# Question 1

Determine the validity of the statements:

$\boldsymbol{S}$ **:** If $\vec{F}$ is a vector field, then $\mathrm{div}\,\vec{F}$ is a vector field.

$\boldsymbol{T}$ **:** If $\vec{F}$ is a vector field, then $\mathrm{curl}\,\vec{F}$ is a vector field.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 2

Determine the validity of the statements:

***S*** **:** If $f$ has continuous partial derivatives of all orders on $\mathbb{R}^3$, then $\operatorname{div}(\operatorname{curl} \nabla f) = 0$.

***T*** **:** If $f$ has continuous partial derivatives of all on $\mathbb{R}^3$ and $C$ is any circle, then $\int\limits_C \nabla f \cdot d\vec{r} = 0$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 3

Determine the validity of the statements:

$\boldsymbol{S}$ : if $\vec{F} = P\vec{i} + Q\vec{j}$ and $P'_y = Q'_x$ in an open region $D$, then $\vec{F}$ is conservative.

$\boldsymbol{T}$ : if $\vec{F} = P\vec{i} + Q\vec{j}$ and $P'_y \neq Q'_x$ in an open region $D$, then $\vec{F}$ is conservative.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 4

Determine the validity of the statements:

$\boldsymbol{S}$ **:** $\vec{F} = P\vec{i} + Q\vec{j}$ is a vector field on an open simply-connected region $D$, where $P$ and $Q$ have continuous first-order derivatives and $\dfrac{\partial P}{\partial y} \neq \dfrac{\partial Q}{\partial x}$ throughout $D$, then $\vec{F}$ is conservative.

$\boldsymbol{T}$ **:** $\vec{F} = P\vec{i} + Q\vec{j}$ is a vector field on an open simply-connected region $D$, where $P$ and $Q$ have continuous first-order derivatives and $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ throughout $D$, then $\vec{F}$ is conservative.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 5

Determine the validity of the statements:

**$S$ :** The work done by a conservative force filed in moving a particle around a closed path is zero.

**$T$ :** If $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ is a vector field on an open simply-connected region $D$, where $P$, $Q$ and $R$ have continuous first-order partial derivatives and $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ throughout $D$, then $\vec{F}$ is conservative.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 6

Determine the validity of the statements:

$\boldsymbol{S}$ **:** $\int\limits_{-C} f(x,y)ds = -\int\limits_{C} f(x,y)ds$

$\boldsymbol{T}$ **:** The work done by a conservative force field in moving a particle around a closed path is not zero.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 7

Determine the validity of the statements:

***S*** **:** If $\vec{F}$ and $\vec{G}$ are vector fields and $\operatorname{div}\vec{F} = \operatorname{div}\vec{G}$, then $\vec{F} = \vec{G}$.

***T*** **:** $\int\limits_{-C} f(x,y)ds = \int\limits_{C} f(x,y)ds$

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 8

Determine the validity of the statements:

***S*** **:** If $\vec{F}$ and $\vec{G}$ are vector fields in which the appropriate partial derivatives exist and are continuous, then $\operatorname{curl}(\vec{F}+\vec{G}) = \operatorname{curl}\vec{F} + \operatorname{curl}\vec{G}$

***T*** **:** If $\vec{F}$ and $\vec{G}$ are vector fields in which the appropriate partial derivatives exist and are continuous, then $\operatorname{curl}(\vec{F}\cdot\vec{G}) = \operatorname{curl}\vec{F} \cdot \operatorname{curl}\vec{G}$

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

## Question 9

Determine the validity of the statements:

$\boldsymbol{S}$ : There is a vector field $\vec{F}$ such that $\operatorname{curl} \vec{F} = x \sin y\, \vec{i} + \cos y\, \vec{j} + (z - xy)\vec{k}$.

$\boldsymbol{T}$ : There is a vector field $\vec{F}$ such that $\operatorname{curl} \vec{F} = x \sin y\, \vec{i} + \cos y\, \vec{j} + (2 - xy)\vec{k}$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 10

Determine the validity of the statements:

***S*** **:** There is a vector field $\vec{F}$ such that $\operatorname{curl}\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$.

***T*** **:** Any vector field of the form $\vec{F}(x,y,z) = f(x)\vec{i} + g(y)\vec{j} + h(z)\vec{k}$, where $f$, $g$, $h$ are differentiable functions, is irrotational.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 11

Determine the validity of the statements:

***S*** **:** $\int\limits_{\Gamma} \vec{F} \cdot d\vec{r}$ is independent of path in $D$ if and only if $\int\limits_{C} \vec{F} \cdot d\vec{r} = 0$ for every closed path $C$ in $D$.

