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Logic Gates

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Objective

- Basics of Logic Gates



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Logic gate



- Building blocks of digital electronic circuits.
- Boolean functions are implemented in digital circuits using these **logic gates**.
- Most logic gates have 2 inputs and 1 output.
- At any time, every terminal will be in one of the two binary conditions LOW (FALSE; 0;0V) or HIGH (TRUE;1;+5V).
- The function of each logic gate will be represented by Boolean expression

Logic gate

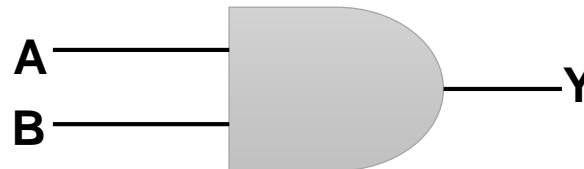
- Logic gates are classified into:
 - Basic gates : OR, AND, NOT
 - Universal gate : NAND, NOR
 - Special purpose gates: EX-OR, EX-NOR
- Truth table explains all the outputs of a logic circuit for all possible inputs to that circuits.



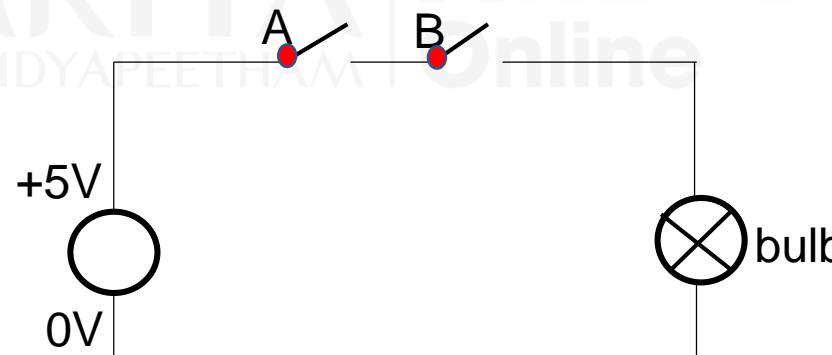
AND gate



- The realization of logical AND operator .
- Logic expression, $Y = A \cdot B$ or AB
- The circuit will give high output (1) if both inputs are high otherwise, the output is low



Logic symbol



Switching circuit

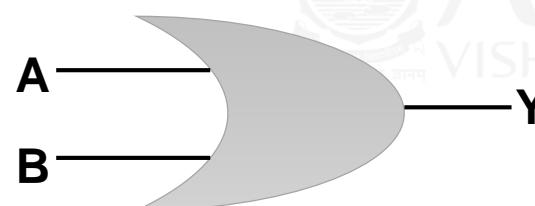
inputs		output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

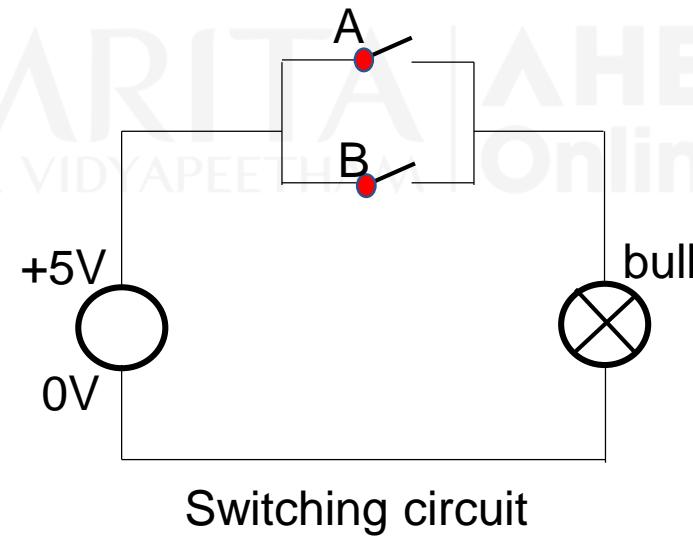
OR Gate



- Logic expression, $Y=A+B$
- The circuit will give high output if any one input is high otherwise the output is low.



Logic symbol



Switching circuit

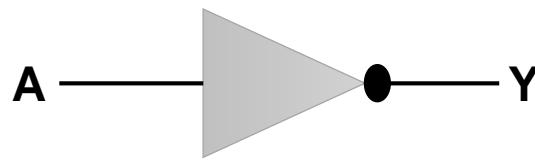
inputs		output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

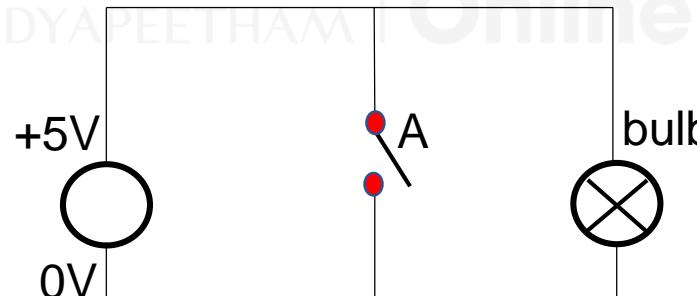
NOT Gate



- Single input single output gate.
- Also called Inverter because it inverts the given binary input.
- Logic expression, $Y = A' = \bar{A}$



Logic symbol



Switching circuit

input	output
A	Y
0	1
1	0

Truth table

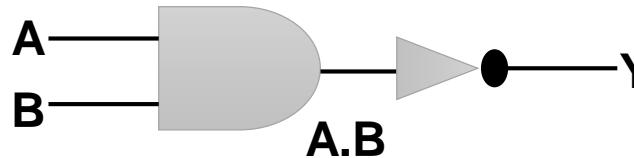
NAND Gate



- NAND is a combination of AND gate and NOT gate
NOT + AND = NAND

- The circuit will give high (1) output if any one input is low (0) otherwise, the output is low (0).

- Logical expression, $Y = \overline{A \cdot B}$



or



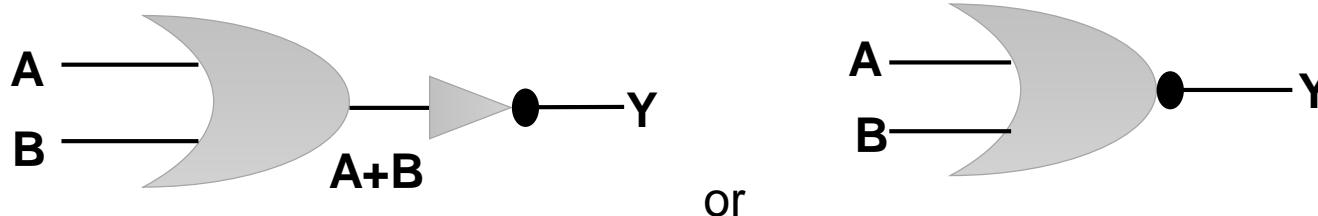
inputs		output
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

Truth table

NOR Gate



- NOR is a combination of OR gate and NOT gate.
NOT + OR = NOR
- The circuit will give high (1) output if both inputs is low (0) otherwise the output is high (1).
- Logical expression, $\text{Y} = \overline{A+B}$



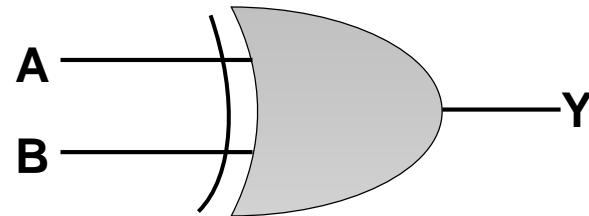
inputs		output
A	B	$\text{Y} = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

XOR Gate



- Exclusive-OR gate
- The output is high if the circuit input has odd number of 1's otherwise the output is low i.e., with even number of 1's.
- Logical expression $Y = A \oplus B = A'B + AB'$ (SOP)



inputs		output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

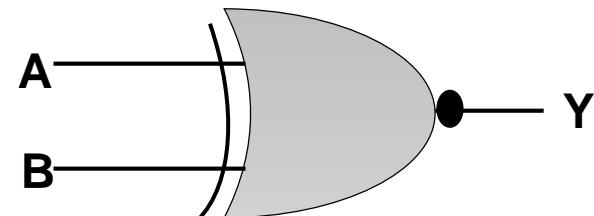
Truth table

X-NOR Gate



- Exclusive NOR gate or equivalence.
- The output is high if the circuit input has even number of 1's otherwise the output is low.
- Logical expression $Y = A \odot B = A'B' + AB$

