Modeling Market Mechanism with Minority Game

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I. Motivation and Problem Description

The ability to predict the movement of a financial market has always been a highly valued and heavily researched area since people can receive huge financial rewards if a model applies to the financial market and makes correct predictions. In real market, the amount of investment in a product is unstable and usually fluctuates: sometimes it goes up, and sometimes it goes down. A lot of underlying factors drives the fluctuation of market, including government policies, natural environment, and people's considerations [1].

We are going to focus on one important factor causing the market to fluctuate: the basic relationship between industry profitability and competition. The more people investing their money in a certain area, the more intense competition for customers they have to face, and thus the harder they can benefit from their investments. Investors who failed in the previous time period have to think of which new area to invest where they can gain more profit, while investors who gained a lot have to consider whether this area is still good for investing. Investors' decisions vary all the time, making the market fluctuate up and down. To more closely observe how market mechanism works, we introduce the concept of the Minority Game (which will be introduced in detail in the following section) to our model, because the nature of our model is similar to the Minority Game that the agents on the minority side are the winners [2].

In real market, there are a lot of products for agents to choose from and investors use different strategies to maximize their own profits. To simplify this problem, we make several assumptions in our model:

a. There are three products in the market.
b. Agents make decisions by predicting how many agents will invest in each product using "strategies".
c. Each agent has finite number of strategies to choose from and does not come up with new strategies during the process.
d. Only the record of previous several rounds could affect an agent's decisions.
e. Each agent chooses to use the strategy that makes the best prediction for the next round based on the record of previous rounds in the history.

We list several research questions to lead our research:

a. How does the market fluctuate?

b. Is the fluctuation periodic or not?

c. If the fluctuation is periodic, what does period depend on?

d. Is the market full of randomness?

e. How do the default profits of products affect people's decisions?

f. How can we come up with better strategies?

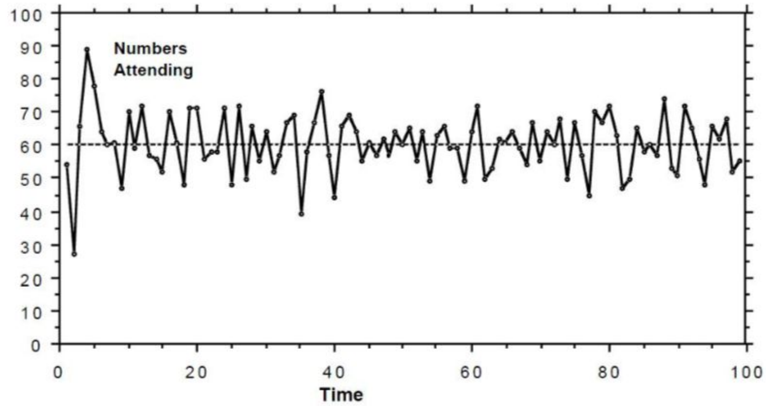
The significant parameters for the problem include the Number of Strategies, Memory Size, and Default Prices for each product. We expect to find the periodic pattern of the fluctuation in the simulations. Then, based on that, we hope to investigate how the three parameters listed above affect the periodic fluctuation pattern. Finally, we want to develop a new strategy that takes the pattern of periodic fluctuation into consideration than can make more profits on average.

In the following sections of this report, we are going to show the background information, mathematical model, simulation model, experiments and results of our project.

II. Background

1. El Farol Model

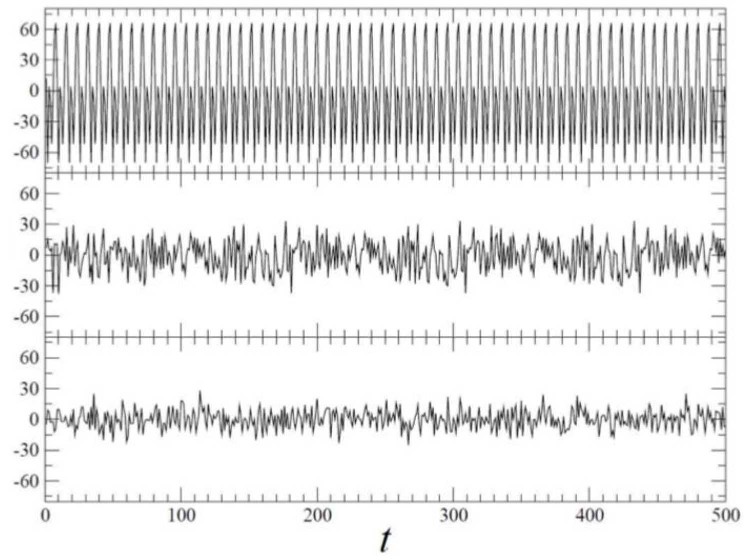
The El Farol Model was created by W. Brian Arthur in 1994. N people decide independently every Thursday night whether to go to a bar in Santa Fe, New Mexico. Because the bar is very small, it is not fun to go there if it's too crowded. Arthur defined that if more than xN people go to the bar, where $x \in (0, 1)$, then they will have a worse time at the bar than stay at home; otherwise, they will have a better time than stay at home. All these people are required to make decisions at the same time so that their decisions are independent of each other. There is no way to tell the numbers coming for sure in advance, so a person goes to the bar if he believes fewer than xN people to go, or stays at home if he believes more than xN people will be at the bar. The only information available for these N people is the attendance numbers of people in the past few weeks. In order to reduce the complexity of the game, all the people are provided with a limited number of strategies. When they play the game, they would evaluate the strategies and adjust their decisions accordingly. The result of this game shows that the number of people go to the bar finally evolves to the critical value xN, as is seen on the plot below, where the value of x is 0.6 [3]:



In the El Farol Model, we do not see any periodic pattern, because the sample is very small and going to the bar or not is influenced by many other factors, people do not particularly use some precise strategies to make decisions. Still, we observe the fluctuation pattern in the plot and average numbers of people attending the bar is around 60, which is the threshold in this problem.

