Assignment 2

Control Theory group 48

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1 – DC Motor Velocity Controller

(a)

Our primary goals in designing the controller is to have zero steady state error and minimise the effect of noise on the system. Given that all we need for zero steady state error is an integrator, we chose a PI controller for our purposes. The one drawback for this controller is phase lag, but it won't be a problem for us as we have enough of a phase margin.

(b)

Our design process is outlined as follows:

. (1) Our first estimation we make as if we had a second order function.

. (2) We extract T_i and K from this estimation.

. $D(s)G(s) = 1$ at ω_c
. $\phi = -180°$+PM+($10°$ to $15°$)
. $T_i\omega_c = tan(90° - (10°$ to $15°)$
. $T_i = \frac{tan(90° - (10°...15°)}{\omega_c}$

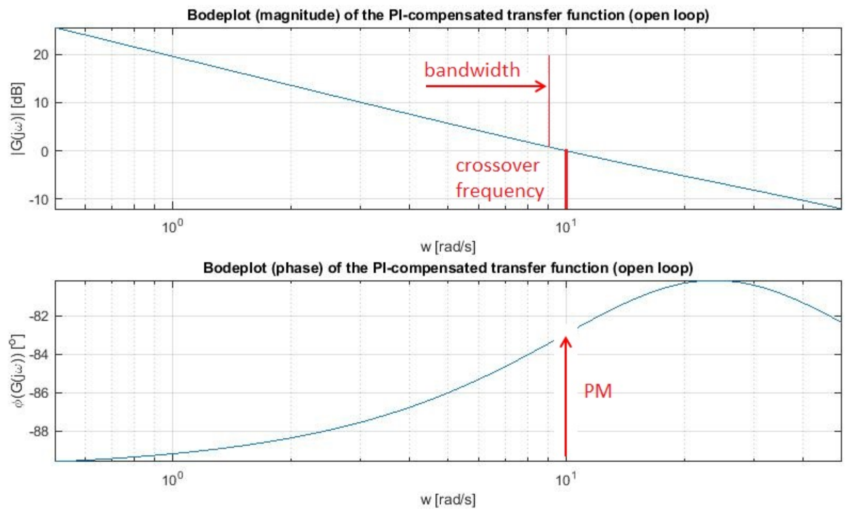
. We use $15°$ and ω_c of 10 Hz. We choose this as in our identification we have
. noise at higher frequencies than 10 Hz. Thus, our first estimate is $T_i = 0.3732$.
. K is found using the fact that $|D(j\omega)G(j\omega)| = 1$ at ω_c. Thus $K = 0.4713$

. (3) Our tests show that the rise time is too long.

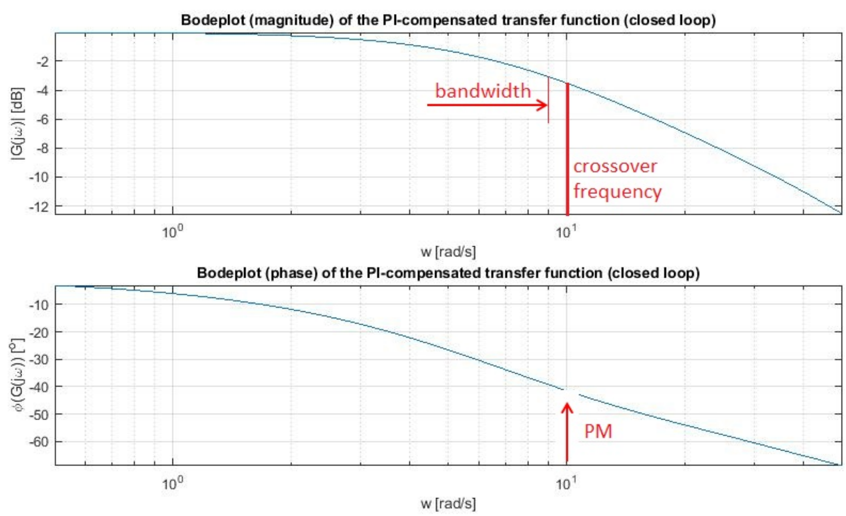
. (4) We then try other values of T_i from 0.1s to 0.05s and adjust K as necessary
. for the motor/cart. Uncoupling T_i from ω_c will increase the phase lag at ω_c but we
. are not worried as we are sufficiently far away from $-180°$ at this point.

. (5) Finally we check phase margin to make sure our results are satisfactory.

We eventually settle on a T_i of 0.05s with motor gain (K) as 0.2173 and cart gain (K) as 0.2506 (depending on whether we were testing the free-spinning wheels or the whole cart on the ground). We constantly adjust K according to T_i and G. Below we can see a Bode Plot of the open loop serial connection of the PI controller and the System:



As we can see, the cutoff frequency is about 10 rad/s and the phase stays significantly far from $-180°$. Now, with the loop closed, the bode plot shifts to the following:



As we can see we don't have a gain margin in our case as the phase never crosses $-180°$. Our final values are tabulated below:

	motor	cart
K	0.2173	0.2506
T_i	0.05 s	0.05 s
ω_c	10 Hz	10 Hz

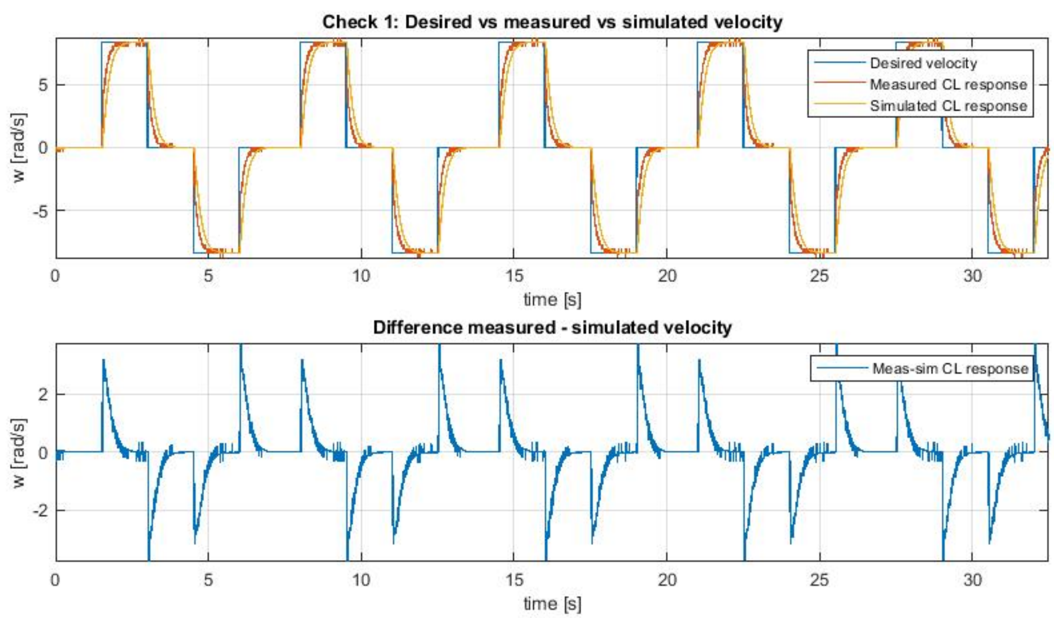
(c)

We could design a controller with a higher bandwidth by increasing the gain in our system. We chose not to do this so that we avoid the influence of noise. This is a practical constraint. There is also a theoretical constraint which is that the bandwidth can't be higher than half of our sampling frequency. This is due to the implementation of the controller (software + micro controller) in discrete time. Overall, an increase in K or a decrease in T_i maps to a higher bandwidth, but with each comes drawbacks. With a further decrease in T_i we see a lot of overshoot and a longer settling time.

