AMRITA VISHWA VIDYAPEETHAM

AHEAD Online

CALCULUS -3

Dr. Ajay Vadakkepatt
Adjunct Professor
Department of Computer Science
Amrita Vishwa Vidyapeetham

FUNCTIONS

- Objective
  - Learn and understand functions formally
  - Definitions
  - Classifications
  - Functional transformations
- Recap
  - Data observations
  - Defined variables of interest
  - Plotted the data appropriately
  - Output variables (dependent) are functions of input variables (independent)

One dimensional functions

One-dimensional functions take a single input value and output a single evaluation of the input.

Let x be the independent variable

Let y be the function on x or be the dependent variable

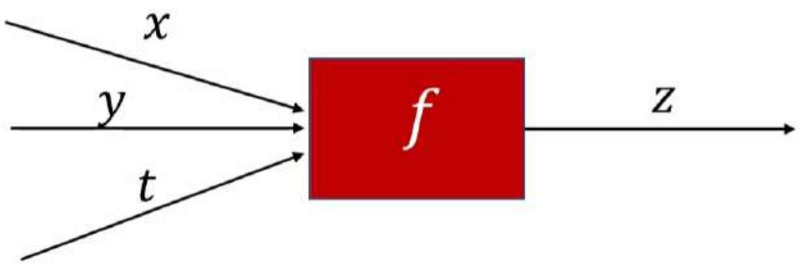
Therefore,
$$y = f(x)$$

Functions of many variables

It is up to us to choose what are the independent/input variables and what are dependent/output variables

Let the generic function be f. It can be a function of variables in many ways

$$x = f(t)$$
$$y = f(x)$$
$$y = f(x,t)$$
$$z = f(x,y)$$
$$z = f(x,y,t)$$



AMRITA VISHWA VIDYAPEETHAM

AHEAD Online

CALCULUS -4

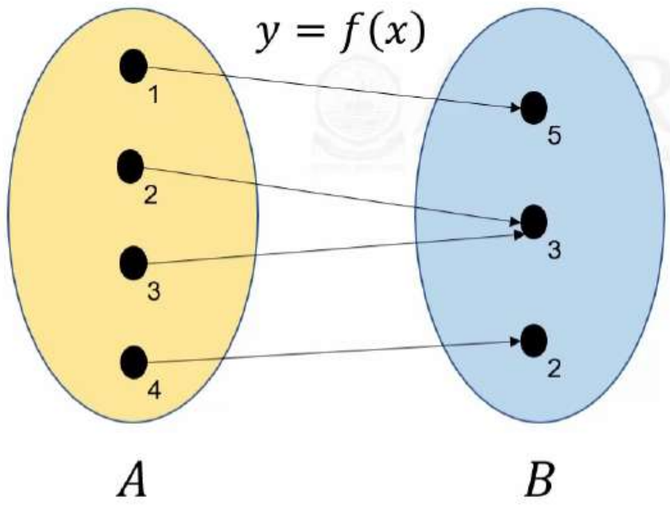
Dr. Ajay Vadakkepatt
Adjunct Professor
Department of Computer Science
Amrita Vishwa Vidyapeetham

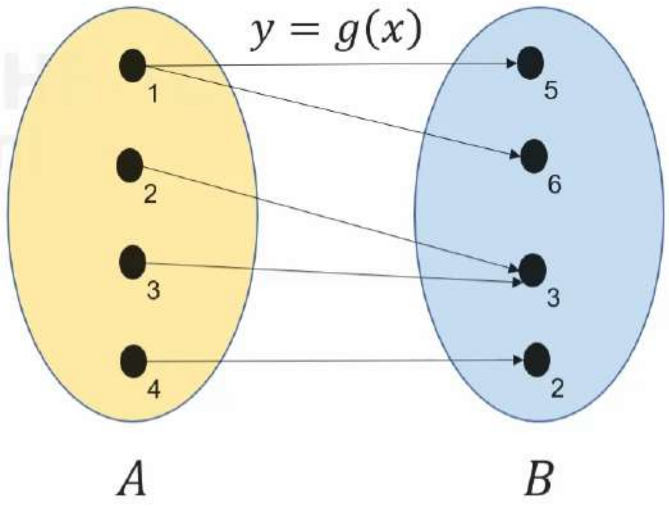
Functions - Definition

For now, consider two examples

$$y = f(x)$$
$$y = g(x)$$

Consider two sets A and B. A function is a rule that assigns to each element x in A exactly one element y in B.