2. Minority Game

Based on the basic El Farol Model, Yi-Cheng Zhang and Damien Challet proposed a formal definition of Minority Game, which clarifies the definition of agents' strategies. The basic Minority Game models market interactions between a population of N agents competing in repeated games, where N is an odd integers. At each round of the game, every agent has to choose between one of two possible actions, either "buy" or "sell". These two bids are represented by "-1" and "1". The minority wins the round and all the winning agents are rewarded. With N agents, the attendance can take N possible values each week in the past M weeks. The parameter M is known as the memory of the agnets. At the start of the game, S strategies are randomly drawn by the agents from a strategy pool. These strategies help the agents in their decision making throughout the game and there are no strategies that are better or preferred. Each strategy is represented by an array of random number between -1 and 1. Each agent choose the best strategy to make decisions, which gives the best prediction for the next round. The strategy giving the minimum error with the data in memory is chosen to be the best strategy and this is continuously updated through the whole process [4]. Many researchers did experiments on the Minority Game, and the results align with their expectation. The plot shown below is from one of such experiments [5]:



The result shows that after the initial unstable period, the pattern does have a periodic fluctuation pattern (t= 280 to t= 300 and t= 300 to t= 420). The average level is around 0 because the two actions are "-1" and "1", which aligns with the expectation.

Our model applies the same idea with the Minority game, but make some variations on that. The model approaches will be illustrated in detail in the next two sections.

III. Mathematical Model

1. Constants

Our model is actually an variation of the basic Minority Game model. Instead of choosing from two options, "buy" or "sell", agents in our model choose from three options, each representing a product, Product 1, Product 2, or Product 3. The Default Profits of the three products are denoted by P_1, P_2, P_3. The Number of Strategies each agent has is denoted by S. The Memory Size is denoted by M.

2. Variables

The variables in our model include the numbers of agents choosing the three products and the profits that agents gains. The numbers of agents choosing the three products are determined by the choices of individual agents, which are controlled by basic artificial intelligence. The profit that agents gain from one product depends on the Default Profit of the product and the number of agents, which is calculated using the formula: $Profit = (Default\ Profit) \times (Total\ Number\ of\ Agents) / 3 / (Number\ of\ Agents\ on\ This\ Product)$. This formula means that the actual profit earned from a product is negatively related to the number of agents investing on the profit. If the number of agents investing on that product is exactly one third of the total number of agents, then the actual profit is the same as the Default Profit; the more agents investing, the less profit gained; and the fewer agents investing, the more profit gained.

3. Modeling Agents' Artificial Intelligence

   We model the agents to have basic abilities of predicting the market. The agents predict by taking a linear combination of the records in the several previous rounds of investments, and select the product with the highest predicted profit. And the agents try to optimizes the coefficients in their linear combinations by applying these linear combinations to previous rounds of investments and choosing to use the linear combination that yields the minimum error between the observations in the history and the corresponding predictions.

IV. Simulation Model and Implementation

1. Simulation Environment

   We choose NetLogo as our simulation environment since we are doing agent-based modeling, and NetLogo also provides convenient interfaces for controlling parameters and displaying real-time data.

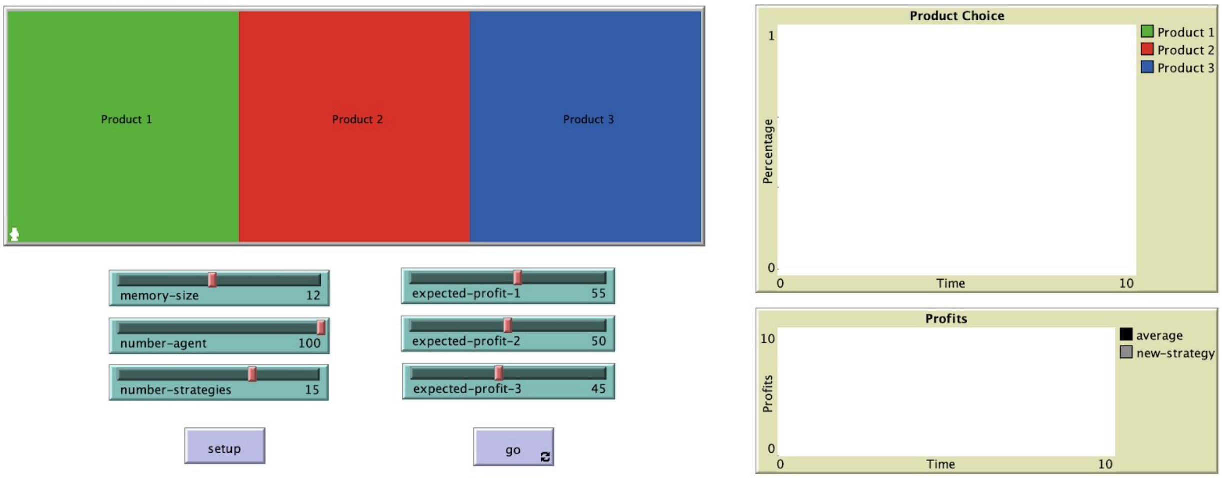
2. Interface

   To control the parameters in our model, we have 6 sliders: 3 for the Default Prices of the 3 products, 1 for Memory Size, 1 for the Number of Agents, and 1 for the Number of Strategies. The user can drag the slider to change the parameters before starting simulation.

