Logic Gates

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Objective

- Basics of Logic Gates

Logic gate

- Building blocks of digital electronic circuits.
- Boolean functions are implemented in digital circuits using these **logic gates**.
- Most logic gates have 2 inputs and 1 output.
- At any time, every terminal will be in one of the two binary conditions LOW (FALSE; 0;0V) or HIGH (TRUE;1;+5V).
- The function of each logic gate will be represented by Boolean expression

Logic gate

- Logic gates are classified into:

  Basic gates : OR, AND, NOT

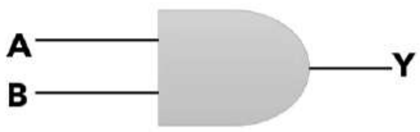
  Universal gate : NAND, NOR

  Special purpose gates: EX-OR, EX-NOR

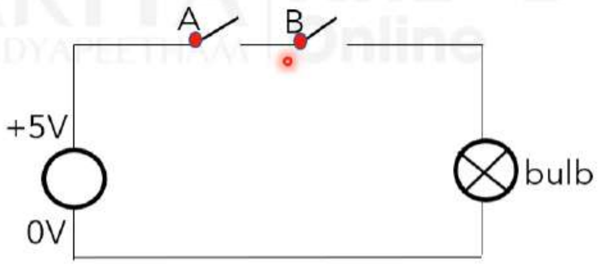
- Truth table explains all the outputs of a logic circuit for all possible inputs to that circuits.

AND gate

- The realization of logical AND operator .
- Logic expression, **Y =A.B** or **AB**
- The circuit will give high output (1) if both inputs are high otherwise, the output is low



Logic symbol



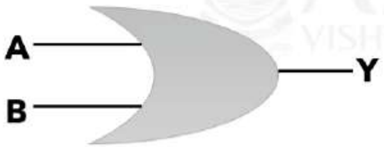
Switching circuit

inputs		output
A	**B**	**Y**
0	0	0
0	1	0
1	0	0
1	**1**	**1**

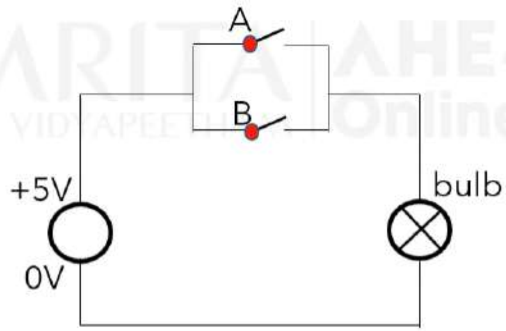
Truth table

OR Gate

- Logic expression, **Y=A+B**
- The circuit will give high output if any one input is high otherwise the output is low.



Logic symbol



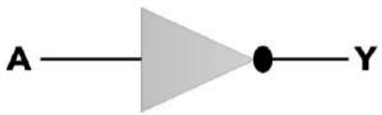
Switching circuit

inputs		output
A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	1

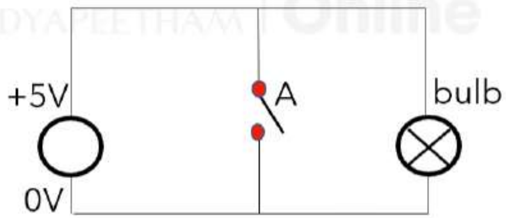
Truth table

NOT Gate

- Single input single output gate.
- Also called Inverter because it inverts the given binary input.
- Logic expression, **$Y=A' =\overline{A}$**



Logic symbol



Switching circuit

input	output
A	**Y**
0	1
1	0

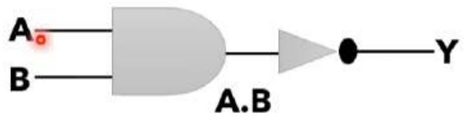
Truth table

NAND Gate

- NAND is a combination of AND gate and NOT gate
  NOT + **AND** = **NAND**

- The circuit will give high (1) output if any one input is low (0) otherwise, the output is low (0).

- Logical expression, $\mathbf{Y = \overline{A.B}}$



or

inputs		output
A	**B**	**$Y=\overline{A.B}$**
0	0	1
0	1	1
1	0	1
1	1	0

Truth table

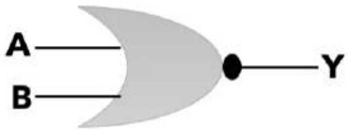
NOR Gate

- NOR is a combination of OR gate and NOT gate.
  NOT + **OR** = **NOR**

- The circuit will give high (1) output if both inputs is low (0) otherwise the output is high (1).

