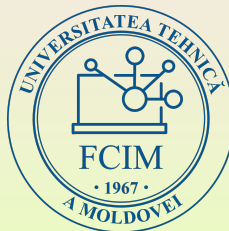
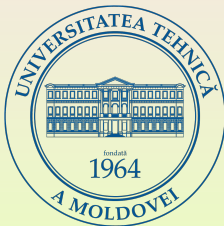


# Mathematical Analysis II

**Conf. univ., dr. Elena COJUHARI**

*[elena.cojuhari@mate.utm.md](mailto:elena.cojuhari@mate.utm.md)*

Technical University of Moldova



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## HIGHER-ORDER

### Differential Equations

$$F(x, y, y', y'', \dots, y^{(n)}) = 0$$

(1)

$$y^{(n)} = f(x, y, y', y'', \dots, y^{(n-1)})$$

(1')

Th. If in the equation  $y^{(n)} = f(x, y, y', y'', \dots, y^{(n-1)})$

the function  $f(x, y, y', y'', \dots, y^{(n-1)})$  and its partial derivatives with respect to the arguments  $y, y', y'', \dots, y^{(n-1)}$  are continuous in some region containing the values  $x=x_0, y=y_0, y'=y'_0, \dots, y^{(n-1)}=y_0^{(n-1)}$ , then there is one and only one solution,  $y=y(x)$ , of the equation that satisfies the conditions

$$\left\{ \begin{array}{l} y_{x=x_0} = y_0 \\ y'_{x=x_0} = y'_0 \\ \vdots \\ y^{(n-1)}_{x=x_0} = y_0^{(n-1)} \end{array} \right. \quad (2) \quad \text{initial conditions}$$

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$$y'' = f(x, y, y')$$

the geometrical meaning

$y=y_0, y'=y'_0$ , where  $x_0, y_0, y'_0$  are given numbers

Def. The general solution of a dif. eq. of nth order

$$y = \varphi(x, C_1, C_2, \dots, C_n) \quad (3)$$

which is dependent of n arbitrary constants  $C_1, C_2, \dots, C_n$  and

- .) it satisfies the equation for any values of the constants  $C_1, C_2, \dots, C_n$ ;
- ..) for specified initial conditions

$$y|_{x=x_0} = y_0$$

$$y'|_{x=x_0} = y'_0$$

$$y^{(n-1)}|_{x=x_0} = y_0^{(n-1)}$$

the constants  $C_1, C_2, \dots, C_n$  may be chosen so that the function (3) will satisfy these conditions...

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the particular solution  $y = \varphi(x, C_1^0, C_2^0, \dots, C_n^0)$  integral curve

ODE of order  $n$  which admit

A LOWERING OF THE ORDER

AN EQUATION OF THE FORM

$$y^{(n)} = f(x)$$

$$y^{(n-1)} = \int f(x) dx + C_1 = F_1(x) + C_1$$

$$y^{(n-2)} = \int (F_1(x) + C_1) dx = F_2(x) + C_1 x + C_2$$

...

$$y = F_n(x) + \frac{C_1}{(n-1)!} x^{n-1} + \frac{C_2}{(n-2)!} x^{n-2} + \dots + C_{n-1} x + C_n$$

where

$$F_n(x) = \underbrace{\int \int \dots \int}_{n \text{ times}} f(x) \underbrace{dx \dots dx}_{n \text{ times}}$$

Example.  $y'' = x + \sin x$

$$y' = \frac{x^2}{2} - \cos x + C_1$$

$$y = \frac{x^3}{6} - \sin x + C_1 x + C_2$$

the general solution

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Exercises:

$$y^{(4)} = \cos^2 x, \quad y(0) = 1/32, y'(0) = 0, y''(0) = 1/8, y'''(0) = 0$$

$$y''' = x \sin x, \quad y(0) = 0, y'(0) = 0, y''(0) = 2$$

$$y'' = 2 \sin x \cos^2 x - \sin^3 x$$

$$F(x, y^{(k)}, \dots, y^{(n)}) = 0$$

does not contain  $y$

$$y^{(k)}(x) = z(x)$$

$$F(x, z, z', \dots, z^{(n-k)}) = 0$$

of order  $n-k$



Example  $y'' - 2y' \operatorname{ctg} x = \sin^3 x$

$$z(x) = y' \Rightarrow z' = y'' \quad z' - 2z \operatorname{ctg} x = \sin^3 x$$

$$\dots$$
$$y' = z = C_1 \sin^2 x - \sin^2 x \cos x$$
$$y = \frac{C_1}{4} (2x - \sin 2x) - \frac{1}{3} \sin^3 x + C_2$$

Exercises:

$$y'' - 3 \frac{y'}{x} = x$$
$$xy'' = y' \ln \frac{y'}{x}$$

$$y'''(x-1) - y'' = 0, \quad y(2)=2, \quad y'(2)=1, \quad y''(2)=1$$

$$F(y', y'', \dots, y^{(n)}) = 0$$

first

, substitution

$$y = z(x)$$

and then take  $z$  as independent variable

$$y'''(1+y'^2) - 3y'y''^2 = 0$$

$$y' = z(x)$$

$$z''(1+z^2) - 3z(z')^2 = 0$$



see next page

$$F(y, y', y'', \dots, y^{(n)}) = 0$$

$$y' = p$$

, consider  $p$  as a function and  
take  $y$  as an independent variable

$$y'' = p \frac{dp}{dy}$$

$$y''' = p \left( \frac{dp}{dy} \right)^2 + p^2 \frac{d^2 p}{dy^2}$$

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Ex.  $y'' = 2yy'$

$$yy'' - 2y'^2 = 0$$

$$y'' = y' e^y, \quad y(0) = 0, \quad y'(0) = 1$$

DE (1) homogeneous with respect  $y, y', \dots, y^{(n)}$

$$z(x) = \frac{y'}{y} \quad \text{or} \quad y = e^{\int z(x) dx}$$

reduce the DE (1) to the DE of order  $n-1$

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Ex

$$y'^2 + 2yy'' = 0$$

$$xyy'' - xy'^2 - yy' = 0$$

$$3y'^2 = 4yy'' + y^2.$$

