   There are two buttons at the bottom of the interface, which are "setup" and "go". Setup Button, which initializes the simulation environment according to the parameters controlled by the sliders, should be pressed before each simulation. Pressing Go Button starts the simulation, and pressing it again pauses the simulation.

   The two plots on the right hand side are used to show the real time data in the simulation. The first plot with title "Product Choice" shows the proportions of agents choosing the 3 products as functions of time. The x-axis represents simulation time, and the y-axis represents the proportions, which range from 0 to 1. The other plot with title "Profits" draws the curve of the profit gained by the agents using our new strategy instead of random strategies versus the curve of the average profit gained by all agents.

   Though agents in our model do not interact in space, we want to have a visual representation of how agents select products. We make three blocks of different colors to represent the three product, and agents are randomly put on an empty patch in the block that corresponds to their selected product. We ensure that agents do not overlap each other, so that the density of agents in one block can precisely reflect the popularity of the corresponding product.



3. Code Implementation
   a. Globals
      - Memory-size: integer value for the number of previous rounds that agents take into consideration
      - Market: 3 by 1 vector storing the numbers of agents investing on the three products for the current round
      - History: 3 by (2 * memory-size) matrix storing the numbers of agents investing on the three products for recent (2 * memory-size) rounds
      - Default-profits: 3 by 1 vector storing the Default Profits for the three products
   b. Agent Properties
      - Strategies: list of strategies that the agent can choose from
      - Best-strategy: one strategy from the list strategies which makes the best prediction to records in history
      - Prediction: 3 by 1 vector storing the agent's predictions on how many agents will invest on the three products for the this round
      - Choice: the product that the agent chooses for this round
   c. Setup Method

      Setup Method is used to initialize the simulation. In this method, agents are created and they are assigned lists of random strategies.
   d. Go Method

Go Method is basically the method that controls the update of the simulation. It is called once for each round of investment. It asks the agents to make predictions based on their best strategies and make choices on investing which product. It then calculates the actual profits for each product according to the number of agents for each product. It also updates the globals, including Market and History. Agents are also asked to update their best strategies by looking at the updated Market and History.

e. Strategy Implementation

Predictions are made by taking linear combinations of the records in the previous rounds, and strategies determines how the linear combinations should be taken. One strategy is composed of a 3 by memory-size matrix containing coefficients that are used in the linear combinations for market prediction. The three rows stand for the coefficients put on the three products, and the columns stand for the coefficients put on previous rounds. For example, the element in the second row and the third column represents the coefficient for product 2 on the round that is 3 rounds before the current round.

The agents determine which strategy is the best by looking for the strategy with the minimum "mean squared error", which is calculated by the average of the squared differences between the predictions made using the strategy and their corresponding observations in the history.

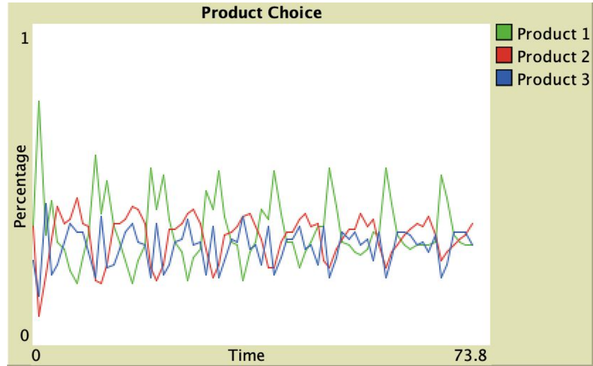
V. Experimental Studies and Results

1. Memory Size (Experiment 1)

To investigate how Memory Size (M) affects the pattern of fluctuation, we need to make the other parameters P_1, P_2, P_3, N, S constants for each set of simulations. We do 3 sets of simulations for this. In each set of simulation, we increase the Memory Size (M) from 6, 8, 10, to 12.

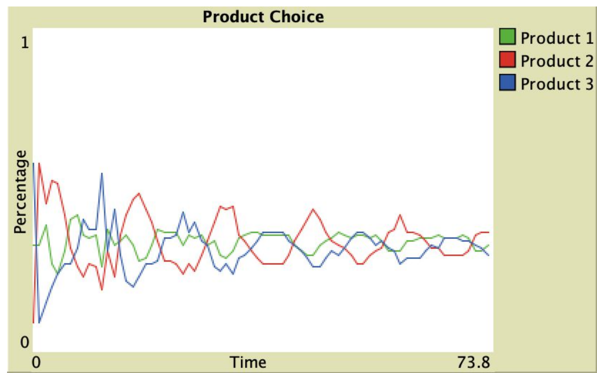
a. $P_1 = 55, P_2 = 50, P_3 = 45, N = 100, S = 15$

