AM II

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LINE INTEGRALS

Stewart,
chapter 16, section 16.2

16.2 Exercises, p.1072-1075

1, 2, 3, 4,
9, 10, 11, 12,

33,
35, 36, 38

5, 6, 7, 8,
13, 14, 15, 16,

32, 39, 40, 41, 42

Advanced Engineering Mathematics
,
chapter 10,

Line integral of a vector field

Stewart,
chapter 16, section 16.2

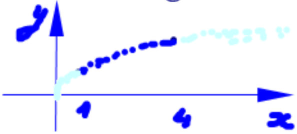
16.2 Exercises, p.1072-1075

5, 6, 7, 8,
13, 14, 15, 16,

32, 39, 40, 41, 42

Ex.5, p.1072 (Stewart)

Evaluate the line integral



$$\int_C (x^2 y^3 - \sqrt{x})\,dy$$

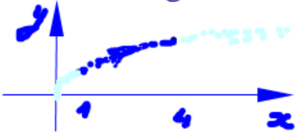
where C is the arc of the curve $y = \sqrt{x}$ from (1,1) to (4,2).

$$\int_C (x^2 y^3 - \sqrt{x})\,dy = \int_1^4 \left(x^2 x^{3/2} - \sqrt{x}\right)\left(\sqrt{x}\right)' dx =$$

$$= \frac{1}{2} \int_1^4 (x^3 - 1)\,dx = \frac{1}{2}\left(4^3 - 4 + \frac{3}{4}\right.$$

Ex.5, p.1072 (Stewart)

Evaluate the line integral



$$\int_C (x^2y^3-\sqrt{x})\,dy$$

where C is the arc of the curve $y=\sqrt{x}$ from (1,1) to (4,2).

$x=y^2$

$$\int_C (x^2y^3-\sqrt{x})\,dy = \int_1^4 (x^2x^{3/2}-\sqrt{x})(\sqrt{x})'\,dx =$$

OR

$$=\frac{1}{2}\int_1^4 (x^3-1)\,dx=\frac{1}{2}\left(4^3-4+\frac{3}{4}\right.$$

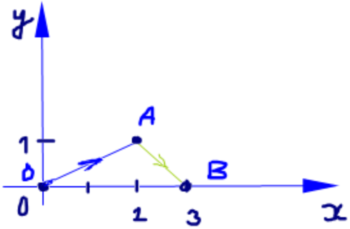
$$\int_C (x^2y^3-\sqrt{x})\,dy=\int_1^2 (y^4 y^3-y)\,dy=\frac{1}{8}(2^8-1)-\frac{1}{2}(2^2-1)= \ldots$$

Ex.7, p.1072 (Stewart)

Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy,$$

C consists of line segments from $(0,0)$ to $(2,1)$ and from $(2,1)$ to $(3,0)$



OA:

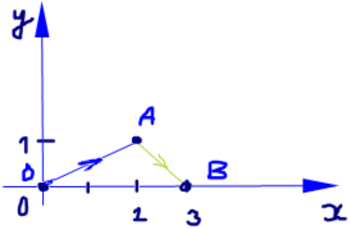
AB:

Ex.7, p.1072 (Stewart) (CONT.)

Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy\,,$$

C consists of line segments from (0,0) to (2,1) and from (2,1) to (3,0)



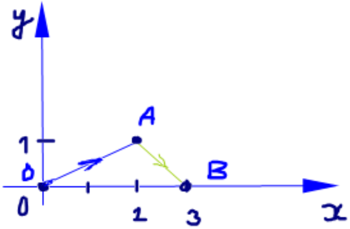
$$OA: \begin{cases} x = 2t \\ y = t \end{cases}, \quad 0 \leq t \leq 1$$

$$AB: \begin{cases} x = 2+t \\ y = 1-t \end{cases}, \quad 0 \leq t \leq 1$$

Ex.7, p.1072 (Stewart) (CONT.) Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy =$$

