Homework 1

Due 18:00, September 14, 2023
Submit HW via ELSE platform

Problem 1.1

Give possible sample space Ω for each of the following experiments:

a) A fair coin is tossed four times.

b) A student is chosen at random in your class.

c) A 4-sided fair dice is rolled 3 times.

Problem 1.2

Let Ω be a sample space associated to three tosses of a coin, and let

a) $E_1 = \{HHH, HHT, HTH, THH\}$

b) $E_2 = \{HHH, TTT\}$

c) $E_3 = \{THT, TTH, THH, TTT\}$

Describe in words events E_i, $i = 1, 2, 3$. What is $P(E_1 \cup E_2)$, and $P(E_2 \cap E_3)$?

Problem 1.3

Let $\Omega = \{A, B, C\}$ be a sample space. Let $m(A) = 1/3$, and $m(B) = 1/4$. Find the probabilities of all possible events (subsets) of sample space Ω.

Problem 1.4

Let A and B be events such that $P(A \cap B) = 1/4$, $P(A^c) = 1/3$ and $P(B) = 1/2$. What is $P(A \cup B)$?

Problem 1.5

Prove that for any events A and B we have $P(A \cap B) \geqslant P(A) + P(B) - 1$.

Problem 1.6

Consider the experiment of tossing a fair 5-sided dice two times. Let X be the result of the first toss and Y the result of the second. Compute probabilities:

a) $P\big(X \text{ is odd }\big)$

b) $P\big(X + Y \leqslant 7\big)$

c) $P\big(\max(X, Y) \leqslant 3\big)$

d) $P\big(|X - Y| = 2\big)$

Problem 1.7

A die is loaded (hence it is not a fair dice) in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.) What is the probability of getting an even number in one throw?