Linear Algebra

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Introduction

- Representing Objects
- Type of Matrices
- Operations
- Properties
- Determinants
- Solving $Ax = b$ by Elimination
- Eigen Values

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- **Representing Objects**
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Matrix

- A matrix is an array of numbers (one or more rows, one or more columns)
- Consider a set of equations

$$\boxed{\begin{aligned} x + 2y + 3z + 5t &= 0 \\ 4x + 2y + 5z + 7t &= 0 \\ 3x + 4y + 2z + 6t &= 0 \end{aligned}}$$

3 equations
4 variables

- The matrix A can be formed by writing the coefficients of x, y, z, t in a rectangular array in rows and columns

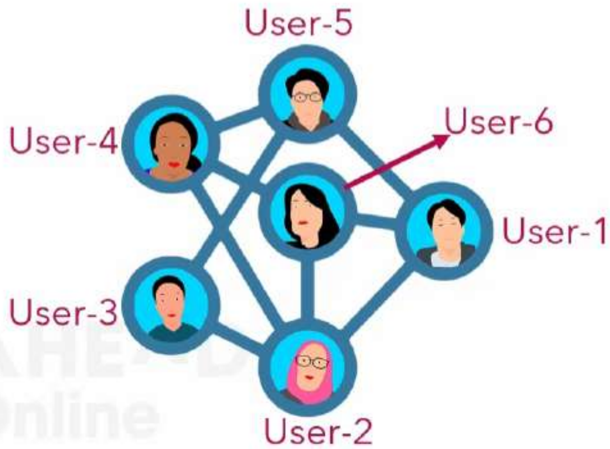
$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 4 & 2 & 5 & 7 \\ 3 & 4 & 2 & 6 \end{bmatrix}$$

coefficients of variables
x, y, z, t in linear system

- Order of matrix A is 3×4

Representing Objects

$$
S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0
\end{bmatrix}_{6\times 6} \end{array}
$$



$$
R = \begin{bmatrix}
125 & 125 & 124 & \dots & \dots & 118 \\
124 & 124 & 125 & \dots & \dots & 117 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
63 & 63 & 62 & .. & .. & 56
\end{bmatrix}_{256\times 256}
$$

Types of Matrices

- Row Matrix : Matrix has only one row and any number of columns

$$A = \begin{bmatrix} 2 & 7 & 3 & 9 \end{bmatrix}$$

1 row and 4 columns

- Column Matrix: Matrix has one column and any number of rows

$$A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

3 rows and 1 column

- Null Matrix/Zero Matrix: All the elements are zeros

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Null matrix of size 2

Types of Matrices

- Square Matrix : Number of rows is equal to the number of columns

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Square matrix of size 2

- Diagonal Matrix: Square Matrix is a diagonal matrix if all its non-diagonal elements are zero

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

Diagonal matrix of size 2

- Unit/Identity Matrix: Square Matrix is called a unit matrix if all the diagonal elements are unity and non diagonal elements are zero

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity matrix of size 2

Conclusion

- What is a matrix?
- Representing objects using matrices
- Type of Matrices
  - Row matrix
  - Column Matrix
  - Null Matrix
  - Square Matrix
  - Diagonal Matrix
  - Identity Matrix

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Transpose of a matrix

- The transpose of a matrix is obtained by changing its rows into columns (or equivalently, its columns into rows)

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}$$

Order is 3 × 2

Order is 2 × 3

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A = A^T \qquad A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Order is 3 ×3

Order is 3 ×3

Matrix Addition

- If A and B are matrices of the same order, then they can be added

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}$$

$A + B$ is not a valid expression

Order is 3 × 2

Order is 2 × 3

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix} \qquad A + B = \begin{bmatrix} 2+1 & 1+1 \\ 3+0 & 3+3 \\ 4+1 & 1+2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3 & 6 \\ 5 & 3 \end{bmatrix}$$

Order is 3 × 2

Order is 3 × 2

Matrix Subtraction

- If A and B are matrices of the same order, then they can be subtracted

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}$$

$A - B$ is not a valid expression

Order is 3×2

Order is 2×3

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix} \qquad A - B = \begin{bmatrix} 2-1 & 1-1 \\ 3-0 & 3-3 \\ 4-1 & 1-2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \\ 3 & -1 \end{bmatrix}$$

Order is 3×2

Order is 3×2

Scalar Multiplication

- If a matrix A is multiplied by a scalar quantity n then each element of A is multiplied by n

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \qquad n = 3 \qquad nA = \begin{bmatrix} 6 & 3 \\ 9 & 9 \\ 12 & 3 \end{bmatrix}$$

Order is 3 × 2

Order is 2 × 3

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad n = 1000 \qquad nA = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