C consists of line segments from (0,0) to (2,1) and from (2,1) to (3,0)



$$= \int_{OA} (x+2y)\,dx + x^2\,dy +$$

$$+ \int_{AB} (x+2y)\,dx + x^2\,dy =$$

$$OA: \begin{cases} x = 2t \\ y = t \end{cases}, \quad 0 \le t \le 1$$

$$= \int_0^1 \left[(2t+2t)\cdot 2 + (2t)^2 \cdot 1 \right] dt +$$

$$+ \int_0^1 \left[(2+t+2(1-t))\cdot 1 + (2+t)^2 (-1) \right] dt$$

$$AB: \begin{cases} x = 2+t \\ y = 1-t \end{cases}, \quad 0 \le t \le 1$$

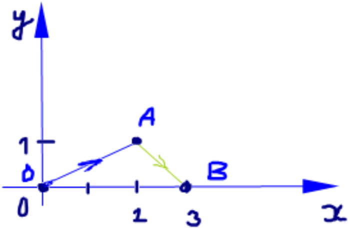
$$= \int_0^1 (8t+4t^2)\,dt + \int_0^1 (4-t-4-4t-t^2)\,dt =$$

$$= 4\int_0^1 (2t+t^2)\,dt - \int_0^1 (5t+t^2)\,dt =$$

$$= 4\cdot\left(t^2+\frac{t^3}{3}\right)\Big|_0^1 - \left(5\frac{t^2}{2}+\frac{t^3}{3}\right)\Big|_0^1 =$$

$$= 4\left(1+\frac{1}{3}\right) - \left(\frac{5}{2}+\frac{1}{3}\right) =$$

$$= 4+1-\frac{5}{2} = \frac{5}{2}$$



$OA: \begin{cases} x = 2t \\ y = t \end{cases}, \; 0 \le t \le 1$

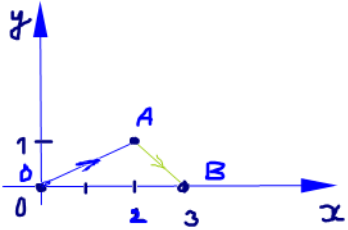
$AB: \begin{cases} x = 2+t \\ y = 1-t \end{cases}, \; 0 \le t \le 1$

Ex.7, p.1072 (Stewart)

Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy\,,$$

C consists of line segments from (0,0) to (2,1) and from (2,1) to (3,0)



$OA: \; y=\frac{1}{2}x \;, \; 0\le x\le 2$

$AB: \; y=-x+3, \; 2\le x\le 3$

Ex.7, p.1072 (Stewart)

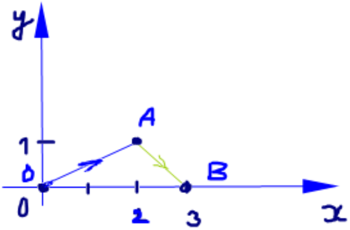
Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy =$$

$$= \int_{OA} (x+2y)\,dx + x^2\,dy +$$

$$+ \int_{AB} (x+2y)\,dx + x^2\,dy =$$

$$=$$



$OA: \; y = \frac{1}{2}x, \; 0 \le x \le 2$

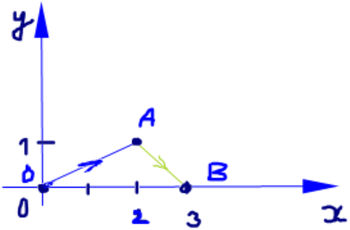
$AB: \; y = -x + 3, \; 2 \le x \le 3$

Another approach (CONT.)

$$\int_{OA} (x+2y)\,dx + x^2\,dy$$

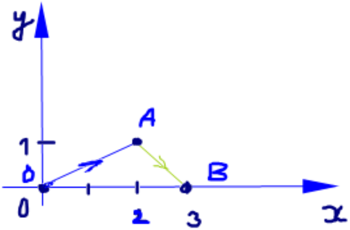
$$= \int_0^2 \left((x+x) + x^2 \cdot \frac{1}{2} \right) dx =$$

$$= \int_0^2 \left(\frac{x^2}{2} + 2x \right) dx = \left(\frac{x^3}{6} + x^2 \right) \Big|_0^2 = \frac{4}{3} + 4 = \frac{16}{3}$$



$OA: \ y = \frac{1}{2} x, \ 0 \le x \le 2$

Another approach (CONT.)



$$\int_{AB} (x+2y)\,dx + x^2\,dy =$$

$$= \int_2^3 \left[(x+2(-x+3)) + x^2\cdot(-1) \right] dx = \int_2^3 (-x^2 - x + 6)\,dx =$$

AB: $y = -x+3,\ 2 \le x \le 3$

$$= \left(-\frac{x^3}{3} - \frac{x^2}{2} + 6x \right)\Big|_2^3 = -3^2 - \frac{3^2}{2} + 18 - \left(-\frac{8}{3} - 2 + 12 \right) = -\frac{17}{6}$$

Ex.7, p.1072 (Stewart)