Complement of XOR



inputs		output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Truth table

Realizing Logic circuit from function



- Boolean Function, $F = x + y'z$

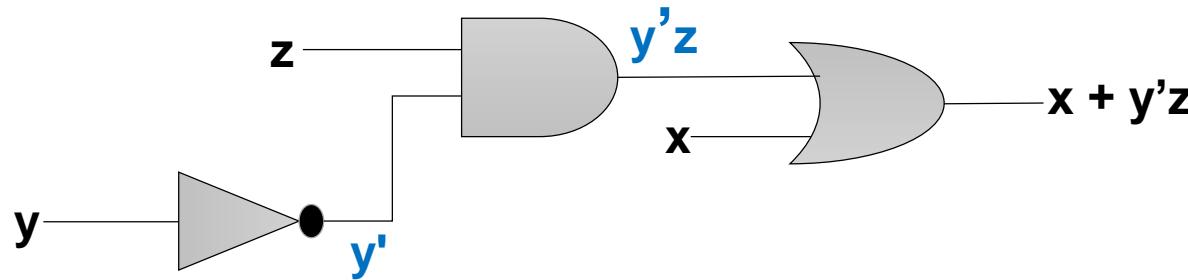
input variable : x, y, z

- Truth table

3 input variables = 2^3 combinations

- Logic circuit

literals – 3 (x, y', z) and 2 terms

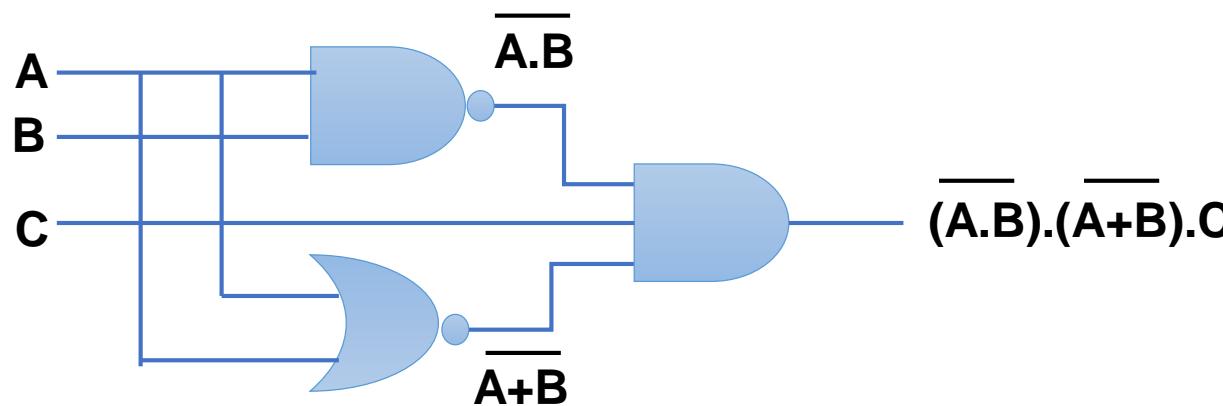
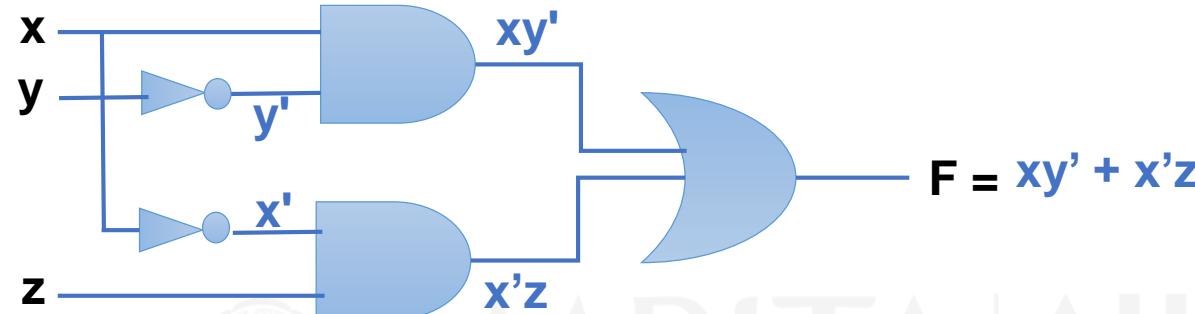


inputs			output
x	y	z	$F=x+y'z$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$\begin{aligned} & \rightarrow x = 0, y = 1, z = 0 \\ & F = 0 + 1'.0 \\ & = 0 + 0.0 \\ & = 0 + 0 \\ & = 0 \end{aligned}$$

$$\begin{aligned} & \rightarrow x = 1, y = 1, z = 0 \\ & F = 1 + 1'.0 \\ & = 1 + 0.0 \\ & = 1 + 0 \\ & = 1 \end{aligned}$$

Realizing function from Logic circuit



Summary

- Basics of Logic gates



Reference

- M Morris Mano - Computer System Architecture - PHI - Third Edition
- Gideon Langholz, Abraha& Joe L Mott - Digital Logic Design - World Scientific Publishing Co Ltd
- Thomas C Bartee - Digital Computer Fundamentals - Tata Mc Graw Hill - Sixth Edition



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Universal Logic Gate

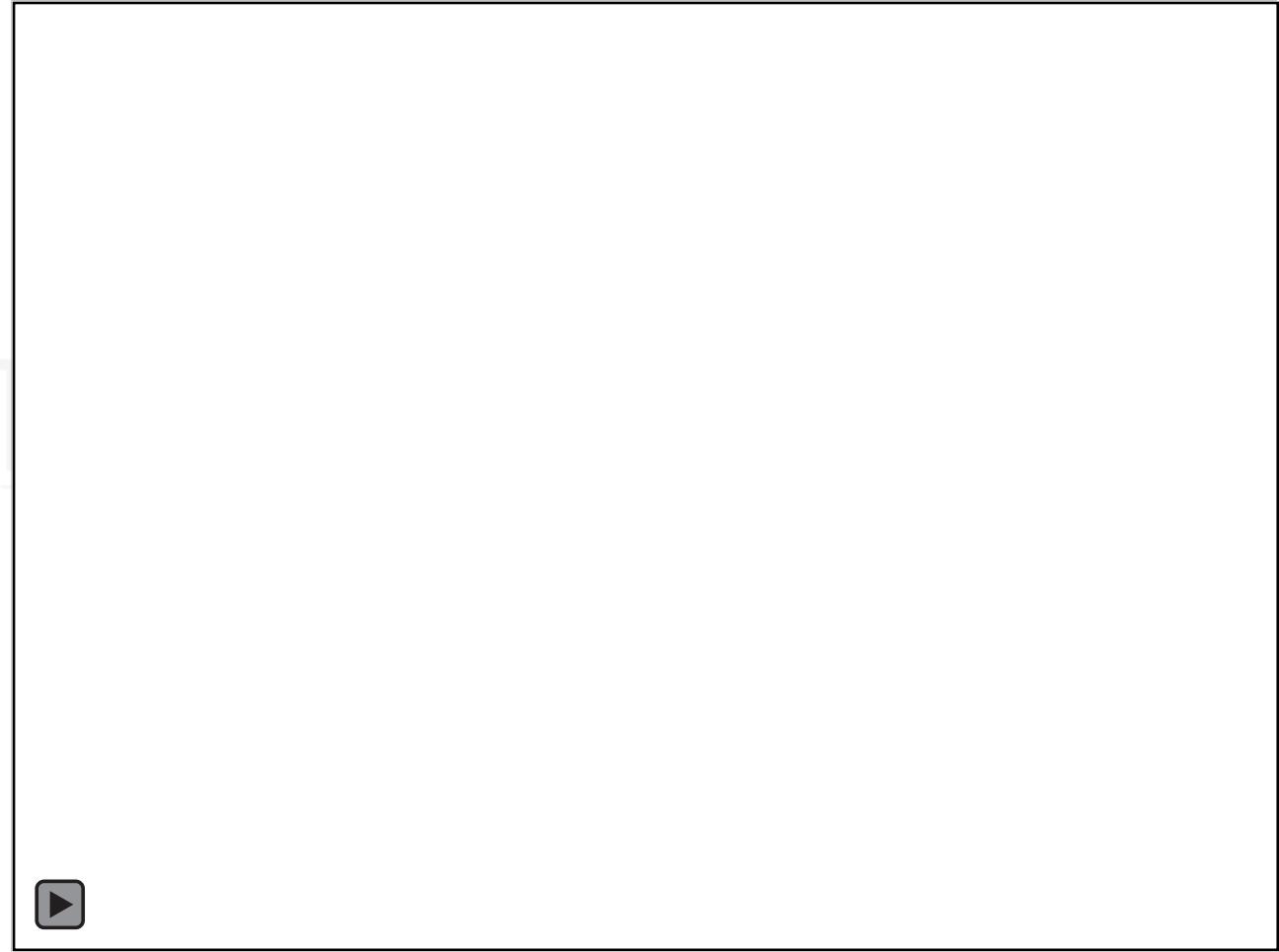
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Objective

