Taylor approximations

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Taylor and Maclaurin Series

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For example, Taylor series of $f(x) = e^x$ at $a = 2.7$ is

$$f(2.7) + f'(2.7)(x-2.7) + \frac{f''(2.7)}{2!}(x-2.7)^2 + \frac{f'''(2.7)}{3!}(x-2.7)^3 + .$$

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For example, Taylor series of $f(x) = e^x$ at $a = 5$ is

$$f(5) + f'(5)(x-5) + \frac{f''(5)}{2!}(x-5)^2 + \frac{f'''(5)}{3!}(x-5)^3 + \dots$$
$$e^5 + e^5(x-5) + \frac{e^5}{2!}(x-5)^2 + \frac{e^5}{3!}(x-5)^3 + \dots$$

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For example, Taylor series of $f(x) = \cos x$ at $a = \frac{\pi}{6}$ is

$$f\left(\frac{\pi}{6}\right) + f'\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) + \frac{f''\left(\frac{\pi}{6}\right)}{2!}\left(x - \frac{\pi}{6}\right)^2 + \frac{f'''\left(\frac{\pi}{6}\right)}{3!}\left(x - \frac{\pi}{6}\right)^3 +$$

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$$\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\left(x - \frac{\pi}{6}\right) - \frac{\cos\frac{\pi}{6}}{2!}\left(x - \frac{\pi}{6}\right)^2 + \frac{\sin\frac{\pi}{6}}{3!}\left(x - \frac{\pi}{6}\right)^3 + \dots$$

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$$\frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{6}\right)^2 + \frac{1}{12}\left(x - \frac{\pi}{6}\right)^3 + \dots$$

Taylor and Maclaurin Series

If $a = 0$ then Taylor series is called **Maclaurin series of function** f and is defined by

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$$\cos 0 - \sin 0 \cdot x - \frac{\cos 0}{2!}x^2 + \frac{\sin 0}{3!}x^3 + \frac{\cos 0}{4!}x^4 + \ldots$$

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$$\cos 0 - \sin 0 \cdot x - \frac{\cos 0}{2!}x^2 + \frac{\sin 0}{3!}x^3 + \frac{\cos 0}{4!}x^4 + \dots$$
$$1 - 0 \cdot x - \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 - \frac{0}{5!}x^5 - \frac{1}{6!}x^6 + \dots$$

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1. Under what conditions will these series converge (absolutely)?

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1. Under what conditions will these series converge (absolutely)?
2. If the series converge, will its sum be $f(x)$?

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$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots \quad \forall x \in \mathbb{R}$$

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\begin{aligned}
e^x &= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots \quad \forall x \in \mathbb{R} \\
\sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots \quad \forall x \in \mathbb{R}
\end{aligned}
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\cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots \quad \forall x \in \mathbb{R} \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \dots, \quad |x| < 1
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\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \dots, \quad |x| < 1 \\
\frac{1}{1+x} &= 1 - x + x^2 - x^3 + x^4 - x^5 + x^6 + \dots, \quad |x| < 1
\end{aligned}
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\ln(1+x) &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots \quad |x| < 1
\end{aligned}$$

Taylor Polynomials

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots + \frac{1}{n!}x^n + \frac{1}{(n+1)!}x^{n+1} + \ldots$$

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$$e^x = \underbrace{1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \ldots + \frac{1}{n!}x^n}_{=T_n(x)} + \underbrace{\frac{1}{(n+1)!}x^{n+1} + \ldots}_{=R_n(x)}$$

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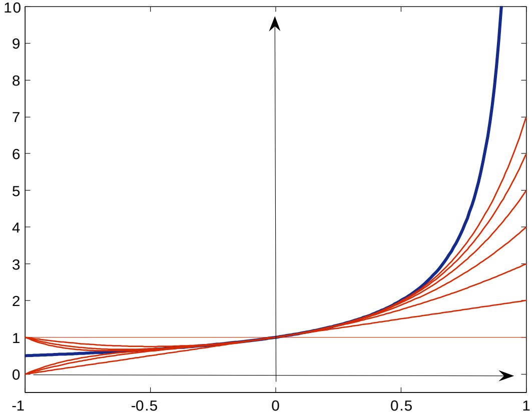
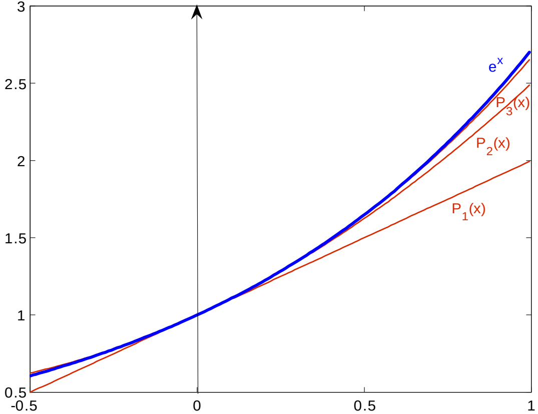
e^x &= T_n(x) + R_n(x)
\end{aligned}
$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \dots, \quad |x| < 1$$