***T*** **:** Let $\vec{F}$ be a vector field that is continuous on an open connected region $D$. If $\int\limits_{C} \vec{F} \cdot d\vec{r}$ is independent of path in $D$, then there exists a function $f$ such that $\nabla f = \vec{F}$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 12

Determine the validity of the statements:

***S*** **:** Any vector field of the form $\vec{F}(x,y,z) = f(x)\vec{i} + g(y)\vec{j} + h(z)\vec{k}$, where $f$, $g$, $h$ are differentiable functions, is not irrotational.

***T*** **:** If $f$ is a function of three variables that has continuous second-order partial derivatives, then $\text{curl}(\nabla f) = \vec{0}$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 13

Determine the validity of the statements:

***S*** **:** Let $\vec{F}$ be a vector field that is continuous on an open connected region $D$. If $\int_C \vec{F} \cdot d\vec{r}$ is independent of path in $D$, then $\vec{F}$ is a conservative vector field on $D$.

***T*** **:** If $F$ is a vector field defined on all of $\mathbb{R}^3$ whose component functions have continuous partial derivatives and $\operatorname{curl} \vec{F} = \vec{0}$, then $\vec{F}$ is a conservative vector field.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 14

Determine the validity of the statements:

$\boldsymbol{S}$ : Suppose $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$ is a vector field that is $P$ and $Q$ have continuous first-order partial derivatives on an open connected region $D$. If $\int\limits_C \vec{F}\cdot d\vec{r}$ is independent of path in $D$, then $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ throughout $D$.

$\boldsymbol{T}$ : If $\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$ is a conservative vector field, where $P$ and $Q$ have continuous first-order partial derivatives on a domain $D$, then $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ throughout $D$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 15

Determine the validity of the statements:

***S*** **:** Let $C$ be a positively oriented, piecewise-smooth, simple closed curve in the plane and let $D$ be the region bounded by $C$. If $P$ and $Q$ have continuous partial derivatives on an open region that contains $D$, then $\int_C Pdx + Qdy = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$

***T*** **:** If $C$ is any smooth simple closed plane curve and $f$ and $g$ are differentiable functions on $\mathbb{R}^3$, then $\int_C f(x)dx + g(y)dy \neq 0$.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 16

Determine the validity of the statements:

***S*** **:** If $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$ is a vector field on $\mathbb{R}^3$ and $P$, $Q$, and $R$ have continuous second-order partial derivatives, then $\operatorname{div}\operatorname{curl}\vec{F} = 0$.

***T*** **:** If $f$ is a scalar field and $\vec{F}$ is a vector field, then the expression $\operatorname{div}(\operatorname{curl}(\text{grad}\, f))$ is meaningful.

**A.** $S$ and $T$ are true.

**B.** only $S$ is true.

**C.** only $T$ is true.

**D.** $S$ and $T$ are false.

# Question 17

What is the divergence of the vector field

$$\vec{F} = 3x^2\vec{i} + 5xy^2\vec{j} + xyz^3\vec{k}$$

at the point $(1, 2, 3)$.

**A.** 87

**B.** 80

**C.** 124

**D.** 100

# Question 18

What is the divergence of the vector field

$$\vec{F} = 3x^2\vec{i} + 5xy^2\vec{j} + xyz^3\vec{k}$$

at the point $(1, -1, 0)$.

**A.** -7

**B.** 8

**C.** 4

**D.** -4

# Question 19

Divergence and curl of a vector field are

**A.** Scalar & Scalar.

**B.** Scalar & Vector.

**C.** Vector & Vector.

**D.** Vector & Scalar.

# Question 20

Which of the following holds for any non-zero vector $\vec{F}$?

**A.** $\nabla \cdot \vec{F} = 0.$

**B.** $\nabla \times \vec{F} = \vec{0}.$

**C.** $\nabla \cdot (\nabla \times \vec{F}) = 0$

**D.** $\nabla (\nabla \times \vec{F}) = \vec{0}$

# Question 21

Gauss-Ostrogradsky theorem uses which of the following operations?

**A.** Gradient

**B.** Curl

**C.** Divergence

**D.** Laplacian

# Question 22

Evaluate the surface integral $\iint\limits_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = 3x\vec{i} + 2y\vec{j}$ and $S$ is the sphere given by $x^2 + y^2 + z^2 = 9$.

**A.** 180

**B.** $180\pi$

**C.** $45\pi$

**D.** $405\pi$

# Question 23

The Divergence Theorem converts

**A.** line to surface integral

**B.** line to volume integral

**C.** surface to line integral

**D.** surface to volume integral

# Question 24

Stokes theorem is used to convert ________ to ________

**A.** Surface Integral, Volume Integral

**B.** Line Integral, Volume Integral

**C.** Line Integral, Surface Integral

**D.** none of the above

# Question 25

Which among the following theorems uses the curl operation?

**A.** Green's Theorem

**B.** Stokes' Theorem

**C.** The Divergence Theorem

**D.** None of the above