- Universal Logic gates
 - NAND gate
 - NOR gate



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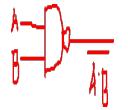
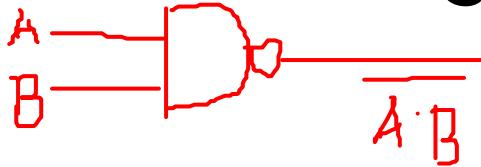


Introduction

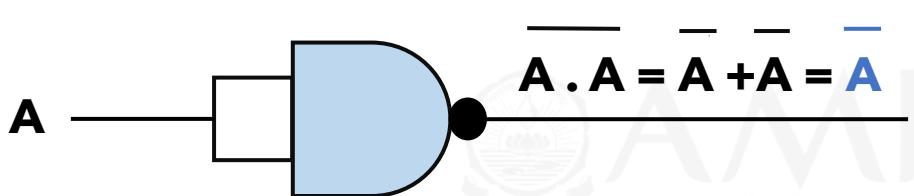


- **NAND & NOR** gate are known as the **universal gates**.
- These gates alone sufficient to implement any Boolean expression and are inexpensive.
- Basic logic gates (AND, OR and NOT) are logically complete.
- Sufficient to show that AND, OR and NOT can be implemented with NAND and NOR gate.

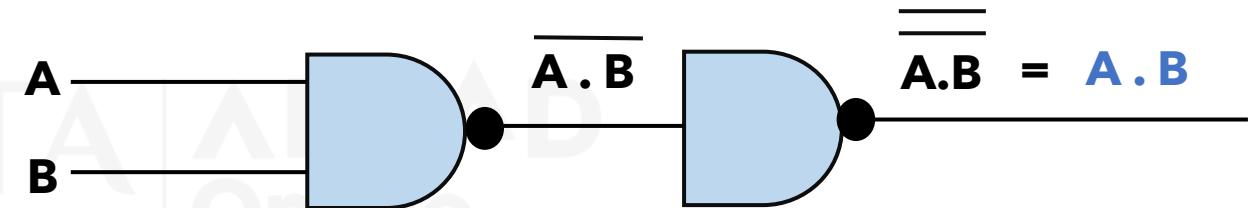
Universal NAND gate



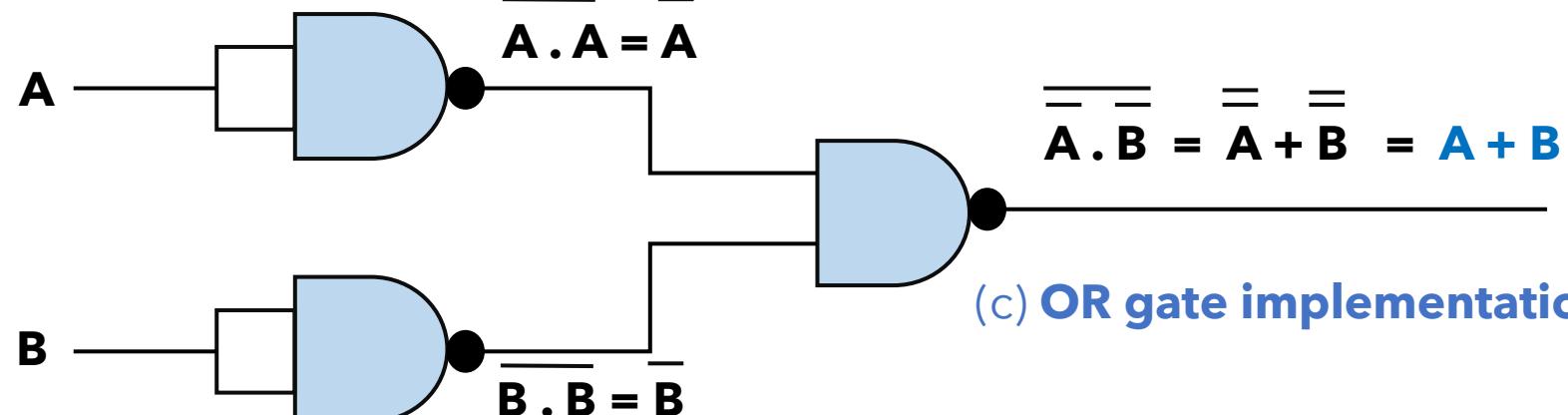
- We can implement NOT, AND and OR gates by combining one or more NAND gates.



