Recitation Session
Continuous & Discrete

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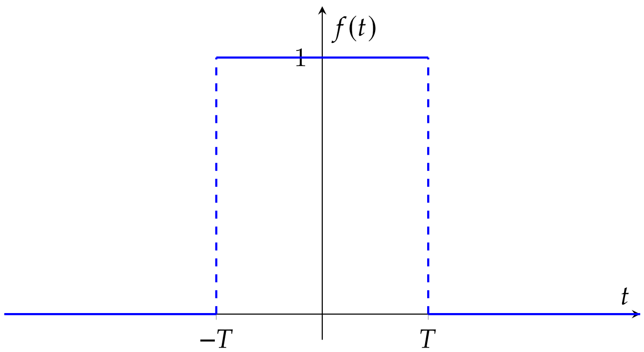
Signals Review

Question 1: Fourier Transform

For the following time-domain function:



Determine the Fourier Transform, $F(\omega)$, of the the signal (symbolically with T).

Solution:

$$f(t) = \begin{cases} 1 & -T \leqslant t \leqslant T \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} F(\omega) &= \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} \\ &= \frac{2\sin(\omega T)}{\omega} \end{aligned}$$

Routh-Hurwitz Criterion

Consider a strictly proper transfer function of form:

$$H(s) = \frac{Y(s)}{U(s)}$$

We would like to determine if the system is strictly stable (i.e. all roots have a real positive component).

Given the denominator polynomial of the n^{th}-degree polynomial

$$U(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_1 s + a_0$$

We can construct the following **Routh array**:

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \ldots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \ldots \end{array}$$

Note:-

You may pad zeros at the end of the array where necessary

We then construct the third row, s^{n-2}:

$$\begin{array}{c|ccccc} s^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \ldots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \ldots \\ s^{n-2} & b_1 & b_2 & b_3 & b_4 & \ldots \end{array}$$

Where b's are a determined by the formula,

$$b_1 = -\frac{\det \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{a_{n-1}},\ b_2 = -\frac{\det \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}}{a_{n-1}},\ b_3 = -\frac{\det \begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}}{a_{n-1}},\ \ldots$$

We continue this process for the following rows:

$$\begin{array}{c|ccccc} s^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \ldots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \ldots \\ s^{n-2} & b_1 & b_2 & b_3 & b_4 & \ldots \\ s^{n-3} & c_1 & c_2 & c_3 & c_4 & \ldots \end{array}$$

$$c_1 = -\frac{\det \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{b_1},\ c_2 = -\frac{\det \begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}}{b_1},\ c_3 = -\frac{\det \begin{vmatrix} a_{n-1} & a_{n-7} \\ b_1 & b_4 \end{vmatrix}}{b_1},\ \ldots$$

Util row s_0 is reached.

Theorem 1 The Routh-Hurwitz Criterion

The number of sign changes in the first column of the Routh Table is equal to the number of roots in the right hand plane.

We can use this to determine the stability of the system.

Question 2: Routh-Hurwitz Criterion

Determine if the system is stable. If not, how many poles are in the right half-plane.

$$\frac{s+15}{s^3+10s^2+31s+1030}$$

Solution:

$$\begin{array}{c|ccc} s^3 & 1 & 31 & 0 \\ s^2 & 10 & 1030 & 0 \\ s^1 & -72 & 0 & 0 \\ s^0 & 103 & 0 & 0 \end{array}$$

There are two sign changes 10→-72 and -72→103, thus two poles in the right plane.

Laplace, Complex Analysis, and Applications

Question 3: Laplace Transform

Find the Laplace Transform of the following function:

$$f(t) = t^2 e^{-3t},\ t \geqslant 0$$

Solution:

$$\mathcal{L}\{t^2 e^{-3t}\} = \frac{2}{(s+3)^3},\ s > -3$$

Question 4: Cauchy-Riemann

Verify whether the following function $f(z)$ given $z = x + jy$ is differentiable over all points $\mathbb{C}$.

$$f(z) = z^2$$

Solution:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -2y$$

Question 5: Inital Value Problem with Laplace

Find $y(t)$ from the following differential equation using Laplace transform,

$$2\ddot{y} - 10\dot{y} + 9y = 5t$$

Given the inital conditions, $y(0) = 0$ and $\dot{y}(0) = -2$

Solution:

$$y(t) = \frac{1}{125}(-96e^{\frac{t}{2}} + 96e^{-2t} - 10te^{-2t} - \frac{25}{2}t^2e^{-2t})$$

Question 6: Bode Plot

Create an approximate bode plot for the following transfer function:

$$H(s) = \frac{500(s+1)}{(s+5)(s+10)}$$

Solution: