Mathematics for Computer Science

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Lecture 20

Joke

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Question: What is common between Shakespeare and monkeys?

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Answer:

Joke

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Answer: Infinite monkey theorem.

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and the **"monkey"** is not an actual monkey, but a metaphor for an abstract device that produces a random sequence of letters and symbols ad infinitum.

The probability of a monkey exactly typing a complete work such as Shakespeare's Hamlet is so tiny that the chance of it occurring during a period of time even a hundred thousand orders of magnitude longer than the age of the universe is extremely low, but not actually zero.

Infinite monkey theorem: history

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To which Borges adds:
"Strictly speaking, one immortal monkey would suffice."

Infinite monkey theorem: popular quotes

In the early 20th century, Emile Borel and Arthur Eddington used the theorem to illustrate the timescales implicit in the foundations of statistical mechanics.

Some other quotes reffering to monkeys and typewriters:

> *It's just the Internet.*
> *A million monkeys with typewriters could run it.*
> *Simon Higgs*

> *Come to think of it, there are already a billion monkeys on a billion typewriters, and Internet is NOTHING like Shakespeare.*

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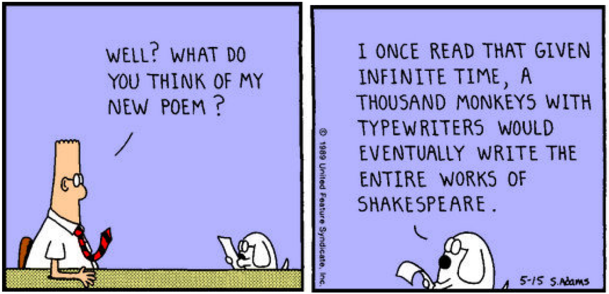

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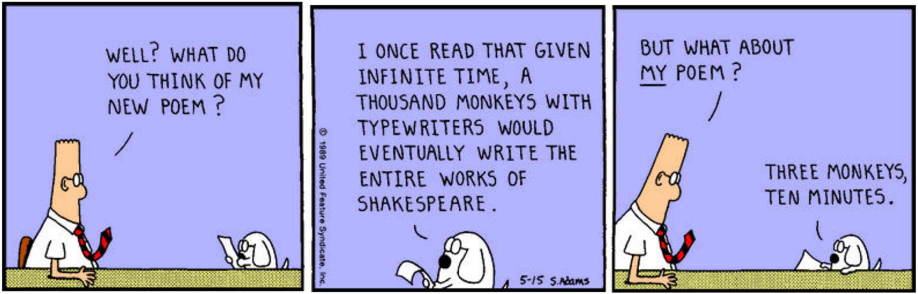

Experiment with monkeys

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As result six monkeys produced five pages,

after a while they started to attack the keyboard with a stone, and continued by urinating.

Experiment with monkeys: monkeys work, pages 1 and 2

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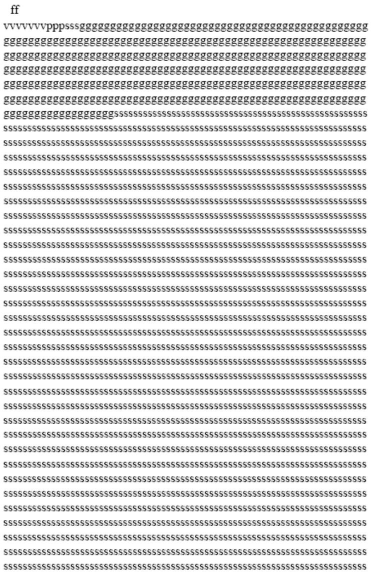
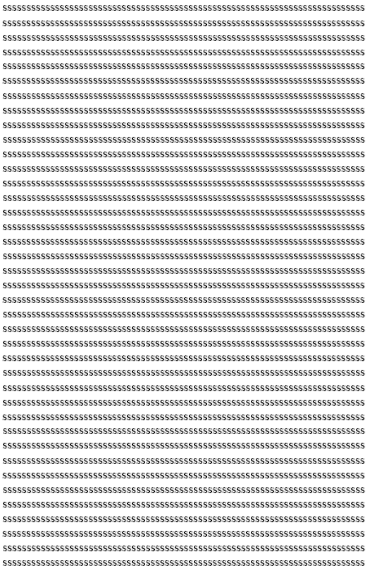
Experiment with monkeys: monkeys work, pages 1 and 2





