

# Learning Functions - Recap until now

Defined functions

Vertical line test

Domain and range of the function

Absolute value of the function

Functional transformations

# Even and Odd functions

A function,  $y = f(x)$ , is **odd** if  $f(-x) = -f(x)$  for all  $x$  in the domain of  $f$ .

Geometrically, odd function is symmetrical about origin(rotational symmetry)

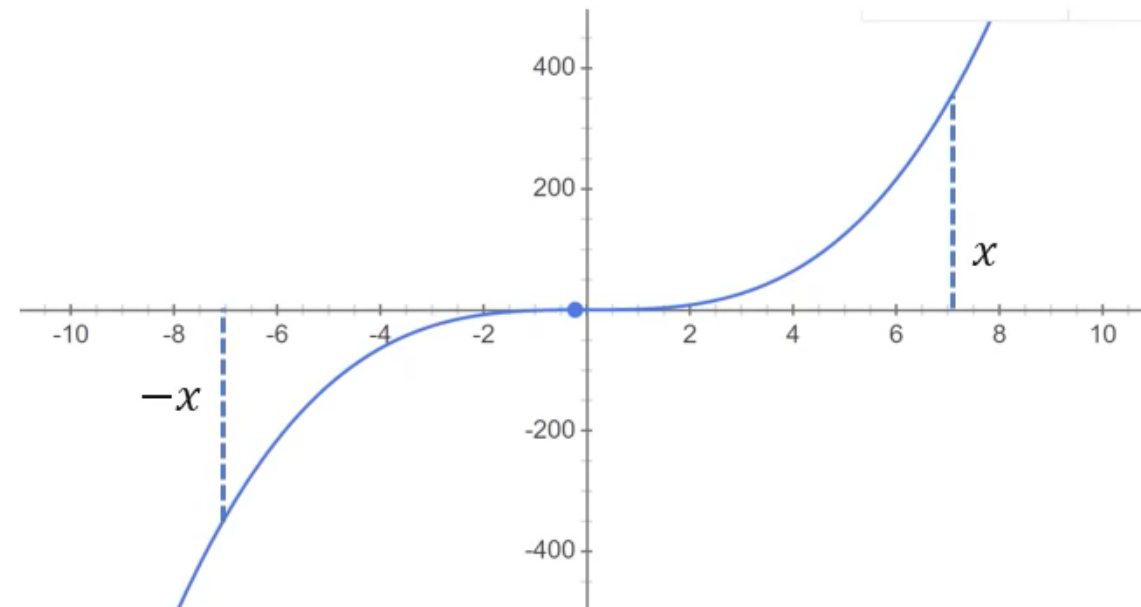
Example

$$f(x) = x^3$$

$$f(-x) = (-x)^3$$

$$= -x \times -x \times -x = -(x^3) = -f(x)$$

$$= -f(x)$$



# Examples - Even and Odd functions

Determine whether the following functions are even or odd or neither both algebraically and graphically

1.  $f(x) = 3x^2 - 4$

2.  $f(x) = \frac{1}{2x}$

3.  $f(x) = x^3 + x^2$



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# Examples - Even and Odd functions

