AMRITA VISHWA VIDYAPEETHAM

AHEAD Online

CALCULUS

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Learning Functions - Recap until now

Defined functions

Vertical line test

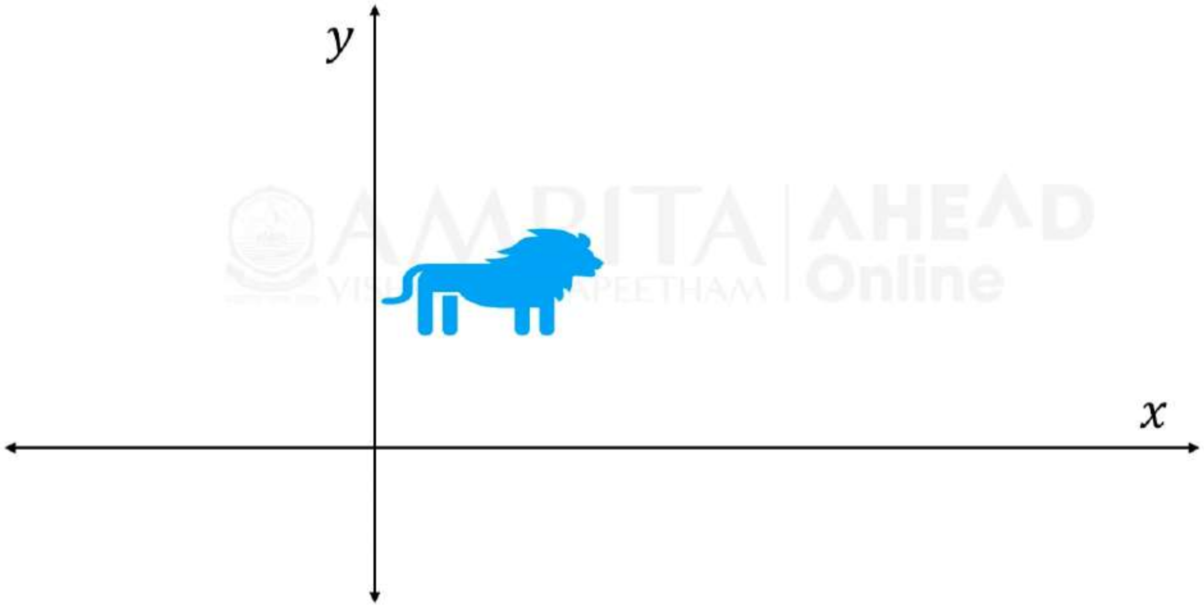
Domain and range of the function

Absolute value of the function

Functional transformations

Learning Functions - Recap until now

Functional transformations



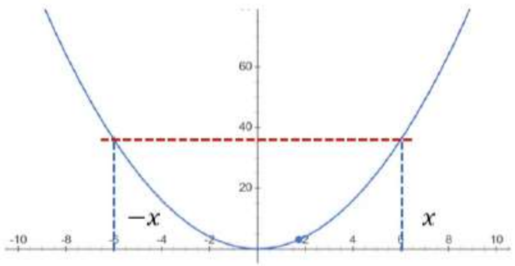
Even and Odd functions

A function, $y = f(x)$, is **even** if $f(x) = f(-x)$ for all x in the domain of f.

Geometrically, even function is symmetrical about y axis (line symmetry)

Example

$$\begin{aligned} f(x) &= x^2 \\ f(-x) &= (-x)^2 \\ &= -x \times -x = x^2 \\ &= f(x) \end{aligned}$$



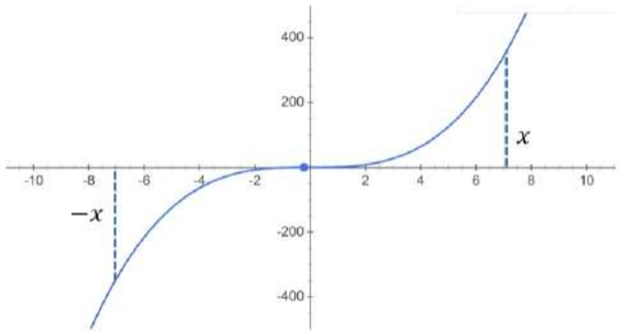
Even and Odd functions

A function, $y = f(x)$, is **odd** if $f(-x) = -f(x)$ for all x in the domain of f.

