AM II

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LINE INTEGRALS

Stewart,
chapter 16, section 16.2

16.2 Exercises, p.1072-1075

1, 2, 3, 4,
9, 10, 11, 12,

33,
35, 36, 38

5, 6, 7, 8,
13, 14, 15, 16,

32, 39, 40, 41, 42

Advanced Engineering Mathematics
,
chapter 10,

LINE INTEGRALS of the first kind

Line integral of a scalar field

Stewart,
chapter 16, section 16.2

16.2 Exercises, p.1072-1075

1, 2, 3, 4,
9, 10, 11, 12,

33,
35, 36, 38

Ex.1, p.1072 (Stewart)

Evaluate the line integral

$$\int_C y^3\,ds \qquad C: \begin{cases} x = t^3 \\ y = t \end{cases}, \quad 0 \le t \le 2$$

Solution.

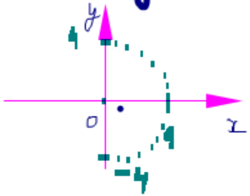
$$\int_C y^3\,ds = \int_0^2 t^3 \cdot \sqrt{(3t^2)^2 + 1^2}\,dt =$$

$$= \int_0^2 t^3 \cdot \sqrt{9t^4+1}\,dt = \left| \begin{array}{l} u = 9t^4+1 \\ du = 36t^3\,dt \end{array} \right| =$$

$$= \frac{1}{36}\int_1^{145} \sqrt{u}\,du = \frac{u^{3/2}}{36 \cdot 3/2}\Bigg|_1^{145} = \frac{1}{18 \cdot 3}\left(145^{3/2} - 1\right) = \frac{145^{3/2} - 1}{54}$$

Ex.3, p.1072 (Stewart) Evaluate the line integral

$\int_C xy^4\,ds$, where C is the right half of the circle $x^2+y^2=16$



$$C:\begin{cases} x = 4\cos t \\ y = 4\sin t \end{cases}, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} 4\cos t\cdot(4\sin t)^4\sqrt{(-4\sin t)^2+(4\cos t)^2}\,dt$$

$$= 4^6\int_{-\pi/2}^{\pi/2}\sin^4 t\cos t\,dt = 4^6\left.\frac{\sin^5 t}{5}\right|_{-\pi/2}^{\pi/2} = \frac{4^6}{5}\left(1^5-(-1)^5\right) = \frac{2}{5}4^6$$

Ex.11, p.1072 (Stewart)

$$\int_C x e^{yz}\, ds$$, C is the line segment from (0, 0, 0) to (1, 2, 3)

Solution. the direction vector: $\vec{v} = (1, 2, 3)$

$$C: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}, \quad 0 \le t \le 1$$

$$\int_C x e^{yz}\, ds = \int_0^1 t\, e^{6t^2} \cdot \sqrt{14}\, dt = \left| \begin{array}{l} a = 6t^2 \\ da = 12t\,dt \end{array} \right| =$$

$$= \frac{\sqrt{14}}{12}\int_0^6 e^a \, da = \frac{\sqrt{14}}{12} e^a \Big|_0^6 = \frac{e^6 - 1}{12}\sqrt{14}.$$

Ex.35, p.1073 (Stewart)

a) $$\bar{x} = \frac{1}{m} \int_C x \rho(x,y,z)\, ds$$

$$\bar{y} = \frac{1}{m} \int_C y \rho(x,y,z)\, ds$$

$$\bar{z} = \frac{1}{m} \int_C z \rho(x,y,z)\, ds$$

where $$m = \int_C \rho(x,y,z)\, ds$$

b) the helix

$$\begin{cases} x = 2\sin t \\ y = 2\cos t \\ z = 3t \end{cases}, \quad 0 \le t \le 2\pi, \qquad \rho(x,y,z) = k$$

$$\bar{x} = \frac{1}{m} \int_C x k \, ds = \frac{k}{m} \int_C x \, ds$$

$$\bar{y} = \frac{k}{m} \int_C y \, ds$$

$$\bar{z} = \frac{k}{m} \int_C z \, ds$$

m — ?

$$m = \int_C \rho(x,y,z) \, ds$$

$$m = k \int_C ds$$

$$m = k\int_C ds = k\int_0^{2\pi} \sqrt{(2\cos t)^2+(-2\sin t)^2+3^2}\,dt =$$

$$= k\int_0^{2\pi} \sqrt{4+9}\,dt = k\sqrt{13}\,t\Big|_0^{2\pi} = 2\pi k\sqrt{13}$$

$$\overline{x} = \frac{k}{m}\int_C x\,ds = \frac{k}{2\pi k\sqrt{13}}\int_0^{2\pi} 2\sin t\cdot\sqrt{13}\,dt =$$

$$= \frac{1}{\pi}\cdot(-\cos t)\Big|_0^{2\pi} =$$

$$= -\frac{1}{\pi}(\cos 2\pi - \cos 0) = 0$$

$$\bar{y} = \frac{k}{m}\int_C y\,ds = \frac{1}{2\pi\sqrt{13}}\int_0^{2\pi} 2\cos t \cdot \sqrt{13}\,dt =$$

$$= \frac{1}{\pi}\sin t\Big|_0^{2\pi} = 0$$

$$\bar{z} = \frac{k}{m}\int_C z\,ds = \frac{1}{2\pi\sqrt{13}}\int_0^{2\pi} 3t\cdot\sqrt{13}\,dt =$$

$$= \frac{3}{2\pi}\int_0^{2\pi} t\,dt = \frac{3}{2\pi}\cdot\frac{t^2}{2}\Big|_0^{2\pi} = \frac{3}{4\pi}\left[(2\pi)^2 - 0\right] =$$

$$= \frac{3}{\pi}\pi^2 = 3\pi$$

$A: \quad (0, 0, 3\pi)$