Functions - Definition

f is a function – every element in A has associated with it one element in B

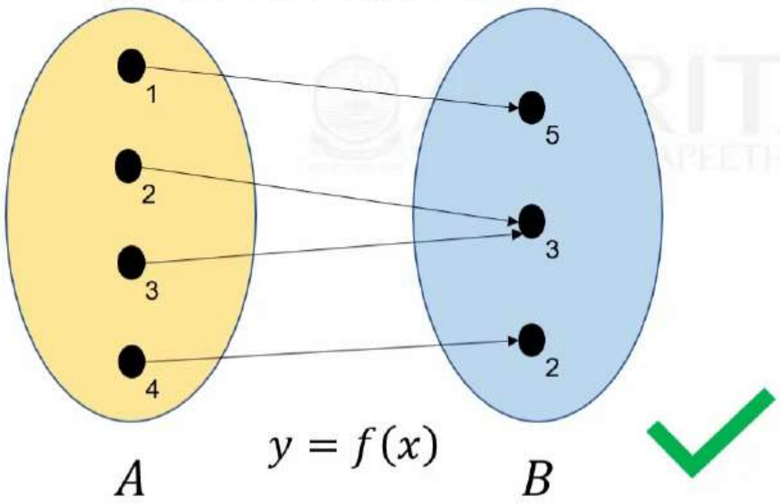
g is not a function - the element 1 in A is assigned two elements 5 and 6 in B

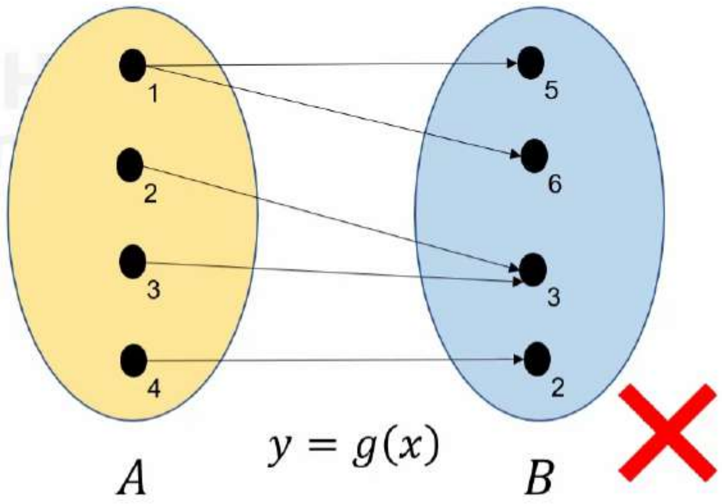
A function is also described as an ordered pair:

$F = \{(1,5), (2,3), (3,3), (4,2)\}$ is a function

Ordered pair: (x, y)