Geometrically, odd function is symmetrical about origin(rotational symmetry)

Example

$$\begin{aligned} f(x) &= x^3 \\ f(-x) &= (-x)^3 \\ &= -x \times -x \times -x = -(x^3) = -(f(x)) \\ &= -f(x) \end{aligned}$$



Examples - Even and Odd functions

Determine whether the following functions are even or odd or neither both algebraically and graphically

1. $f(x) = 3x^2 - 4$
2. $f(x) = \frac{1}{2x}$
3. $f(x) = x^3 + x^2$

Examples - Even and Odd functions

1. $f(x) = 3x^2 - 4$ **Even**

$$f(-x) = 3 \times (-x)^2 - 4$$
$$= 3x^2 - 4 = f(x)$$

2. $f(x) = \frac{1}{2x}$ **Odd**

$$f(-x) = \frac{1}{2(-x)} = \frac{-1}{2(x)} = -f(x)$$

3. $f(x) = x^3 + x^2$ **Neither Even nor Odd**

$$f(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$$
$$f(-x) \neq f(x)$$
$$f(-x) \neq -f(x)$$

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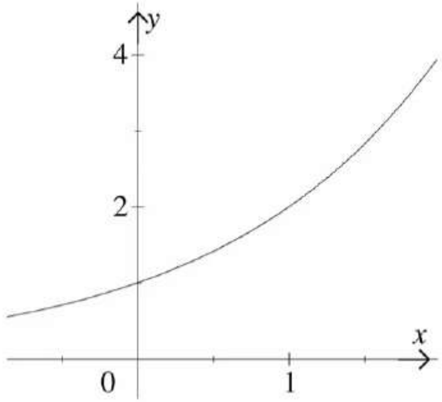
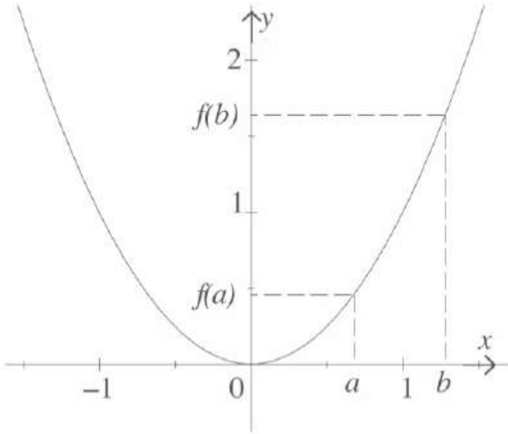
Increasing and decreasing functions

A function, $y = f(x)$, is **increasing** on an interval I, if for all a and b in the interval I such that $a < b$, $f(a) < f(b)$.



The graph of $y = 2^x$. This function is increasing for all real x.

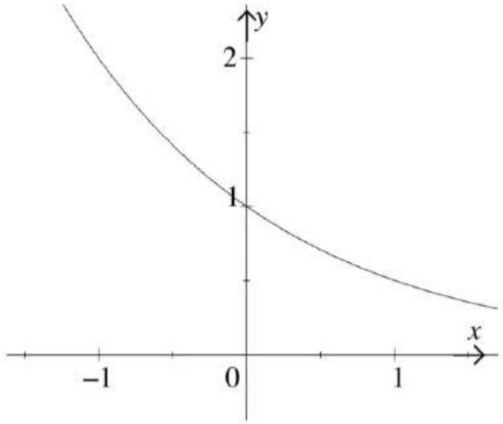


The graph of $y = x^2$. This function is increasing on the interval $x > 0$.

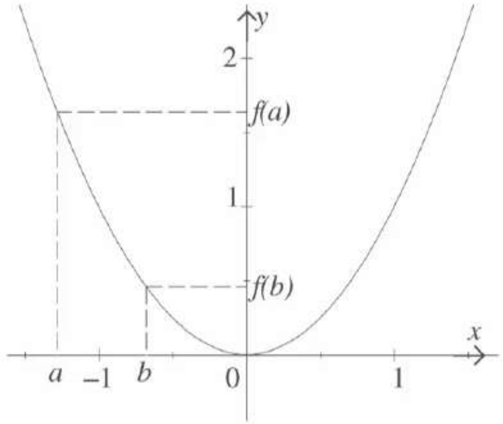
Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

Increasing and decreasing functions

A function, $y = f(x)$, is **decreasing** on an interval I, if for all a and b in the interval I such that $a < b$, $f(a) > f(b)$.



The graph of $y = 2^{-x}$. This function is decreasing for all real x.