$M = 6$, Period = 7



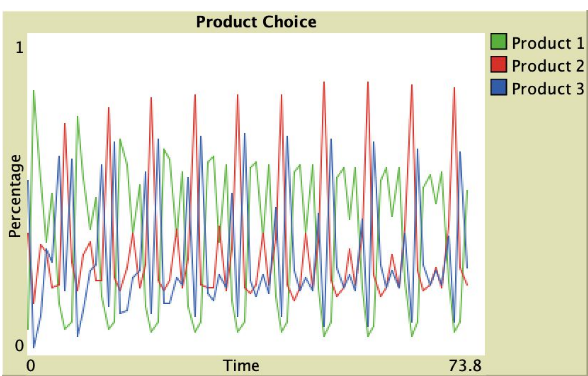
$M = 8$, Period = 9

$M = 10$, Period = 11



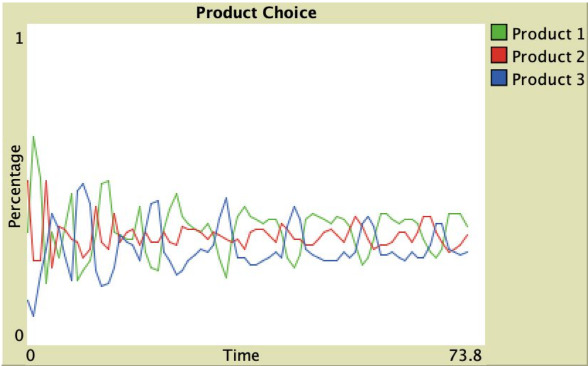
$M = 12$, Period = 13

b. $P_1 = 55, P_2 = 50, P_3 = 45, N = 100, S = 20$



$M = 6$, Period = 7

$M = 8$, Period = 9

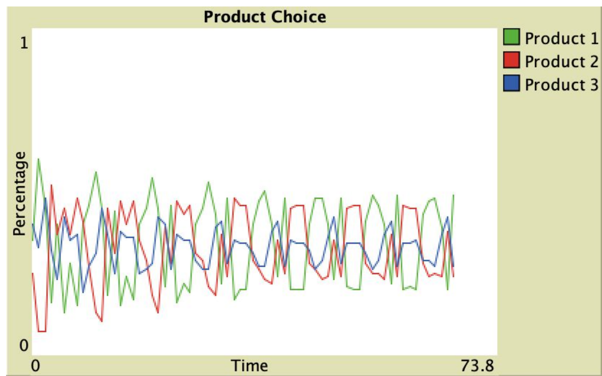


$M = 10$, Period = 11

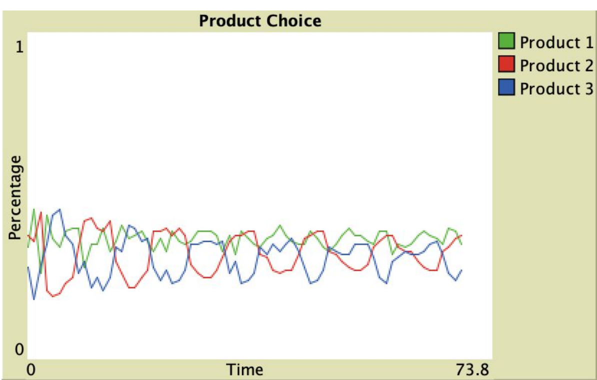
$M = 12$, Period = 13

c. $P_1 = 50, P_2 = 50, P_3 = 50, N = 100, S = 15$

$M = 6$, Period = 7



$M = 8$, Period = 9



$M = 10$, Period = 11

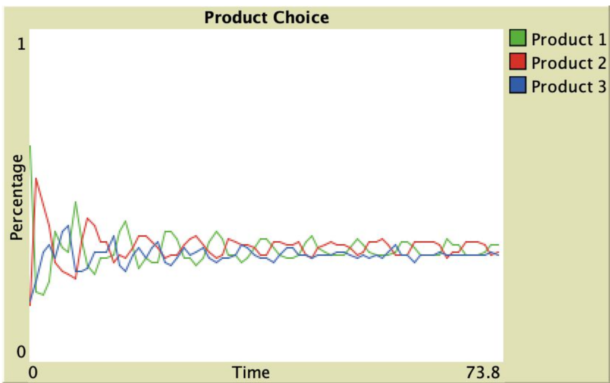
$M = 12$, Period = 13

From these 3 sets of simulations, we can easily find the fluctuation pattern is periodic. The period can be obtained by taking the difference between two adjacent peaks for the same product. Comparing the periods for the 3 set of simulations, we observe that no matter how we change the Default Profits or the Number of Strategies, the periods do not change, and the period only depends on Memory Size. To be more specific, Period = $M + 1$.

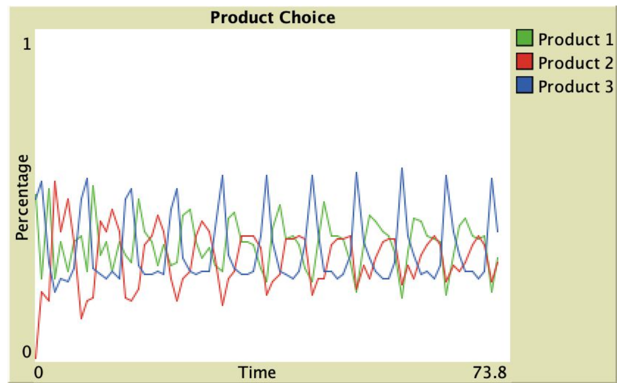
2. Number of Strategies (Experiment 2)

To investigate how Number of Strategies (S) affects the pattern of fluctuation, we need to make the other parameters P_1, P_2, P_3, N, M constants for each set of simulations. We do 3 sets of simulations for this. In each set of simulation, we increase the Number of Strategies (S) from 5, 10, 15, to 20.