Order is 3 ×3

Order is 3 ×3

Matrix Vector Multiplication

- In the multiplication $A\boldsymbol{b}$ between a matrix A and a vector $\boldsymbol{b}$, the number of columns in A equals the number of rows in $\boldsymbol{b}$

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix}$$

Order is 3×2

$$\boldsymbol{b} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Order is 1×3

Ab is not a valid expression

bA is a valid

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix}$$

Order is 3×2

$$\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Order is 2×1

$$A\boldsymbol{b} = \begin{bmatrix} 2 \times 1 + 1 \times 2 \\ 3 \times 1 + 3 \times 2 \\ 4 \times 1 + 1 \times 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 6 \end{bmatrix}$$

Order is 3×1

Matrix-Matrix Multiplication

- Two matrices A and B can be multiplied only if the number of columns in A is equal to number of rows in B
- Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Then the product AB will be an $m \times p$ matrix

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 40 & 49 \\ 64 & 79 \\ 94 & 116 \end{bmatrix}$$

COLAB Demo

```
from sympy import *
A = Matrix([[1,2,3,5], [4,2,5,7], [3,4,2,6]])
A
```

```
[ ] C=A.transpose()
    C
```

```
[ ] from sympy import *
    A = Matrix([[2,3,4], [4,5,6], [6,7,9]])
```

COLAB Demo

[2]

$$\begin{bmatrix} 2 & 2 & 4 \\ 3 & 5 & 2 \\ 5 & 7 & 6 \end{bmatrix}$$

```
from sympy import *
A = Matrix([[2,3,4], [4,5,6], [6,7,9]])
A
```

[ ]

```
C=3*A
C
```

COLAB Demo

[4]

$$\begin{bmatrix} 12 & 15 & 18 \\ 18 & 21 & 27 \end{bmatrix}$$

```
from sympy import *
A = Matrix([[2,3,4], [4,5,6], [6,7,9]])
B = Matrix([[2,3],[4,5],[6,7]])
```

[ ]

```
C=A*B
C
```

COLAB Demo

```
C=A*B
C
```

$$\begin{bmatrix} 40 & 49 \\ 64 & 79 \\ 94 & 116 \end{bmatrix}$$

```
[ ] A+B
```

Conclusion

- Transpose
- Matrix Addition
- Matrix Subtraction
- Matrix Multiplication
  - Scalar Multiplication
  - Matrix Vector Multiplication
  - Matrix-Matrix Multiplication
- Operations in Python

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- **Determinants**
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Matrix Addition is Commutative

- If A and B are matrices of the same order, then they can be added in any order

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 1+2 & 1+1 \\ 0+3 & 3+3 \\ 1+4 & 2+1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3 & 6 \\ 5 & 3 \end{bmatrix}$$

$$A + B = B + A$$

$$B = \begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+1 & 1+1 \\ 3+0 & 3+3 \\ 4+1 & 1+2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3 & 6 \\ 5 & 3 \end{bmatrix}$$

Matrix Addition is Associative

- If A, B and C are matrices of the same order, then they can be added by grouping in any order

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(A+B)+C = \left(\begin{bmatrix} 2+1 & 1+1 \\ 3+0 & 3+3 \\ 4+1 & 1+2 \end{bmatrix} \right) + \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 3 & 6 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 7 \\ 6 & 3 \end{bmatrix}$$

$$\boxed{A + (B + C) = (A + B) + C}$$

$$A + (B + C) = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} + \left(\begin{bmatrix} 1+0 & 1+2 \\ 0+-1 & 3+1 \\ 1+1 & 2+0 \end{bmatrix} \right) = \begin{bmatrix} 2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 7 \\ 6 & 3 \end{bmatrix}$$

Determinant

- It is a special number that can be calculated from any square matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \times d - b \times c$$

2 × 2 Case

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = + \, a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

3 × 3 Case

- For a 4 × 4 matrix the pattern will be **+ - + -**

Computing Determinant-Example

$$\begin{vmatrix} 2 & 1 \\ -6 & 3 \end{vmatrix} = 2 \times 3 - (1 \times -6) = 6 + 6 = 12$$

$$\begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix} = 2 \times 3 - (1 \times 6) = 6 - 6 = 0$$

$$\begin{vmatrix} 2 & 3 & 1 \\ 4 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 2\begin{vmatrix} -1 & 2 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= 2(-1 \times 3 - (2 \times 2)) - 3\big(4 \times 3 - (2 \times 1)\big) + 1(4 \times 2 - (-1 \times 1))$$

$$= 2(-3 - (4)) - 3\big(12 - (2)\big) + 1(8 - (-1))$$

$$= 2(-7) - 3(10) + 1(9)$$

$$= -14 - 30 + 9 = -35$$

COLAB Demo

```
import numpy as np
from scipy import linalg
a = np.array([[2,1], [-6,3]])
linalg.det(a)
```

```
[ ] import numpy as np
from scipy import linalg
a = np.array([[2,1], [6,3]])
linalg.det(a)
```

