



Homework 2

Due 18:00, Thursday September 28, 2023

Problem 2.1

A die is rolled twice. What is the probability that the sum of the faces is greater than 5 (event E), given that

- a) the first outcome was a 3?
- b) the first outcome was greater than 4?
- c) the first outcome was 1?
- d) the first outcome was less than 5?

Problem 2.2

There are two urns. In urn I there are 5 white balls and 6 black balls, in urn II there are 4 white balls and 3 black balls. 2 balls from urn I are picked at random and transferred in urn II . Then, two balls are chosen at random from urn II . What is the probability that both balls are black? What is the probability that balls of the same color were transferred from I to urn II if balls chosen from urn II are of different color?

Problem 2.3

What is the probability that a family of three children has

- a) two boys given that it has at least one boy?
- b) two boys given that the first child is a girl?
- c) three girls, given that the first child is a girl?

Problem 2.4

Examphobia is a rare disease in which the victim has the delusion that he or she is being subjected to an intense mathematical examination. A person selected uniformly at random has examphobia with probability $1/50$. A person with examphobia has shaky hands with probability $7/10$. A person without examphobia has shaky hands with probability $3/20$. What is the probability that a person selected uniformly at random has examphobia, given that he or she has shaky hands? What is probability that a person doesn't have shaky hands given that he or she has examphobia?

Problem 2.5

Each of the four engines of an airplane are functioning correctly on a given flight with probability of 0.99, and the engines function independently of each other. Assume that the plane can make a safe landing if at least two of its engines are functioning properly. What is the probability that the engines will allow a safe landing?

Problem 2.6

BONUS PROBLEM.

Two cowboys A and B decide to solve a dispute with a duel. Cowboy A hits his target $1/3$ of the time. Cowboy B hits the target $2/3$ of the time. It is decided that A will take the first shot, cowboy B will take the second shot (if still alive). This will continue until there is only one left alive. Also, a cowboy can not shoot two times in a row. What are cowboy A chances of winning the duel?

Remark: You might need the formula for the sum of infinite geometric progression, known as geometric series:

$$1 + q + q^2 + q^3 + q^4 + \dots = \frac{1}{1 - q}, \quad \forall q \text{ such that } |q| < 1$$
$$q + q^2 + q^3 + q^4 + \dots = \frac{q}{1 - q}, \quad \forall q \text{ such that } |q| < 1$$