$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots, \quad x \in \mathbb{R}$$



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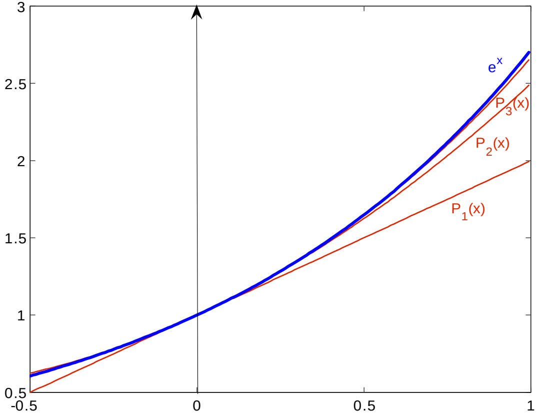

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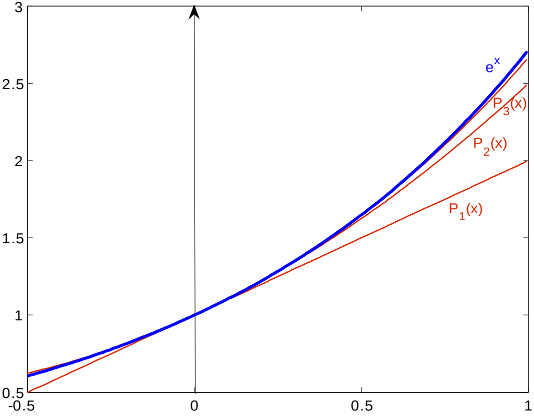

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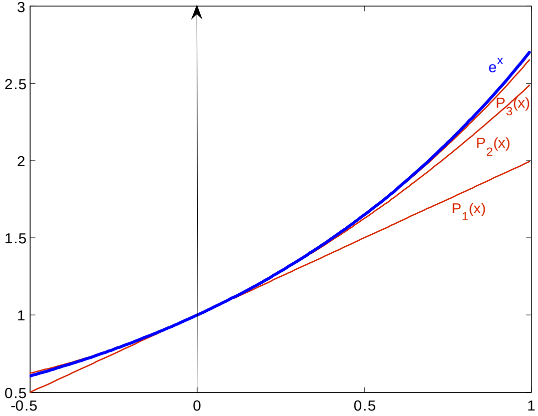
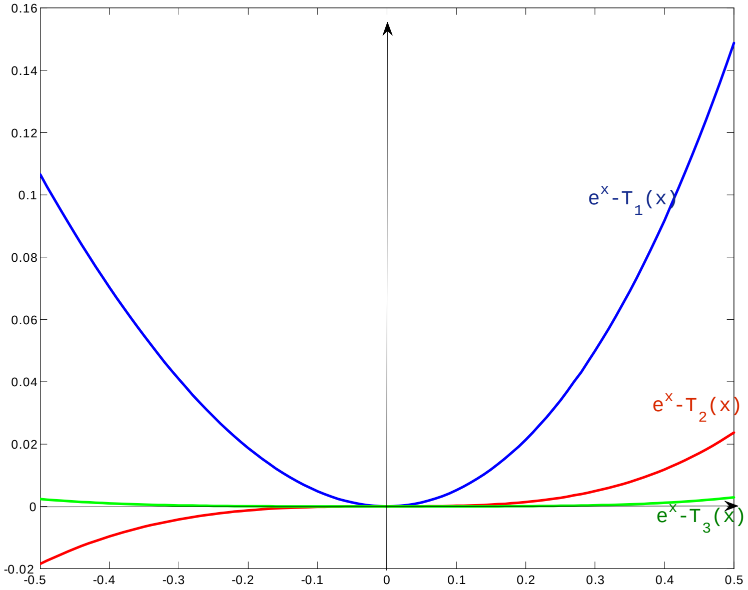
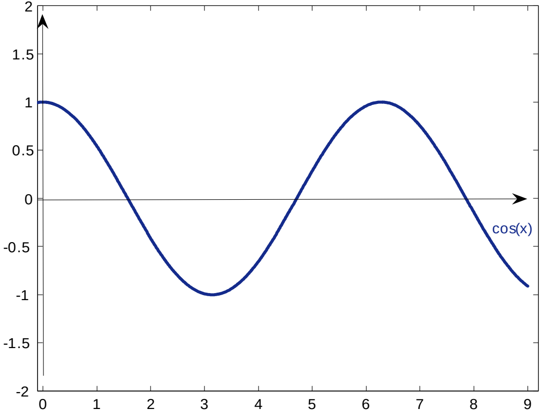