(a) NOT gate implementation



(b) AND gate implementation

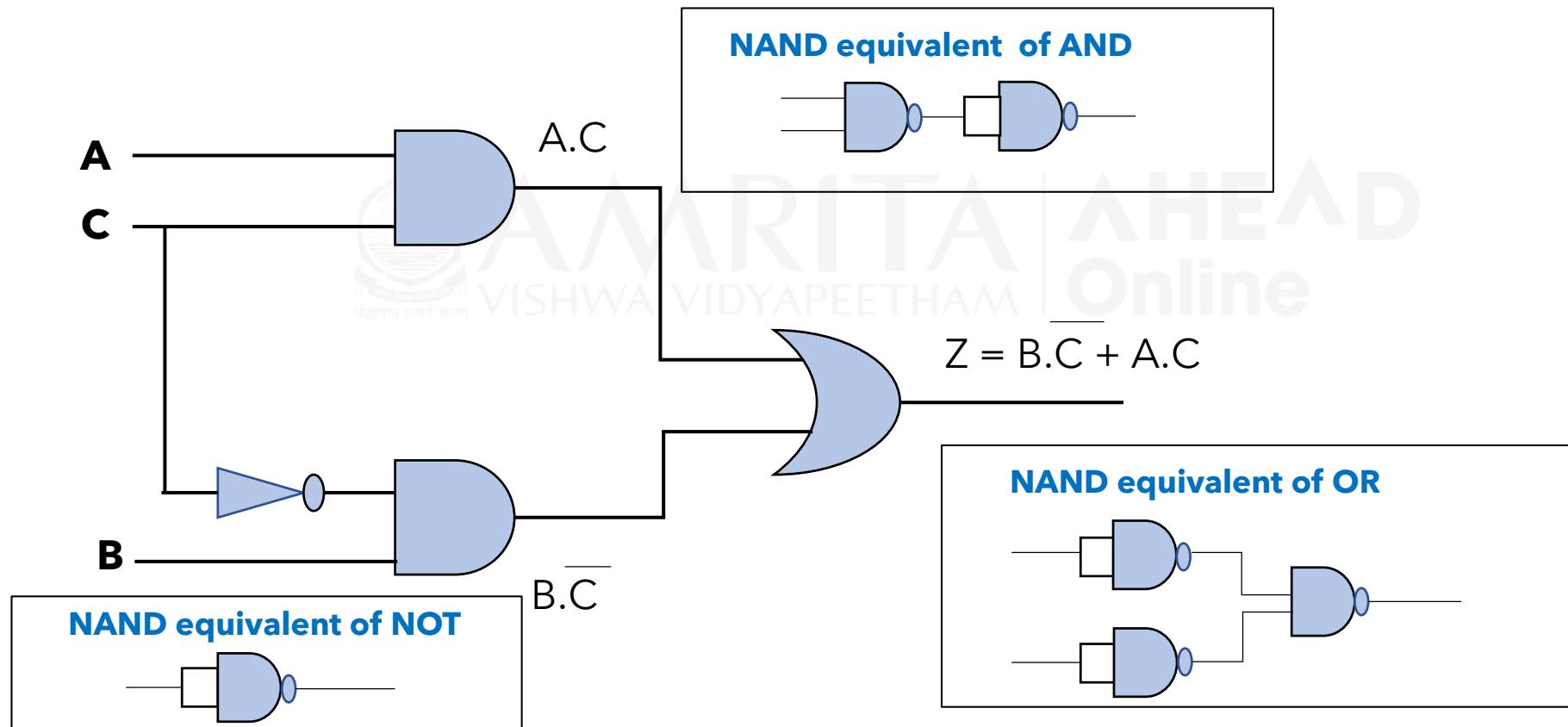


(c) OR gate implementation

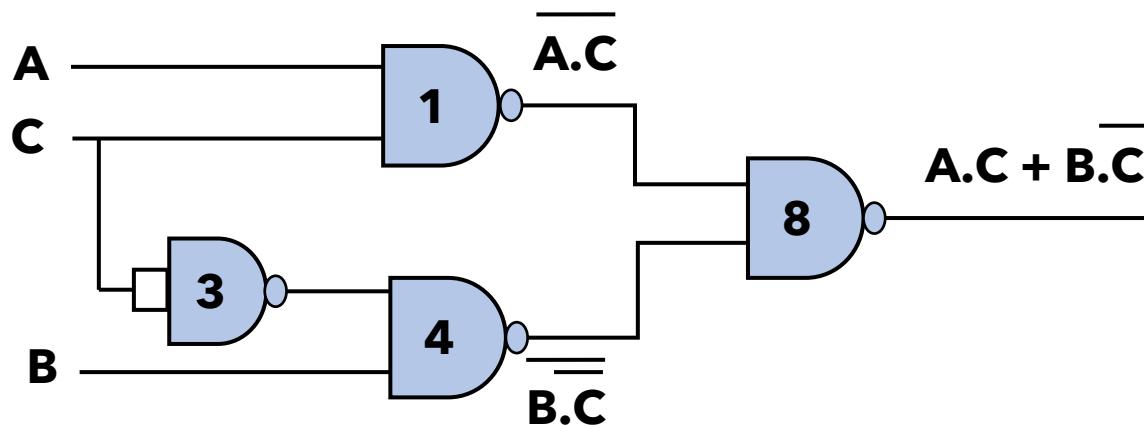
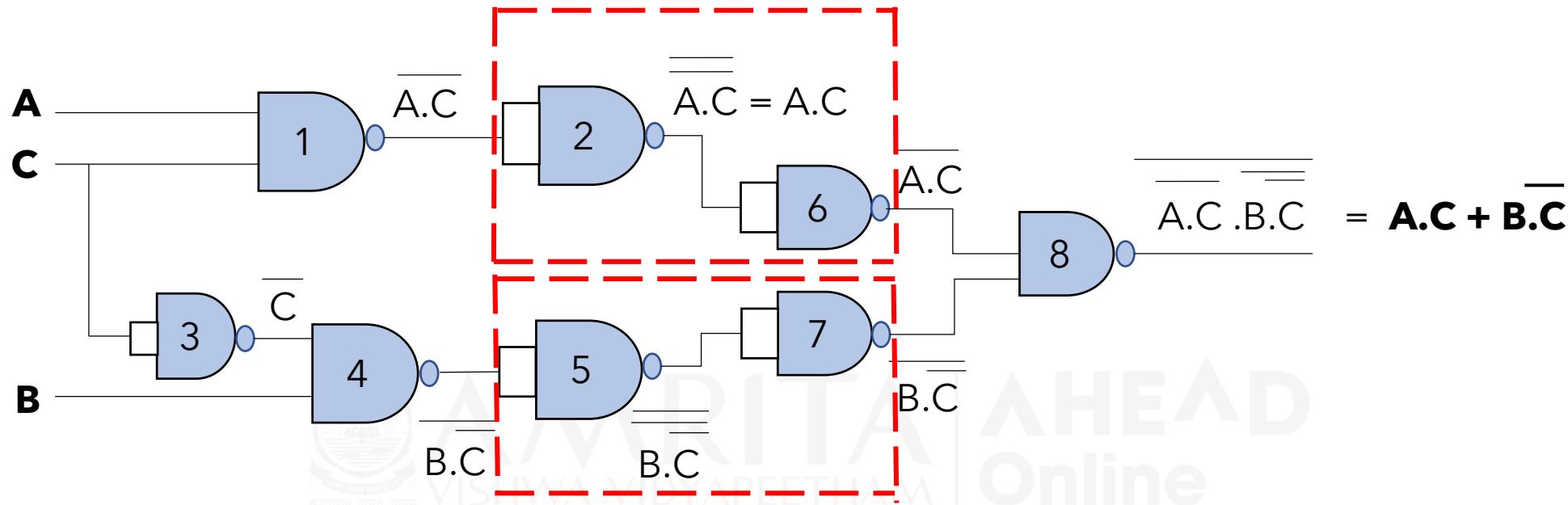
NAND Logic circuit



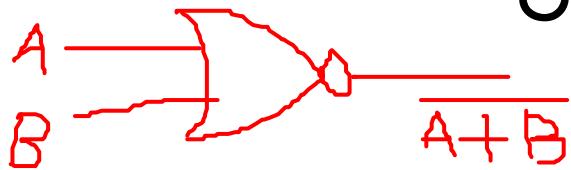
- Design NAND logic circuit for the Boolean expression $Z = \overline{BC} + AC$



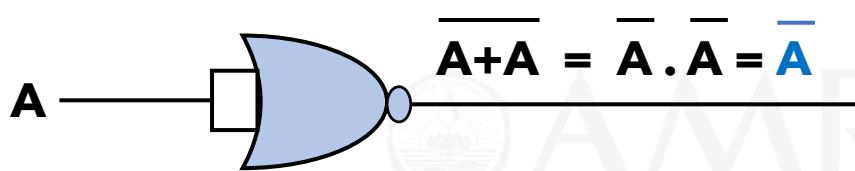
NAND logic circuit



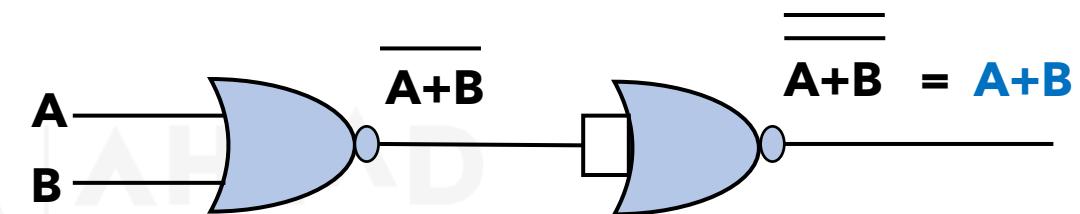
Universal NOR gate



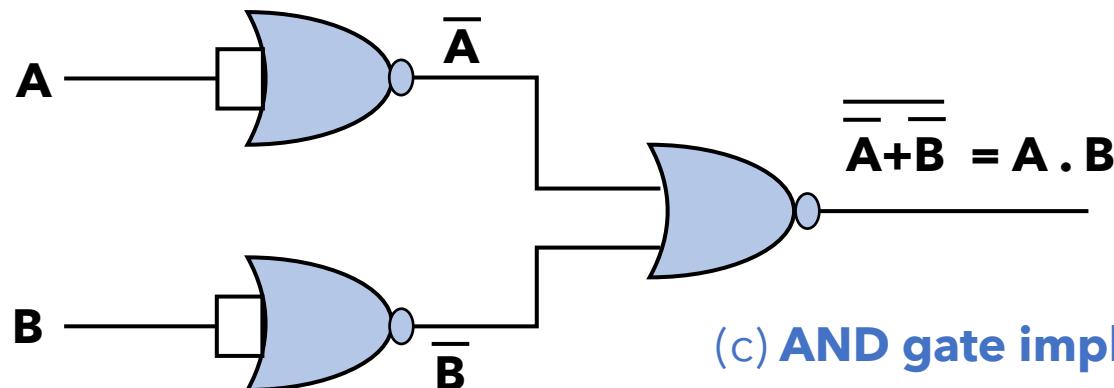
- Implement NOT, AND and NOR gates by combining one or more NOR gates.