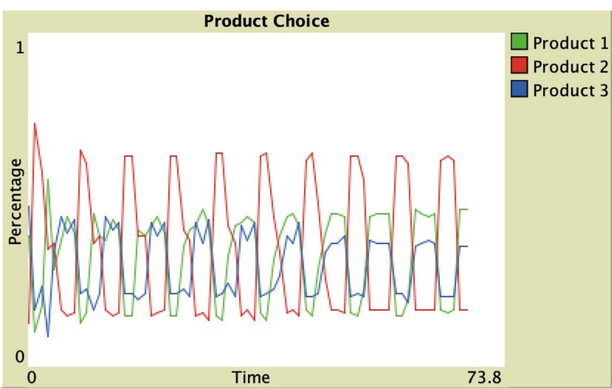
a. $P_1 = 50, P_2 = 50, P_3 = 50, N = 100, M = 6$



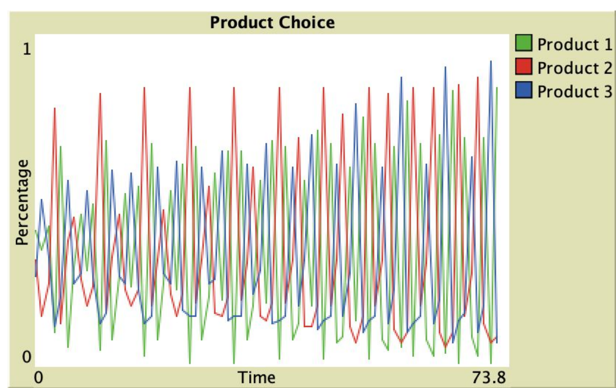
$S = 5$



$S = 10$

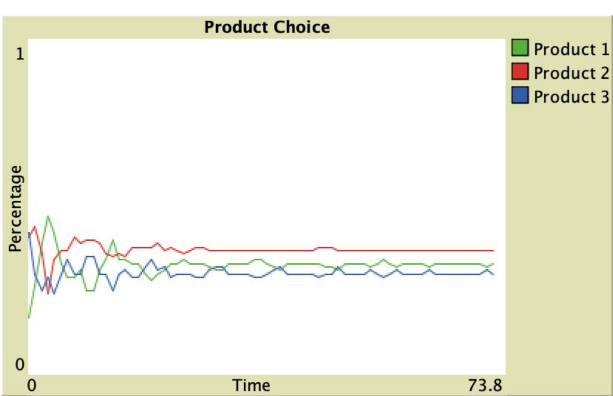


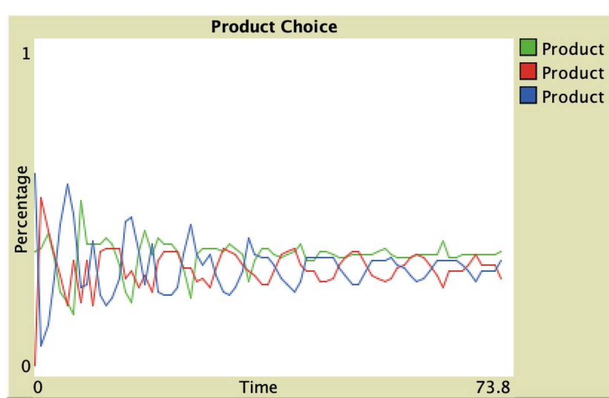
$S = 15$



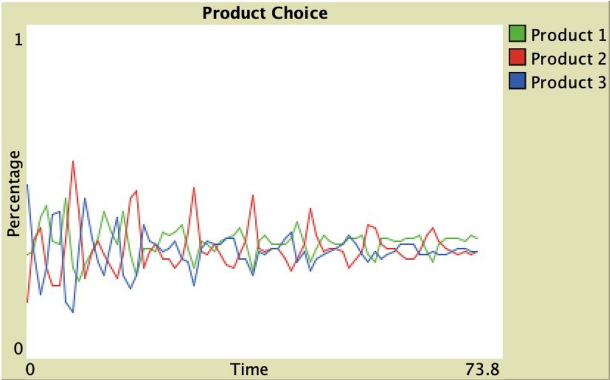
$S = 20$

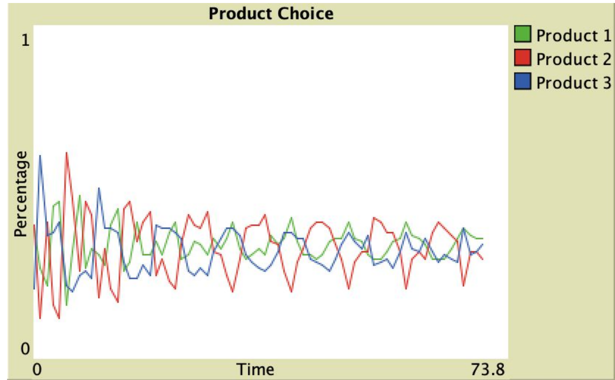
b. $P_1 = 50, P_2 = 50, P_3 = 50, N = 100, M = 8$