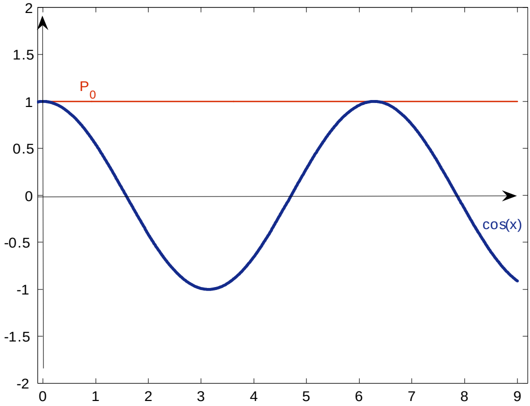


0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
$e^x - T_1(x)$
$e^x - T_2(x)$
$e^x - T_3(x)$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \frac{x^{16}}{16!} - \frac{x^{18}}{18!} + \dots$$



$$\cos x = \color{red}{1} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!} - \frac{x^{14}}{14!} + \frac{x^{16}}{16!} - \frac{x^{18}}{18!} + \dots$$



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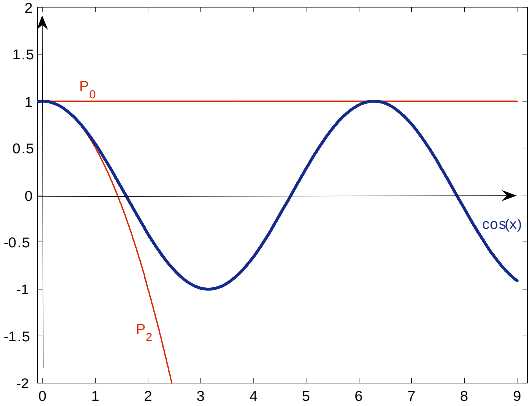

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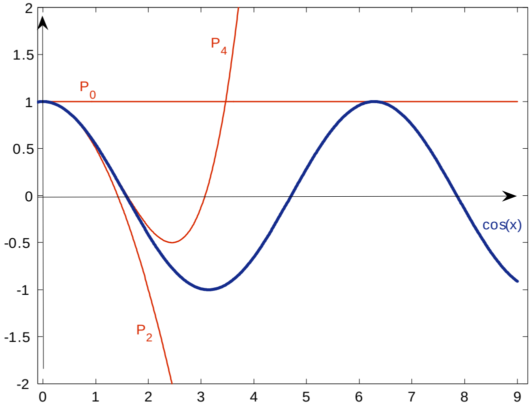

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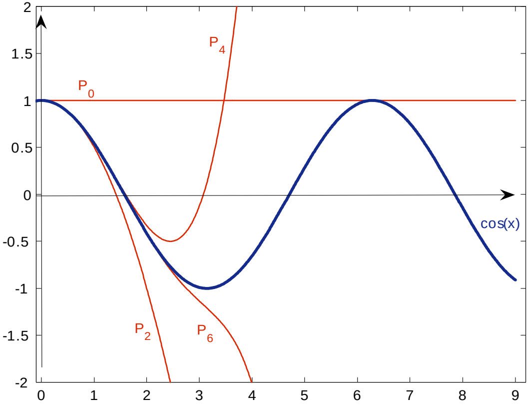

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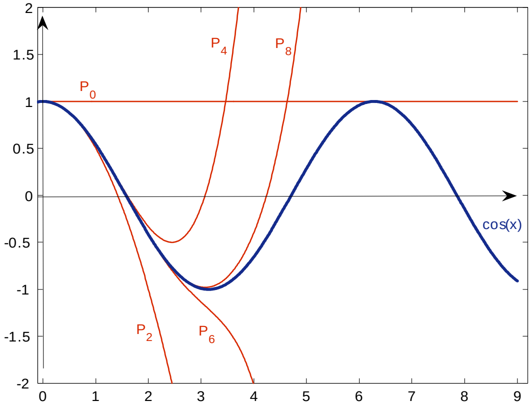

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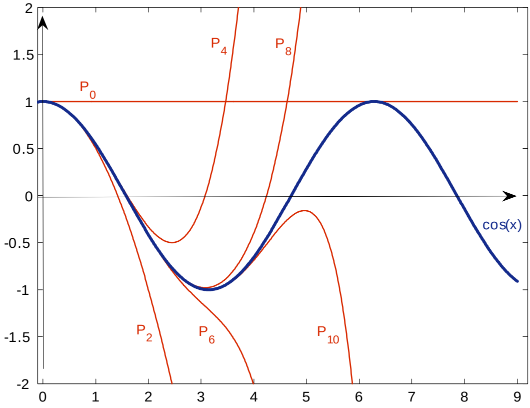

