AMRITA VISHWA VIDYAPEETHAM

AHEAD Online

Probability and Statistics

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Why Probability and Statistics ?

- Probability is an essential tool in applied mathematics and mathematical modelling
  - Random experiment
  - Sample space
  - Counting Techniques
  - Conditional Probability
- Statistics means collection, study and summarizing data
  - Assume the context is analysing performance of students in an admission test
  - How to collect data?
  - What type of study to be performed?
  - How to summarize ?

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Objective

- To learn the concept of Random Experiment
- To Identify Sample Space for simple experiments
- To perform Set Operation on Events
- To learn about Mutually Exclusive Events

Random Experiment

Throwing a die

Choosing a card from deck of cards

https://www.pexels.com/photo/person-holding-playing-cards-102107/

An experiment that can result in different outcomes

Tossing a coin

Monitoring vitals of a patient

https://www.pxfuel.com/en/free-photo-jrfwk/download/2560x1600-

Sample Space

- Set of all possible outcomes of a random experiment is called the sample space, denoted as S
- Tossing a coin

$$S = \{H, T\}$$

- Tossing a coin 2 times

$$S = \{HH, HT, TT, TH\}$$

- Rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

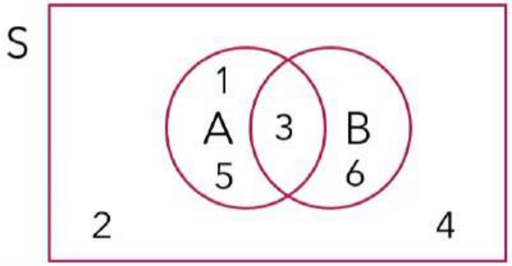
Events

- Event is a subset of the sample space
- **Venn diagrams** can be used to represent a sample space and events in a sample space

Let the random experiment be rolling a die

A - Odd number turns up in die

B - Number divisible by 3 turns up in dice



$$A = \{1,3,5\}$$
$$B = \{3,6\}$$

Complement

$$\bar{A} = \{2,4,6\}$$
$$\bar{B} = \{1,2,4,5\}$$

Union

$$A \cup B = \{1,3,5,6\}$$

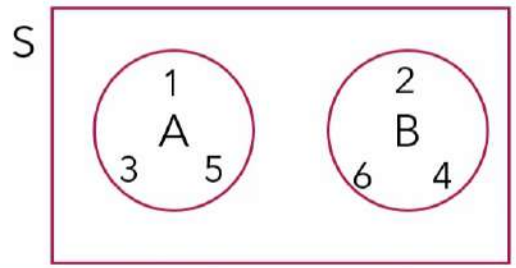
Intersection

$$A \cap B = \{3\}$$

Mutually Exclusive Events

- Events that cannot occur simultaneously

Let the random experiment be rolling a die
A - Odd number turns up in die
B - Number divisible by 2 turns up in die



$$A = \{1,3,5\}$$
$$B = \{2,4,6\}$$

Complement

$$\bar{A} = \{2,4,6\}$$
$$\bar{B} = \{1,3,5\}$$

Union

$$A \cup B = \{1,2,3,4,5,6\}$$

Intersection

$$A \cap B = \varnothing$$

Example

- A weighing machine can record weights up to 100 kg. Let A be the event that weight exceeds 50 kg, B denote the event that a weight is less than or equal to 75 kg, C be the event that a weight is greater than or equal to 40 kg and less than or equal to 60 kg. Find the following

$$A = \{51,52,..,100\},\ B = \{0,1,..,75\},\ C = \{40,41,..,60\}$$

$A \cup C = \{40, 41, .. 100\}$

$$\begin{aligned}
A \cup B &= \{0,1,2,..,100\} & \overline{(A \cup C)} &= \{0,1,2,..,39\} \\
A \cap B &= \{51,52,..,75\} & A \cap B \cap C &= \{51,52,..,60\} \\
\bar{A} &= \{0,1,2,..,50\} & \bar{B} \cap C &= \emptyset \\
A \cup B \cup C &= \{0,1,2,..,100\} & A \cup (B \cap C) &= \{40,41,..,100\}
\end{aligned}$$