$G = \{(1,5), (4,2), (2,3), (3,3), (1,6)\}$ is not a function

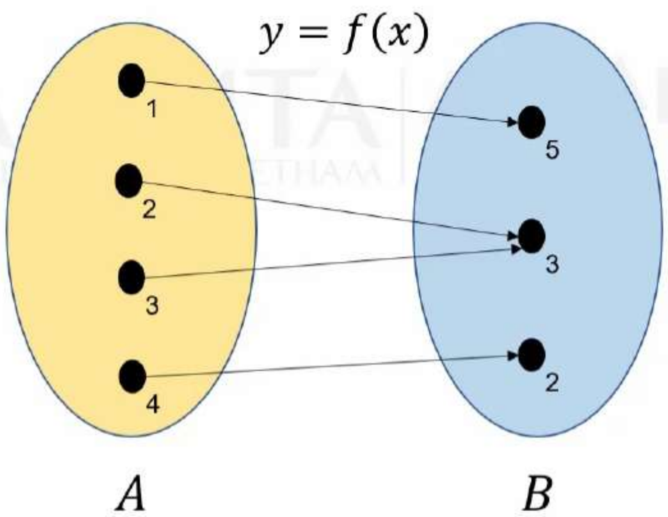




Functions – Domain and Range

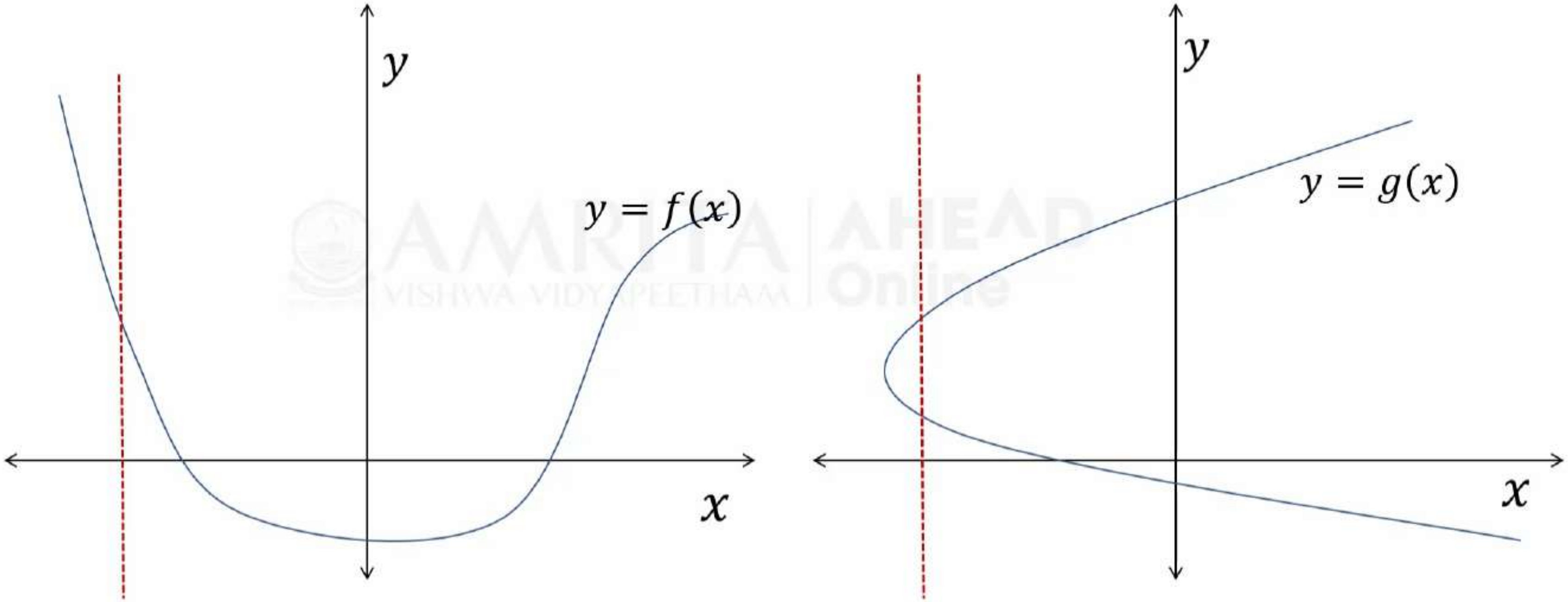
Domain of the function $y = f(x)$ is the set A

Range of the function $y = f(x)$ is the set B



Functions – Vertical line test

The vertical line test states that it should not be possible to draw a vertical line that cuts the graph in more than one point. If so, the graph is a function.



AMRITA
VISHWA VIDYAPEETHAM

AHEAD Online

CALCULUS

Dr. Ajay Vadakkepatt
Adjunct Professor
Department of Computer Science
Amrita Vishwa Vidyapeetham