The graph of $y = x^2$. This function is decreasing on the interval $x < 0$.

Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

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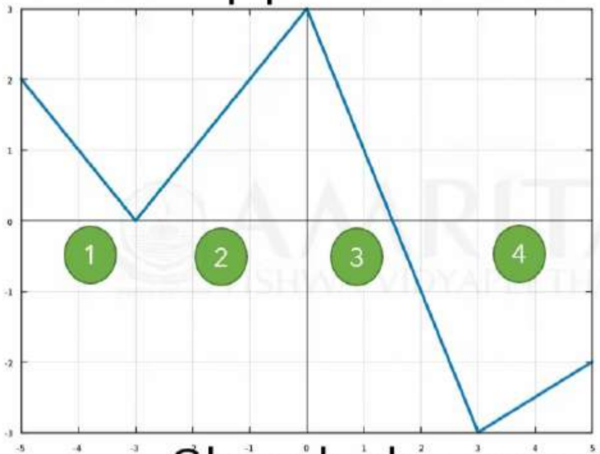
Piecewise functions

A function defined by multiple sub-functions, where each sub-function applies to a different interval in the domain.



$$f(x) = \begin{cases} -3 - x & \text{if} \quad x \le -3 \\ x + 3 & \text{if} \quad -3 \le x \le 0 \\ 3 - 2x & \text{if} \quad 0 \le x \le 3 \\ 0.5x - 4.5 & \text{if} \quad 3 \le x \end{cases}$$

Sketch the graph of the function

$f(x) = 1 - x$ for $x < 0$

$f(x) = 1 - x^2$ for $x \geq 0$

Piecewise functions

Plot: $f(x) = 1 - x$ for $x < 0$, $f(x) = 1 - x^2$ for $x \geq 0$

+ Code + Text

RAM Disk Editing

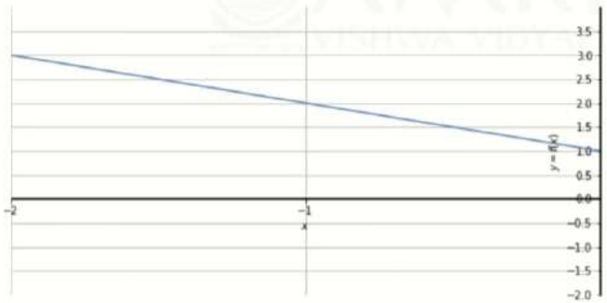
```python
import matplotlib.pyplot as plt
import numpy as np
###########################################
x1_lower_limit = -2 #Edit this variable
x1_upper_limit = 0 #Edit this variable
x1 = np.linspace(x1_lower_limit, x1_upper_limit, num=100)
y1 =  1 - x1 #Code up the equation
y_lower_limit = -2 #Edit this variable
y_upper_limit = 4 #Edit this variable
###########################################
fig, ax = plt.subplots()
ax.plot(x1, y1)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x1_lower_limit,x1_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x1_lower_limit, x1_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

3.5

Piecewise functions

Plot: $f(x) = 1 - x$ for $x < 0$, $f(x) = 1 - x^2$ for $x \geq 0$

```
plt.axvline(color='black')
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x1_lower_limit,x1_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x1_lower_limit, x1_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```