(a) NOT gate implementation



(b) OR gate implementation

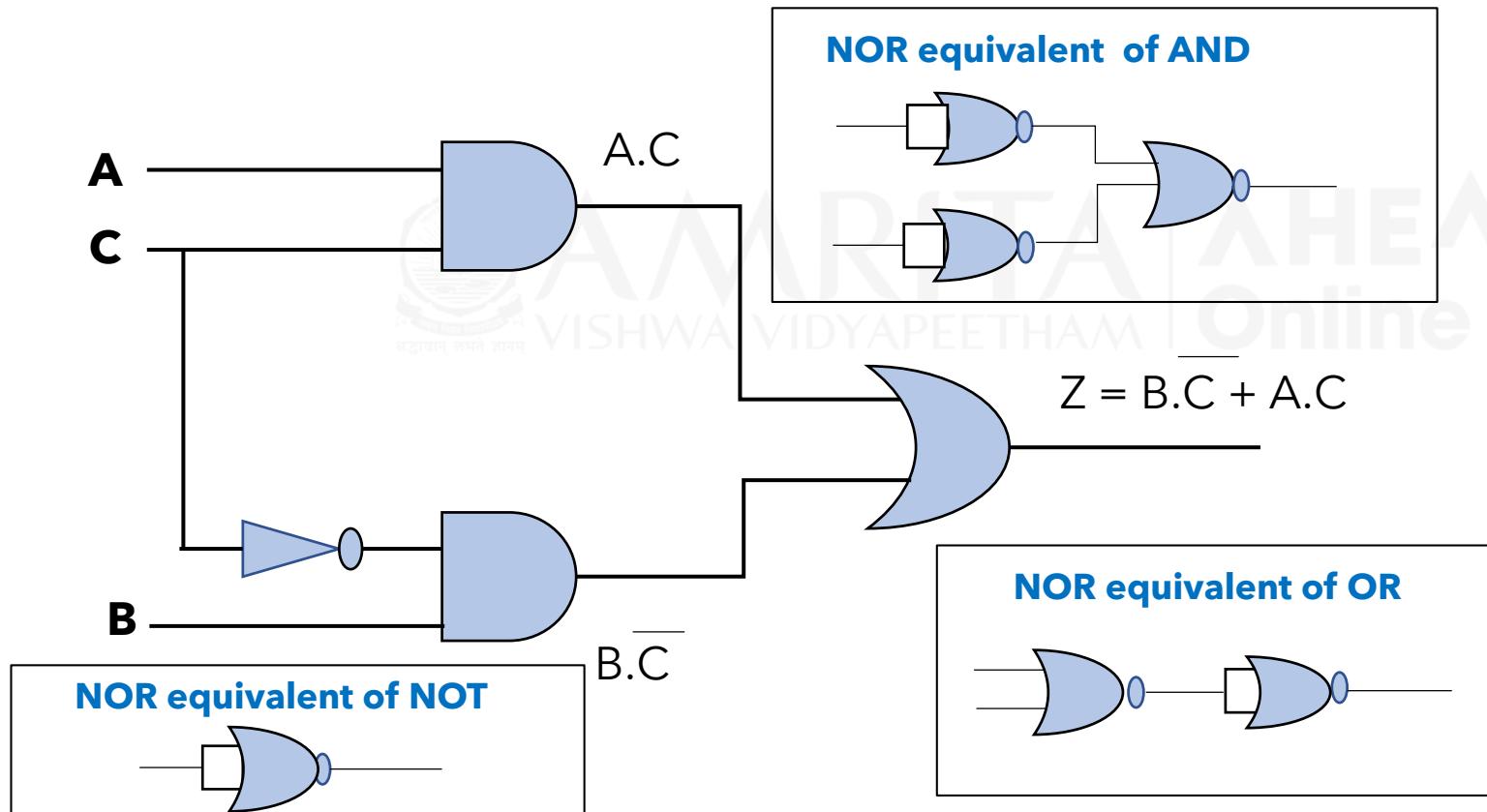


(c) AND gate implementation

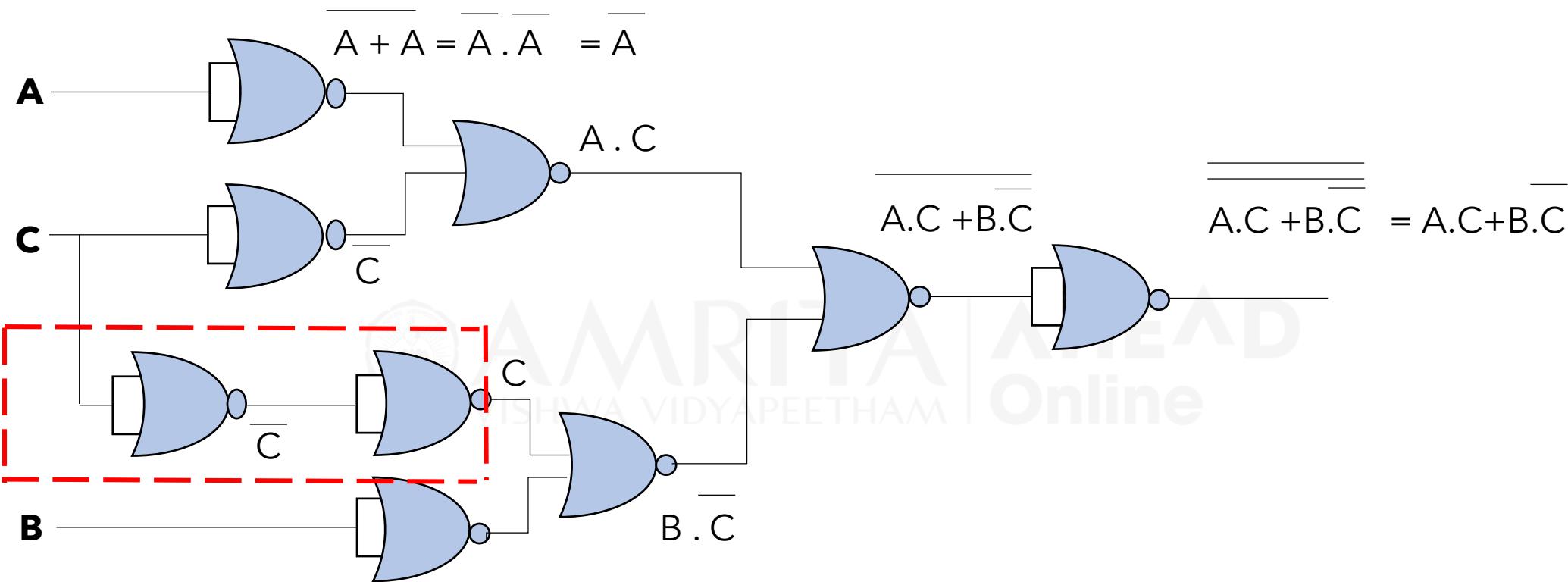
NOR Logic circuit



- Design NOR logic circuit for the Boolean expression $Z = \overline{BC} + AC$

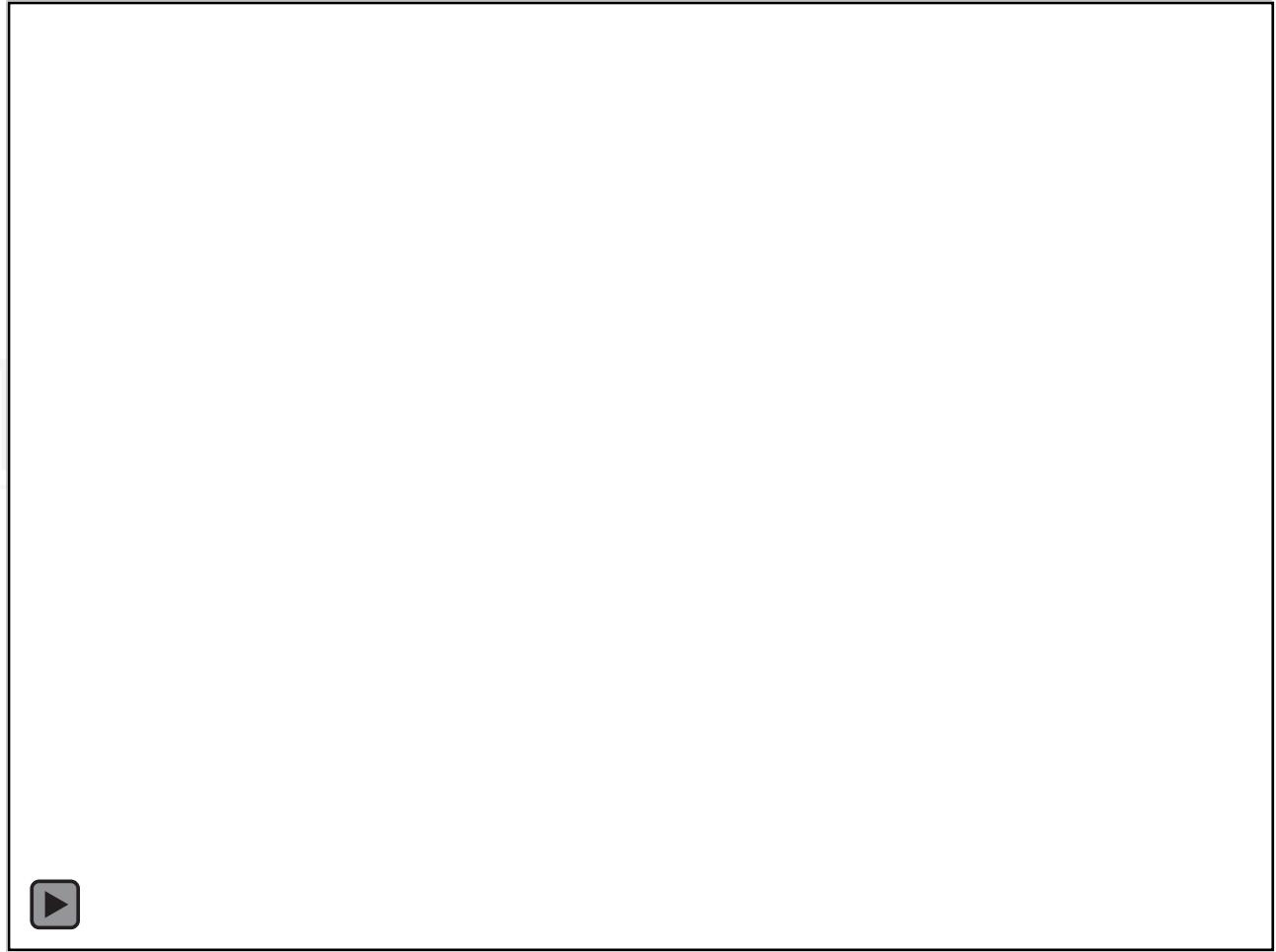


NOR Logic circuit



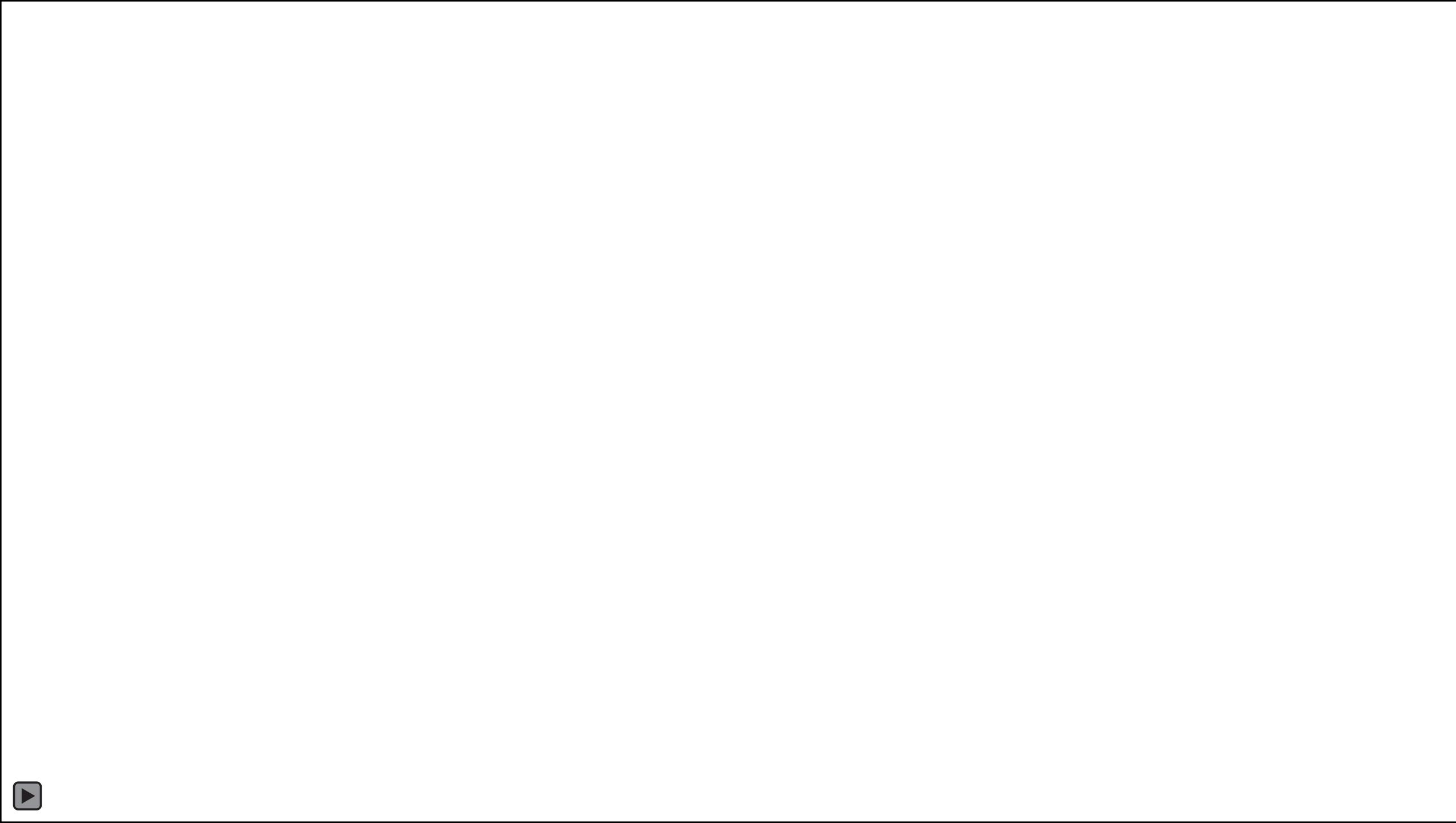
Summary

- Universal Logic gates



Reference

- M Morris Mano - Computer System Architecture - PHI - Third Edition
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Canonical & Standard Forms

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Objective

Representation of Boolean Function

- Standard form
- Canonical Form



Introduction



- Boolean functions can be expressed in **canonical** and **standard forms**
- In the Boolean expression binary variable will be either in normal form (x) or in its complement form (x').

$F(A,B,C,D)$ is a function :

- **Product term** : Logical ANDed among the literals.
e.g., AB , $A'B$, ABC'
- **Sum term** : Logical ORed among the literals.
e.g., $A+B$, $A'+B$, $A+B+C'$

Minterm : m_j