Example – Domain and Range

Find the domain and range of the function

$$y = f(x) = \sqrt{x + 4}$$

Analytical

Square root is defined only for positive numbers,

$$\Rightarrow x + 4 \geq 0$$

$$\Rightarrow x \geq -4$$

Square root functions are always positive,

$$\Rightarrow y \geq 0$$

Domain: $x \geq -4$

Range: $\mathrm{y} \geq 0$

Example – Domain and Range

Find the domain and range of the function

$$y = \sqrt{x+4}$$

Graphical

+ Code + Text

▾ Domain and Range of $y = f(x) = \sqrt{x+4}$

```
import matplotlib.pyplot as plt
import numpy as np
############################################
x_lower_limit = -10 #Edit this variable
x_upper_limit = 2 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  np.sqrt(x+4) #Code up the equation
y_lower_limit = -2 #Edit this variable
y_upper_limit = 4 #Edit this variable
############################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

Example – Domain and Range

Find the domain and range of the function

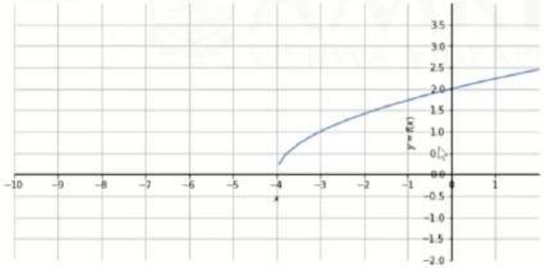
$$y = \sqrt{x+4}$$

Graphical

+ Code + Text

```
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

```
/usr/local/lib/python3.7/dist-packages/ipykernel_launcher.py:7: RuntimeWarning: invalid value encountered in sqrt
  import sys
```



Domain and Range of $(x^2 - 6x)/3$

```
import matplotlib.pyplot as plt
import numpy as np
```

Example – Domain and Range

Find the domain and range of the function

$$y = \sqrt{x+4}$$

Graphical

+ Code + Text

Domain and Range of $y = f(x) = \sqrt{x+4}$

```python
import matplotlib.pyplot as plt
import numpy as np
###########################################
x_lower_limit = -4 #Edit this variable
x_upper_limit = 2 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  np.sqrt(x+4) #Code up the equation
y_lower_limit = -2 #Edit this variable
y_upper_limit = 4 #Edit this variable
###########################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

Example – Domain and Range

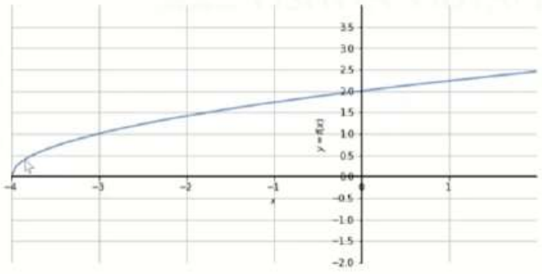
Find the domain and range of the function

$$y = \sqrt{x + 4}$$

Graphical

+ Code + Text

```
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 1))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```



▾ Domain and Range of $(x^2 - 6x)/3$

Example – Domain and Range

Find the domain and range of the function
$$y = (x^2 - 6x)/3$$
Graphical - Domain: $x \in \mathbb{R}$, Range: $y \geq -3$

Example – Domain and Range

Find the domain and range of the function
$$y = (x^2 - 6x)/3$$

Graphical - Domain: $x \in \mathbb{R}$, Range: $y \geq -3$

+ Code + Text

RAM
Disk
Editing

Domain and Range of $(x^2 - 6x)/3$

```
import matplotlib.pyplot as plt
import numpy as np
##########################################
x_lower_limit = -2 #Edit this variable
x_upper_limit = 8 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  (x**2 - 6*x)/3 #code up the equation
y_lower_limit = -4 #Edit this variable
y_upper_limit = 3 #Edit this variable
##########################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

Example – Domain and Range

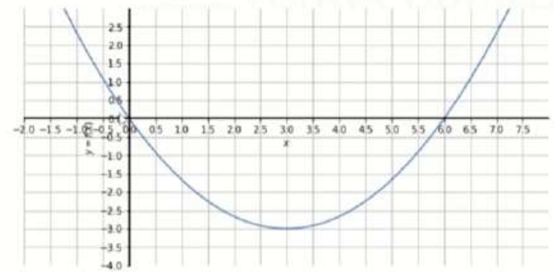
Find the domain and range of the function

$$y = (x^2 - 6x)/3$$

Graphical - Domain: $x \in \mathbb{R}$, Range: $y \geq -3$

+ Code + Text