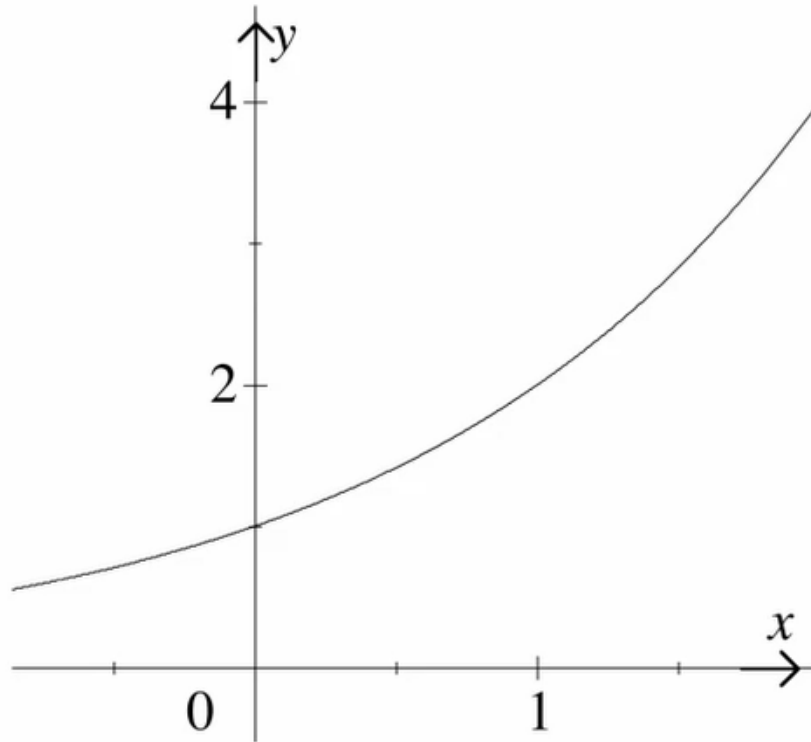
$$\begin{aligned} 1. \quad f(x) &= 3x^2 - 4 \quad \textbf{Even} \\ f(-x) &= 3 \times (-x)^2 - 4 \\ &= 3x^2 - 4 = f(x) \end{aligned}$$

$$\begin{aligned} 2. \quad f(x) &= \frac{1}{2x} \quad \textbf{Odd} \\ f(-x) &= \frac{1}{2(-x)} = \frac{-1}{2(x)} = -f(x) \end{aligned}$$

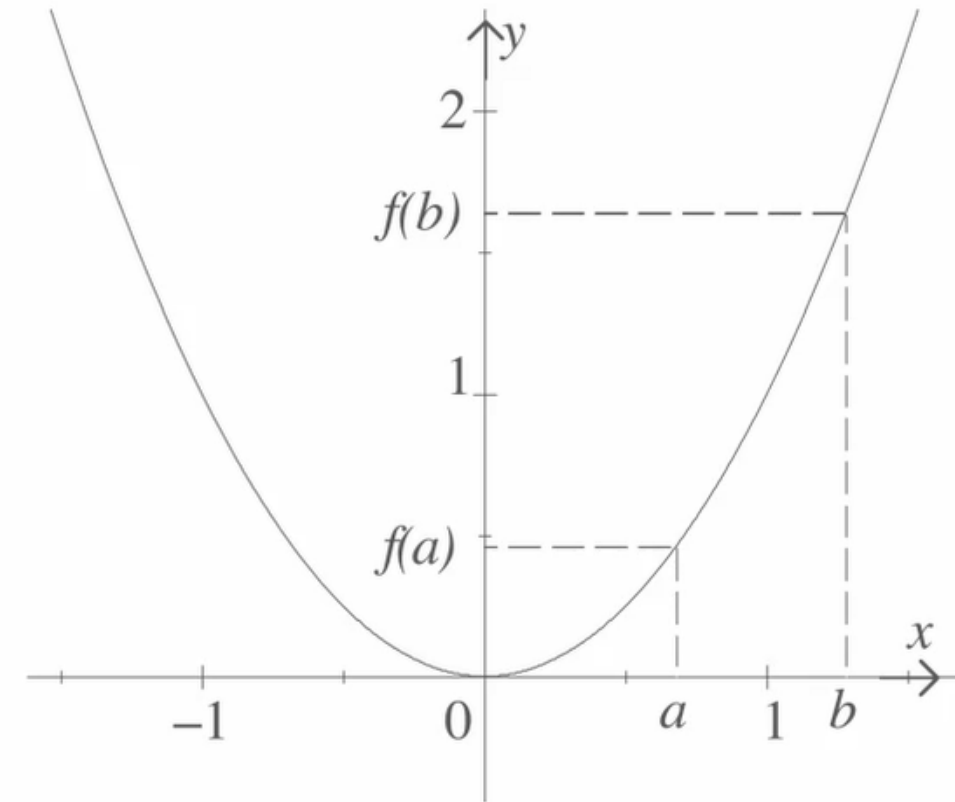
$$\begin{aligned} 3. \quad f(x) &= x^3 + x^2 \quad \textbf{Neither Even nor Odd} \\ f(-x) &= (-x)^3 + (-x)^2 = -x^3 + x^2 \\ f(-x) &\neq f(x) \\ f(-x) &\neq -f(x) \end{aligned}$$

# Increasing and decreasing functions

A function,  $y = f(x)$ , is **increasing** on an interval  $I$ , if for all  $a$  and  $b$  in the interval  $I$  such that  $a < b$ ,  $f(a) < f(b)$ .



