane Transit Authority.

If I were to do this problem in a future scenario, I would probably think more about my goals before starting to make the model. I wasted a good amount of time by creating versions of a model only to realize that it didn't work or didn't do what I wanted. This could have easily been avoided if I thought about these things before implementing them. As for the solution to the model, it did take a while to code the subtour constraints. If the bus stop matrix was 10×10 or more, this model would definitely take too long to code. I realized during the last solution example that I could run the model without these constraints, and create them for any subtours that showed up. This method certainly saved a lot of time and would help someone using this in a future scenario.

I am very proud of my work on this project and I think the experience was really valuable. I will definitely point to this project as a solid example of my work and knowledge in future interviews.