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Objective

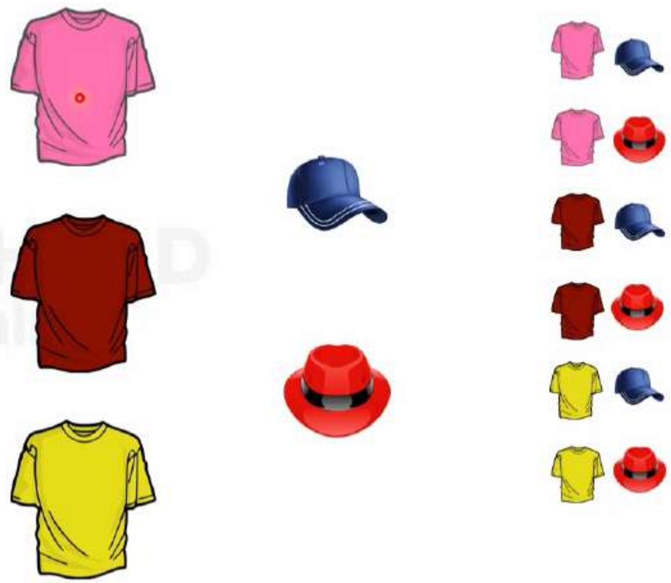
- To learn counting techniques

Multiplication Rule

- If a task can be described as a sequence of k steps and

Number of ways completing step 1	p
Number of ways completing step 2	q
....	.....
Number of ways completing step k	r

- The total number of ways of completing the task is $p \times q \times \cdots r$



Permutations

- Ordered sequence of the elements
- Number of permutations of n different elements is $n!$
- Number of permutations of subset of r elements selected from a set of n different element is $P_r^n = \frac{n!}{(n-r)!}$
- Number of permutations of $n = n_1 + n_2 + \cdots + n_r$ objects where n_1 are of one type, n_2 are of a second type ….. and n_r are of $r'th$ type is $\frac{n!}{n_1!n_2!....n_r!}$

Permutations-Example 1

- If $S = \{a, b, c, d\}$, number of permutations is $4! = 24$

$abcd$	$bacd$	$cabd$	$dabc$
$abdc$	$badc$	$cadb$	$dacb$
$acbd$	$bcad$	$cbad$	$dbac$
$acdb$	$bcda$	$cbda$	$dbca$
$adbc$	$bdac$	$cdab$	$dcab$
$adcb$	$bdca$	$cdba$	$dcba$

- If $S = \{1,2,3\}$, number of permutations is $3! = 6$

123	213	312
132	231	321

Permutations-Example 2

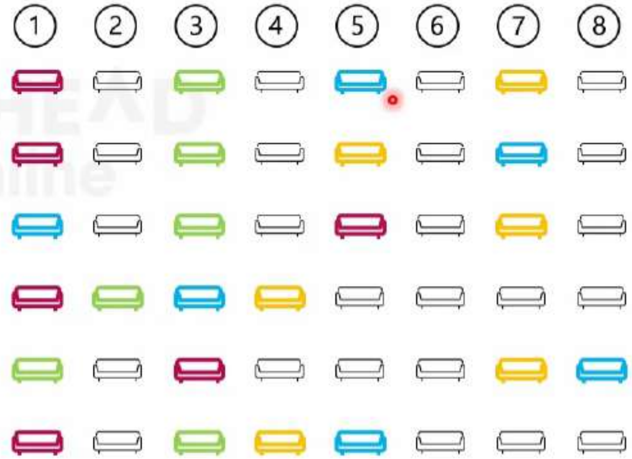
- Eight seats are there in a row. If four celebrities are to be seated on the row, how many different arrangements are possible ?

$$P_4^8 = \frac{8!}{4!} = \underline{1680}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}}$$

$$= 8 \times 7 \times 30$$

$$= 56 \times 30 = \underline{1680}$$



- 8 locations and 4 components

Permutations-Example 3

- How many different barcodes can be generated from 2 thick lines and 3 thin lines ?

$$\frac{5!}{2!\,3!} = 10$$

$$\frac{5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times 1}{2 \times 1 \times \cancel{3} \times \cancel{2} \times 1} = \underline{\underline{10}}$$

1 2 3 4 5 6 7 8 9 10 11

- In how many ways the word MISSISSIPPI can be arranged ?