```
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```



AMRITA VISHWA VIDYAPEETHAM

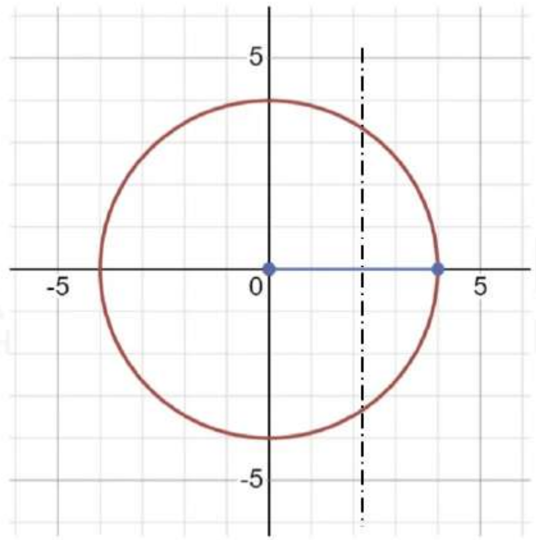
AHEAD Online

CALCULUS

Dr. Ajay Vadakkepatt
Adjunct Professor
Department of Computer Science
Amrita Vishwa Vidyapeetham

Example - Vertical line test

Graph $x^2 + y^2 = 16$. Is the graph a function?



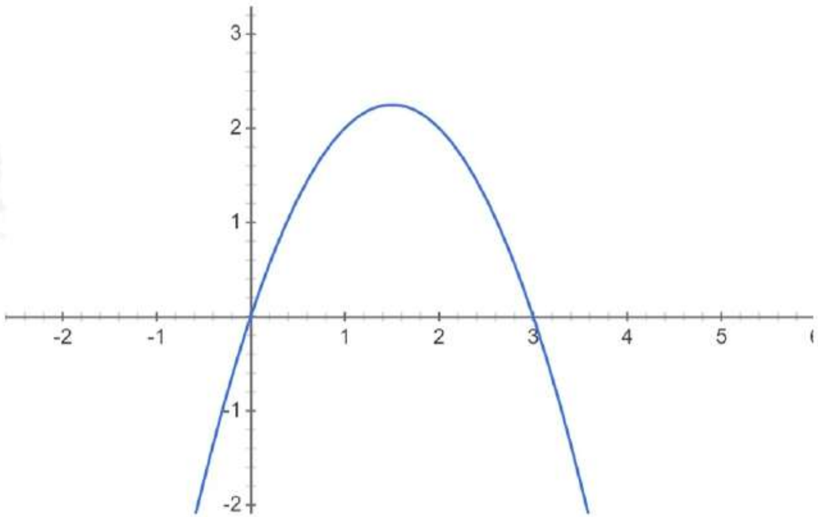
The graph fails the vertical line test. Therefore, not a function

Example – Domain and Range

Find the domain and range of the function

$$y = f(x) = 3x - x^2$$

Graphical

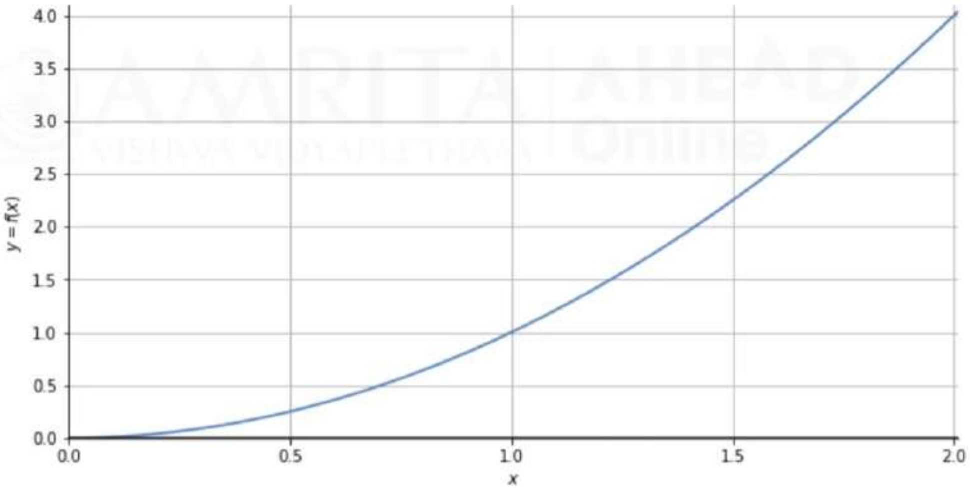


Specifying the domain of the function

Find the domain and range of the function
$y = x^2$ for $0 \leq x \leq 2$

x is restricted to be in $0 \leq x \leq 2$

Therefore, y takes the value $0 \leq y \leq 4$



Absolute value function

Absolute value of x

$$y = |x|$$
$$y = x \; if \; x \geq 0$$
$$y = -x \; if \; x < 0$$

Consider another absolute value function

$$y = |x - 2|$$
$$y = x - 2 \, if \; x \geq 2$$
$$y = -(x - 2) \; if \; x < 2$$

Consider another absolute value function

$$y = |2x + 4|$$
$$y = 2x + 4 \; if \; 2x + 4 \geq 0$$
$$y = -(2x + 4) if \; 2x + 4 < 0$$