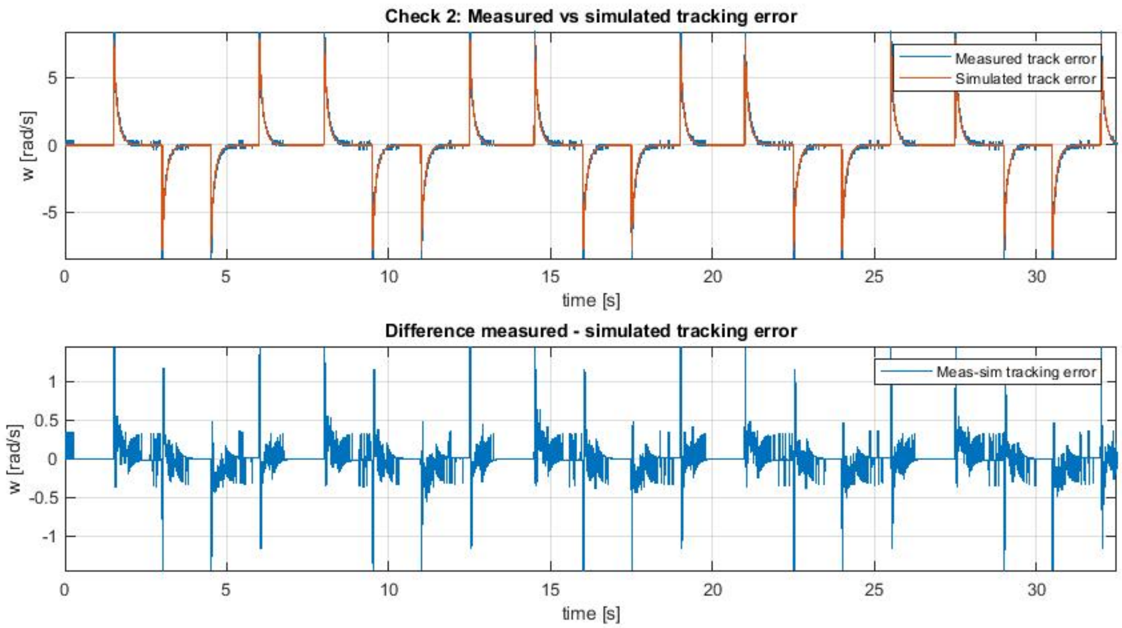
2 – Controller Validation

(a)

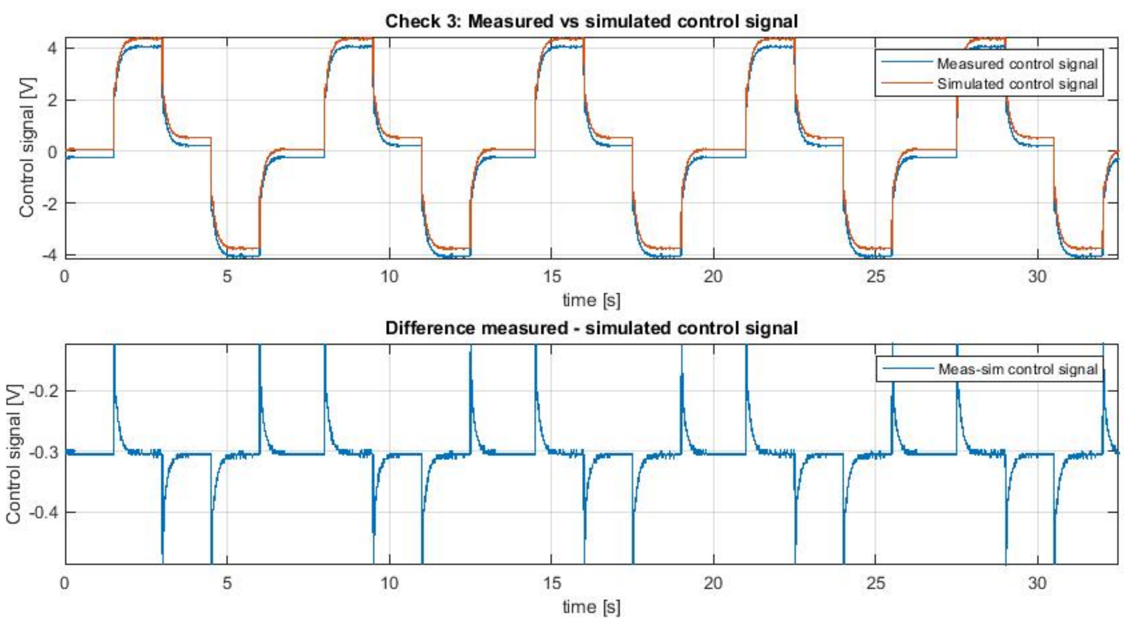
In converting open to closed systems we used the MATLAB command `feedback` and it was not clear in this particular case why for some of the input zeros there is a difference in the measured and control signals. If there was a steady state difference at those zeros we would have also expected to see a measured angular frequency difference. For the motor with the specified values for K and T_i above we see the following simulated and measured responses:



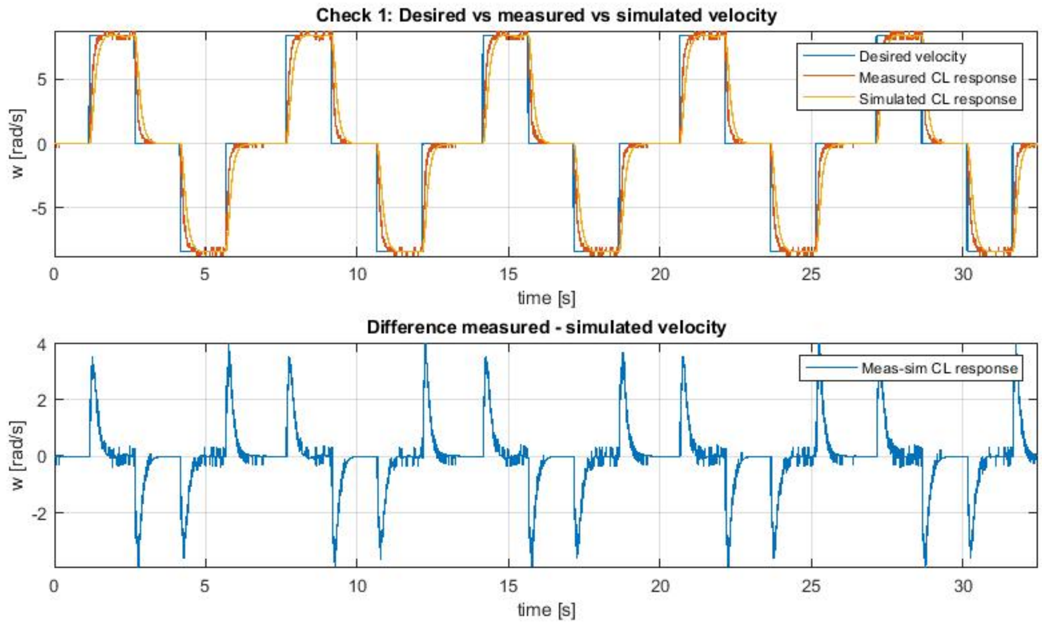
With the simulated and measured tracking errors:



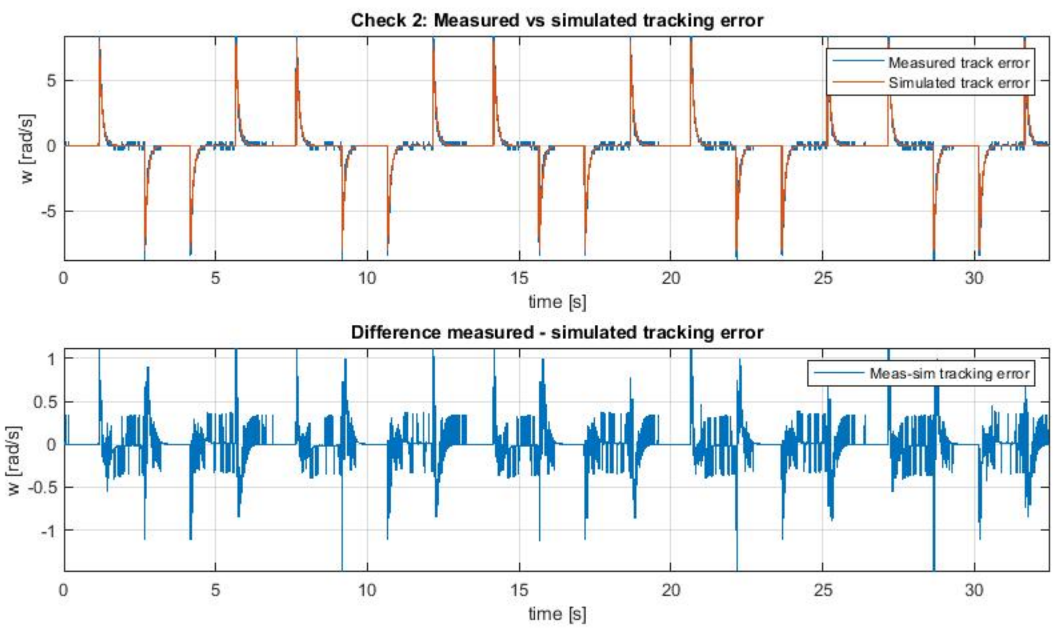
And the following simulated and measured control signals (note our unexplained constant offset error):