- Logical expression, $Y = \overline{A+B}$

or

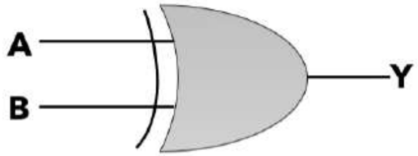


inputs		output
A	B	$Y=\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

XOR Gate

- Exclusive-OR gate
- The output is high if the circuit input has odd number of 1's otherwise the output is low i.e., with even number of 1's.
- Logical expression **Y= A⊕B = A'B+AB'** (SOP)



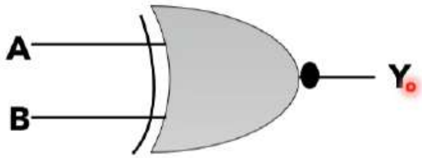
inputs		output
A	**B**	**Y**
0	0	0
0	1	1
1	0	1
1	1	0

Truth table

X-NOR Gate

- Exclusive NOR gate or equivalence.
- The output is high if the circuit input has even number of 1's otherwise the output is low.
- Logical expression **$Y = A \odot B = A'B' + AB$**

Complement of XOR



inputs		output
A	**B**	**Y**
0	0	1
0	1	0
1	0	0
1	1	1

Truth table

Realizing Logic circuit from function

- Boolean Function, **F = x + y'z**

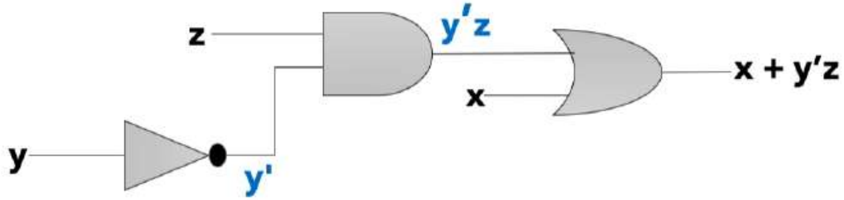
  input variable : x, y, z

- Truth table

  3 input variables = 2^3 combinations

- Logic circuit

  literals - 3 (x, y', z) and 2 terms

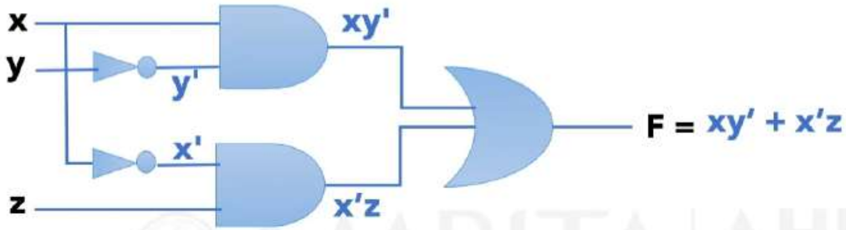


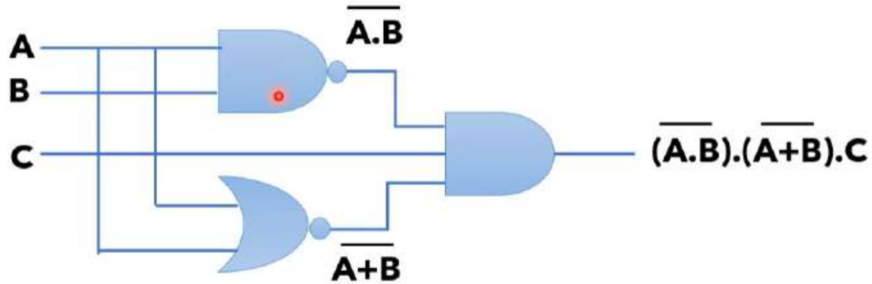
inputs			output
x	**y**	**z**	**F=x+y'z**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x = 0, y = 1, z = 0
F = 0 + 1'.0
= 0 + 0.0
= 0 + 0
= 0

x = 1, y = 1, z = 0
F = 1 + 1'. 0
= 1 + 0.0
= 1 + 0
= 1

Realizing function from Logic circuit





Summary

- Basics of Logic gates

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Universal Logic Gate

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Objective

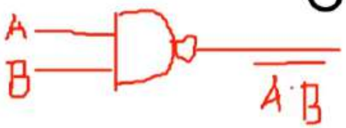
➢ Universal Logic gates

- NAND gate
- NOR gate

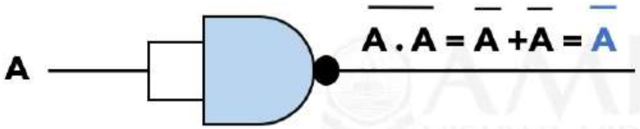
Introduction

- **NAND** & **NOR** gate are known as the **universal gates**.
- These gates alone sufficient to implement any Boolean expression and are inexpensive.
- Basic logic gates (AND, OR and NOT) are logically complete.
- Sufficient to show that AND, OR and NOT can be implemented with NAND and NOR gate.

