Mathematical Analysis II

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Differential Equations

Stewart, ch.9

9.1 Exercises
p.584 ex. 3, 5, 7,9, 15

9.2 Exercises
p.592 ex. 1, 3, 5, 7, 11, 18, 19, 25, 27

9.3 Exercises

p. 600, ex. 1-10,
11-18,
23,
33

9.2 Exercises

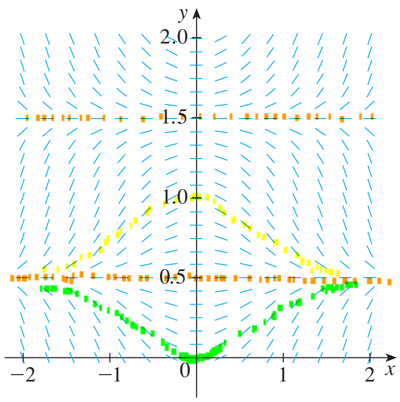
1. A direction field for the differential equation $y' = x \cos \pi y$ is shown.

(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

(i) $y(0) = 0$ (ii) $y(0) = 0.5$

(iii) $y(0) = 1$ (iv) $y(0) = 1.6$

(b) Find all the equilibrium solutions.



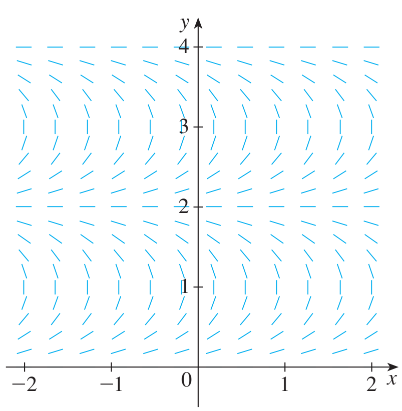
2. A direction field for the differential equation $y' = \tan\left(\frac{1}{2}\pi y\right)$ is shown.

(a) Sketch the graphs of the solutions that satisfy the given initial conditions.

(i) $y(0) = 1$ (ii) $y(0) = 0.2$

(iii) $y(0) = 2$ (iv) $y(1) = 3$

(b) Find all the equilibrium solutions.



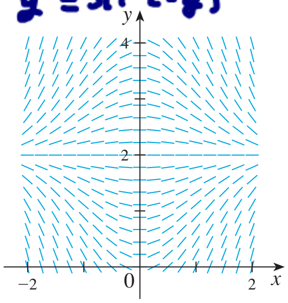
3–6 Match the differential equation with its direction field (labeled I–IV). Give reasons for your answer.

3. $y' = 2 - y$ **4.** $y' = x(2 - y)$

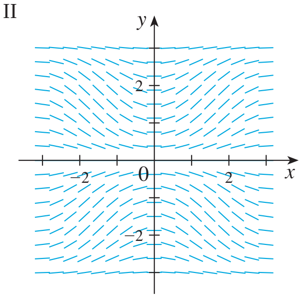
5. $y' = x + y - 1$ **6.** $y' = \sin x \sin y$

$y' = x(2-y)$

I

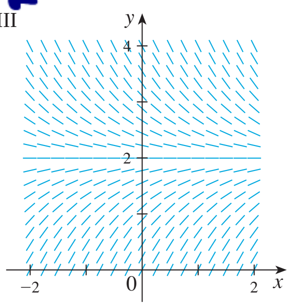


II

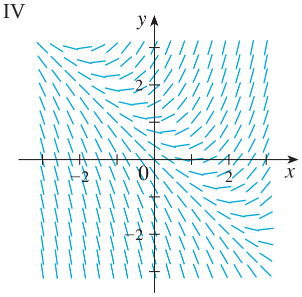


$y' = 2-y$

III



IV



7. Use the direction field labeled II (above) to sketch the graphs of the solutions that satisfy the given initial conditions.

(a) $y(0) = 1$ (b) $y(0) = 2$ (c) $y(0) = -1$

8. Use the direction field labeled IV (above) to sketch the graphs of the solutions that satisfy the given initial conditions.

(a) $y(0) = -1$ (b) $y(0) = 0$ (c) $y(0) = 1$

9–10 Sketch a direction field for the differential equation. Then use it to sketch three solution curves.

9. $y' = \frac{1}{2}y$ **10.** $y' = x - y + 1$

11–14 Sketch the direction field of the differential equation. Then use it to sketch a solution curve that passes through the given point.

11. $y' = y - 2x$, $(1, 0)$ **12.** $y' = xy - x^2$, $(0, 1)$

13. $y' = y + xy$, $(0, 1)$ **14.** $y' = x + y^2$, $(0, 0)$

CAS **15–16** Use a computer algebra system to draw a direction field for the given differential equation. Get a printout and sketch on it the solution curve that passes through $(0, 1)$. Then use the CAS to draw the solution curve and compare it with your sketch.

