Homework 3

Due March 12, 19:00

Problem 3.1

A robot arm with a rapid laser is used to do a quick quality check, such as the radius of hole, on six holes located on a rectangular plate. Coordinates of the centers of the holes are given below

x	2.1	4.6	5.25	7.82	9.2	10.6
y	7.3	7.0	6.0	5.1	3.5	5.0

The path of the robot going from one point to another point needs to be smooth so as to avoid sharp jerks in the arm that can otherwise create premature wear and tear of the robot arm.

a) One such path can be the interpolating polynomial $P_5(x)$ that passes those points. Find it.

b) Other paths can be obtained by cubic spline interpolation. Find several cubic splines (see Answers to homework 3) interpolating these data, plot them together with $P_5(x)$ and propose the one with the shortest path.

Problem 3.2

The gamma function is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$$

and it can be shown (integrate by parts) that $\Gamma(n) = (n-1)!\ \forall n \in \mathbb{N}$.
a) Write a computer programm that will calculate Newton's divided differences and then find (using Newton's formula) the interpolating polynomial $P_4(x)$ of the Γ function at the following points:

n	1	2	3	4	5
$\Gamma(n)$	1	1	2	6	24

b) Find a natural cubic spline $S(x)$ (using an available spline routine) that interpolates the same data.

c) Another approximation can be obtained by first calculating the polynomial $Q_4(x)$ that interpolates points $(n, \log\Gamma(n))$:

n	1	2	3	4	5
$\log\Gamma(n)$	0	0	$\log 2$	$\log 6$	$\log 24$

(here log denotes the natural logarithm) and then, consider $q(x) = e^{Q_4(x)}$.

d) Plot in the same figure the graphs of $\Gamma(x)$, $P_4(x)$, $S(x)$ and $q(x)$ on $[1, 5]$.

e) Compute with accuracy of at least 3 significant digits

$$\max_{x\in[1,5]} \Big|\Gamma(x) - P_4(x)\Big|, \quad \max_{x\in[1,5]} \Big|\Gamma(x) - S(x)\Big| \text{and} \quad \max_{x\in[1,5]} \Big|\Gamma(x) - q(x)\Big|.$$

Which of the 3 approximations is more accurate on $[1, 5]$?

HINTS. In MATLAB or GNU Octave the Γ function is known as `gamma`. In Python it is available in the `scipy.special` library. In order to complete d) you can compute the maximum from the graph, just make sure that sampling is done at sufficiently dense points that will ensure the needed accuracy.

Problem 3.3

Consider function $f(x) = \sqrt{x+1}$ on interval $[-1, 1]$. Find the near minimax polynomial approximation of degree 7 m_7 for this function. Also, find the polynomial interpolant $P_7(x)$ on evenly spaced points. Plot together function $f(x), m_7(x)$ and $P_7(x)$.