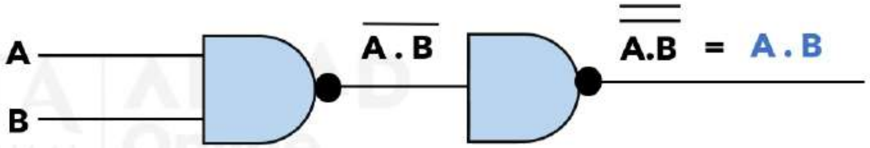
Universal NAND gate



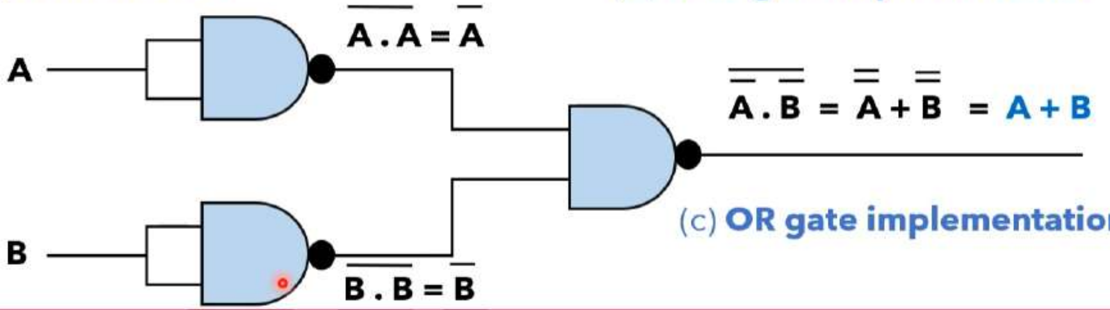
- We can implement NOT, AND and OR gates by combining one or more NAND gates.