15. $y' = x^2 \sin y$ **16.** $y' = x(y^2 - 4)$

Graphing calculator or computer required CAS Computer algebra system required **1.** Homework Hints available at stewartcalculus.com

Ex.1, p.600 (Stewart)

$$\frac{dy}{dx} = xy^2$$

$y = 0$?

$$dy = xy^2 dx, \quad y \neq 0$$

$$y^{-2} dy = x\, dx$$

$$\int y^{-2} dy = \int x\, dx$$

$$-\frac{1}{y} = \frac{x^2}{2} + C$$

the general solution:

$$\boxed{\begin{gathered} y = \frac{1}{-\frac{x^2}{2} + C} \\ y = 0 \end{gathered}}$$

Ex.4, p.600 (Stewart)

$$(y^2 + xy^2)\, y' = 1$$

$$y^2(1+x)\frac{dy}{dx} = 1$$

$1+x \neq 0$

$$y^2\, dy = \frac{dx}{1+x}$$

$$\frac{y^3}{3} + C = \ln|1+x|$$

$$\ln|1+x| = \frac{y^3}{3} + c,$$

$$|1+x| = e^{\frac{y^3}{3}+c}, \quad c \in \mathbb{R}$$

$$1+x = \pm e^{c} \cdot e^{\frac{y^3}{3}}$$

$$1+x = c \cdot e^{\frac{y^3}{3}}$$

$$x = -1 + c \cdot e^{\frac{y^3}{3}}, \quad c \in \mathbb{R}^*$$

Ex.7, p.600 (Stewart)

$$\frac{dy}{dt} = \frac{t}{ye^{y+t^2}}$$

$$dy = \frac{t}{y e^y \cdot e^{t^2}} \cdot dt$$

$$\int y e^y \, dy = \int t e^{-t^2} dt$$

$$ye^y - e^y + C = -\frac{1}{2} e^{-t^2}$$

the general solution:

Ex.8, p.600 (Stewart)

$$\frac{dy}{d\theta} = \frac{e^{y}\sin^2\theta}{y\sec\theta}$$

$$\int y e^{-y}\,dy = \int \sin^2\theta\cdot\cos\theta\,d\theta$$

$$-ye^{-y} - e^{-y} + C = \frac{1}{3}\sin^3\theta$$

Ex.14, p.600 (Stewart)

$$y' = \frac{xy\sin x}{y+1}, \qquad y(0) = 1$$

$$\frac{dy}{dx} = \frac{y\, x\sin x}{y+1} \qquad : \frac{y}{y+1} \text{ or } \cdot \frac{y+1}{y}$$

$$\left(1+\frac{1}{y}\right)dy = x\sin x\, dx$$

$$y + \ln|y| = -x\cos x + \sin x + C$$

$$y = 0$$

$$y(0) = 1 \Rightarrow C = 1,$$

the particular solution:

y +ln|y| = -x cos x + sin x + 1

Ex.16, p.600 (Stewart)

$$\frac{dP}{dt} = \sqrt{Pt} \quad , \quad P(1) = 2$$

$$P^{-\frac{1}{2}} dP = t^{\frac{1}{2}} dt$$

$$2\sqrt{P} = \frac{2}{3} t\sqrt{t} + C$$

$$\sqrt{P} = \frac{1}{3} t\sqrt{t} + c$$

$$\sqrt{2} = \frac{1}{3} + c \qquad \sqrt{P} = \frac{1}{3}\left(t\sqrt{t} + 3\sqrt{2} - 3\right)$$

$$c = \frac{3\sqrt{2} - 3}{3}$$

Ex.17, p.600 (Stewart)

$$y' \operatorname{tg} x = a + y, \quad y\left(\frac{\pi}{3}\right) = a, \quad 0 < x < \frac{\pi}{2}$$

$$\frac{dy}{dx} \operatorname{tg} x = a + y$$

$y = -a$?

$$\frac{dy}{a+y} = \operatorname{ctg} x \, dx$$

$$\ln|a+y| = \ln|\sin x| + C$$

$$|a+y| = e^{\ln|\sin x| + C}$$

$a + y = c \cdot \sin x$ $\qquad y = -a$

$y = -a + c\sin x, \quad c \in \mathbb{R}$

$y\left(\frac{\pi}{3}\right) = a \qquad c - ?$

$a = -a + c\sin\frac{\pi}{3}$

$2a = c \cdot \frac{\sqrt{3}}{2}$

$c = \frac{4}{\sqrt{3}}a$

$y = -a + \frac{4a\sqrt{3}}{3}\sin x$ the particular

Ex.23a, p.600 (Stewart)

$$y' = 2x\sqrt{1-y^2}$$

$$\frac{dy}{dx} = 2x\sqrt{1-y^2}$$

$$\frac{dy}{\sqrt{1-y^2}} = 2x\,dx$$

$$\arcsin y = x^2 + C$$

$$y = \pm 1$$

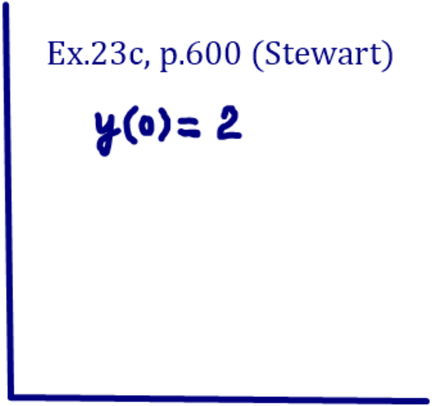
Ex.23b, p.600 (Stewart)

$$y' = 2x\sqrt{1-y^2}$$

$$y(0) = 0$$

Ex.23c, p.600 (Stewart)

$$y(0) = 2$$



$$\arcsin y = x^2 + C$$

$$y = \sin(x^2 + C)$$

$$y = \pm 1$$