Homework 1

Due 18:00 Wednesday, September 20, 2023
submit via ELSE platform

Problem 1.1

Read the 1-st Chapter "The way of program" from the book "How to think like a computer scientist" and read the section "Advice from an old programmer" from the book "Learn Python the hard way".
See references on the next page.

Problem 1.2

Find 2^A and 2^{2^A}, where

a) $A = \{a, b\}$;

b) $A = \{\emptyset\}$;

c) $A = \{a, \{a\}\}$.

Problem 1.3

Prove the following set inclusions or equalities for any sets A, B, and C.

a) If $A \subset B$, then $(C \setminus B) \subset (C \setminus A)$.

b) $(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C)$.

c) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

Problem 1.4

Given n sets $A_1, A_2, \ldots, A_n$ prove the following set equality (DeMorgan identityt for n sets)

$$\Big(\bigcup_{i=1}^{n} A_i\Big)^c = \bigcap_{i=1}^{n} A_i^c.$$

Problem 1.5

Let $X = \{a, b\}$ and $Y = \{a, c, d\}$. List all elements of

a) $X \times Y$;

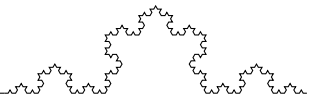
b) X^3;

c) $Y \times X \times Y$.

Problem 1.6

Prove the following property of the Cartesian product:

$$A \times B = B \times A \text{ if and only if } A = B \text{ or if } A \times B = \emptyset.$$



Problem 1.7

Consider the following functions defined from integers to integers:

1. $f(x) = x^2$.
2. $g(x) = x + 2$.
3. $h(x) = 2x$.
4. $k(x) = -x$.
5. $m(x) = [x/2]$, that is, the quotient of x divided by 2.

For each function, indicate whether it is

a) a bijection;

b) a surjection, but not a bijection;

c) an injection, but not a bijection;

d) neither an injection nor a surjection.

Problem 1.8

For each of the following sets indicate (with reasonable arguments) whether it is finite, countably infinite or uncountable.

a) The set of solutions to the equation $x^5 - x = -0.1$.

b) The set of natural numbers that are perfect squares.

c) The set of complex numbers $\mathbb{C}$.

d) The set of words in Romanian no more than 10 letters long.

e) The set of all bijections from an arbitrary set A to a set B.

f) A set $A \cup B$, where A is countable and B is uncountable.

g) The set of all lines in the plane with the same slope.

h) The set of all words in English.

j) The set of all circles in plane $\mathbb{R}^2$.

i) The set $\{0,1\}^{10^{10}}$ of all bit strings of length 10^{10}.

k) The set $\{0,1\}^*$ of all finite bit strings.

Problem 1.9

Give an example of two uncountable sets A and B such that the intersection $A \cap B$ is

a) finite;

b) countably infinite;

c) uncountable.

Problem 1.10

Let $\mathbb{N}^\infty$ be the set of infinite sequences on non negative integers. For example, some sequences of this kind are:

$$(2, 4, 6, 8, 10, \dots)$$
$$(0, 1, 2, 3, 5, 8, \dots)$$
$$(1, 10, 100, 1000, 10000, \dots)$$
$$(17, 2, 9, 21, 5, 33, \dots)$$

Prove that this set is uncountable.

References

[P1] Peter Wentworth, Jeffrey Elkner, Allen B. Downey, and Chris Meyers, **How to think like a computer scientist**, 3rd Ed. `https://openbookproject.net/thinkcs/python/english3e/`

[P2] Zed A. Shaw, **Learn Python the hard way**, 3rd Ed., `https://learnpythonthehardway.org/python3/`