Mathematics for Computer Science

Prof.dr.hab. Viorel Bostan

Technical University of Moldova

viorel.bostan@adm.utm.md

Pre Lecture 2

Definition

Mathematical logic is concerned with formalizing and analyzing the kinds of reasoning used in mathematics.

Recall the elementary geometry course from high school.

Part of the problem with formalizing mathematical reasoning is the necessity of precisely specifying the language(s) in which it is to be done.

The natural languages spoken by humans won't do:

Natural Language Limitations

Natural languages are complex and continually changing as to be impossible to pin down completely. Moreover, the ambiguities inherent in everyday language can be a real problem.

Natural Languages

Definition (What is a language?)

Language is the (human) capacity for acquiring and using complex systems of communication, or a specific instance of such a system of complex communication.

Languages can be **natural** (English, Romanian, Japanese, Zulu, Esperanto, etc) or **formal.**

Definition (Formal Language)

A language is **formal** if it is provided with a mathematically rigorous representation of its:

- alphabet of symbols,
- formation rules specifying which strings of symbols count as well-formed.

Ambiguity of Natural Language

Example

A boy touches the girl with the flower.

Consider some sequences taken from English language:

1. "You may have cake or you may have ice cream."
2. "If you don't clean your room, then you won't play Counter Strike!"
3. "If pigs can fly, then you can understand the Banach-Tarsky Theorem."
4. "If you can solve any problem we come up with in this class, then you get grade 10 for this course."
5. "Every human has a dream."

Can you find ambiguities, if any, in the above expressions?

Mathematical Logic in Natural Languages

Some uncertainty is tolerable in normal conversation. But ...

Imperativ!

We need to formulate ideas precisely as in mathematics.
The ambiguities inherent in everyday language become a real problem!

We can't hope to make an exact argument if we're not sure exactly what the individual words mean.

To get around the ambiguity of English, mathematicians have devised a special language for talking about logical relationships.

This language mostly uses ordinary English words and phrases such as **"and"**, **"or"**, **"implies** and **"for all"**.

But mathematicians endow these words with definitions more precise than those found in an ordinary English dictionary.

Definition

A proposition is a statement that is either true or false.

Proposition

$2 + 3 = 5.$

Propositional Logic

Proposition A

All Greeks are human.

Proposition B

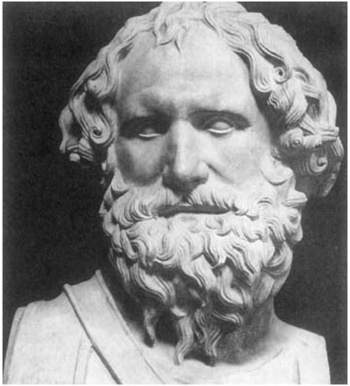
All humans are mortal.

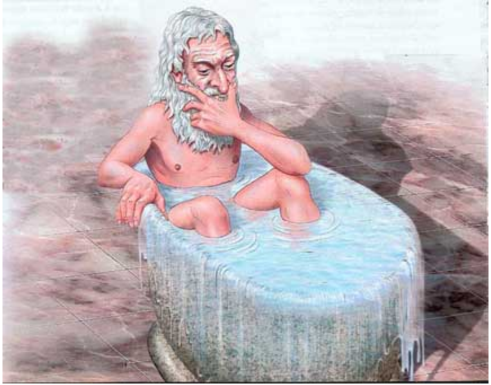
Proposition C

All Greeks are mortal.

Archimedes spent some time playing with such sentences (called logic syllogisms) in the $3rd$ century BC.

If **A** is true, and **B** is true, then **C** is also true!





Archimedes of Syracuse, 287–212 BC

Archimedes developed an early form of logic that he applied in proving rigorously geometrical theorems including the area of a circle, the surface area and volume of a sphere.

Theorem, lemma, corollary, conjecture

Definition

Proposition is a statement that is either true or false.

Definition

Important propositions are called **theorems**.

Definition

A **lemma** is a preliminary proposition useful for proving later propositions/theorems.

Definition

A **corollary** is a proposition that follows in just a few logical steps from a theorem.

Definition

A **conjecture** is a proposition that is not yet proved or disproved.

What is a proof?