Piecewise functions

Plot: $f(x) = 1 - x$ for $x < 0$, $f(x) = 1 - x^2$ for $x \geq 0$

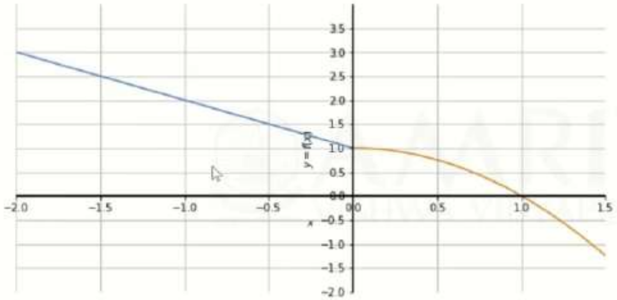
+ Code + Text

```
x1_upper_limit = 0 #Edit this variable
x1 = np.linspace(x1_lower_limit, x1_upper_limit, num=100)
y1 =  1 - x1 #Code up the equation

x2_lower_limit = 0 #Edit this variable
x2_upper_limit = 2 #Edit this variable
x2 = np.linspace(x2_lower_limit, x2_upper_limit, num=100)
y2 =  1 - x2**2 #Code up the equation

y_lower_limit = -2 #Edit this variable
y_upper_limit = 4 #Edit this variable
##########################################
fig, ax = plt.subplots()
ax.plot(x1, y1)
ax.plot(x2, y2)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x1_lower_limit,x1_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x1_lower_limit, x2_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

Piecewise functions

Plot: $f(x) = 1 - x$ for $x < 0$, $f(x) = 1 - x^2$ for $x \geq 0$

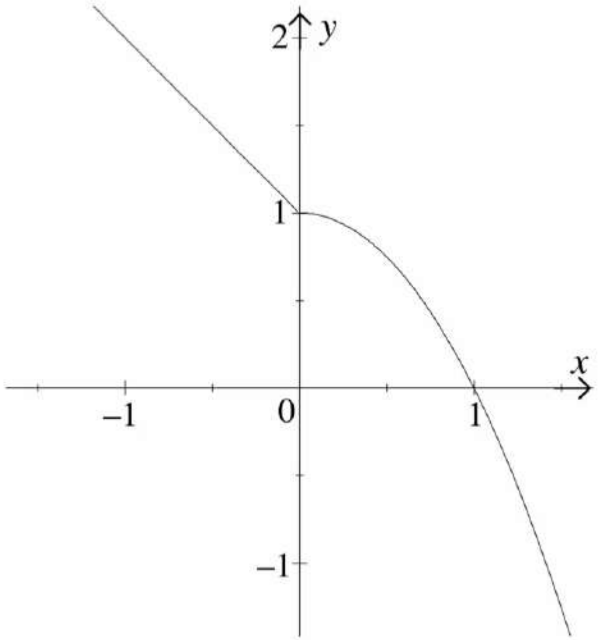
```
ax.set_xticks(np.arange(x1_lower_limit, x2_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```



Piecewise functions

We plott

$$f(x) = \begin{cases} 1 - x^2 & \text{for } x \geq 0 \\ 1 - x & \text{for } x < 0 \end{cases}$$



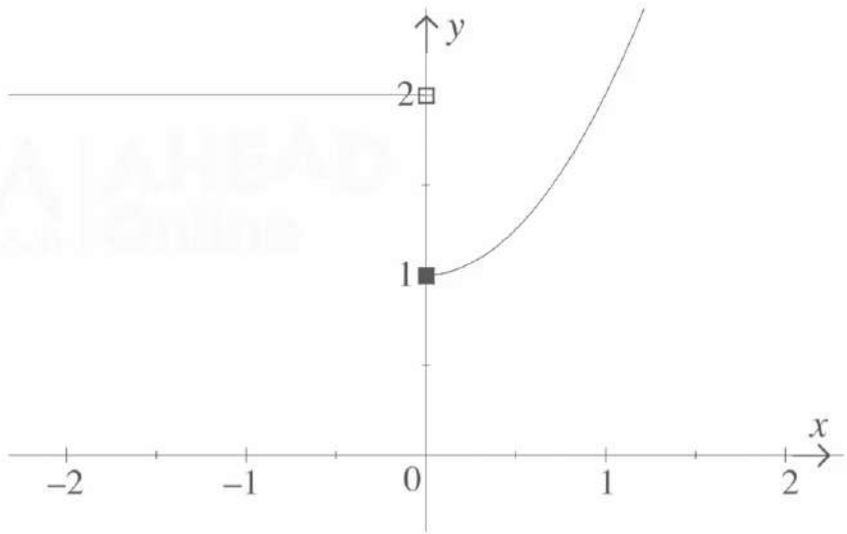
Piecew

Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

Piecewise functions

Plot the function

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x \geq 0 \\ 2 & \text{for } x < 0 \end{cases}$$



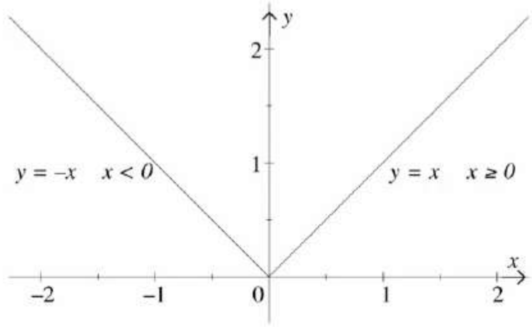
Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

Piecewise functions

Absolute function $f(x) = |x|$

$$f(x) = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

Piecewise continuous



The graph of $y = |x|$.

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