The graph of  $y = 2^x$ . This function is increasing for all real  $x$ .

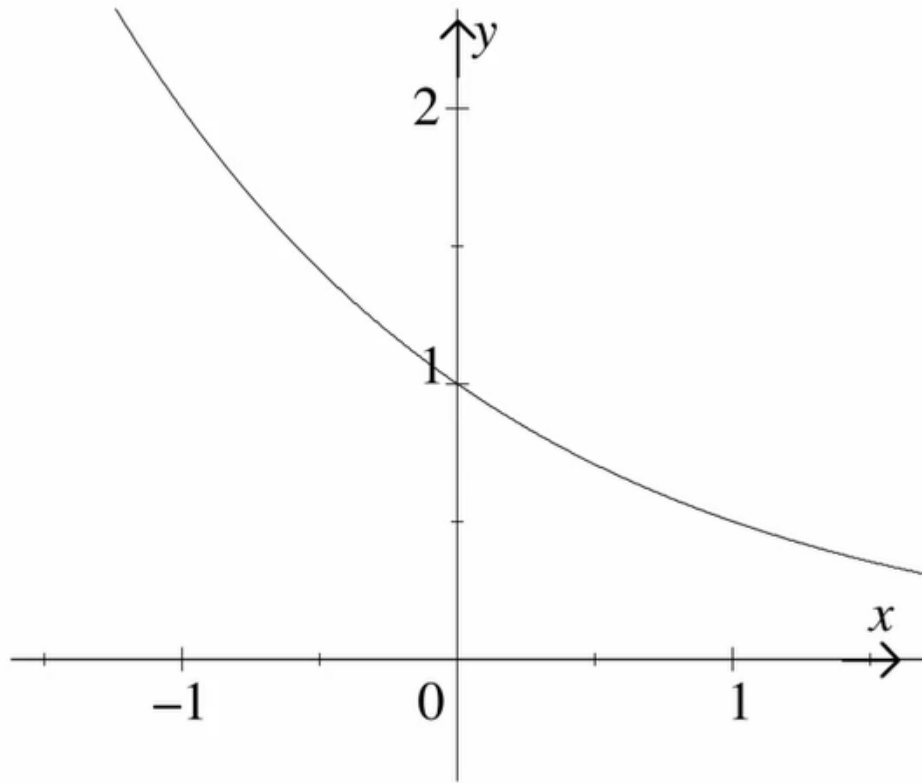


The graph of  $y = x^2$ . This function is increasing on the interval  $x > 0$ .