(a) **NOT gate implementation**



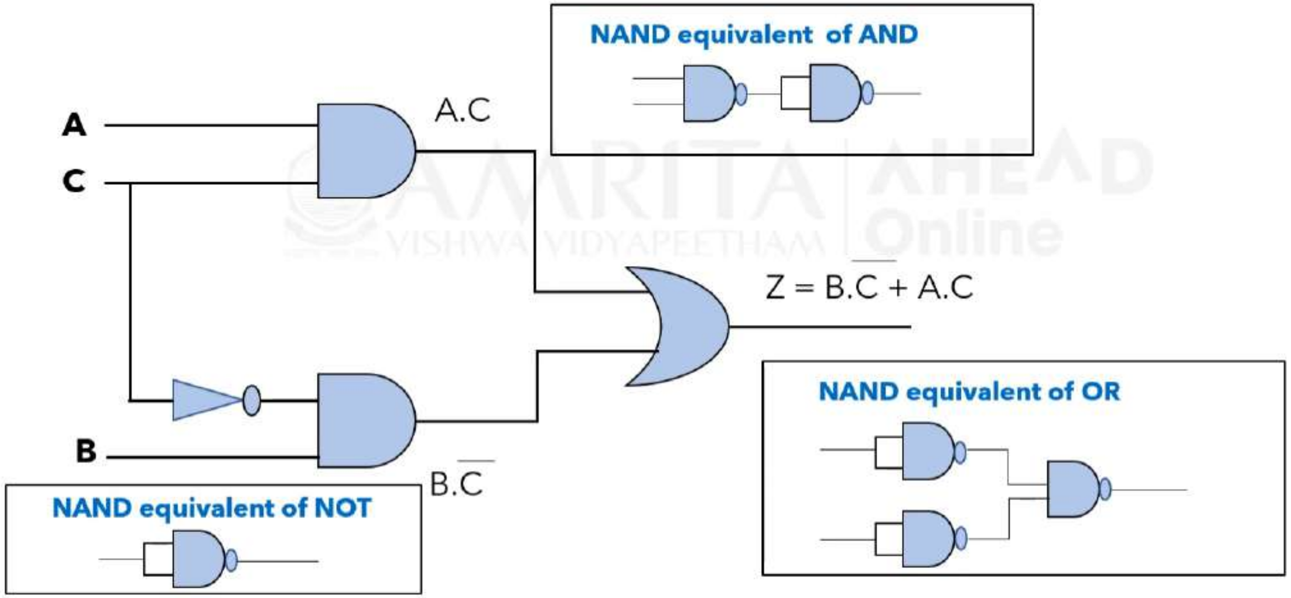
(b) **AND gate implementation**



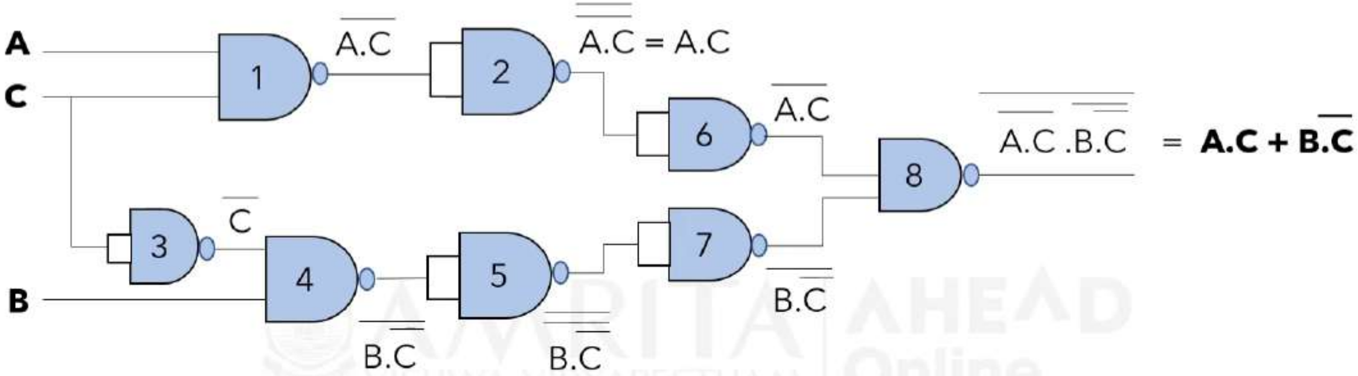
(c) **OR gate implementation**

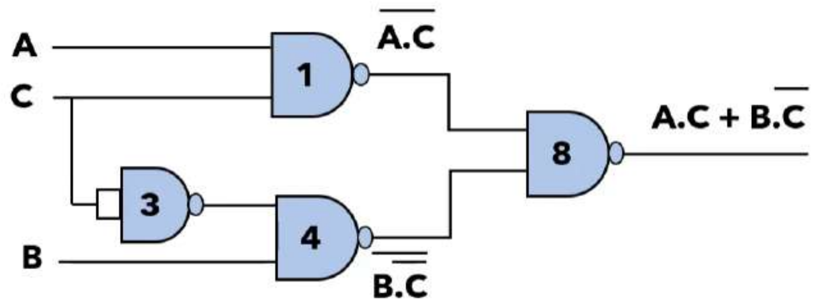
NAND Logic circuit

- Design NAND logic circuit for the Boolean expression $Z = B\overline{C} + AC$

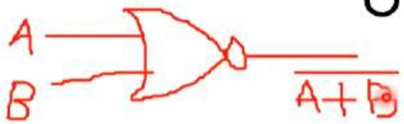


NAND logic circuit

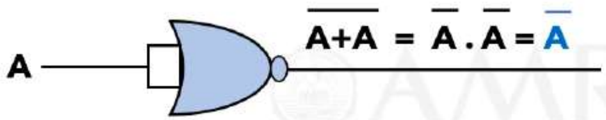




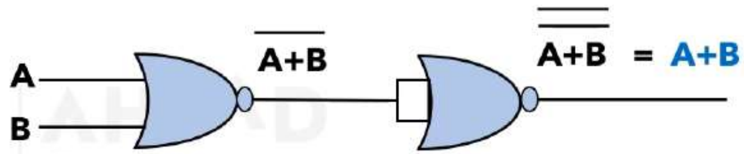
Universal NOR gate



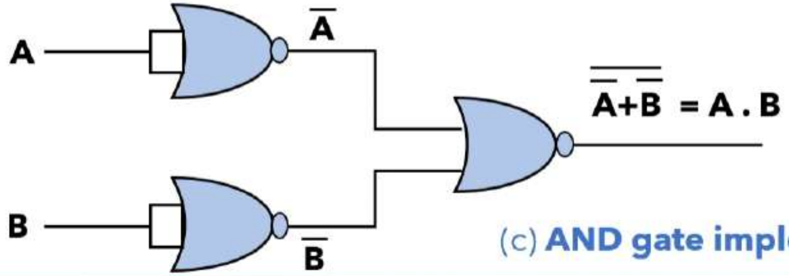
- Implement NOT, AND and NOR gates by combining one or more NOR gates.



