AM II

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Vector Fields

<u>Stewart, ch. 16</u>

16.1 Exercises, p.1061

1-10 (at least one ex.),
11-14,
15-18 (opt), 19,
21-24,
33, 35

Adv. Eng. Math.
section 9.4
pp.375-378

Exercises 15-21, p.381

Ex. 4, p.1061 (Stewart) $\vec{F}(x,y) = y\,\vec{i} + (x+y)\,\vec{j}$

$\vec{F}(0,0) = \vec{0}$

$\vec{F}(1,0) = \vec{j}$

$\vec{F}(a,0) = a\vec{j}$

$\vec{F}(0,1) = \vec{i} + \vec{j}$

$\vec{F}(0,b) = b\vec{i} + b\vec{j}$

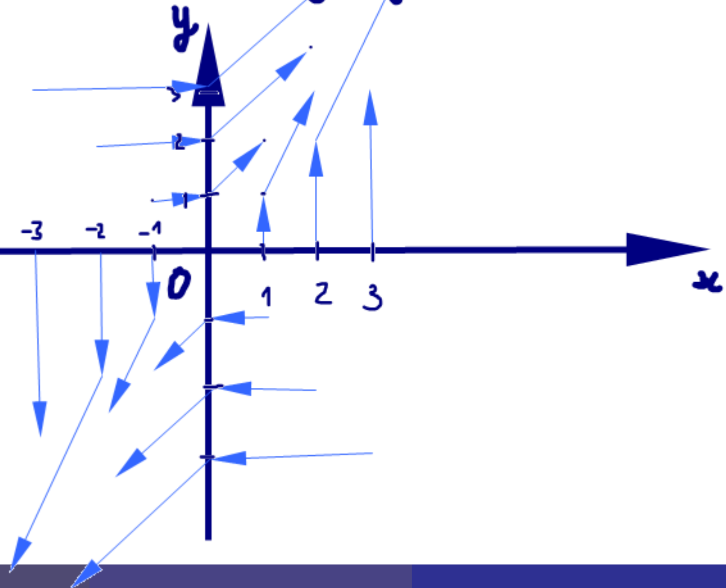
$\vec{F}(1,1) = \vec{i} + 2\vec{j}$

$\vec{F}(1,-1) = -\vec{i}$

$F(a,-a) = -a\,i$

$\vec{F}(-1,1) = \vec{i}$

$F(-a,a) = a\vec{i}$



Ex. 5, p.1061 (Stewart)

