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AHEAD Online

Linear Algebra

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Introduction to Linear Algebra

- Probably the most important branch of mathematics
- Specifically important for computer science professionals, ML and data science
- Confluence of
  - Algorithms
  - Data structures
  - Theory
  - Applications
  - Abstraction
  - Visualization

Prof. Gilbert Strang

In other words, if you wanna start thinking, learn how to think straight (linear) first. 😎

Linear Algebra and Computing

- Data structures
  - Vectors, Matrices, Tensors
  - Dense matrix and Sparse matrix
- Algorithms
  - LU decomposition
  - QR decomposition
  - Eigen value computation
  - SVD
  - Iterative methods and Krylov spaces
- Linear algebra software libraries
  - LAPACK, BLAS, SciPy

https://blogs.mathworks.com/cleve/2013/06/24/the-linpack-benchmark/

A quote from Prof. Strang

- "I personally believe that many more people need linear algebra than calculus. Isaac Newton might not agree! But he isn't teaching mathematics in the 21st century (and maybe he wasn't a great teacher, but we will give him the benefit of the doubt). Certainly, the laws of physics are well expressed by differential equations. Newton needed calculus—quite right. But the scope of science and engineering and management (and life) is now so much wider, and linear algebra has moved into a central place." Prof. Gilbert Strang

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Two fundamental problems in Linear Algebra

1. Linear system of equations

$$Ax = b$$

- Solution of linear equations
- LU decomposition
- QR decomposition, orthogonalization
- Least square fitting, linear regression

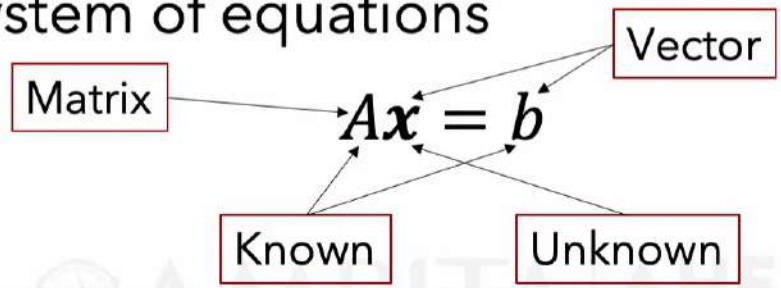
2. Eigen value problem

$$Ax = \lambda x$$

- Eigen value decomposition, computation
- Principal component analysis
- Singular value decomposition

Two fundamental problems in Linear Algebra

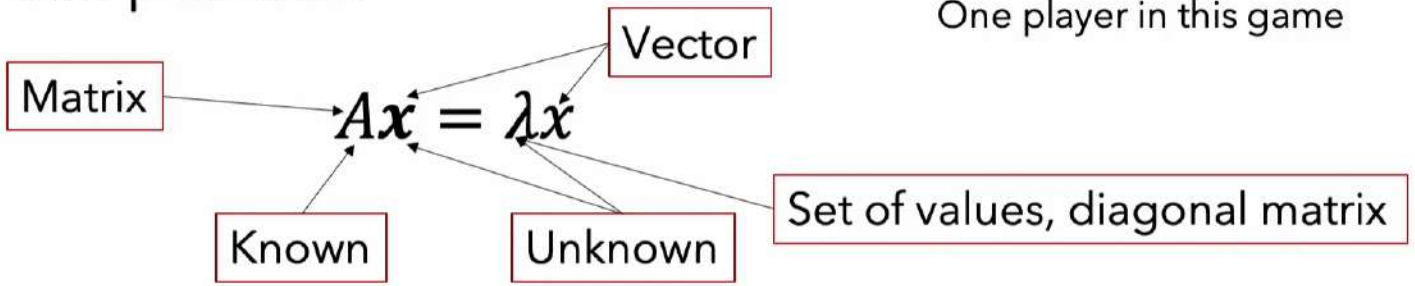
1. Linear system of equations



Two players in the game

2. Eigen value problem

One player in this game



Two fundamental problems in Linear Algebra

1. Linear system of equations

Linear problem

$$\begin{aligned} 4x - 2y + 3z &= 1 \\ x + 3y - 4z &= -7 \\ 3x + y + 2z &= 5 \end{aligned} \Rightarrow A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & -4 \\ 3 & 1 & 2 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, b = \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix} \Rightarrow \boldsymbol{Ax} = b$$

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2. Eigen value problem

Non-Linear problem

$$\begin{aligned} 4x - 2y + 3z &= \lambda_1 x \\ x + 3y - 4z &= \lambda_2 y \\ 3x + y + 2z &= \lambda_3 z \end{aligned} \Rightarrow A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & -4 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\lambda_1, \boldsymbol{x_1} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}; \lambda_2, \boldsymbol{x_2} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}; \lambda_3, \boldsymbol{x_3} = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \Rightarrow \boldsymbol{Ax} = \boldsymbol{\lambda x}$$

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Solution of Equations

1. Linear system of equations

$$Ax = b$$

- A, b → Knowns
- x → Unknown
- Linear problem

2. Eigen value problem

$$Ax = \lambda x$$

- A → Known
- λ, x → Unknowns
- Non-Linear problem

Solution of equations?

- Both problems are trying to seek solutions for x or λ, x
- May not be able to solve using Sympy
- Need specialized algorithms for solving real life linear system and eigen value problem

First problem $Ax = b$

Applications

- Differential equations→ Numerical Solution (Finite element, finite volume methods, etc) → $Ax = b$
- Operations research → $Ax = b$
- Management applications → $Ax = b$
- Graphs and Networks → $Ax = b$
- Linear regression, curve fitting → $A\acute{x} = \acute{b}$
- Many many more

Scale

- A, x, b can be from 10 to 1000s to billions

Need specialized algorithms to solve $Ax = b$

Reference – Order 3

$$A = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 3 & -4 \\ 3 & 1 & 2 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, b = \begin{bmatrix} 1 \\ -7 \\ 5 \end{bmatrix}$$

Second problem $Ax = \lambda x$

Applications

- Hidden Markov models
- Powers of Matrix – computation of A^n
- PCA and dimensionality reduction
- Image processing
- Data science
- Many more …

Scale

- A, x, λ can be from 10s to 1000s to billions

Need specialized algorithms to solve
$Ax = \lambda x$

Real life examples

1. Linear system of equations – Heat conduction along a rod

$$Ax = b$$

2. Eigen value problem – Simple face recognition system

$$Ax = \lambda x$$

Let's build the basics!