- **Minterm /Standard product** : Product term containing all the variables in normal or as complements.
e.g., $F(A,B,C) : ABC'$, ~~AB~~
- For n- variable function have 2^n possible minterms.
- While obtaining minterm, each variable is complemented if the corresponding bit of binary number is 0 and uncomplemented if 1.

x	y	minterm	symbol
0	0	$x'y'$	m_0
0	1	$x'y$	m_1
1	0	xy'	m_2
1	1	xy	m_3

A minterm equals 1 at exactly one input combination and is equal to 0 otherwise.
e.g., $x'y' = 1$ only when $x = 0, y = 0$

Minterms with 2 binary variables x and y
2 variable - $2^2 = 4$ minterms
x is primed if $x=0$; x is unprimed if $x=1$

Minterms

- 3 variable function with $2^3 = 8$ possible minterms



A	B	C	minterm	symbol		A=0	B=0	C=0
0	0	0	$A'B'C'$	m_0	→	A'	B'	C'
0	0	1	$A'B'C$	m_1				
0	1	0	$A'BC'$	m_2				
0	1	1	$A'BC$	m_3				
1	0	0	$AB'C'$	m_4	→	A	B'	$C=0$
1	0	1	$AB'C$	m_5				
1	1	0	ABC'	m_6				
1	1	1	ABC	m_7				

Expressing functions as Sum of Minterms



- Boolean function can be expressed algebraically from the truth table
 - Select the minterms that produces a 1 in the function.
 - Take OR of those selected minterms.