COLAB Demo

```
0.0
```

```
import numpy as np
from scipy import linalg
a = np.array([[2,3,1], [4,-1,2],[1,2,3]])
linalg.det(a)
```

Conclusion

- Matrix Addition is
  - Commutative
  - Associative
- What is determinant ?
- Computing determinant
  - 2×2 matrix
  - 3×3 matrix
- Determinant in Python

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- **Solving $\mathbf{Ax} = \mathbf{b}$ by Elimination**
- Eigen Values

Solving $Ax = b$ by Elimination

- Matrix form of linear system
  - Coefficient matrix
  - Augmented matrix
  - Vector of unknowns
  - Vector of constants
- Elementary row operations
  - Swap
  - Multiply a row by a scalar
  - Add a multiple of one row to another row
- Row echelon form
- Gaussian Elimination

Matrix Form

$$\begin{bmatrix} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{bmatrix}$$

Augmented Matrix

Consider the following Linear System

$$\boxed{\begin{aligned} x - 3y + z &= 4 \\ 2x - 8y + 8z &= -2 \\ -6x + 3y - 15z &= 9 \end{aligned}}$$

Vector of Unknowns

$$\underbrace{\begin{bmatrix} 1 & -3 & 1 \\ 2 & -8 & 8 \\ -6 & 3 & -15 \end{bmatrix}}_{\text{Coefficient Matrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 4 \\ -2 \\ 9 \end{bmatrix}}_{\text{Vector of Constants}}$$

MATRIX FORM

Elementary row operations

$$\begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \end{array}\begin{bmatrix} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{bmatrix}$$

$$\xrightarrow{\text{Swap Row 1 and Row 2}} \begin{bmatrix} 2 & -8 & 8 & -2 \\ 1 & -3 & 1 & 4 \\ -6 & 3 & -15 & 9 \end{bmatrix}$$

$$\xrightarrow{\text{Row 2} \times \frac{1}{2}} \begin{bmatrix} 1 & -3 & 1 & 4 \\ 1 & -4 & 4 & -1 \\ -6 & 3 & -15 & 9 \end{bmatrix}$$

$$\xrightarrow{\text{Row 2 is Row 2 - (2} \times \text{Row 1)}} \begin{bmatrix} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ -6 & 3 & -15 & 9 \end{bmatrix}$$

1. Swap
2. Multiply a row by a scalar
3. Add a multiple of one row to another row

Row Echelon form

A matrix is in row echelon form if

1. All rows consisting of only zeroes (if any) are at the bottom
2. The leading coefficient of a nonzero row is always strictly to the right of leading coefficient of row above it

Which of the following satisfies above properties ?

$$\begin{bmatrix} 2 & -8 & 8 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Leading nonzero in row 3 is to the left of row 2

Row 1 consists of only zeroes

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 2 & 3 \\ 0 & -3 & 1 & 4 \\ 0 & 0 & -15 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 2 & 3 \\ 0 & -3 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Conclusion

- Matrix form of a linear system
- Elementary Row operations
- Row Echelon Form

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Solving $Ax = b$ by Elimination

- Matrix form of linear system
  - Coefficient matrix
  - Augmented matrix
  - Vector of unknowns
  - Vector of constants
- Elementary row operations
  - Swap
  - Multiply a row by a scalar
  - Add a multiple of one row to another row
- Row echelon form
- **Gaussian Elimination**

Gaussian Elimination

- It is an algorithm to solve linear systems
- Elementary row operations are performed to augmented matrix to convert it to row echelon form
- Procedure for reducing matrix to row echelon form
  1. Locate the leftmost column that does not consist entirely of zeros
  2. Interchange the top with another row, if necessary, to bring a nonzero entry to the top of the column found in Step 1
  3. Add suitable multiples of top row to the rows below so that all entries below the leading nonzero entry become zeros
  4. Now cover the top row in the matrix and begin again with Step 1 applied to the sub-matrix that remains. Continue in this way until the entire matrix is in row echelon form

COLAB Demo

1. Locate the leftmost column that does not consist entirely of zeros
2. Interchange the top with another row, if necessary, to bring a nonzero entry to the top of the column found in Step 1
3. Add suitable multiples of top row to the rows below so that all entries below the leading nonzero entry becomes zeros
4. Now the cover the top row in the matrix and begin again with Step 1 applied to the sub matrix that remains. Continue in this way until the entire matrix is in row echelon form

COLAB Demo

Solve

$$x - 3y + z = 4$$
$$2x - 8y + 8z = -2$$
$$-6x + 3y - 15z = 9$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{array}\right]$$

COLAB Demo

Augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 2 & -8 & 8 & -2 \\ -6 & 3 & -15 & 9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ -6 & 3 & -15 & 9 \end{array}\right] R_2 = -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \end{array}\right.$$

