

Homework 5

Due April 28, 19:00

Assignment 5.1

Solve the following tridiagonal system:

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ -x_1 + 2x_2 + x_3 &= 2 \\ -x_2 + 2x_3 + x_4 &= 2 \\ -x_3 + 2x_4 + x_5 &= 2 \\ -x_4 + 2x_5 &= 1 \end{aligned}$$

Assignment 5.2

Using Gauss elimination method without pivoting, solve the linear system $Ax = b$, with

$$A = \begin{bmatrix} 3 & -1 & 3 \\ 1 & 2 & -2 \\ 1 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 2 \\ 0 \end{bmatrix}.$$

Also, give the LU factorization of A .

Assignment 5.3

Consider the following iteration method:

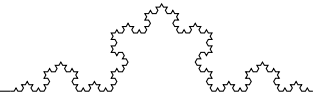
$$\begin{pmatrix} x_1^{(m+1)} \\ x_2^{(m+1)} \\ x_3^{(m+1)} \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} \\ 5 \\ 4 \end{pmatrix} - \frac{1}{10} \begin{pmatrix} 0 & 3 & 1 \\ -2 & 0 & -3 \\ 1 & 4 & 0 \end{pmatrix} \begin{pmatrix} x_1^{(m)} \\ x_2^{(m)} \\ x_3^{(m)} \end{pmatrix}$$

Does $x^{(m)}$ converge as $m \rightarrow \infty$ for any choice of the initial guess $x^{(0)}$?

Assignment 5.4

Solve the following set of equations both by means of a direct method and iterative method. Describe the methods used and why you chose them.

$$\begin{aligned} x_2 + 5x_3 - 7x_4 + 23x_5 - x_6 + 7x_7 + 8x_8 + x_9 - 5x_{10} &= 10 \\ 17x_1 - 24x_3 - 75x_4 + 100x_5 - 18x_6 + 10x_7 - 8x_8 + 9x_9 - 50x_{10} &= -40 \\ 3x_1 - 2x_2 + 15x_3 - 78x_5 - 90x_6 - 70x_7 + 18x_8 - 75x_9 + x_{10} &= -17 \\ 5x_1 + 5x_2 - 10x_3 - 72x_5 - x_6 + 80x_7 - 3x_8 + 10x_9 - 18x_{10} &= 43 \\ 100x_1 - 4x_2 - 75x_3 - 8x_4 + 83x_6 - 10x_7 - 75x_8 + 3x_9 - 8x_{10} &= -53 \\ 70x_1 + 85x_2 - 4x_3 - 9x_4 + 2x_5 + 3x_7 - 17x_8 - x_9 - 21x_{10} &= 12 \\ x_1 + 15x_2 + 100x_3 - 4x_4 - 23x_5 + 13x_6 + 7x_8 - 3x_9 + 17x_{10} &= -60 \\ 16x_1 + 2x_2 - 7x_3 + 89x_4 - 17x_5 + 11x_6 - 73x_7 - 8x_9 - 23x_{10} &= 100 \\ 51x_1 + 47x_2 - 3x_3 + 5x_4 - 10x_5 + 18x_6 - 99x_7 - 18x_8 + 12x_{10} &= 0 \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 &= 100 \end{aligned}$$

**Assignment 5.5**

Solve the following system of equations by Gauss-Jordan and Gauss-Seidel iteration starting with an initial guess of $x = y = z = 1$.

$$8x + 3y + 2z = 20.00$$

$$16x + 6y + 4.001z = 40.02$$

$$4x + 1.501y + z = 10.01$$

Comment on the accuracy of your solution and the relative efficiency of the two methods.