i.e., sum of all minterms

$$f = x'y'z + xy'z' + xyz$$

$$= m_1 + m_4 + m_7$$

$$= \Sigma(1,4,7)$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

x'y'z ← **xy'z'** ← **xyz** ←

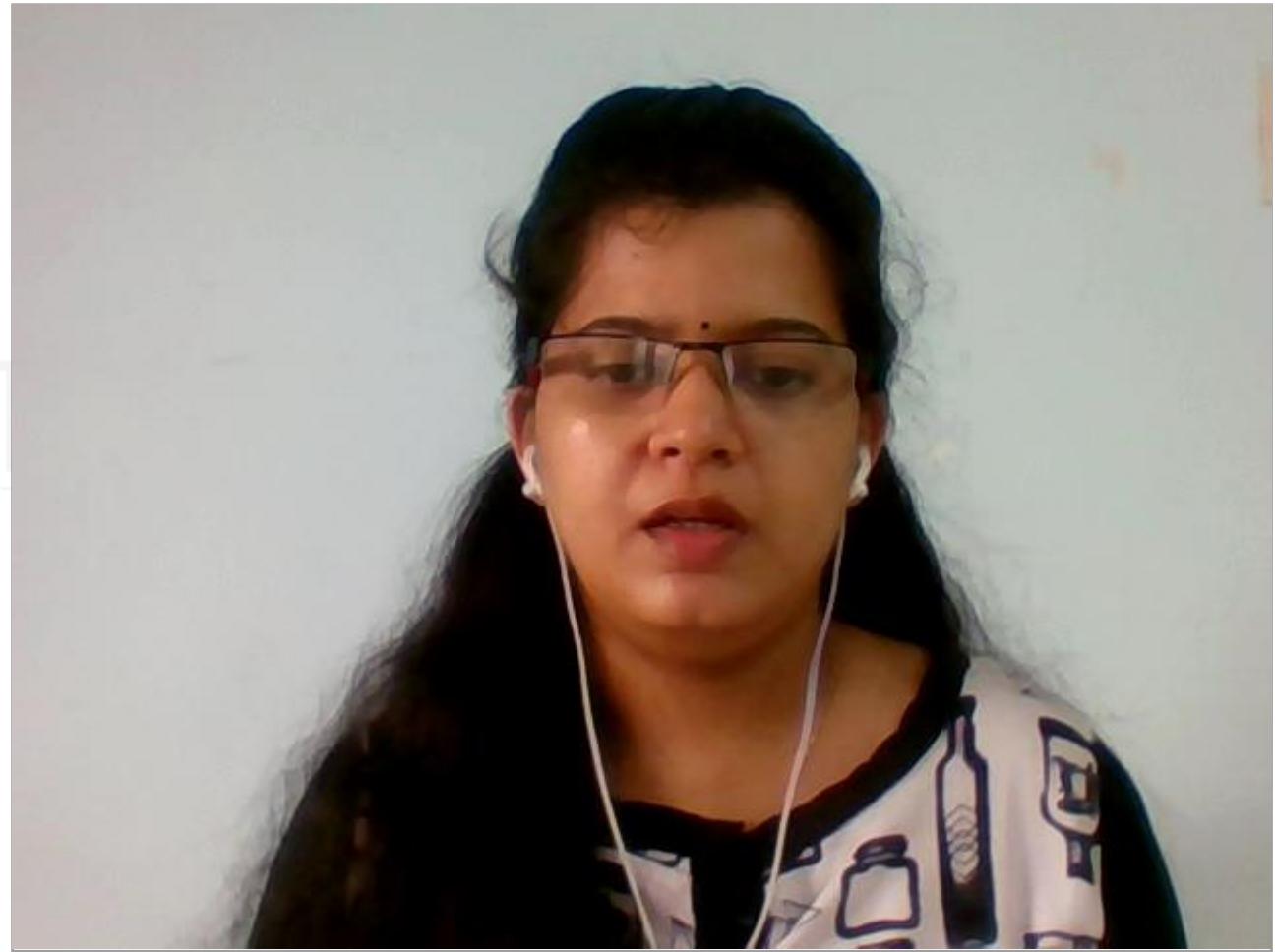
Expressing functions as Sum of Minterms



- Any function can be expressed by ORing all minterms (m_i) corresponding to input combinations (i) at which the function has a value of 1.
- Resulting expression is referred to as SUM of minterms and is expressed as $F = \Sigma(2, 4, 5, 7)$ where Σ indicates ORing of the indicated minterms.
- $F = \Sigma(2, 4, 5, 7) = (m_2 + m_4 + m_5 + m_7)$

Summary

- Canonical forms : Sum of Minterms



Reference

- M Morris Mano - Computer System Architecture - PHI - Third Edition
- Gideon Langholz, Abraha& Joe L Mott - Digital Logic Design - World Scientific Publishing Co Ltd
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Canonical & Standard form-II

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Objective

- Canonical form and standard form using maxterms



Maxterm : M_j



- **Maxterm /Standard sums** : Sum term containing all the variables or their complements.
e.g., $F(A,B,C) : A+B+C'$, $A'+C$
- For n - variables can have 2^n possible maxterms.
- For maxterm, each variable is primed if the corresponding bit of binary number is 1 and unprimed if 0

x	y	maxterm	symbol
0	0	$x+y$	M_0
0	1	$x+y'$	M_1
1	0	$x'+y$	M_2
1	1	$x'+y'$	M_3

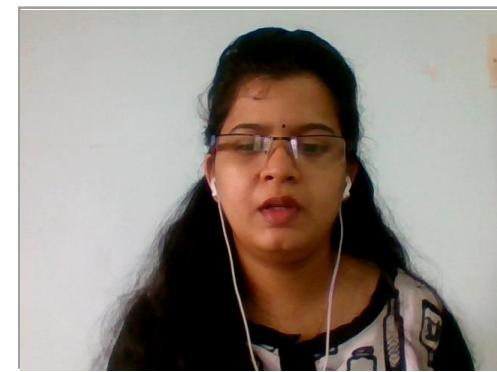
A maxterm equals 0 at exactly one input combination and is equal to 1 otherwise.

e.g., $x+y' = 0$ only when $x = 0, y = 1$

**Maxterms with 2 binary variables x and y
2 variable - $2^2 = 4$ maxterms
x is primed if $x=1$; x is unprimed if $x=0$**

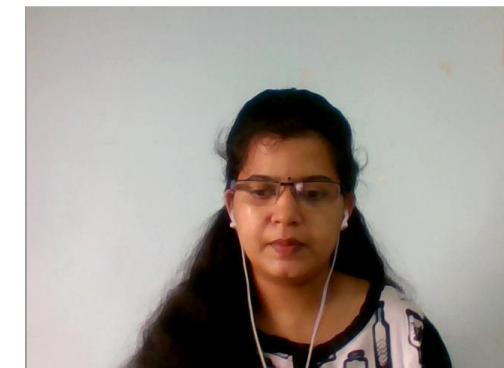
Maxterms

- 3 variable function with $2^3 = 8$ possible maxterms



A	B	C	maxterm	symbol		
0	0	0	$A+B+C$	M_0	$\rightarrow A=0$	$B=0$
0	0	1	$A+B+C'$	M_1	\downarrow	C
0	1	0	$A+B'+C$	M_2	A	\downarrow
0	1	1	$A+B'+C'$	M_3	\downarrow	B
1	0	0	$A'+B+C$	M_4	$\rightarrow A=1$	$B=0$
1	0	1	$A'+B+C'$	M_5	\downarrow	C
1	1	0	$A'+B'+C$	M_6	A'	\downarrow
1	1	1	$A'+B'+C'$	M_7	B	\downarrow

Expressing functions as Product of Maxterms



- Select maxterms that produces a 0 in the function
- Take AND of those selected maxterm
i.e., product of all maxterms

$$f = (x+y+z)(x+y'+z)(x+y'+z')(x'+y+z')(x'+y'+z)$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

$$= \prod(0, 2, 3, 5, 6)$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Maxterms identified:

- $x+y+z$ (Row 1)
- $x+y'+z$ (Row 2)
- $x+y'+z'$ (Row 3)
- $x'+y+z'$ (Row 5)
- $x'+y'+z$ (Row 6)

Maxterm from Minterm



- Minterm and Maxterm are complement to each other.
- Complement of minterm = maxterm

Example : $F(x, y, z) = \Sigma (1, 4, 5, 6, 7)$

$$F'(x, y, z) = \Sigma(0, 2, 3) = (m_0 + m_2 + m_3)$$

Take complement of F' using De-morgan's law,

$$F(x, y, z) = (m_0 + m_2 + m_3)' = (m_0'.m_2'.m_3') = (M_0.M_2.M_3) = \Pi(0, 2, 3)$$

x	y	z	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Canonical & Standard Form



- The sum of minterms and product of maxterms are called the **canonical forms**.
- In canonical form, each minterm or maxterm contain all variables in complemented or uncomplemented form.
e.g., $F(A, B, C) = ABC + A'B'C'$
- **Standard form** : simplified versions of canonical forms
The terms that form the function contain one, two, or more literals each.
e.g., $F(A, B, C) = (A + B + C)(A' + B')$

Canonical V/s Standard Form



Canonical Form	Standard Form
Representation that helps to describe the Boolean functions	A simplified version of Canonical form
Divides into Canonical SoP (Sum of minterms) and Canonical PoS (Product of maxterms)	Divides into Standard SOP and Standard POS
More complex	Simple
E.g. $F(a,b,c) = a'b'c + ab'c + abc$	E.g. $F(a,b,c) = a + b'c$

Summary

- Canonical and Standard forms : POS



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Karnaugh map

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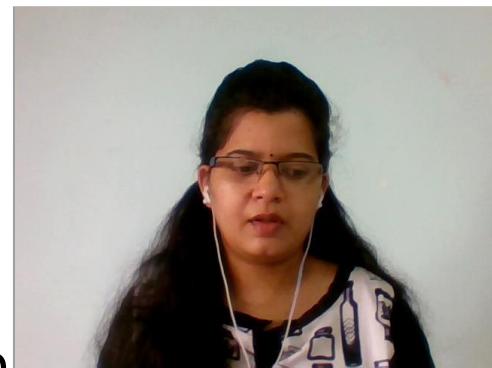
Objective

- Systematic method of simplifying Boolean Expression - **Karnaugh map**



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Introduction



- Pictorial representation of writing Boolean expression.
- The map method modified by Karnaugh is called **K-map** or **Karnaugh map**.
- Objective of K-map is the minimization of Boolean function with a simple procedure.
- K-map : Diagram made up of squares ; n-variable 2^n squares
 - Each square represents one minterm of the function
 - Visual diagram of a function in standard form(SOP or POS)

K-maps



- K-maps use 2-dimensional tables to simplify the Boolean expression.