(a) **NOT gate implementation**



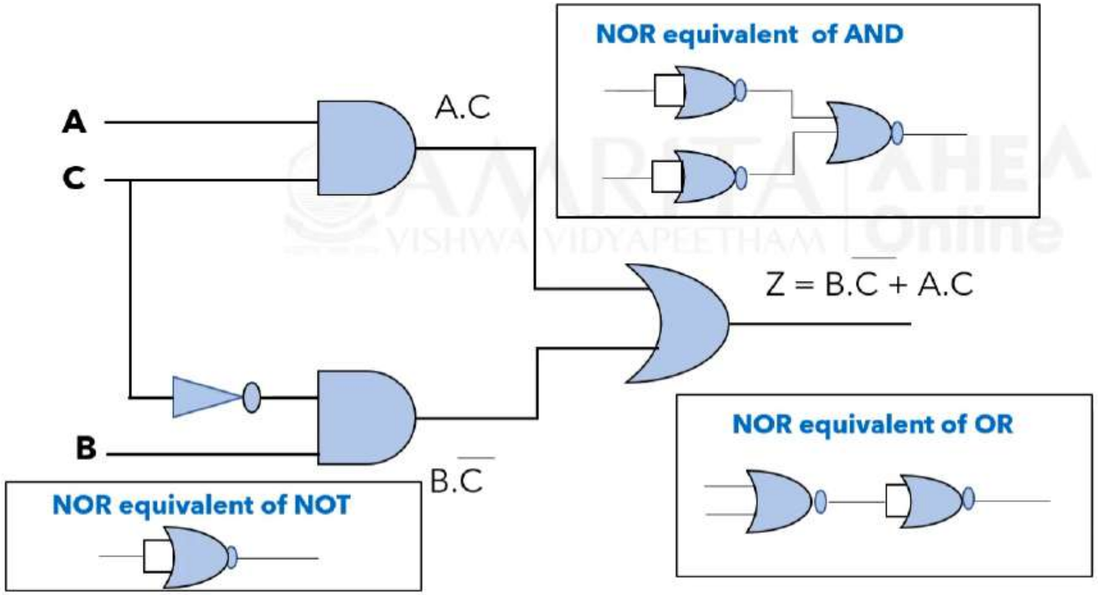
(b) **OR gate implementation**



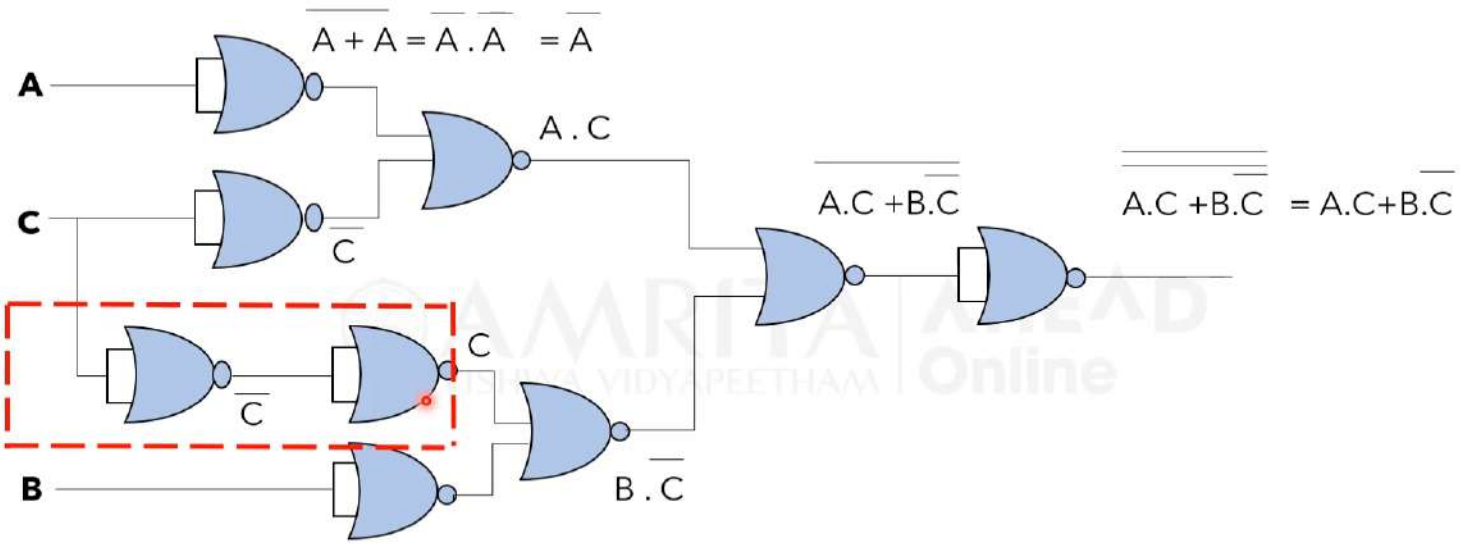
(c) **AND gate implementation**

NOR Logic circuit

- Design NOR logic circuit for the Boolean expression $Z = B\overline{C} + AC$



NOR Logic circuit



Summary

➢ Universal Logic gates

Canonical & Standard Forms

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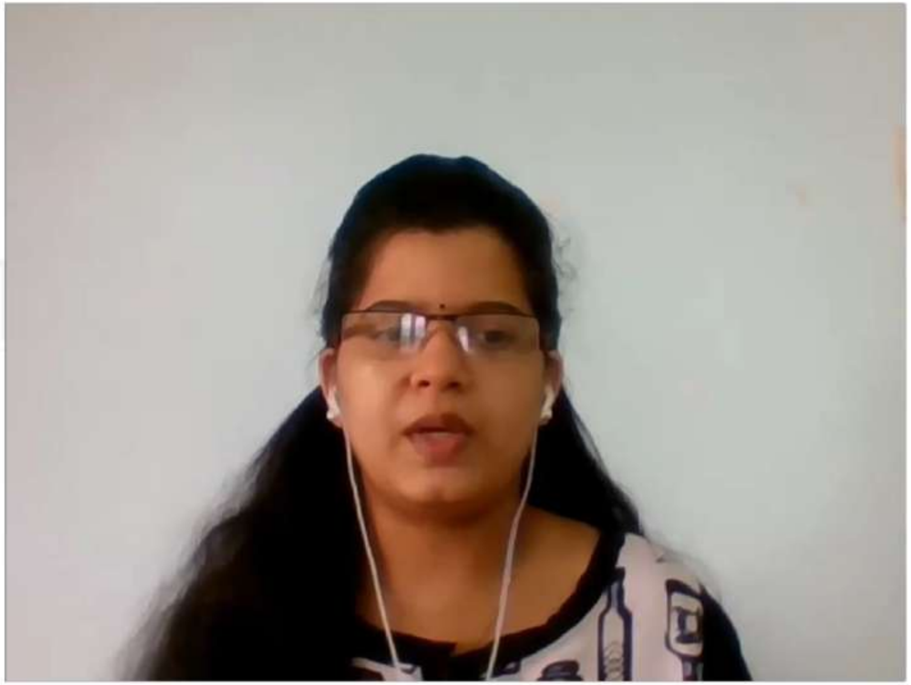
Objective

Representation of Boolean Function

- ➢ Standard form
- ➢ Canonical Form



Introduction

- Boolean functions can be expressed in **canonical** and **standard forms**

- In the Boolean expression binary variable will be either in normal form (x) or in its complement form (x').

  F(A,B,C,D) is a function :
- **Product term** : Logical ANDed among the literals.
  e.g., AB, A'B, ABC'

- **Sum term** : Logical ORed among the literals.
  e.g., A+B, A'+B, A+B+C'

Minterm : m_j

- **Minterm /Standard product** : Product term containing all the variables in normal or as complements.
  e.g.,F(A,B,C) : ABC' , ~~AB~~
- For n- variable function have 2^n possible minterms.
