Mathematical Analysis II

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Adv. Eng. Math.

section 1.4

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Exercises, p. 26-27

EXACT ODE

M(x,y) dx + N(x,y) dy = 0

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$V(x,y) = C : \frac{\partial V}{\partial x} = M(x,y)$$

$$\frac{\partial V}{\partial y} = N(x,y)$$

Ex.1

$$x\,dx + y\,dy = 0$$

$$x^2 + y^2 = C$$

Ex. 2

$$(2x+3x^2y)\,dx+(x^3-3y^2)\,dy=0$$

$$M(x,y)=2x+3x^2y$$

$$N(x,y)=x^3-3y^2$$

Ex. 2

$$(2x+3x^2y)\,dx+(x^3-3y^2)\,dy=0$$

$$M'_y(x,y)=(2x+3x^2y)'_y = 3x^2$$

$$\Vert \Rightarrow \text{Exact DE}$$

$$N'_x(x,y)=(x^3-3y^2)'_x = 3x^2$$

$V(x,y)=?$

$$\frac{\partial V}{\partial x}=M(x,y)$$

$$\frac{\partial V}{\partial y}=N(x,y)$$

Ex. 2

$$(2x+3x^2y)\,dx+(x^3-3y^2)\,dy=0$$

$$\frac{\partial V}{\partial x}=2x+3x^2y$$

$$V(x,y)=x^2+x^3y+\varphi(y)$$

$$\frac{\partial V}{\partial y}=N(x,y)=x^3-3y^2$$

$$\frac{\partial V}{\partial y}=\left(x^2+x^3y+\varphi(y)\right)'_y=$$

$$\frac{\partial V}{\partial y}=0+x^3+\varphi'(y)=x^3-3y^2\Rightarrow\varphi'(y)=-3y^2$$

CONT.

$$\varphi'(y) = -3y^2$$

$$\varphi(y) = -y^3 + C$$

$$V(x,y) = x^2 + x^3 y + \varphi(y)$$

the general solution:

$$x^2 + x^3 y - y^3 = C$$

Ex.3 Verify that the DE

$$(2x+\sin y - ye^{-x})dx + (x\cos y + \cos y + e^{-x})dy = 0$$

is exact and find its solution curves.

Solution.

$$M'_y(x,y) = (2x+\sin y - ye^{-x})'_y =$$

$$= 0 + \cos y - e^{-x}$$

$$N'_x(x,y) = (x\cos y + \cos y + e^{-x})'_x = \cos y + 0 - e^{-x}$$

Exact DE

$V(x,y)-?$ $\quad dV=$ the left part of the given equation

$V'_x = M(x,y)$, $V'_y = N(x,y)$

$V'_x = 2x+\sin y - ye^{-x}$

$V(x,y) = x^2 + x\sin y + ye^{-x} + \varphi(y)$

$V'_y(x,y) = x\cos y + e^{-x} + \varphi'(y)$

$\varphi'(y) = \cos y$

$\Downarrow$

$\varphi(y) = \sin y + c$

$\|$

$N(x,y) = x\cos y + \cos y + e^{-x}$

A: $x^2 + x\sin y + ye^{-x} + \sin y = C.$

Ex.12, p.26 Adv. Eng. Math.

$$(2xy\,dx + dy)\,e^{x^2} = 0, \quad y(0) = 2$$

$$M'_y(x,y) = \left(2xy\,e^{x^2}\right)'_y = 2xe^{x^2}$$

$$\parallel \qquad \text{Exact DE}$$

$$N'_x(x,y) = \left(e^{x^2}\right)'_x = 2xe^{x^2}$$

$$\frac{\partial V}{\partial x} = 2xy\,e^{x^2} \Rightarrow V(x,y) = ye^{x^2} + h(y)$$

$$V'_y(x,y) = \left(ye^{x^2} + h(y)\right)'_y = \underbrace{e^{x^2} + h'(y)} = e^{x^2} \Rightarrow$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = C \Rightarrow A: \quad ye^{x^2} = C$$

Ex.12, p.26 Adv. Eng. Math.

$$(2xy\,dx + dy)\,e^{x^2} = 0, \quad y(0) = 2$$

the general solution is $\quad ye^{x^2} = C$

$$2 \cdot e^0 = C \Rightarrow C = 2$$

the particular solution is

$$ye^{x^2} = 2 \,.$$