Homework 3

Due 18:00, Friday October 6, 2023

Problem 3.1

Five people get on the elevator that stops at five floors. In how many ways they can get off? For example, one person gets off at the first floor, two will get off at the third, and the remaining two at the fifth floor. In how many ways they can get off at different floors? Now, consider that people in elevator have names, say A, B, C, D, and E, assuming that, for example, the case A on the first floor is different from the case B on the first floor. Answer the previous questions with this assumption.

Problem 3.2

A lady wishes to color her fingernails on one hand using at most two of the colors: red, yellow, and blue. In how many ways she can do it?

Problem 3.3

Consider the word

$$EFFERVESCENCE$$

a) How many different words (including nonsense words) can you make by rearranging the letters of this word?
b) How many different 4 letter words (including nonsense words) can be made from the letters of the above word, if letters cannot be repeated?
c) How many different 4 letter words (including nonsense words) can be made from the letters of the above word, if letters can be repeated?

Problem 3.4

On a computer system the valid password should start with a letter (26 letters at all), it is case sensitive (upper case is different form lowercase), its length is from 4 up to 8, and symbols are
{1,2,3,..,9,0,a,b,c,..z,A,B,C,..,Z,?,!,#,%,&,*,/}. How many such passwords can be created?

Problem 3.5

An urn contains 13 balls of which 9 are red and 4 are white. 4 balls are selected from the urn. Answer the following questions:
a) How many (different) samples (of size 4) are possible?
b) How many samples (of size 4) consist entirely of red balls?
c) How many samples have 2 red and 2 white balls?
d) How many samples (of size 4) have exactly 3 red balls?
e) How many samples (of size 4) have at least 3 red?
f) How many samples (of size 4) contain at least one red ball?

Problem 3.6

Solve the following problems using the Pigeonhole Principle. For each problem, try to identify the pigeons, the pigeonholes, and a rule assigning each pigeon to a pigeonhole.

a) In every set of 100 integers, there exist two whose difference is a multiple of 37.

b) For any 5 points inside a unit square (not on the boundary), there are 2 points at distance less than $\frac{1}{\sqrt{2}}$.

c) Show that if $n + 1$ numbers are selected from $\{1, 2, 3, \ldots, 2n\}$, two must be consecutive, that is, equal to k and $k + 1$ for some k.

Problem 3.7

How many of the billion numbers in the range from 1 to 10^9 contain the digit 1?

Problem 3.8

Here are the solutions to the next 8 questions, in no particular order.

$$n^m, \quad m^n, \quad \frac{n!}{(n-m)!}, \quad \binom{n+m}{m}, \quad \binom{n-1+m}{m}, \quad \binom{n-1+m}{n}, \quad 2^{mn}$$

Answer all the questions. Justify your answers.

(a) How many solutions over the natural numbers are there to the inequality

$$x_1 + x_2 + \ldots + x_n \leq m \quad ?$$

(b) How many length m words can be formed from an $n-$letter alphabet, if no letter is used more than once?

(c) How many length m words can be formed from an $n-$letter alphabet, if letters can be reused?

(d) In how many ways you can connect elements from set A to elements from set B when $|A| = m$ and $|B| = n$? One element from A can be connected to as many elements from B as possible.

(e) How many injections are there from set A to set B, where $|A| = m$ and $|B| = n$, and $n > m$?

(f) How many ways are there to place a total of m **distinguishable** balls into n **distinguishable** urns, with some urns possibly empty or with several balls?

(g) How many ways are there to place a total of m **indistinguishable** balls into n **distinguishable** urns, with some urns possibly empty or with several balls?

(h) How many ways are there to put a total of m **distinguishable** balls into n **distinguishable** urns with at most one ball in each urn?

Problem 3.9

Bonus Problem (for extra credit)

How many times will the instruction B in the following code be executed?

```
For k := 1 to n
     For j := 1 to k
         For i := 1 to j
               B(i,j,k)
         end for
     end for
end for
```