Homework 7

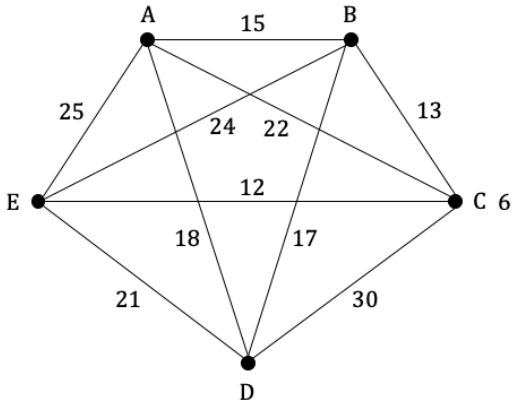
Due December 3rd, 2023

Problem 7.1

Find a spanning tree for the cube graph and for K_4. How many different spanning trees there are for K_4?

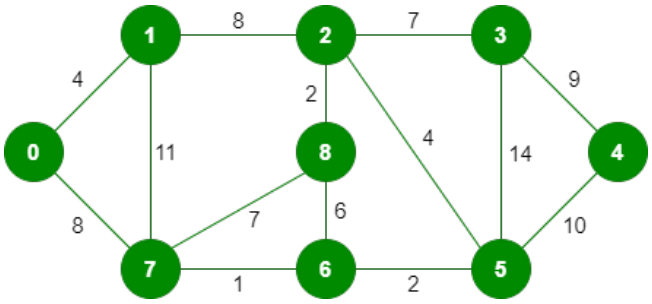
Problem 7.2

Apply Prim's algorithm to find the minimal spanning tree for the weighted graph presented below.



Problem 7.3

Apply Kruskal's algorithm to find the minimal spanning tree for the weighted graph presented below.



Problem 7.4

Consider 5 cities numbered from 1 to 5. The cost of construction of a road between city i and city j is a_{ij}, where a_{ij} are given in the following adjacency matrix. Note that $a_{34} = a_{43} = +\infty$ since no road is considered between city 3 and city 4. Your goal is to find the minimum cost of a road network connecting these cities with each other.

$$\begin{bmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & +\infty & 10 \\ 11 & 9 & +\infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{bmatrix}$$

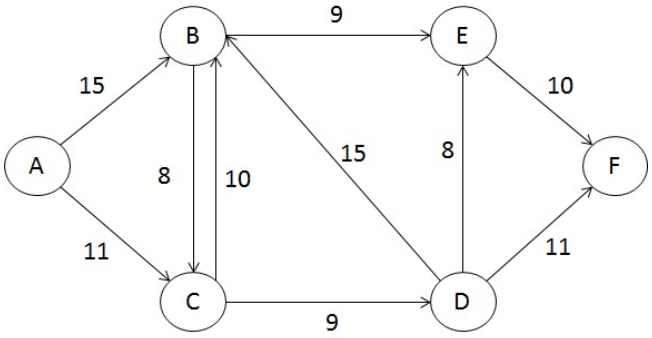
Apply Prim's and Kruskal's algorithms to compute the minimal spanning tree, that will provide you the sought network of roads.

Problem 7.5

Find the maximal flow using Fold Fulkerson algorithm for the following network:
$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_4), (v_2, v_5), (v_3, v_4), (v_3, v_5), (v_4, v_2), (v_4, v_6), (v_5, v_3), (v_5, v_6)\}$
and capacities
$c_{12} = 1, c_{13} = 4, c_{24} = 1, c_{25} = 3, c_{34} = 3, c_{35} = 1, c_{42} = 1, c_{46} = 1, c_{53} = 1, c_{56} = 4$.

Problem 7.6

Apply Ford-Fulkerson algorithm to find the maximal flow in the network presented below.



Problem 7.7

Apply Ford-Fulkerson algorithm to find the maximal flow in the network presented below.