- While obtaining minterm, each variable is complemented if the corresponding bit of binary number is 0 and uncomplemented if 1.

x	y	minterm	symbol
0	0	x'y'	m_0
0	1	x'y	m_1
1	0	xy'	m_2
1	1	xy	m_3

A minterm equals 1 at exactly one input combination and is equal to 0 otherwise.
e.g., x'y' = 1 only when x = 0, y = 0

Minterms with 2 binary variables x and y
2 variable - 2^2 = 4 minterms
x is primed if x=0 ; x is unprimed if x=1

Minterms

- 3 variable function with $2^3 = 8$ possible minterms

A	B	C	minterm	symbol
0	0	0	A'B'C'	m_0
0	0	1	A'B'C	m_1
0	1	0	A'BC'	m_2
0	1	1	A'BC	m_3
1	0	0	AB'C'	m_4
1	0	1	AB'C	m_5
1	1	0	ABC'	m_6
1	1	1	ABC	m_7

A=0 B=0 C=0

$\downarrow$ $\downarrow$ $\downarrow$

A' B' C'

A=1 B=0 C=0

$\downarrow$ $\downarrow$ $\downarrow$

A B' C'

Expressing functions as Sum of **Minterms**

- Boolean function can be expressed algebraically from the truth table
  - ➢ Select the minterms that produces a 1 in the function.
  - ➢ Take OR of those selected minterms.

i.e., sum of all minterms

$$\mathbf{f} = x'y'z + xy'z' + xyz$$

$$= m_1 + m_4 + m_7$$

$$= \Sigma(1,4,7)$$

	x	y	z	f
	0	0	0	0
x'y'z ←	0	0	1	**1**
	0	1	0	0
	0	1	1	0
xy'z' ←	1	0	0	**1**
	1	0	1	0
	1	1	0	0
xyz ←	1	1	1	**1**

