Homework 2

Due 18:00, Friday, September 29, 2023

Problem 2.1

Prove the following statement by proving its contrapositive: if r is irrational, then $r^{1/5}$ is irrational. Be sure to state the contrapositive explicitly.

Problem 2.2

Prove by contradiction that $\log_4 6$ is irrational.

Problem 2.3

Identify exactly where the bugs are in the following bogus proof:
Bogus Claim: If a and b are two equal real numbers, then $a = 0$.
Proof:

$$\begin{aligned} a &= b \\ a^2 &= ab \\ a^2 - b^2 &= ab - b^2 \\ (a-b)(a+b) &= (a-b)b \\ a+b &= b \\ a &= 0. \end{aligned}$$

Problem 2.4

Write out $((\alpha \vee \beta) \wedge (\beta \to \alpha)$ using only $\neg$ and $\to$.

Problem 2.5

Write the truth table for the following formulas (keep in mind the precedence rules):

a) $((A_1 \wedge A_2) \to A_3) \to (A_1 \vee A_2)$;

b) $(\neg A_5 \to A_2) \to (A_5 \wedge A_2)$;

c) $(A_1 \wedge A_2 \to A_3 \wedge A_4) \longleftrightarrow (\neg(A_1 \vee A_3) \to \neg(A_2 \vee A_4))$.

Problem 2.6

Show that $(\alpha \to \beta) \vee (\beta \to \alpha)$ is a tautology.

Problem 2.7

Prove Contrapositive and Composed Contrapositive Inference Rules.

Problem 2.8

Is the following logical deduction true?
If detective Jones haven't seen Smith last night , then either Smith is the murderer, or Jones is lying. If Smith is not the killer, then Jones haven't seen Smith last night and the murder happened after midnight. If the murder happened after midnight, then either Smith is the murderer or Jones is lying. Therefore, Smith is the murderer.

Problem 2.9

Answer the following questions:

a) Write the negation of the statement: "All prime numbers are odd."

b) Write the contrapositive of the statement: "If a set is finite, then it is countable."

c) Write the converse of the statement: "You pass the exam, if you study hard."

d) Write down the negation of the statement: "There are people who do not study Special Math."

e) Write down the negation of the statement: $\forall x \in \mathbb{R} \quad \exists y \in \mathbb{Z}$ such that $(x + y) \geq 0$. Is the original statement true or false?