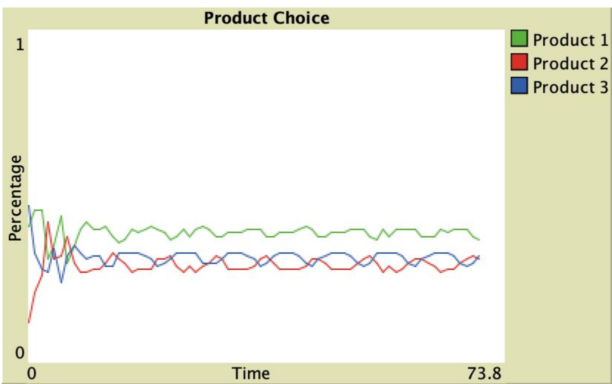
$S = 5$ $\qquad\qquad$ $S = 10$



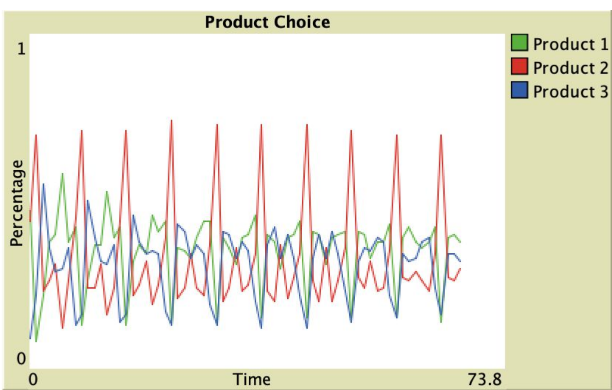


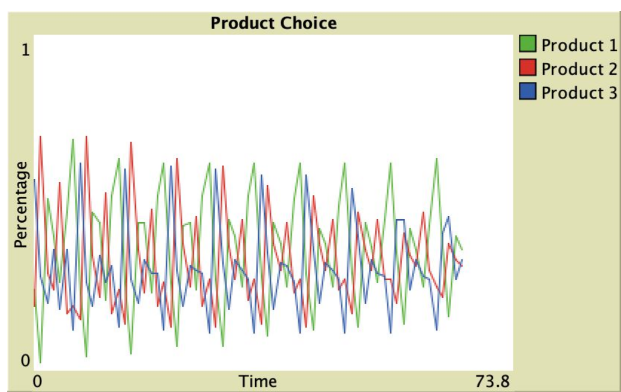
$S = 15$ $\qquad\qquad$ $S = 20$

c. $P_1 = 55, P_2 = 50, P_3 = 45, N = 100, M = 6$



$S = 5$ $\qquad\qquad$ $S = 10$





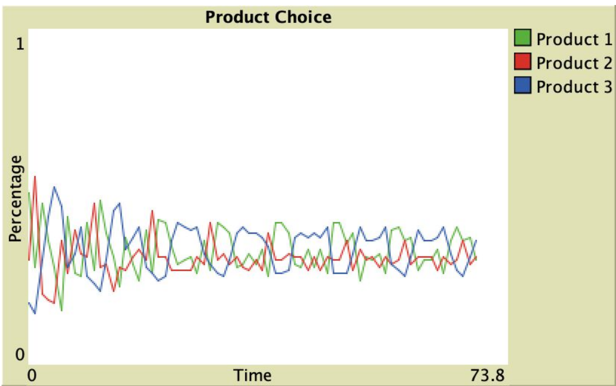
$S = 15$ $\qquad\qquad$ $S = 20$

From these 3 sets of simulations, we observe that the overall amplitude of fluctuation becomes larger as the Number of Strategies becomes larger, no matter how we change the Default Profits or the Memory Size. This means that when people have more strategies to choose from, the fluctuation of the market will be larger. The randomness of every agent's behavior increases, which causes the whole market to fluctuate more heavily.

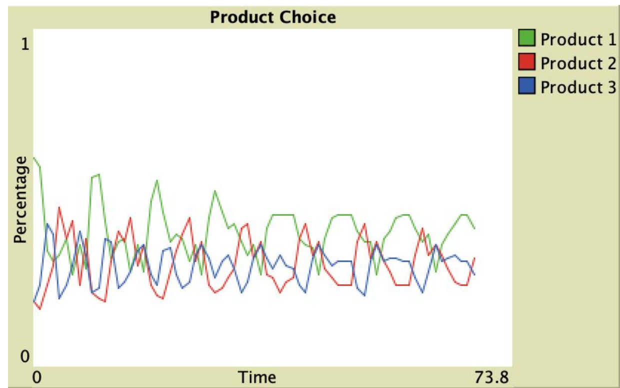
3. Default Profits for 3 products (Experiment 3)

To investigate how Default Profits (P_1, P_2, P_3) affect the pattern of fluctuation, we need to make the other parameters S, N, M constants for each set of simulations. We do 3 sets of simulations for this. In each set of simulation, we set different set of values for Default Profits (P_1, P_2, P_3).