Expressing functions as Sum of **Minterms**

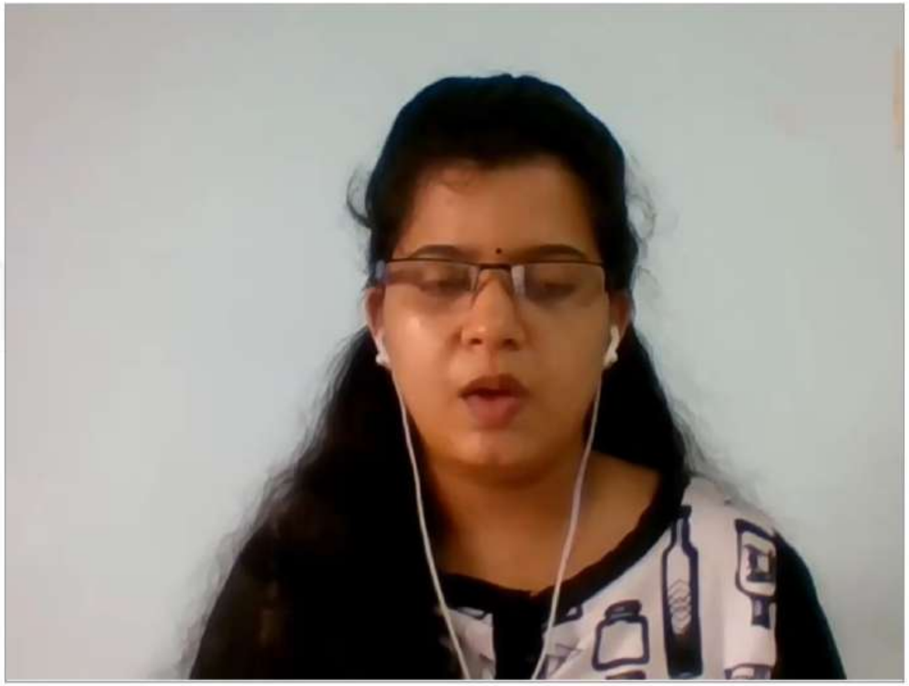
- Any function can be expressed by **ORing all minterms (m_i)** corresponding to input combinations (i) at which the function has a value of 1.

- Resulting expression is referred to as SUM of minterms and is expressed as F = Σ(2, 4, 5, 7) where Σ indicates ORing of the indicated minterms.

- F = Σ(2, 4, 5, 7) = (m2 + m4 + m5 + m7)

Summary

➢ Canonical forms : Sum of Minterms



Canonical & Standard form-II

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Objective

- Canonical form and standard form using maxterms

Maxterm : M_j

- **Maxterm /Standard sums** : <u>Sum term</u> containing all the variables or their complements.

  e.g., F(A,B,C) : A+B+C' , ~~A'+C~~
- For n – variables can have 2^n possible maxterms.
- For maxterm, each variable is primed if the corresponding bit of binary number is 1 and unprimed if 0

x	y	maxterm	symbol
0	0	x+y	M_0
0	1	x+y'	M_1
1	0	x'+y	M_2
1	1	x'+y'	M_3

A maxterm equals 0 at exactly one input combination and is equal to 1 otherwise.
e.g., x+y' = 0 only when x = 0, y = 1

Maxterms with 2 binary variables x and y
2 variable - 2^2 = 4 maxterms
x is primed if x=1 ; x is unprimed if x=0

Maxterms

- 3 variable function with $2^3 = 8$ possible maxterms

A	B	C	maxterm	symbol
0	0	0	A+B+C	M_0
0	0	1	A+B+C'	M_1
0	1	0	A+B′+C	M_2
0	1	1	A+B′+C'	M_3
1	0	0	A′+B+C	M_4
1	0	1	A′+B+C'	M_5
1	1	0	A′+B′+C	M_6
1	1	1	A′+B′+C'	M_7

A=0 B=0 C=0
$\downarrow$
A B C

A=1 B=0 C=0
$\downarrow$
A' B C

Expressing functions as Product of **Maxterms**

- Select maxterms that produces a 0 in the function
- Take AND of those selected maxterm

i.e., <u>product of all maxterms</u>

f = (x+y+z)(x+y′+z)(x+y′+z′)(x′+y+z′)(x′+y′+z)

$= M_0.M_2.M_3.M_5.M_6$

$= \Pi(0, 2, 3, 5, 6)$

	x	y	z	f
x+y+z ←	**0**	**0**	**0**	**0**
	0	0	1	1
x+y′+z ←	**0**	**1**	**0**	**0**
x+y′+z′ ←	**0**	**1**	**1**	**0**
	1	0	0	1
x′+y+z′ ←	**1**	**0**	**1**	**0**
x′+y′+z ←	**1**	**1**	**0**	**0**
	1	1	1	1

Maxterm from Minterm

- Minterm and Maxterm are complement to each other.
- Complement of minterm = maxterm

Example : $F(x, y, z) = \Sigma\,(1, 4, 5, 6, 7)$

$F'(x, y, z) = \Sigma(0, 2, 3) = (m0 + m2 + m3)$

x	y	z	F	F′
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

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Karnaugh map

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Objective

- Systematic method of simplifying Boolean Expression – **Karnaugh map**

Introduction

- Pictorial representation of writing Boolean expression.
- The map method modified by Karnaugh is called **K-map** or **Karnaugh map**.
- Objective of K-map is the minimization of Boolean function with a simple procedure.
- K-map : Diagram made up of squares ; n-variable 2^n squares

  Each square represents one minterm of the function

  Visual diagram of a function in standard form(SOP or POS)

K-maps

- K-maps use 2-diamensional tables to simplify the Boolean expression.