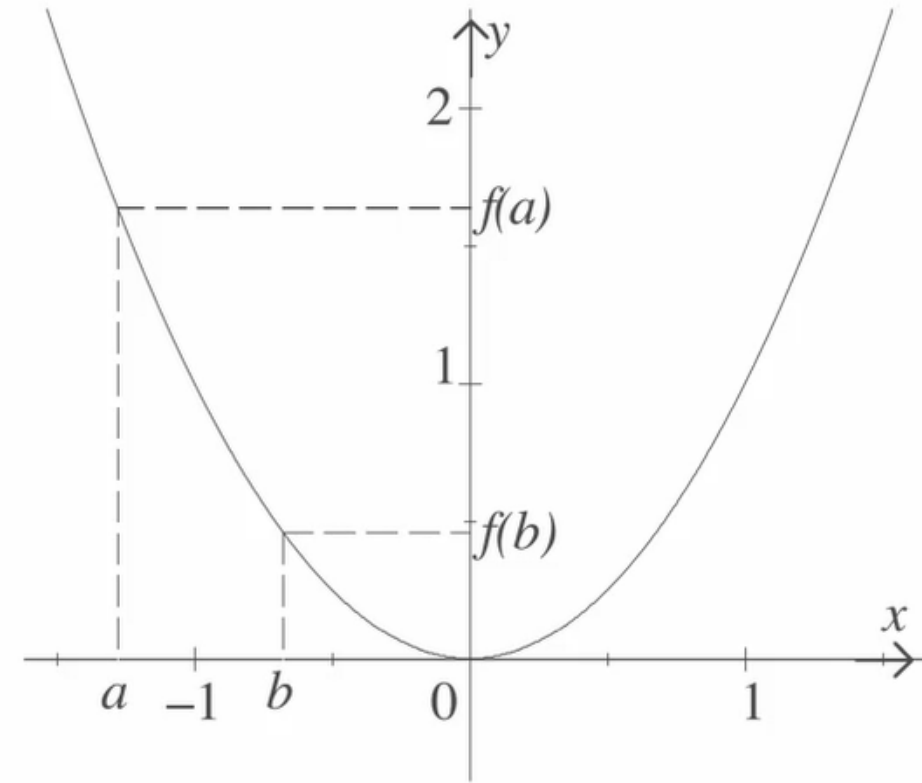
Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

# Increasing and decreasing functions

A function,  $y = f(x)$ , is **decreasing** on an interval  $I$ , if for all  $a$  and  $b$  in the interval  $I$  such that  $a < b$ ,  $f(a) > f(b)$ .



The graph of  $y = 2^{-x}$ . This function is decreasing for all real  $x$ .



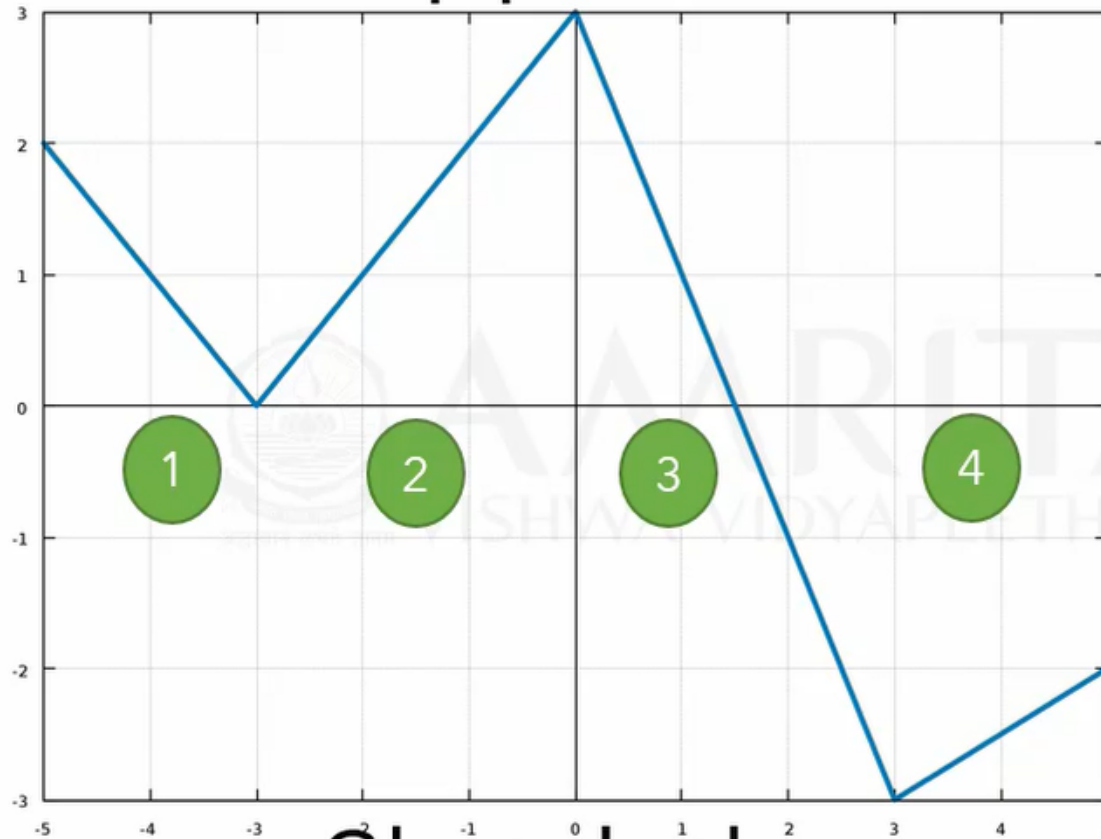
The graph of  $y = x^2$ . This function is decreasing on the interval  $x < 0$ .

Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney



# Piecewise functions

A function defined by multiple sub-functions, where each sub-function applies to a different interval in the domain.



$$f(x) = \begin{cases} -3 - x & \text{if } x \leq -3 \text{ (1)} \\ x + 3 & \text{if } -3 \leq x \leq 0 \text{ (2)} \\ 3 - 2x & \text{if } 0 \leq x \leq 3 \text{ (3)} \\ 0.5x - 4.5 & \text{if } 3 \leq x \text{ (4)} \end{cases}$$

Sketch the graph of the function

$$f(x) = 1 - x \text{ for } x < 0$$

$$f(x) = 1 - x^2 \text{ for } x \geq 0$$

# Piecewise functions

Plot:  $f(x) = 1 - x$  for  $x < 0$ ,  $f(x) = 1 - x^2$  for  $x \geq 0$

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+ Code + Text
RAM
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Editing

import matplotlib.pyplot as plt
import numpy as np
#####
x1_lower_limit = -2 #Edit this variable
x1_upper_limit = 0 #Edit this variable
x1 = np.linspace(x1_lower_limit, x1_upper_limit, num=100)
y1 = 1 - x1 #Code up the equation

x2_lower_limit = 0 #Edit this variable
x2_upper_limit = 2 #Edit this variable
x2 = np.linspace(x2_lower_limit, x2_upper_limit, num=100)
y2 = 1 - x2**2 #Code up the equation