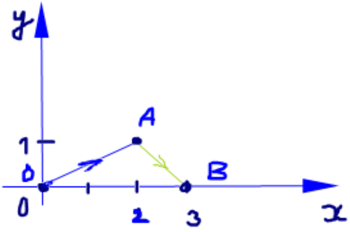
Evaluate the line integral

$$\int_C (x+2y)\,dx + x^2\,dy =$$

$$= \int_{OA} (x+2y)\,dx + x^2\,dy +$$

$$+ \int_{AB} (x+2y)\,dx + x^2\,dy =$$

$$= \frac{16}{3} - \frac{17}{6} = \frac{15}{6} = \frac{5}{2}.$$



OA: $y = \frac{1}{2}x$, $0 \le x \le 2$

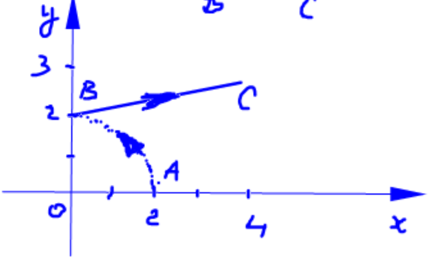
AB: $y = -x + 3$, $2 \le x \le 3$

Ex.8, p.1072 (Stewart)

Evaluate the line integral

$$\int_C x^2\,dx + y^2\,dy,$$

C: $x^2+y^2=4$ from A (2,0) to B (0,2) folowed by the line segment from B (0,2) to C (4,3)



$$\int_C \ldots = \int_{AB} \ldots + \int_{BC} \ldots$$

$$\overrightarrow{BC} = \vec{v} = (4,1)$$

$BC: \begin{cases} x = 4t \\ y = 2+t \end{cases}$ $\quad 0 \le t \le 1$

$AB: \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}$ $\quad 0 \le t \le \frac{\pi}{2}$

$$\int_C x^2dx + y^2dy =$$

$$= \int_{AB} x^2dx + y^2dy + \int_{BC} x^2dx + y^2dy =$$

$$\overrightarrow{BC} = \vec{v} = (4,1)$$

$$BC: \begin{cases} x = 4t \\ y = 2 + t \end{cases}$$

$$0 \leq t \leq 1$$

$$AB: \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}$$

$$0 \leq t \leq \frac{\pi}{2}$$

$$\int_{AB} x^2 dx + y^2 dy =$$

$$= \int_0^{\pi/2} \left[(2\cos t)^2 \cdot (2\cos t)' + (2\sin t)^2 \cdot (2\sin t)' \right] dt =$$

$$AB: \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}$$

$$0 \le t \le \frac{\pi}{2}$$

$$\int_{AB} x^2\,dx + y^2\,dy =$$

$$= \int_0^{\pi/2} \left[(2\cos t)^2 \cdot (2\cos t)' + (2\sin t)^2 \cdot (2\sin t)' \right] dt =$$

$$= -8 \int_0^{\pi/2} \cos^2 t \sin t \, dt + 8 \int_0^{\pi/2} \sin^2 t \cos t \, dt =$$

$$= 8 \left. \frac{\cos^3 t}{3} \right|_0^{\pi/2} + 8 \left. \frac{\sin^3 t}{3} \right|_0^{\pi/2} = \frac{8}{3}(0-1) + \frac{8}{3}(1-0) = 0$$

$$\int_{BC} x^2\,dx + y^2\,dy =$$

$$= \int_0^1 \left[(4t)^2 \cdot 4 + (2+t)^2 \cdot 1\right] dt =$$

$$= \int_0^1 (65t^2 + 4t + 4)\,dt = \left(\frac{65}{3}t^3 + 2t^2 + 4t\right)\Big|_0^1$$

$$\vec{BC} = \vec{v} = (4,1)$$

$$= \frac{65}{3} + 2 + 4 = \frac{83}{3}$$

$$BC: \begin{cases} x = 4t \\ y = 2+t \end{cases}$$

$$0 \le t \le 1$$

$$\int_C x^2\,dx + y^2\,dy =$$

$$= \int_{AB} x^2\,dx + y^2\,dy + \int_{BC} x^2\,dx + y^2\,dy = 0 + \frac{83}{3} =$$

$$= \frac{83}{3}$$

$$\overrightarrow{BC} = \vec{v} = (4,1)$$

$$BC: \begin{cases} x = 4t \\ y = 2 + t \end{cases} \quad 0 \le t \le 1$$

$$AB: \begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases} \quad 0 \le t \le \frac{\pi}{2}$$