A \ B	0	1
0	m_{00}	m_{01}
1	m_{10}	m_{11}

2-variable map

2 variable = f(A,B)

2^2minterm = 4 cell map

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

3-variable map

3-variable = f(A,B,C)

2^3 minterm = 8 cell map

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

4-variable map

Adjacent cells differ by just one single bit

K- map Simplification

- First fill the appropriate cells with the value based on the output variable.

- Group the maximum number of 1's (SOP) and 0's (POS).

- Group need to be in power of 2 and done in decreasing order.
  If 8 cell K-map, try grouping for 8 ($=2^3$), then for 4 ($=2^2$), next for 2 ($=2^1$) and last consider single bit.

- Later each group is expressed in terms of common input variable in the rows and column.

Grouping Rules

- Group the cells with largest number of <u>consecutive 1's </u>and no 0's .

correct

0	0	1	1
1	1	0	0

Incorrect

0	1	1	0

- Grouping must be done in <u>decreasing order </u>and the number of 1's in the group must be a power of 2.

incorrect

0	1	1	1

correct

0	1	1	1

incorrect

1	1	1	1
1	1	1	1

1	1	1	1
1	1	1	1

Grouping Rules

- Focus is on increasing the size of the group, so same elements can be repeated in multiple groups.

incorrect

1	1	1	1
0	1	1	0

correct

1	1	1	1
0	1	1	0

- Diagonal grouping is not permitted and elements around the edges can be grouped (circular property).

incorrect

1	0
0	1

correct

1	0	0	1
1	0	0	1

Summary

- Introduction to K-map with the simplification procedures

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K-map simplification

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Objective

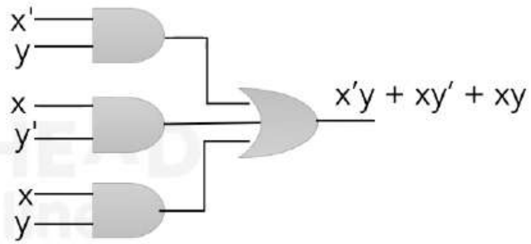
- K-map simplification examples

K-map Simplification

- $F(x,y) = x'y + xy' + xy$ (In SOP x' = 0; x =1)

  01 **10** **11**

With this expression we need 3 AND gates and 1 OR gate



x \ y	0	1
0	0 (0)	1 (1)
1	1 (2)	1 (3)

Group1

x **y**
0 **1**
1 **1**

$\Big]$ **y** — **Common term is y**

Group2

x y
1 0
1 1

$\Big]$ **x** — **Common term is x**

Simplified expression, $F(x, y) = x+y$

Simplified expression need only 1 OR gate

Example(SOP)

- $Y(A,B,C,D) = \sum(0, 2, 5, 7, 8, 10, 13, 15)$

AB \ CD	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

$Y = A'B'C'D' + A'B'CD' + A'BC'D + A'BCD + AB'C'D' + AB'CD' + ABC'D + ABCD$

Group1

A	B	C	D
0	1	0	1
0	1	1	1
1	1	0	1
1	1	1	1

→ **BD**

Group2

A	B	C	D
0	0	0	0
0	0	1	0
1	0	0	0
1	0	1	0

→ **B'D'**

- Simplified expression $Y(A,B,C,D) = BD + B'D'$

Example(POS)

- F(A,B,C,D) = ∑(0,1,2,5,8,9,10)
  Given SOP but for simplification using POS group the0's

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

Group1

A	B	**C**	**D**
0	0	**1**	**1**
0	1	**1**	**1**
1	1	**1**	**1**
1	0	**1**	**1**

→ **C′+D'**

Group2

A	**B**	C	D
1	**1**	0	0
1	**1**	0	1
1	**1**	1	1
1	**1**	1	0

→ **A'+B'**

Group3

A	**B**	C	**D**
0	**1**	0	**0**
1	**1**	0	**0**
0	**1**	1	**0**
1	**1**	1	**0**

→ **B′+D**

- Simplified expression F(A,B,C,D) = (C′+D′)(A′+B′)(B′+D)

Example(Don't care)

- Functions can have unspecified output for some input combination, we don't care the value of those unspecified terms.

➢ $F(A,B,C,D) = \sum(1,3,5,7,9) + d(6,12,13)$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	X
11	X	X	0	0
10	0	1	0	0

Group1

A B C D
0 0 0 1
0 0 1 1
0 1 0 1
0 1 1 1

→ **A'D**

Group2

A B C D
0 0 0 1
0 1 0 1
1 1 0 1
1 0 0 1

→ **C'D**

F(A,B,C,D) = A'D+C'D

Without don't care
F(A,B,C,D) = A'D+B'C'D

Summary

- Explained how to perform K-map simplification with examples