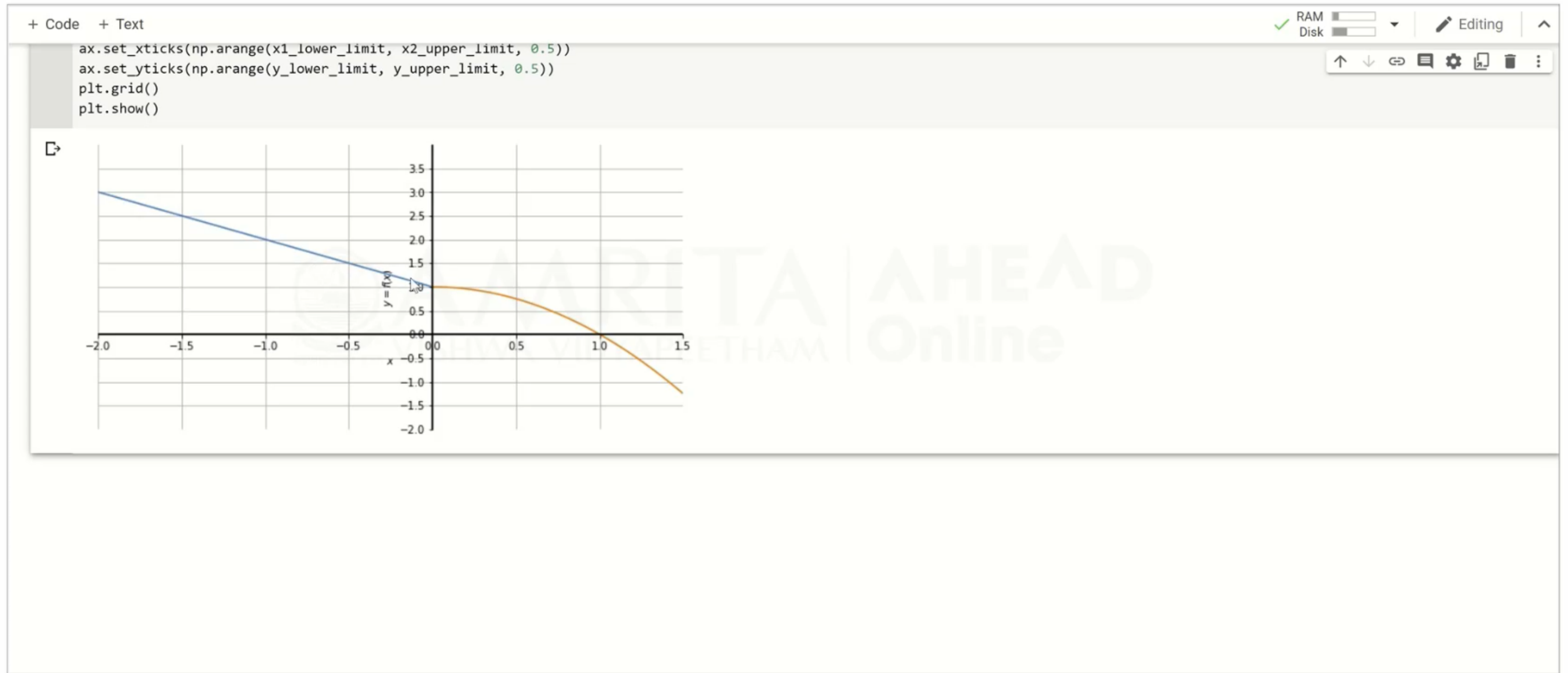
y_lower_limit = -2 #Edit this variable
y_upper_limit = 4 #Edit this variable
#####
fig, ax = plt.subplots()
ax.plot(x1, y1)
ax.plot(x2, y2)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x1_lower_limit, x1_upper_limit])
ax.set_ylim([y_lower_limit, y_upper_limit])

```



# Piecewise functions

Plot:  $f(x) = 1 - x$  for  $x < 0$ ,  $f(x) = 1 - x^2$  for  $x \geq 0$