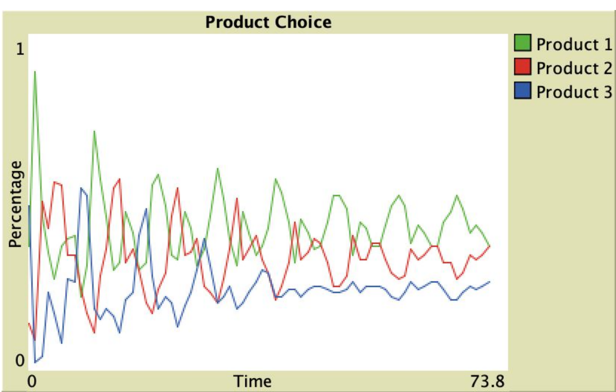
a. $S = 15, N = 100, M = 8$



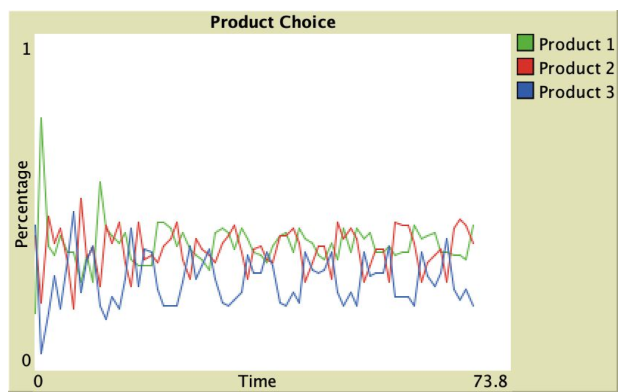
$P_1 = 50, P_2 = 50, P_3 = 50$



$P_1 = 60, P_2 = 50, P_3 = 40$



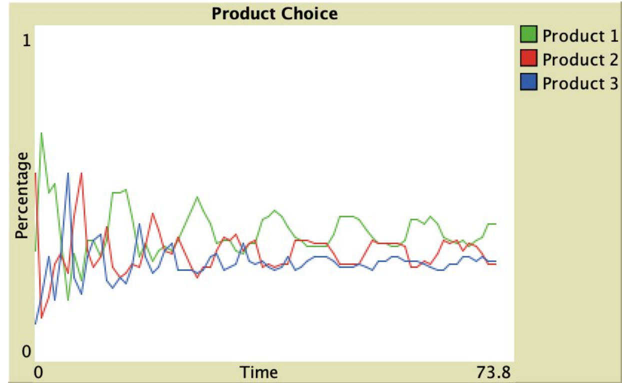
$P_1 = 70, P_2 = 50, P_3 = 30$



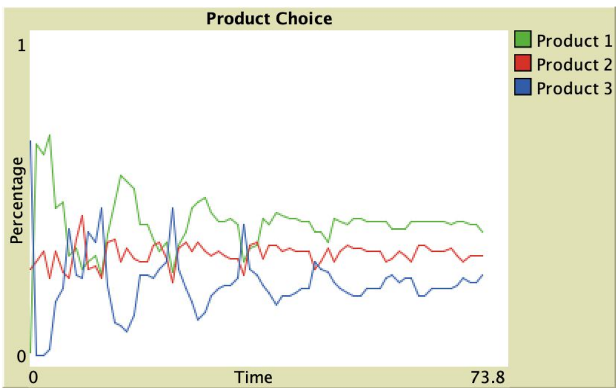
$P_1 = 60, P_2 = 60, P_3 = 40$

b. $S = 15, N = 100, M = 10$

$P_1 = 50, P_2 = 50, P_3 = 50$



$P_1 = 60, P_2 = 50, P_3 = 40$



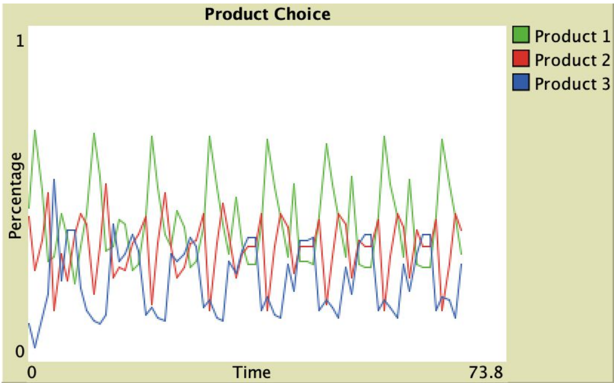
$P_1 = 70, P_2 = 50, P_3 = 30$

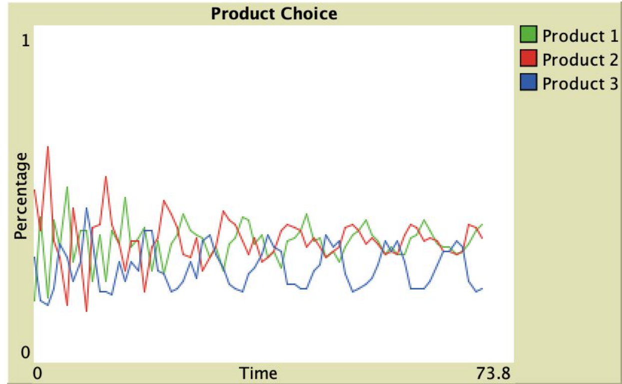
$P_1 = 60, P_2 = 60, P_3 = 40$

c. $S = 20, N = 100, M = 8$

$P_1 = 50, P_2 = 50, P_3 = 50$

$P_1 = 60, P_2 = 50, P_3 = 40$





$P_1 = 70, P_2 = 50, P_3 = 30$ $P_1 = 60, P_2 = 60, P_3 = 40$

Each set of simulation consists of 4 simulations with different Default Profits. When the Default Profits are the same ($P_1 = 50, P_2 = 50, P_3 = 50$), the average percentage of agents choosing each products are the same. When three Default Profits are not the same ($P_1 = 60, P_2 = 50, P_3 = 40$ and $P_1 = 70, P_2 = 50, P_3 = 30$), the average percentage of agents choosing Product 1 is higher than that choosing Product 2, which is again higher than the average percentage of agents choosing Product 3. In addition, the differences between the average percentages for three products become larger when the differences between Default Profits become larger. When two of the three Default Profits are the same, we can find that these two products have the same average percentage of agents, but the one with smaller Default Profits is obviously less chosen by the agents. Comparing the three set of simulations, we also find that, no matter how we change the Memory Size and the Number of Strategies, the same regular pattern apply for all the simulations.

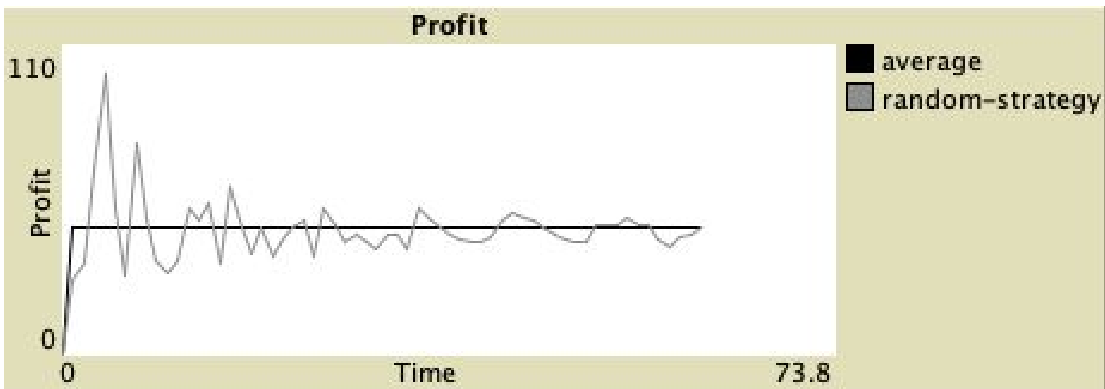
4. New Strategy (Experiment 4)

Since we confirm that there exists a periodic fluctuation pattern, we make our new strategy have the first round in the previous period as a dominating factor, which means the weight in the strategy for the first record in the memory is larger. This makes sense because the market tends to be the same condition as one period before, and increasing the weight put on that round can better predict the market.

Though the pattern for the new strategy is highly random and differs from one simulation to another, we still conclude that our new strategy can generally make more profit than the average, but the profit it gains is also periodic and sometimes it fails to make good predictions.

Below are two plots that compares our new strategy with a random strategy. The first plot is new-strategy versus average, and the second plot is random-strategy versus average. Obviously, random-strategy can only earn profits that fluctuates around the average, but new-strategy is usually above the average.