Experiment with monkeys: monkeys work, pages 3 and 4

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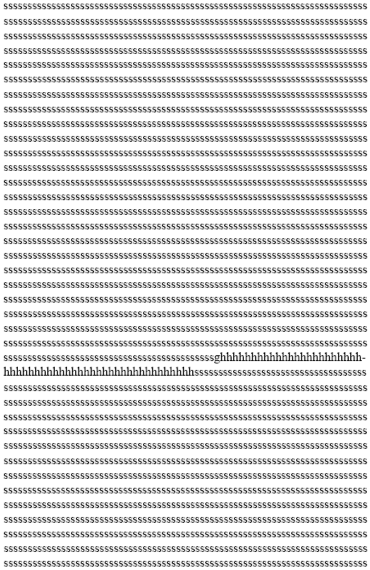
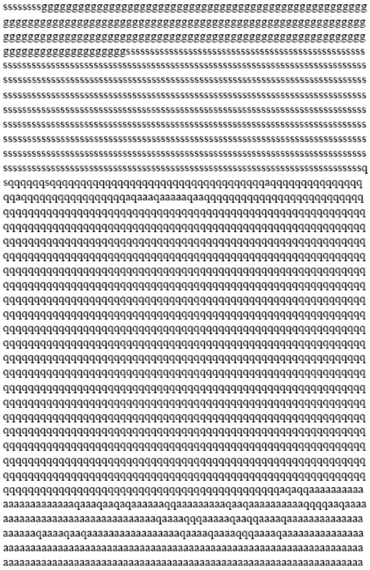
Experiment with monkeys: monkeys work, pages 3 and 4





Experiment with monkeys: monkeys work, page 5

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Experiment with monkeys

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Conclusion of this practical experiment:

Experiment with monkeys

Conclusion of this practical experiment:

monkeys have poor keyboard skills

Infinite monkey theorem

The proof of this theorem is straightforward:

Recall that if two events are independent, then the probability of both happening equals the product of the probabilities of each one happening independently:

$$P(A \cap B) = P(A) \cdot P(B)$$

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Therefore, the chance of the first six letters matching **'banana'** is

$$\frac{1}{50} \cdot \frac{1}{50} \cdot \frac{1}{50} \cdot \frac{1}{50} \cdot \frac{1}{50} \cdot \frac{1}{50} = \left(\frac{1}{50}\right)^6 = \frac{1}{15625000000}$$

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For the same reason, the probability that the next 6 letters match '**banana'** is also $(1/50)^6$, and so on.

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From the above, the probability of **not typing** banana in a given block of 6 letters is $1 - (1/50)^6$.

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Because each block is typed independently, the chance X_n of not typing **banana** in any of the first n blocks of 6 letters is

$$X_n = \left(1 - \frac{1}{50^6}\right)^n.$$

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From the above, the probability of **not typing** banana in a given block of 6 letters is $1 - (1/50)^6$.

Because each block is typed independently, the chance X_n of not typing **banana** in any of the first n blocks of 6 letters is

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As n approaches infinity, the probability X_n approaches zero.

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When we consider $100 \cdot 10^9$ monkeys, the probability falls to 0.17%,

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the probability of the monkeys replicating even a short book is nearly zero!.

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Because the probability shrinks exponentially, at 20 letters it already has only probability

$$\frac{1}{26^{20}} = \frac{1}{19928148895209409152340197376} \approx 5 \cdot 10^{-29}$$

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In the case of the entire text of Hamlet, the probabilities are so small that they can barely be conceived in human terms.

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Even if the observable universe were filled with monkeys the size of atoms typing from now until the heat death of the universe, their total probability to produce a single instance of Hamlet would still be a great many orders of magnitude less than $10^{-183,800}$.

Infinite monkey theorem

If complete Hamlet is too big, how about something shorter?

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Therefore probability of not typing the famous phrase in one attempt is $1 - \frac{1}{32^{41}}$

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- 1 line per second
- 60 seconds per minute= 60 lines per minute
- 60 minutes per hour $=$ 3600 lines per hour
- 24 hours per day= 86400 lines per day
- 365.days per year $= 31,556,736$ lines per year

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- for a year straight $= \left(\left(\left(\left(1 - \frac{1}{32^{41}}\right)^{60}\right)^{60}\right)^{24}\right)^{365}$

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How big is this number:

$$\left(\left(\left(\left(1-\frac{1}{32^{41}}\right)^{60}\right)^{60}\right)^{24}\right)^{365} = \left(1-\frac{1}{32^{41}}\right)^{31{,}536{,}000}$$

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Suppose age of universe is $2^{34} = 17$ billions years. Actually it is 14 billions.

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0.999999999999465759379507781960794856828386656482
6413218810429932659614297586787965691641697343362 8

Infinite monkey theorem

It's about 99.999999999995% **sure that they would fail to produce the sentence.**

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This is from their textbook on thermodynamics, the field whose statistical foundations motivated the first known expositions of typing monkeys.