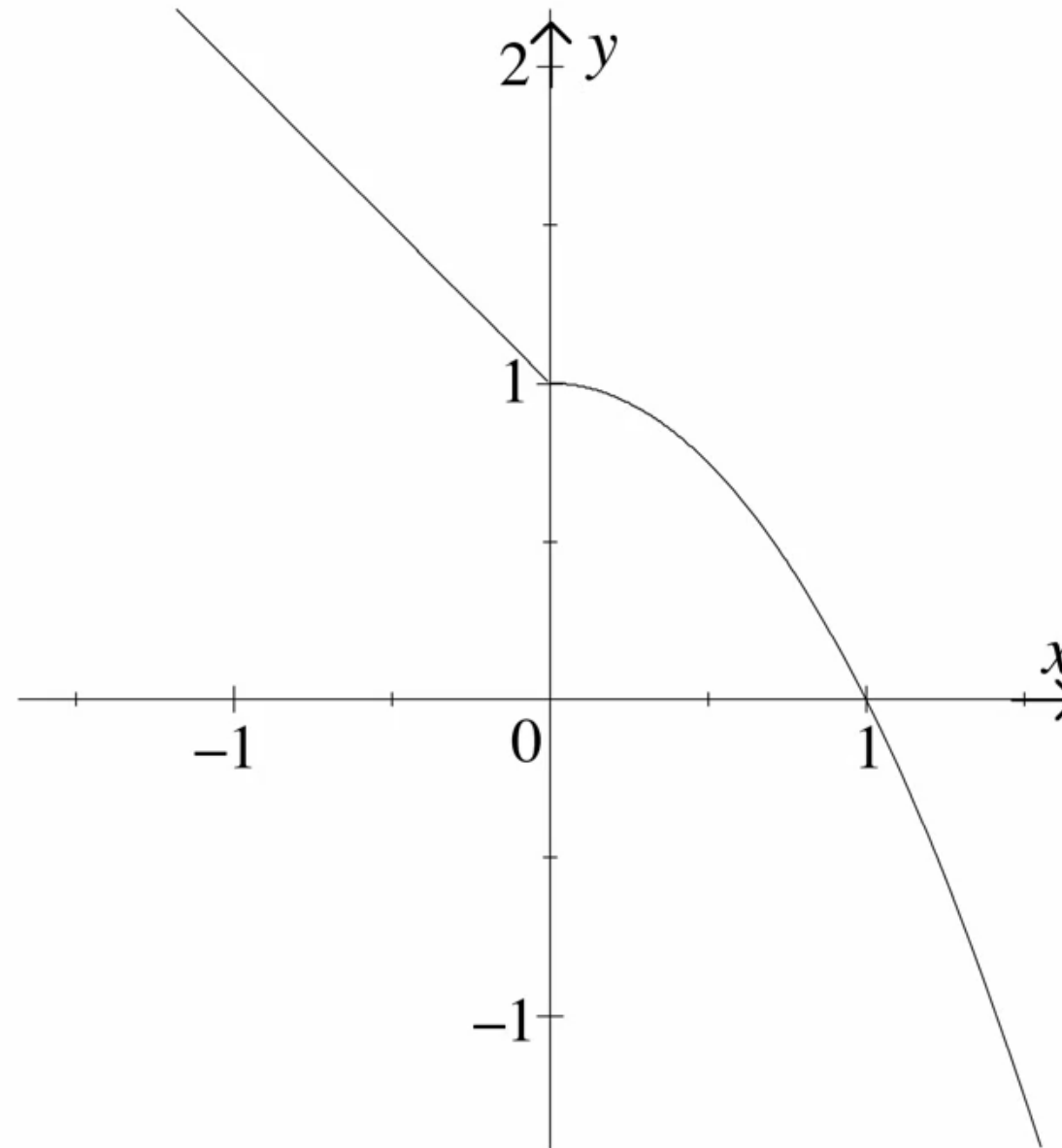


# Piecewise functions

We plott

$$f(x) = \begin{cases} 1 - x^2 & \text{for } x \geq 0 \\ 1 - x & \text{for } x < 0 \end{cases}$$

Piecew

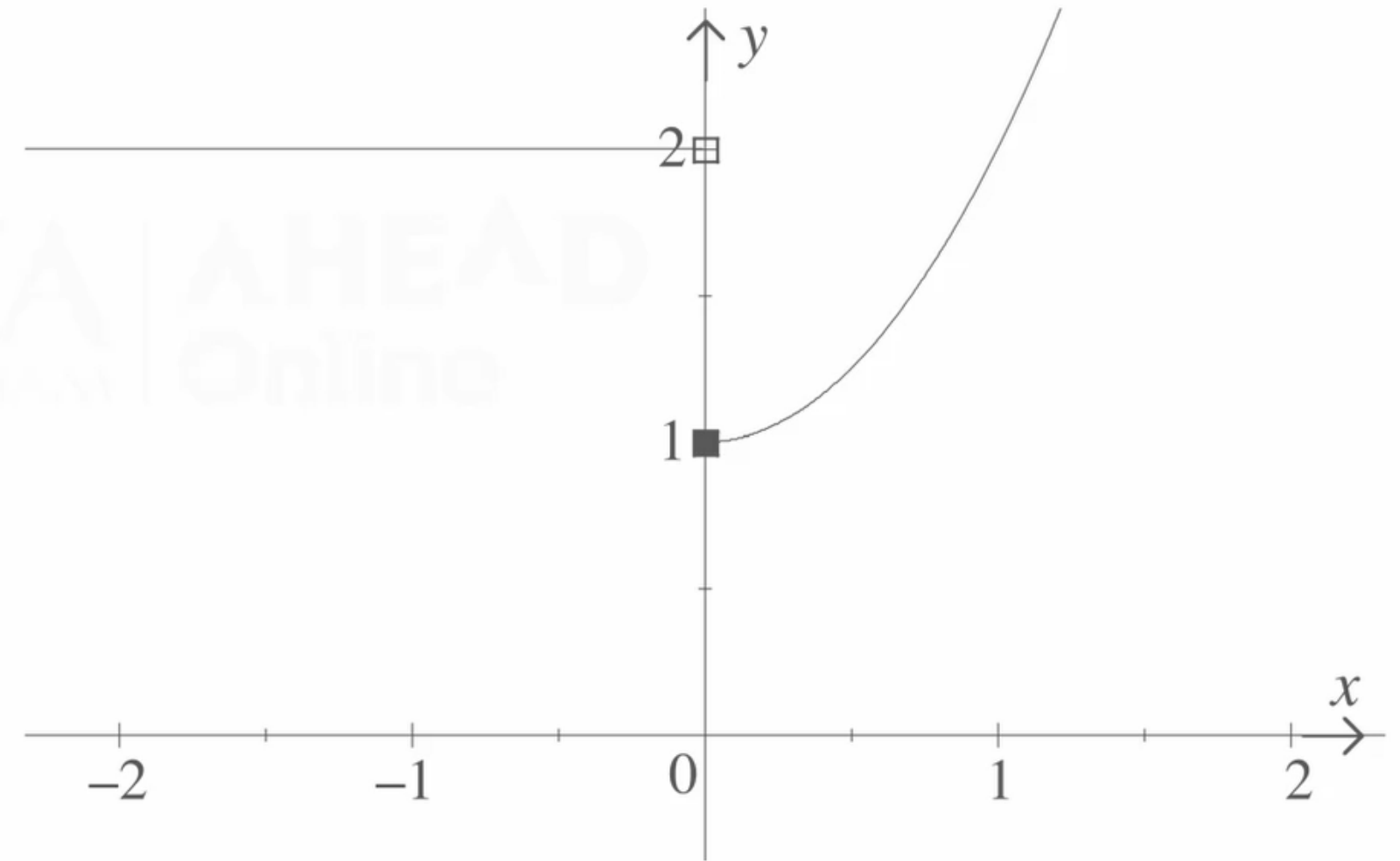


Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

# Piecewise functions

Plot the function

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x \geq 0 \\ 2 & \text{for } x < 0 \end{cases}$$