$P_1 = 40, P_2 = 45, P_3 = 50,$ *New Strategy*



$P_1 = 40, P_2 = 45, P_3 = 50,$ *Random Strategy*

5. Results
   a. The pattern of fluctuation is periodic. Based on Experiment 1, the period is only decided by the Memory Size, which means if every agent takes into account more history, then the period will generally be longer.
   b. The market fluctuates with randomness. Based on Experiment 2, if every agents has more strategies to choose from, then the randomness of the market will be larger and the amplitude of fluctuation for all three products are larger.
   c. Default Prices decide the relative popularity of each products. Based on Experiment 3, products with higher Default Prices are generally more popular than the products with lower Default Prices.
   d. A new strategy that puts more weight on the record one period before can generally makes better predictions and makes more profit than purely random strategies.

VI. Discussion and Conclusion

The project was successful in achieving the purposes that were initially laid out. The results of the experiments align well with our expectation. The fluctuation pattern is periodic and the period is dependent on the Memory Size. Since the pattern is periodic, we can easily predict which product will bring more profits in the next round after we detect the pattern.

We also develop a strategy that is obviously better than the other strategies in our model. Without detecting the exact periodic fluctuation pattern, investors can also make more profits than the average level.

Admittedly, the real situation is much more complicated than our model. First, there are many other factors such as government policies and environment limitations. Second, the number of products in real market is far more than 3, so the periodic pattern will not be that obvious. All these issues limits the application of our model in real life. Nevertheless, if we can develop a more complicated and more intelligent model that incorporates all these factors, we believe it can definitely help us make smart decisions on investments.

Reference:

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Contributions:

- Yuhang Liu

Explored how to build the model in NetLogo. Wrote most of the code in NetLogo. Devised most part of the mathematical model. Contributed to the parts related to modeling approaches and implementations in the Presentation and the Report. Also addressed the expected results, research questions, and assumptions in the Presentation.

- Hunter Wang

  Collected background information and related literature. Did most of the experiments to analyze the relationship between different parameters and the pattern in the model. Contributed to the parts related to generated results in the Presentation and the Report. Did the discussion and conclusion for the Report.