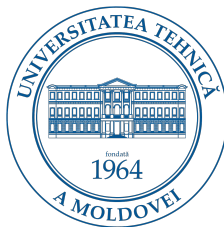


Mathematical Analysis II

conf.univ., dr. Elena Cojuhari

elena.cojuhari@mate.utm.md

Technical University of Moldova



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Adv. Eng. Math.

section 1.4

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Exercises, p. 26-27

EXACT ODE

$$M(x,y) dx + N(x,y) dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$V(x,y) = C : \frac{\partial V}{\partial x} = M(x,y)$$

$$\frac{\partial V}{\partial y} = N(x,y)$$

Ex.1

$$x dx + y dy = 0$$

$$x^2 + y^2 = C$$

Ex. 2

$$(2x + 3x^2y) dx + (x^3 - 3y^2) dy = 0$$

$$M(x, y) = 2x + 3x^2y$$

$$N(x, y) = x^3 - 3y^2$$

Ex. 2

$$(2x + 3x^2y) dx + (x^3 - 3y^2) dy = 0$$

$$M'_y(x, y) = (2x + 3x^2y)'_y = 3x^2$$

$$N'_x(x, y) = (x^3 - 3y^2)'_x = 3x^2 \Rightarrow \text{Exact DE}$$

$$V(x, y) = ? \quad \frac{\partial V}{\partial x} = M(x, y)$$

$$\frac{\partial V}{\partial y} = N(x, y)$$

Ex. 2

$$(2x + 3x^2y) dx + (\underline{x^3 - 3y^2}) dy = 0$$

$$\frac{\partial V}{\partial x} = 2x + 3x^2y$$

$$V(x, y) = x^2 + x^3y + \varphi(y)$$

$$\frac{\partial V}{\partial y} = N(x, y) = x^3 - 3y^2$$

$$\frac{\partial V}{\partial y} = (x^2 + x^3y + \varphi(y))'_y =$$

$$\frac{\partial V}{\partial y} = 0 + x^3 + \varphi'(y) = x^3 - 3y^2 \Rightarrow \varphi'(y) = -3y^2$$

CONT.

$$\varphi'(y) = -3y^2$$

$$\varphi(y) = -y^3 + C$$

$$V(x, y) = x^2 + x^3 y + \varphi(y)$$

the general solution:

$$x^2 + x^3 y - y^3 = C$$

Ex.3

Verify that the DE

$$(2x + \sin y - ye^{-x})dx + (x \cos y + \cos y + e^{-x})dy = 0$$

is exact and find its solution curves.

Solution.

$$M'_y(x, y) = (2x + \sin y - ye^{-x})'_y =$$
$$= \underline{0 + \cos y - e^{-x}}$$

$$N'_x(x, y) = (x \cos y + \cos y + e^{-x})'_x = \underline{\cos y + 0 - e^{-x}}$$

Exact DE

$V(x, y) - ?$ $dV =$ the left part of the given equation

$$V'_x = M(x, y), \quad V'_y = N(x, y)$$

$$V'_x = 2x + \sin y - y e^{-x}$$

$$V(x, y) = x^2 + x \sin y + y e^{-x} + \varphi(y)$$

$$V'_y(x, y) = x \cos y + e^{-x} + \varphi'(y)$$

//

$$\left\{ \begin{array}{l} \varphi'(y) = \cos y \\ \downarrow \\ \varphi(y) = \sin y + C \end{array} \right.$$

$$N(x, y) = x \cos y + \cos y + e^{-x}$$

$$A: \quad x^2 + x \sin y + y e^{-x} + \sin y = C.$$

Ex.12, p.26 Adv. Eng. Math.

$$(2xy dx + dy) e^{x^2} = 0, \quad y(0) = 2$$

$$M'_y(x, y) = (2xy e^{x^2})'_y = 2x e^{x^2} //$$

Exact DE

$$N'_x(x, y) = (e^{x^2})'_x = 2x e^{x^2}$$

$$\frac{\partial v}{\partial x} = 2xy e^{x^2} \Rightarrow v(x, y) = y e^{x^2} + h(y)$$

$$v'_y(x, y) = (y e^{x^2} + h(y))'_y = \underbrace{e^{x^2} + h'(y)}_{e^{x^2}} = e^{x^2} \Rightarrow$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = C \Rightarrow A: y e^{x^2} = C$$

Ex.12, p.26 Adv. Eng. Math.

$$(2xy dx + dy) e^{x^2} = 0, \quad \underline{y(0)=2}$$

the general solution is

$$ye^{x^2} = C$$

$$2 \cdot e^0 = C \Rightarrow C=2$$

the particular solution is

$$ye^{x^2} = 2 .$$