$$y = 2x + 4 \; if \; x \geq -2$$
$$y = -(2x + 4) if \; x < -2$$

Absolute value function

+ Code + Text

Absolute value function

```
import matplotlib.pyplot as plt
import numpy as np
###########################################
x_lower_limit = -4 #Edit this variable
x_upper_limit = 4 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  np.abs(x) #Code up the equation
y_lower_limit = -1 #Edit this variable
y_upper_limit = 4 #Edit this variable
###########################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```

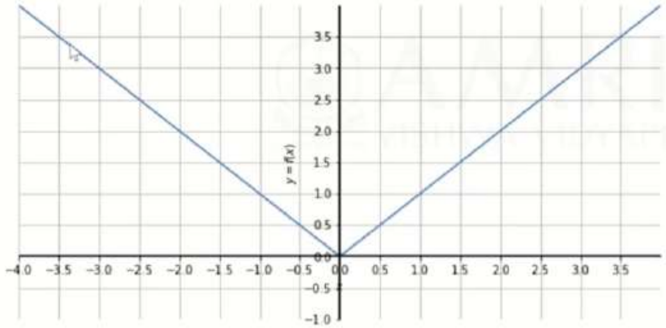
...

Absolute value function

+ Code + Text

RAM Disk Editing

```
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```




```
[ ] import matplotlib.pyplot as plt
    import numpy as np
    ############################################
    x_lower_limit = -10 #Edit this variable
    x_upper_limit = 10 #Edit this variable
```

Absolute value function

+ Code + Text

[ ]

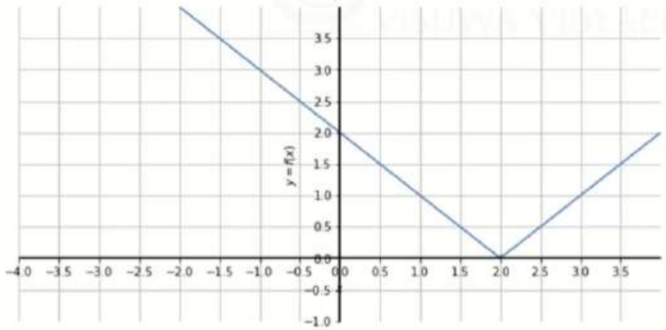
Absolute value function

```
import matplotlib.pyplot as plt
import numpy as np
###########################################
x_lower_limit = -4 #Edit this variable
x_upper_limit = 4 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  np.abs(x-2) #Code up the equation
y_lower_limit = -1 #Edit this variable
y_upper_limit = 4 #Edit this variable
###########################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
```

Absolute value function

+ Code + Text

```
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```



Absolute value function

+ Code + Text

RAM Disk Editing

[ ]

Absolute value function

```
import matplotlib.pyplot as plt
import numpy as np
##########################################
x_lower_limit = -6 #Edit this variable
x_upper_limit = 2 #Edit this variable
x = np.linspace(x_lower_limit, x_upper_limit, num=100)
y =  np.abs(2*x+4) #Code up the equation
y_lower_limit = -1 #Edit this variable
y_upper_limit = 4 #Edit this variable
##########################################
fig, ax = plt.subplots()
ax.plot(x, y)
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
```

Absolute value function

+ Code + Text

RAM Disk Editing

```
plt.rcParams['figure.figsize'] = [10, 5]
plt.axhline(color="black")
plt.axvline(color="black")
plt.xlabel('$x$')
plt.ylabel('$y=f(x)$')
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.set_xlim([x_lower_limit,x_upper_limit])
ax.set_ylim([y_lower_limit,y_upper_limit])
ax.set_xticks(np.arange(x_lower_limit, x_upper_limit, 0.5))
ax.set_yticks(np.arange(y_lower_limit, y_upper_limit, 0.5))
plt.grid()
plt.show()
```