M = 1
I = 4
S = 4
P = 2

$$\frac{11!}{4! \times 4! \times 2!} = \frac{11 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1 \times 2} = 3\cdot$$

Combinations

- Sequence of elements where order does not matter
- The number of combinations of size r that can be selected from a set of n elements is denoted by C_r^n

$$C_r^n = \frac{n!}{r!(n-r)!}$$

- How to select 4 panel members from 8 members?

$$C_4^8 = \frac{8!}{4!\,(8-4)!} = 70$$

Conclusion

- Multiplication Rule
- Permutations
- Combinations

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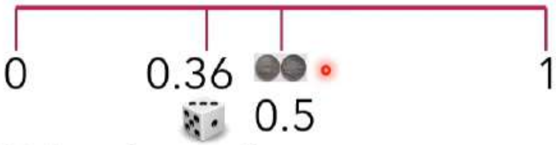
Objective

- To learn the concept of Probability for small examples
- To use counting techniques for probability calculation

Probability

- Probability is used to quantify the chance, that an outcome of a random experiment will occur, denoted by $P(E)$
- If S is the sample space and E is any event in a random experiment
  - $P(S) = 1$
  - $0 \leq P(E) \leq 1$
  - $P(E_1 \cup E_2 \dots) = P(E_1) + P(E_2) + \dots$, where $E_1, E_2, \dots$ are mutually exclusive
- $P(E) = \dfrac{number\ of\ ways\ E\ can\ occur}{total\ number\ of\ possible\ outcomes}$



Experiment : Toss the coin once, report if it heads or tails

$S = \{H, T\}$

$P(H) = \frac{1}{2}, P(T) = \frac{1}{2}$

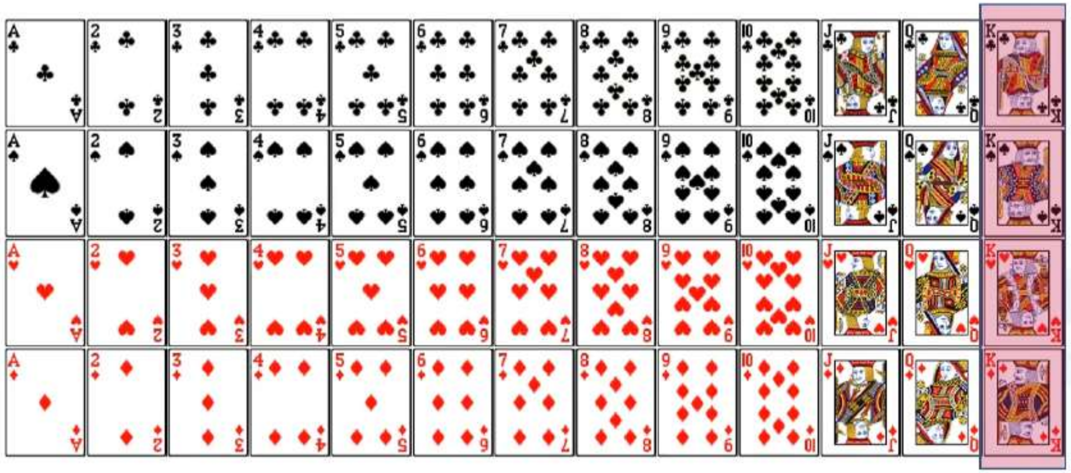
Example-1

Experiment: Roll six sided dice twice. Record the sum of numbers

$$2 - \{(1,1)\}$$
$$3 - \{(1,2), (2,1)\}$$
$$4 - \{(1,3), (2,2), (3,1)\}$$
$$5 - \{(1,4), (2,3), (3,2), (4,1)\}$$
$$6 - \{(1,5), (2,4), (3,3), (4,2), (5,1))\}$$
$$7 - \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1))\}$$
$$8 - \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$
$$9 - \{(3,6), (4,5), (5,4), (6,3)\}$$
$$10 - \{(4,6), (5,5), (6,4)\}$$
$$11 - \{(5,6), (6,5)\}$$
$$12 - \{(6,6)\}$$

Sum	Number of ways	Probability
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Example-2



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Total no of outcomes = 52
Number of kings = 4
Let K be the event

$$P(K) = \frac{4}{52}$$

What is probability of getting a king in a single draw?