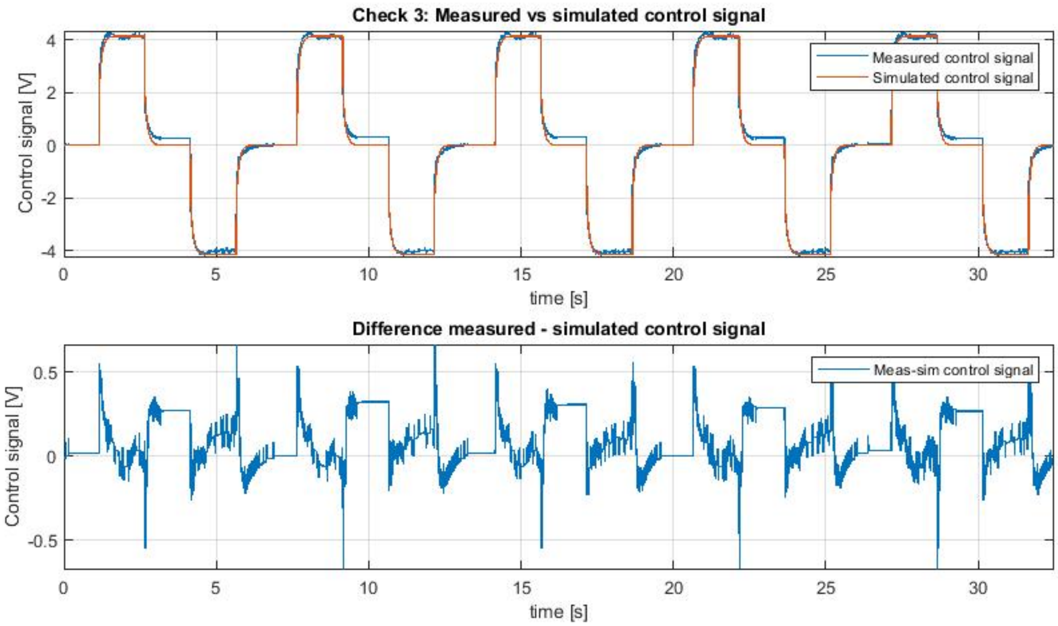
For the cart with the specified values for K and T_i above we see the following simulated and measured responses:



With the simulated and measured tracking errors:

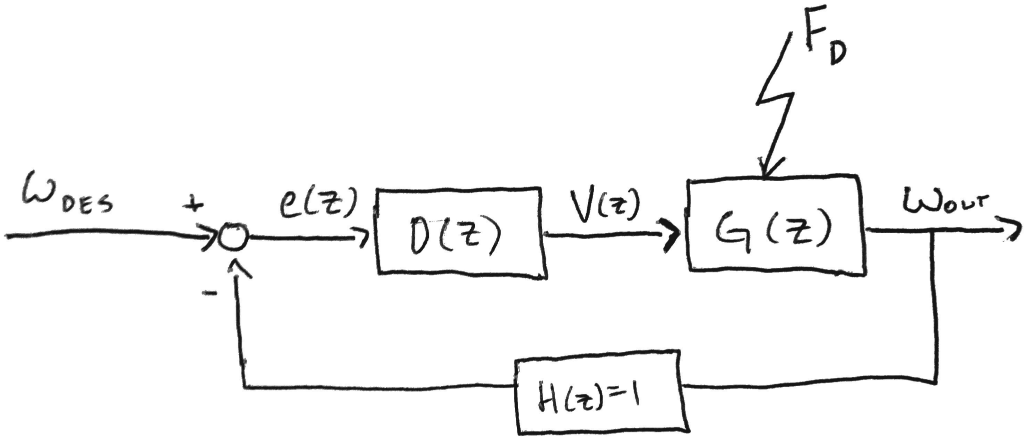


And the following simulated and measured control signals (note the unexplained steady state error when the control voltage returns to 0V from 4V):

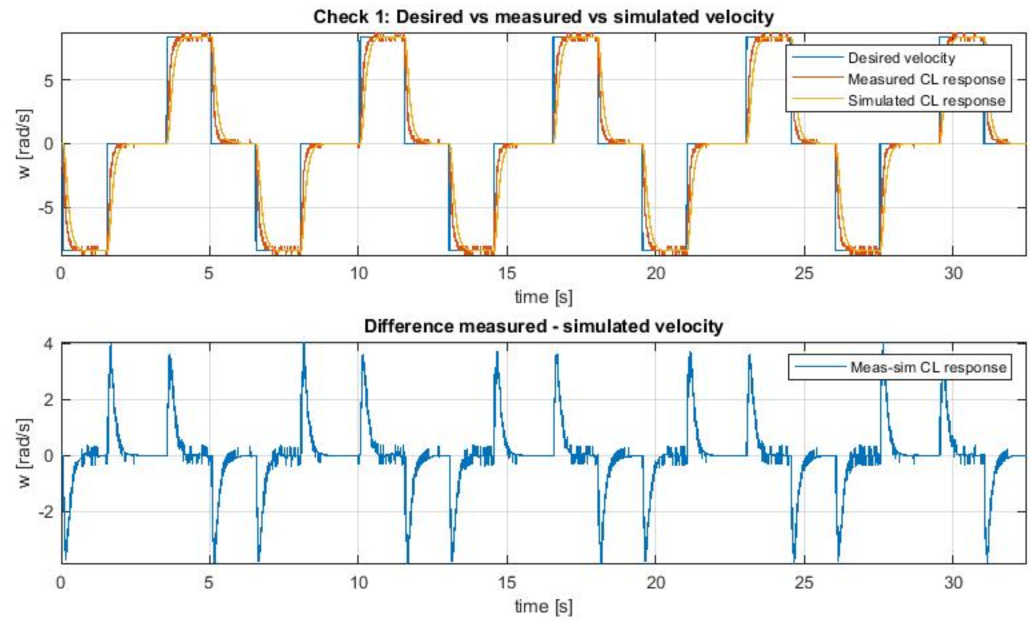


(b)