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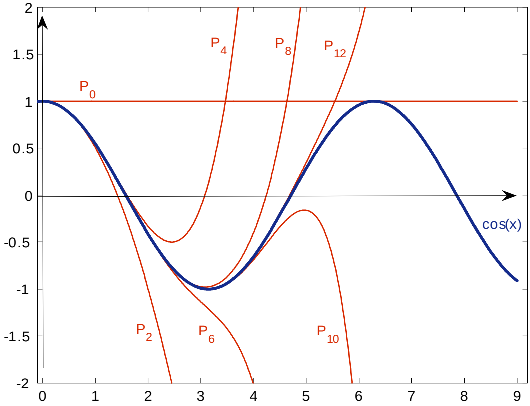

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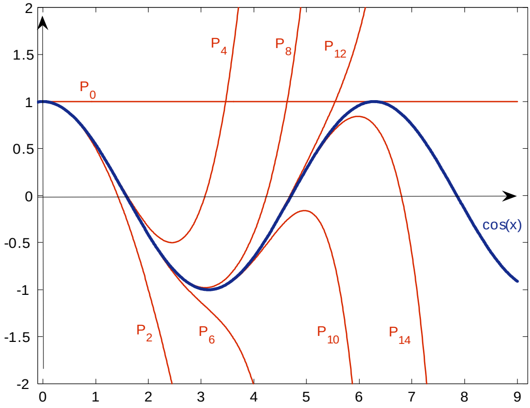

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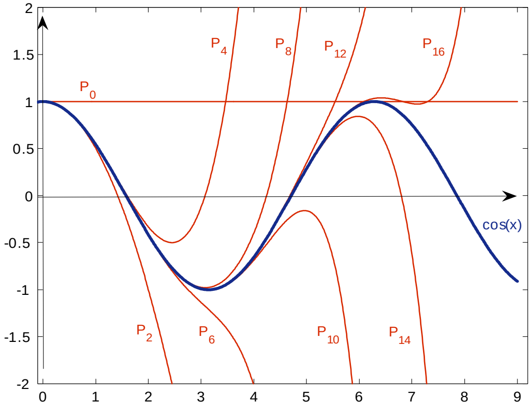

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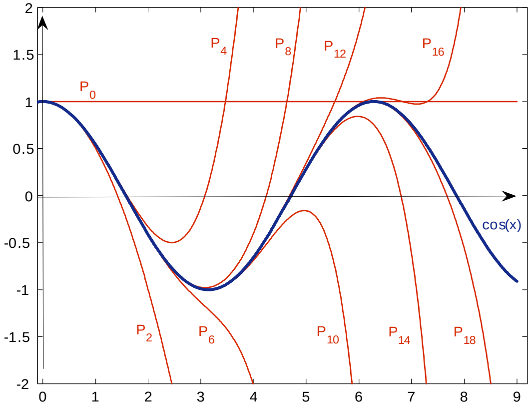
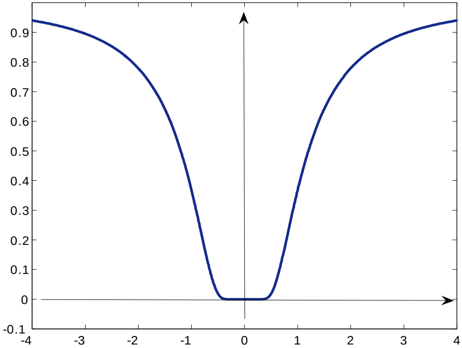


Function whose Taylor series doesn't converge to the function itself

$$f(x) = \begin{cases} 0, & if \quad x = 0 \\ e^{-\frac{1}{x^2}}, & if \quad x \neq 0 \end{cases}$$



Srīnivāsa Aiyangār Rāmānujan

better known as Srinivasa Iyengar Ramanujan
(22 December 1887 – 26 April 1920)

Srīnivāsa Aiyangār Rāmānujan

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$$1 - 5\left(\frac{1}{2}\right)^3 + 9\left(\frac{1\cdot 3}{2\cdot 4}\right)^3 - 13\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3 + \ldots = \frac{2}{\pi}$$

$$1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1\cdot 3}{2\cdot 4}\right)^3-13\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^3+\ldots=\frac{2}{\pi}$$

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\ldots}}}}=3$$

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$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\ldots}}}}=3$$

$$1+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 3\cdot 5}+\frac{1}{1\cdot 3\cdot 5\cdot 7}+\ldots+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{3}{1+\cfrac{4}{1+\ldots}}}}}=\sqrt{\frac{\pi e}{2}}$$

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Generalizations of this idea have spawned the notion of "taxicab numbers".