A	B	0	1
0	m_{00}	m_{01}	
1	m_{10}	m_{11}	

2-variable map

2 variable = $f(A,B)$

2^2 minterm = 4 cell map

A	BC	00	01	11	10
0	m_0	m_1	m_3	m_2	
1	m_4	m_5	m_7	m_6	

3-variable map

3-variable = $f(A,B,C)$

2^3 minterm = 8 cell map

AB	CD	00	01	11	10
00	m_0	m_1	m_3	m_2	
01	m_4	m_5	m_7	m_6	
11	m_{12}	m_{13}	m_{15}	m_{14}	
10	m_8	m_9	m_{11}	m_{10}	

4-variable map

Adjacent cells differ by just one single bit

K- map Simplification



- First fill the appropriate cells with the value based on the output variable.
- Group the maximum number of 1's (SOP) and 0's (POS).
- Group need to be in power of 2 and done in decreasing order.
If 8 cell K-map, try grouping for 8 ($=2^3$), then for 4 ($=2^2$), next for 2 ($=2^1$) and last consider single bit.
- Later each group is expressed in terms of common input variable in the rows and column.

Grouping Rules



- Group the cells with largest number of consecutive 1's and no 0's .

correct

0	0	1	1
1	1	0	0

Incorrect

0	1	1	0

- Grouping must be done in decreasing order and the number of 1's in the group must be a power of 2.

incorrect

0	1	1	1

incorrect

1	1	1	1

correct

0	1	1	1

correct

1	1	1	1
1	1	1	1

Grouping Rules



- Focus is on increasing the size of the group, so same elements can be repeated in multiple groups.

incorrect

1	1	1	1
0	1	1	0

correct

1	1	1	1
0	1	1	0

- Diagonal grouping is not permitted and elements around the edges can be grouped (circular property).

incorrect

1	0
0	1

correct

1	0	0	1
1	0	0	1

Summary

- Introduction to K-map with the simplification procedures



Reference

- M Morris Mano - Computer System Architecture - PHI - Third Edition
- Gideon Langholz, Abraha& Joe L Mott - Digital Logic Design - World Scientific Publishing Co Ltd
- Thomas C Bartee - Digital Computer Fundamentals - Tata Mc Graw Hill - Sixth Edition



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K-map simplification

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Objective

- K-map simplification examples

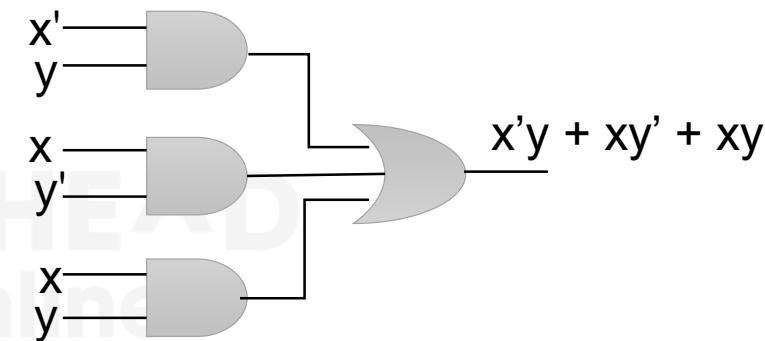
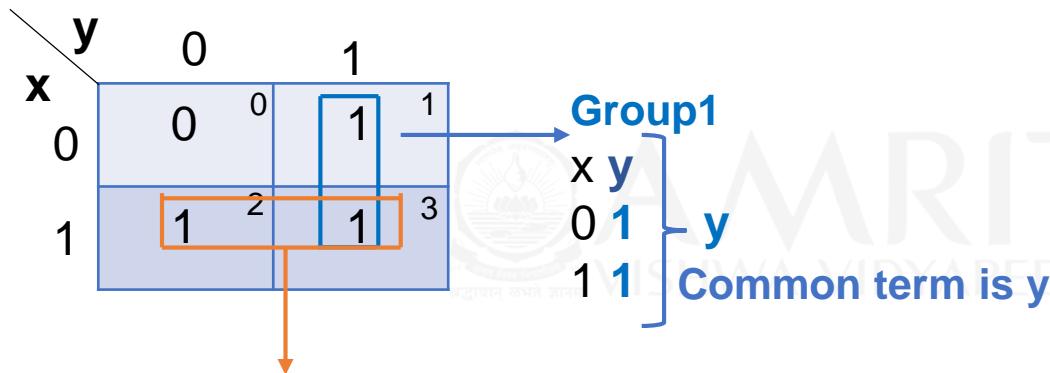


K-map Simplification

➤ $F(x,y) = x'y + xy' + xy$ (In SOP $x' = 0; x = 1$)

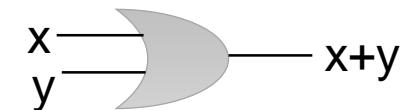
01 10 11

With this expression we need 3 AND gates and 1 OR gate



Group2
Common term is x
Common term is y

Simplified expression, $F(x, y) = x+y$
Simplified expression need only 1 OR gate



Example(SOP)

➤ $Y(A,B,C,D) = \sum(0, 2, 5, 7, 8, 10, 13, 15)$



		CD	00	01	11	10
		AB	1 0	0 1	0 3	1 2
		00	0 4	1 5	1 7	0 6
		01	0 12	1 13	1 15	0 14
		11	1 8	0 9	0 11	1 10

$$Y = A'B'C'D' + A'B'CD' + A'BCD + A'BCD + AB'C'D' + AB'CD' + ABC'D + ABCD$$

Group1
A B C D
0 1 0 1
0 1 1 1
1 1 0 1
1 1 1 1

Group2
A B C D
0 0 0 0
0 0 1 0
1 0 0 0
1 0 1 0

BD B'D'

➤ Simplified expression $Y(A,B,C,D) = BD + B'D'$

Example(POS)



- $F(A,B,C,D) = \sum(0,1,2,5,8,9,10)$
Given SOP but for simplification using POS group the 0's

		CD	00	01	11	10
		AB	00	1	1	0
00	01	AB	00	0	0	0
		AB	01	1	0	0
11	10	AB	11	0	0	0
		AB	10	1	1	0

Group1

A B C D
0 0 1 1
0 1 1 1
1 1 1 1
1 0 1 1

Group2

A B C D
1 1 0 0
1 1 0 1
1 1 1 1
1 1 1 0

Group3

A B C D
0 1 0 0
1 1 0 0
0 1 1 0
1 1 1 0

C'+D'

A'+B'

- Simplified expression $F(A,B,C,D) = (C'+D')(A'+B')(B'+D)$

Example(Don't care)



- Functions can have unspecified output for some input combination, we don't care the value of those unspecified terms.
➤ $F(A,B,C,D) = \sum (1,3,5,7,9) + d(6,12,13)$

AB	CD	00	01	11	10
00	0	1	1	0	
01	0	1	1	X	
11	X	X	0	0	
10	0	1	0	0	

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Group1	Group2
A B C D	A B C D
0 0 0 1	0 0 0 1
0 0 1 1	0 1 0 1
0 1 0 1	1 1 0 1
0 1 1 1	1 0 0 1

$A'D$ $C'D$

$F(A,B,C,D) = A'D + C'D$

Without don't care
 $F(A,B,C,D) = A'D + B'C'D$

Summary

- Explained how to perform K-map simplification with examples



Reference

- M Morris Mano - Computer System Architecture - PHI - Third Edition
- Gideon Langholz, Abraha& Joe L Mott - Digital Logic Design - World Scientific Publishing Co Ltd
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