Week 02 -

2.1 Mathematical Models

Objective: Understand purpose of mathematical models and process of mathematical modeling

2.2 Mathematical Models

- Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena

(Phenomena), (Process), (Object), (Behavior) -> *Representation or describe mathematically* -> Mathematical models

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Volume of a cardboard box

$$V = l \times w \times h$$
$$l = length$$
$$w = width$$
$$h = height$$

- Assumptions
- Accuracy
- Range of applicability of a model

Ideal GAs Law

$$P = \frac{nRT}{V}$$
$$R = \text{Universal gas constant}(8.31\frac{J}{mol.K})$$
$$P = pressure$$
$$n = \text{number of models (quantity of gas)}$$
$$T = temprature$$
$$V = volumeofgas$$

- Is it always true?
    - No, it is a decent model
    - Scientists have developed more accurate models

Real Gas Law

Van der Waals model

$$P = \frac{RT}{Vm - b} - \frac{a}{V^2m}$$

$$R = \text{Universal gas constant}(8.31\frac{J}{mol.K})$$
$$P = pressure$$
$$T = temprature$$
$$Vm = \text{volume of gas in moles}$$
$$\text{a,b are constants}$$

Real Gas Law

Redlich-kwong model

$$F = G\frac{m1m2}{r^2}$$

$$G = \text{Gravitational constant}(6.67 * 10^-11\frac{Nm^2}{kg^2})$$
$$m1 = \text{mass of body 1}$$
$$m2 = \text{mass of body 2}$$
$$r = \text{distance between the mass bodies}$$

Newton's law of gravitation

$$F = G\frac{m1m2}{r^2}$$

$$G = \text{Gravitational constant}(6.67 * 10^-11\frac{Nm^2}{kg^2})$$
$$m1 = \text{mass of body 1}$$
$$m2 = \text{mass of body 2}$$
$$r = \text{distance between the mass bodies}$$

- Is it always true?
  - To great extent
  - Einstein relativistic model superseded

Mathematical Models

$$V = l \times w \times h$$

$$P = \frac{nRT}{V}$$

$$P = \frac{RT}{Vm - b} - \frac{a}{V^2 m}$$

$$P = \frac{RT}{Vm - b} - \frac{a}{\sqrt{TVm(Vm + b)}}$$

$$F = G\frac{m1m2}{r^2}$$

2.3 mathematical models

Logistics population model

$$p(t) = \frac{K}{1 + Ae^{rt}} \text{ Where } A = \frac{K - P0}{P0}$$

- Verhulst - Belgian mathematician
- Assumption: Resources are limited
- P(t) is the size of the population at time t
- p_0 is the initial population
- K is the maximum population environment can support
- r is the constant representing rate of population growth or decay

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Predictions, projections, analysis

$$p(t) = \frac{K}{1 + Ae^{rt}} \text{ Where } A = \frac{K - P0}{P0}$$

- Some Problems:
    1. Compute the population size at a time t
    2. Computer the time for a population will change over time
    3. Predict how a population will change over time
    4. Calculate the r value for a given population
    5. Explain what happens to a population given different caules of r

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Circadian Rhythms

- Suppose a particular species exhibits daily regular fluctuations of a body temperature that can be approximated by the equation,

$$T(t) = 36.8 - 1.3(\frac{\pi}{12}(t + 4))$$

- T(t) = is the temperature at time t in Celsius
- t = time in hours starting at midnight

- Problems
    1. Find the body temperature at midnight.
    2. Find the approximate body temperature at a given time.
    3. Find the period of the function describing body temperature fluctuations.
    4. What time of day does the body temperature reach a maximum.
    5. What time of day does the body temperature reach a minimum.

Tumor Growth

$$\frac{dx}{dt} = ax^a - ax^B$$

- x is size of the tumor in mass or cell at time t
- t = time
- a, b, a, B are constants
- Pioneer in modeling - von Bertalanffy
- Problems
    1. Find the tumor size where growth increases.
    2. Find the size of the tumor when it begins to shrink.
    3. Find the critical size when the tumor stops growing.

- ## 2.4 Mathematical model of projectile motion

Grouping related data together

- Suppose you want to describe a person with height, weight and age.
- Person 1: 180cm, 78 kg, 35 years
- Person 2: 170 cm, 65 kg, 40 years
    - person: $\begin{bmatrix} height \\ weight \\ age \end{bmatrix}$
    - person 1: $\begin{bmatrix} 180 \\ 78 \\ 35 \end{bmatrix}$, Person 2: $\begin{bmatrix} 170 \\ 65 \\ 40 \end{bmatrix}$, person 3: $\begin{bmatrix} 150 \\ 40 \\ 12 \end{bmatrix}$
- grouping data together: Sets, Tuple, Vectors, matrices

Projectile Motion

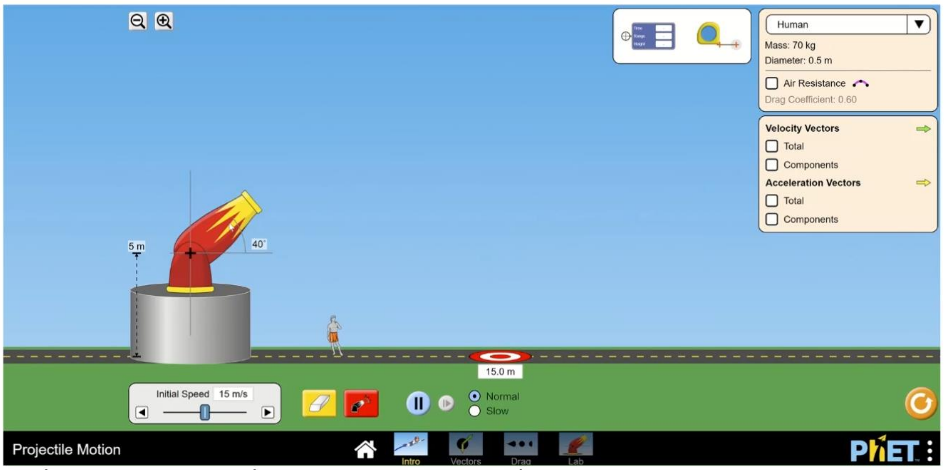
$$x(t) = x0 + voxt$$
$$y(t) = y0 + voyt - \frac{1}{2}gyt^2$$

- t = time
- Position, Vector, Acceleration are vector quantities
- $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ = position of projectile - unknown
- $\begin{bmatrix} x0 \\ y0 \end{bmatrix}$, $\begin{bmatrix} vox \\ voy \end{bmatrix}$ = initial position and velocity
- $\begin{bmatrix} gx \\ gy \end{bmatrix}$, $\begin{bmatrix} 0 \\ 9.81 \end{bmatrix}$ **m/s^2** = acceleration due to gravity - constant
- Assumptions ? is it a;ways true ?
- Model is modified to reflect the effort the efforts of drag on the projectile

- ## 2.5 Mathematical Models

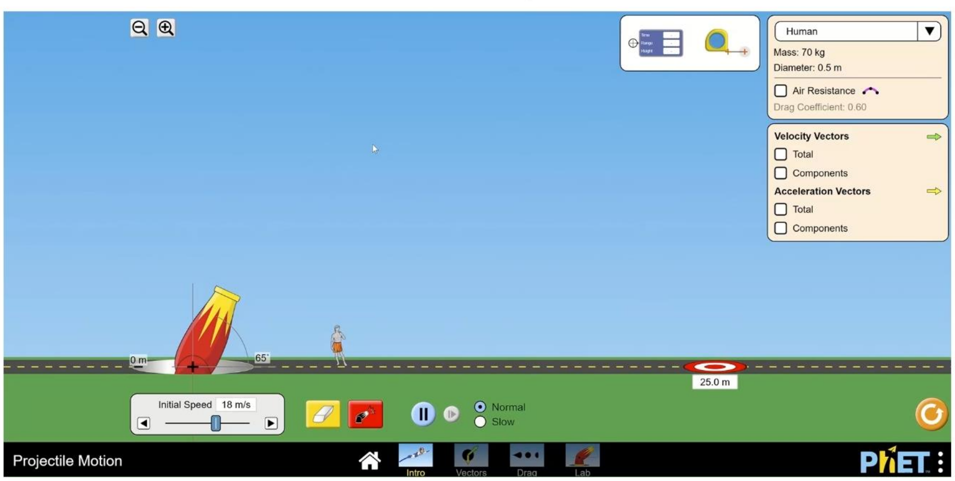
Projectile motion – Playground from PhET

Projectile motion – Describing formally



Control variables	Initial position	Range = horizontal distance traveled	
Initial speed Angle θ Canon height	$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$		At $t = 0$ s, not started At $t = 1$ s, moving upwards At $\sim t = 1.5$ s, reaches peak, starts descending At $\sim t = 3$ s, touches the ground

Projectile motion – Correcting the model for drag



Mathematical model for projectile motion has to be corrected to accommodate the drag force experienced by the body due to atmosphere

Mathematical model for projectile motion has to be corrected to accommodate the drag force experienced by the body due to atmosphere.

Mathematical model should be able to abstract the phenomena or observation.

Mathematical models use mathematics to present, analyze, make predictions or provide insight into real world phenomena.

2.6 Mathematical Models

https://www.youtube.com/watch?v=OmJ-4B-mS-Y&ab_channel=DoS-DomainofScience

https://m3challenge.siam.org/resources/whatismathmodeling