$$\vec{F}(x,y) = \frac{y\vec{i} + x\vec{j}}{\sqrt{x^2+y^2}}$$

$\vec{F}(1,0) = \vec{j}$

$\vec{F}(1,1) = \frac{\sqrt{2}}{2}(\vec{i}+\vec{j})$

$\vec{F}(0,1) = \vec{i}$

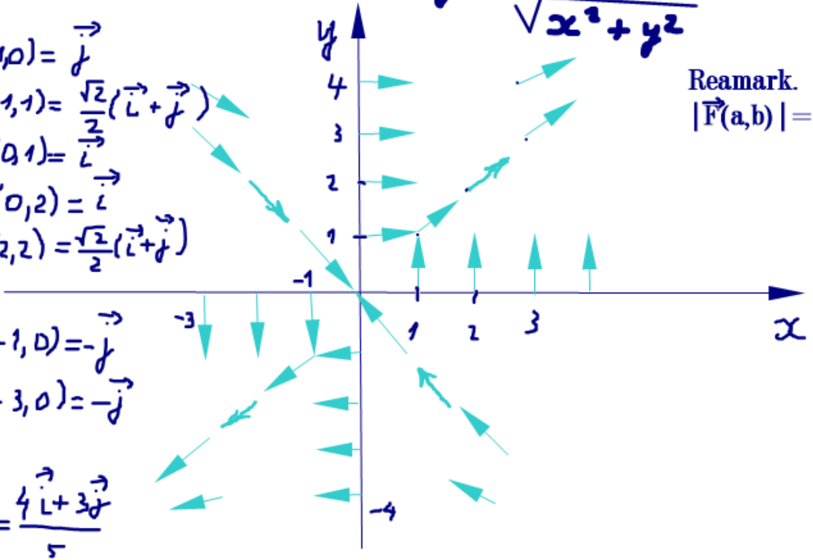
$\vec{F}(0,2) = \vec{i}$

$\vec{F}(2,2) = \frac{\sqrt{2}}{2}(\vec{i}+\vec{j})$

$\vec{F}(-1,0) = -\vec{j}$

$\vec{F}(-3,0) = -\vec{j}$

$\vec{F}(3,4) = \frac{4\vec{i}+3\vec{j}}{5}$



Reamark.
$|\vec{F}(a,b)| = 1$

Ex.6, p.1061

$$\vec{F}(x,y)=\frac{y\vec{i}-x\vec{j}}{\sqrt{x^2+y^2}}$$

$\vec{F}(1,0)=-\vec{j}$; $\vec{F}(a,0)=-\vec{j}$, $a>0$

$\vec{F}(0,1)=\vec{i}$

$\vec{F}(0,b)=\vec{i}$, $b>0$

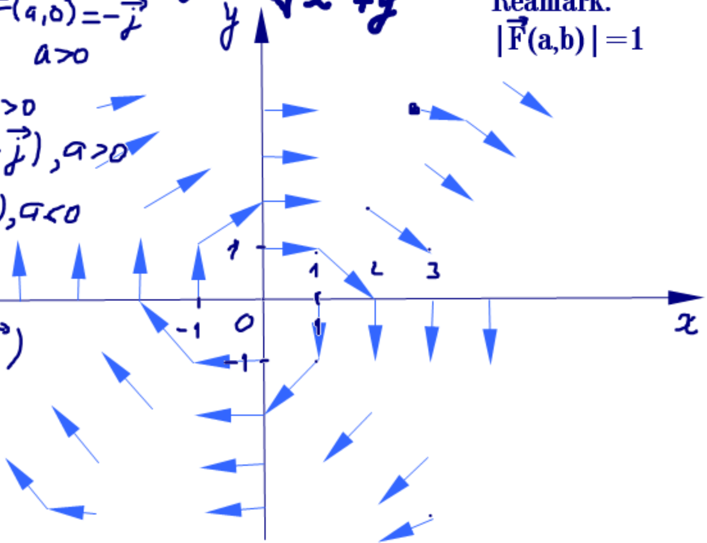
$\vec{F}(a,a)=\frac{1}{\sqrt{2}}(\vec{i}-\vec{j})$, $a>0$

$\vec{F}(a,a)=\frac{1}{\sqrt{2}}(\vec{j}-\vec{i})$, $a<0$

$\vec{F}(1,-1)=\frac{1}{\sqrt{2}}(-\vec{i}-\vec{j})$

$\vec{F}(3,4)=\frac{4}{5}\vec{i}-\frac{3}{5}\vec{j}$

Reamark.
$|\vec{F}(a,b)|=1$



Ex.22, p.1062 Stewart

Find the gradient vector field of $f(x,y) = tg(3x-4y)$

$$\nabla f = \frac{3}{\cos^2(3x-4y)}\vec{i} - \frac{4}{\cos^2(3x-4y)}\vec{j}$$

or

$$\nabla f = \left(\frac{3}{\cos^2(3x-4y)}, -\frac{4}{\cos^2(3x-4y)}\right)$$

Ex.23, p.1062 Stewart

Find the gradient vector field of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\nabla f = \frac{1}{\sqrt{x^2+y^2+z^2}}(x, y, z)$$

or

$$\nabla f = \frac{x}{\sqrt{x^2+y^2+z^2}}\vec{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\vec{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\vec{k}$$

Ex.26, p.1062 Stewart

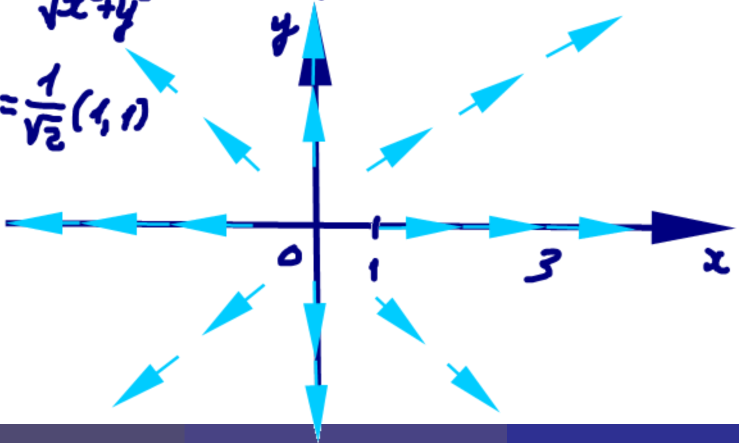
Find the gradient vector field of $f(x,y) = \sqrt{x^2+y^2}$

$(3,4)$

$$\nabla f = \frac{1}{\sqrt{x^2+y^2}}(x,y)$$

and sketch it.

$$\nabla f(1,1) = \frac{1}{\sqrt{2}}(1,1)$$



Ind. W.

Ex.27, p.1062 Stewart

Plot the gradient vector field of

$$f(x,y)=\ln(1+x^2+2y^2)$$

together with a contour map of f.

$$\nabla f=\frac{2x}{1+x^2+2y^2}\vec{i}+\frac{4y}{1+x^2+2y^2}\vec{j}$$

level lines: $f(x,y)=C$

$$\ln(1+x^2+2y^2)=C$$

$$1+x^2+2y^2=e^C$$

$$x^2+2y^2=e^C-1$$

$c=0 \qquad (0,0)$

$c>0$

$e^c-1>0$ ellipses

$a=e^c-1$

$b=\frac{1}{2}(e^c-1)$

Ex.34, p.1062 Stewart

At time t=1, a particle is located at position (1,3).
If it moves in a velocity field $\vec{F}(x,y)=(xy-2, y^2-10)$
find its approximate location at time t=1.05.

Solution. $\vec{F}(x,y)=(xy-2, y^2-10)$

for t=1 $\vec{r}(1)=(1,3)$; $\vec{F}(1,3)=(1,-1)$

$$\vec{r}(1.05)\approx\vec{r}(1)+0.05\cdot\vec{F}(1.3)=$$
$$=(1,3)+0.05\cdot(1,-1)=$$
$$=(1.05, 2.95)$$