Homework 4

DueNovemver 24, 2023

Problem 4.1

Suppose that you roll the fair, ten-sided die. Let the random variable X be the remainder when the number on top is divided by 3, and let the random variable Y be the remainder when the number on top is divided by 4.

a) Are the random variables X and Y independent?

b) Compute $P(X = 2, Y = 1)$, $P(X = 0 \quad | \quad Y = 1)$.

Problem 4.2

Let X be the number of heads after a coin is tossed three times. Let Y denote the face that comes up after rolling a die. Let $Z = X - Y$. Find the expected value $E(Z)$,variance $V(Z)$ and standard deviation $D(Z)$.

Problem 4.3

I propose you a game! You pick a number between 2 and 12. Then you roll two fair dice. The result is the sum of the tosses.

- If your number is not the sum of the tosses, then you lose a dollar.
- If your number is the sum of the tosses, then you win k dollars.

What is the best number to choose initially? What value of k will make this game fair? *Explain your answers.*

Problem 4.4

A baker blends 600 raisins into a dough mix and, from this, makes 500 cookies.

a) Find the probability that a randomly picked cookie will have no raisins.

b) Find the probability that a randomly picked cookie will have exactly two raisins.

c) Find the probability that a randomly chosen cookie will have at least five raisins in it.

Problem 4.5

A shooter takes 10 shots at a target and has probability 0.3 of hitting the target with each shot, independently of all other shots. Let X be the number of successful hits.

a) What is the distribution of X?

b) What is the probability of scoring no hits?

c) What is the probability of scoring more hits than misses?

d) Find the expectation and the variance of X.

e) Suppose the shooter has to pay 3$ to enter the shooting range and he gets 2$ dollars for each hit. Let Y be his profit. Find the expectation and the variance of Y.

f) Now let's assume that the shooter enters the shooting range for free and gets the number of dollars that is equal to the square of the number of hits. Let Z be his profit. Find the expectation of Z.

Problem 4.6

BONUS PROBLEM.

You are throwing a fair coin until you have two heads consecutively. What is the expected number of throws?