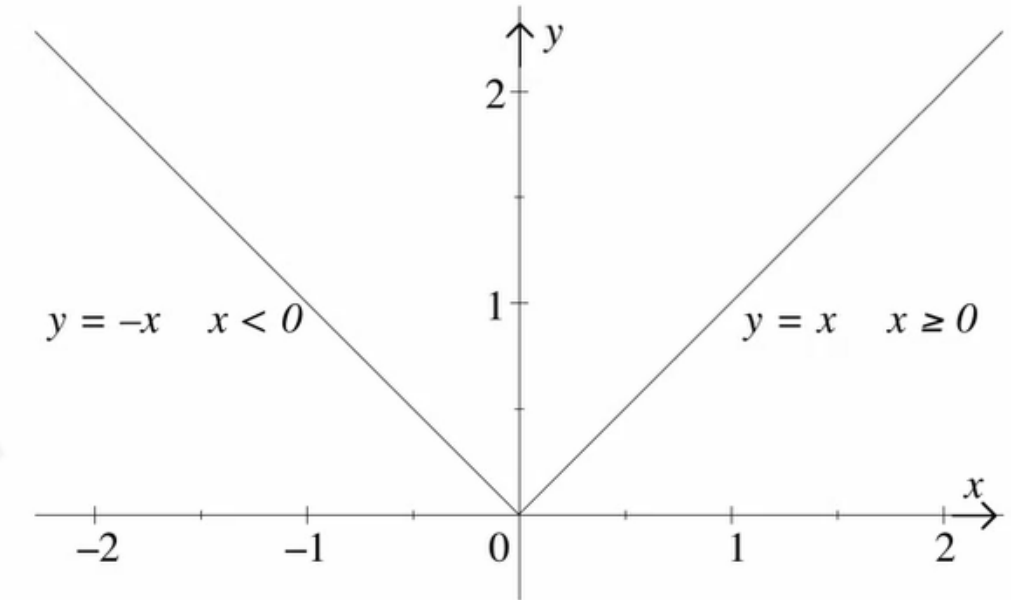
Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney

# Piecewise functions

Absolute function  $f(x) = |x|$

$$f(x) = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

Piecewise continuous



The graph of  $y = |x|$ .

Ack: Jackie Nicholas et al, Functions and their graphs, Mathematical Learning Center, University of Sydney