COLAB Demo

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ -6 & 3 & -15 & 9 \end{array}\right] R_2 = -2R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array}\right] R_3 = -6R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \end{array}\right] R_2 = \frac{1}{2}R_2$$

COLAB Demo

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array}\right] R_3 = -6R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & -15 & -9 & 33 \end{array}\right] R_2 = \frac{1}{2} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \end{array}\right] R_3 = \frac{1}{3} R_3$$

COLAB Demo

$$\left[\begin{array}{ccc|c} 0 & -2 & 6 & -10 \\ 0 & -15 & -9 & 33 \end{array}\right] R_3 = -6R_1 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & -15 & -9 & 33 \end{array}\right] R_2 = \frac{1}{2}R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & -5 & -3 & 11 \end{array}\right] R_3 = \frac{1}{3}R_3$$

COLAB Demo

$$\left[\begin{array}{ccc|c} 0 & -1 & 3 & -5 \\ 0 & -15 & -9 & 33 \end{array}\right] R_2 = \tfrac{1}{2} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & -5 & -3 & 11 \end{array}\right] R_3 = \tfrac{1}{3} R_3$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 1 & 4 \\ 0 & -1 & 3 & -5 \\ 0 & 0 & -18 & 36 \end{array}\right] R_3 = -5R_2 + R_3$$

COLAB Demo

$$\begin{bmatrix} 0 & 0 & -18 & | & 36 \end{bmatrix}$$

From the third row of final matrix we get $-18z = 36 \implies z = -2$

From second row of final matrix $-y + 3z = -5$, Substituting $z = -2$ we get
$-y + (3 \times -2) = -5 \implies y = -1$

From first row of final matrix $x - 3y + z = 4$, Substituting $z = -2, y = -1$ we get
$x - 3(-1) + (-2) = 4 \implies x = 3$

Solution $(x, y, z) = (3, -1, -2)$

COLAB Demo

$$x - 3(-1) + (-2) = 4 \implies x = 3$$

Solution $(x, y, z) = (3, -1, -2)$

```
import numpy as np
a = np.array([[1, -3,1], [2, -8,8],[-6,3,-15]])
b = np.array([4, -2,9])
x = np.linalg.solve(a, b)
x
```

```
array([ 3., -1., -2.])
```

Conclusion

- Gaussian Elimination
- Solving using NumPy

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Matrix transformation

Matrix A	**Vector x**	Ax	**Is Ax a scaled version of x ?**
$\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} -6 \\ -4 \end{bmatrix}$	No
	$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$	$\begin{bmatrix} -2 \\ -2 \end{bmatrix}$	Yes, $-1 \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$
	$\begin{bmatrix} 3 \\ 7 \end{bmatrix}$	$\begin{bmatrix} -23 \\ -15 \end{bmatrix}$	No
	$\begin{bmatrix} 10 \\ 4 \end{bmatrix}$	$\begin{bmatrix} 20 \\ 8 \end{bmatrix}$	Yes, $2 \times \begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 2 \\ 2 \end{bmatrix}}_{x} = \underbrace{-1}_{\lambda} \times \underbrace{\begin{bmatrix} 2 \\ 2 \end{bmatrix}}_{x}$$

$$\underbrace{\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 10 \\ 4 \end{bmatrix}}_{x} = \underbrace{2}_{\lambda} \times \underbrace{\begin{bmatrix} 10 \\ 4 \end{bmatrix}}_{x}$$

Eigen value of a matrix

- Eigen vectors does not change its orientation, but scales by a factor of corresponding eigen value
- How to calculate eigen values ?
  - Determine λ by solving $|A - \lambda I| = 0$, where I is identity matrix
- Example : Find eigen values of $\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$

$$\left|\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0 \Rightarrow \left|\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix}\right| = 0$$

So, $(4-\lambda)(-3-\lambda) + 10 = 0 \Rightarrow -12 - 4\lambda + 3\lambda + \lambda^2 = 0 \Rightarrow \lambda^2 - \lambda - 2 = 0$

Solving we get $\boxed{\lambda = -1, \lambda = 2}$

COLAB Demo

```
from sympy import *
A = Matrix([[4,-5],[2,-3]])
A.eigenvals()
```

```
[ ] A.eigenvects()
```

```
[ ] from sympy import *
    A = Matrix([[3,1,4],[0,2,6],[0,0,5]])
    A
```

COLAB Demo

```
[   1]])])]
```

```
from sympy import *
A = Matrix([[3,1,4],[0,2,6],[0,0,5]])
A
```

```
[ ] A.eigenvals()
```

```
[ ] A.eigenvects()
```

Conclusion

- Computing Eigen values
  - 2×2 matrix
  - 3×3 matrix
- Computing in Python