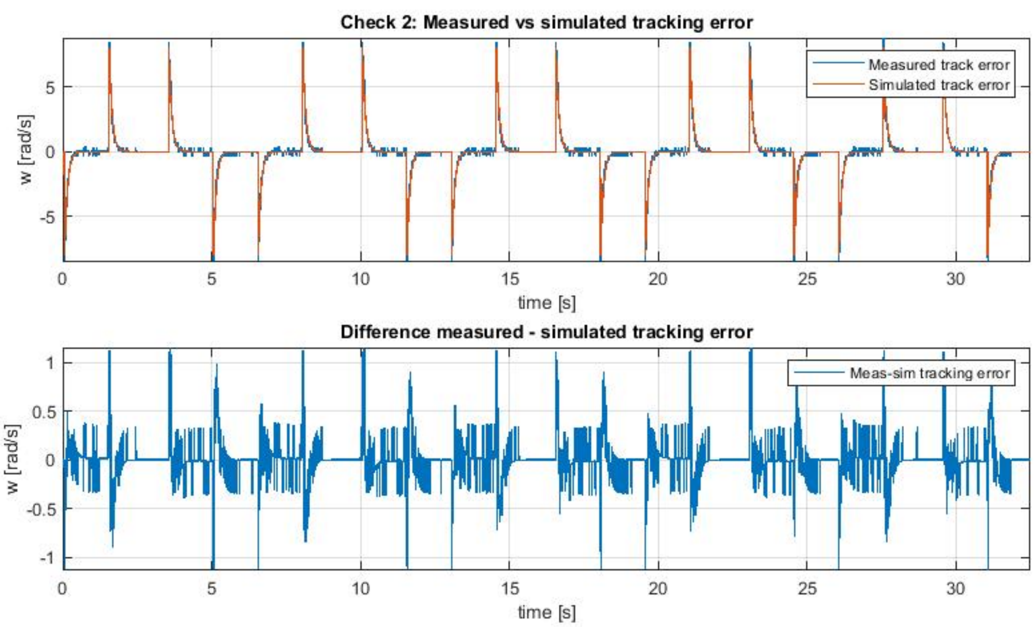
We apply a constant force disturbance to the cart by placing it on a slight incline. We achieve this constant sleight incline by wedging a couple books under the legs of one side of our lab table. The disturbance (F_D) enters the system in the following way:



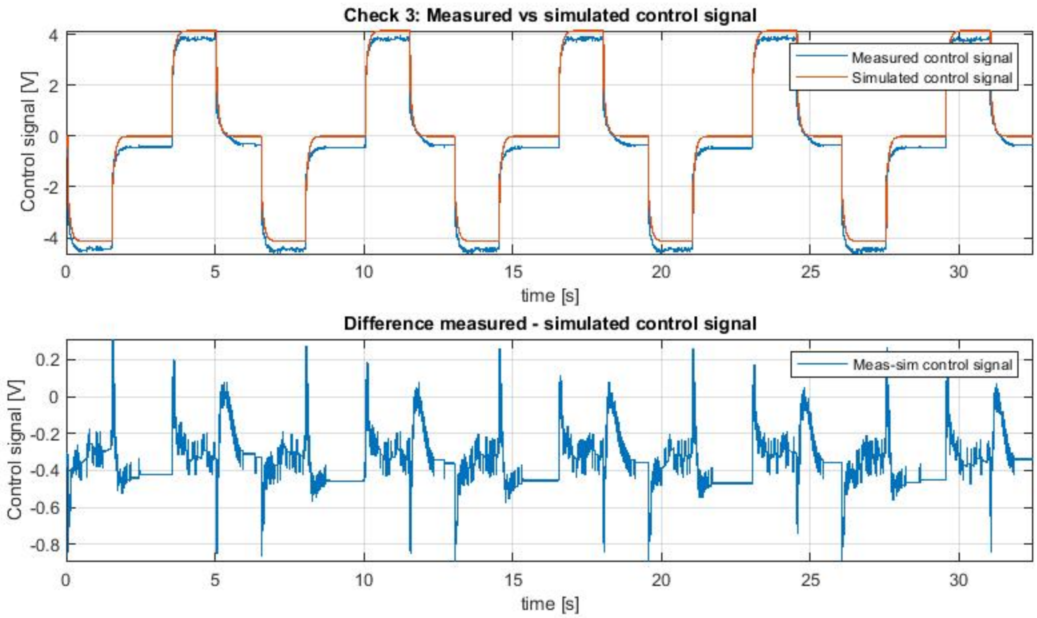
The slight incline is sufficient for the cart to roll downwards with an input voltage of 0V. The incline angle is around 5°. For the cart with the same inputs as the cart above, but with a constant force disturbance, we see the following simulated and measured responses:



With the simulated and measured tracking errors:



And the following simulated and measured control signals:



As we can see there is a constant offset between the measured and simulated control signals of about 0.4V. This would account for the constant force disturbance in one direction. The negative phase corresponds to when the cart is climbing the incline and the positive part corresponds to its descent. Even with this disturbance, the cart tracks its velocity setpoint well.