Example-3 (Using Counting techniques)

- Five cards are drawn from a pack of 52 cards. Find the probability that 3 are tens and 2 are kings

Number of ways in which 3 tens can be selected

Number of ways in which 2 kings can be selected

$$\frac{C_3^4 \; C_2^4}{C_5^{52}} = \frac{1}{108290}$$

Number of ways in which 5 cards can be selected

Conclusion

- To calculate probabilities of different events

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Objective

- To learn the concept of Conditional Probability
- Identify Independent events
- To learn concept of Random variable

Conditional Probability

- Sales at a Store
- Election results
- Weather Prediction
- Events in real life rarely have simple probability

Conditional Probability (2 Events)

- Let A and B be two events and $P(A) > 0$. Probability of B given that A has occurred is denoted by $P(B|A)$

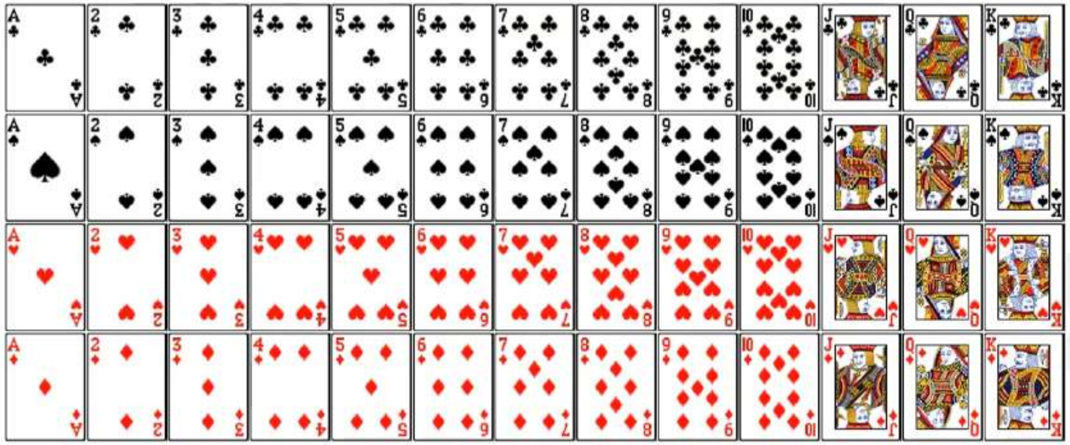
$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- Find probability that a single roll of a die will result in number less than 4
  - If no other information is provided (Answer: $\frac{3}{6}$)
  - Given that rolling resulted in an odd number

B denote the event less than 4
A denote the event resulting odd number

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2}{3}$$

Independent Events



What is probability of getting a 2 kings when 2 cards are draw?

- With replacement
- Without replacement

Let K_1 be the event of getting a king in first draw

Let K_2 be the event of getting a king in second draw

$\frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

With replacement

$$P(K_1 \cap K_2) = P(K_1)P(K_2)$$

Without replacement

$$P(K_1 \cap K_2) = P(K_1)P(K_2|K_1)$$

$\frac{4}{52} \times \frac{3}{51} = \frac{1}{13 \times 17}$

Two events A and B are independent if $P(A \cap B) = P(A)P(B)$

Random Variable (Discrete)

- Function that assigns a real number to each outcome in the sample space of a random experiment
- Denoted by an uppercase letter such as X
- Suppose a coin is tossed twice. Let X represents the number of tails that can come up. Probability function will be as follows

x	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Conclusion

- To calculate conditional probability for events
- To construct probability function for toy examples