Mathematical analysis I

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Subsection 1

Functions of Several Variables

Functions of Several Variables

Stewart, ch.14

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}$

$\mathcal{D} \subseteq \mathbb{R}^2$

$V = \pi R^2 H$

- A **function** f **of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set $\mathcal{D}$ a unique real number $f(x, y)$.
- The set $\mathcal{D}$ is the **domain** of f and its **range** is the set of values that f takes on, i.e., the set $\{f(x, y) : (x, y) \in \mathcal{D}\}$. $\subseteq \mathbb{R}$
- The variables x, y are called **independent variables** and $z = f(x, y)$ is the **dependent variable**.
- If $f(x, y)$ is specified by a formula, then the domain is understood to be the set of all pairs (x, y) for which the given formula yields a well defined real number.

Finding and Graphing the Domain

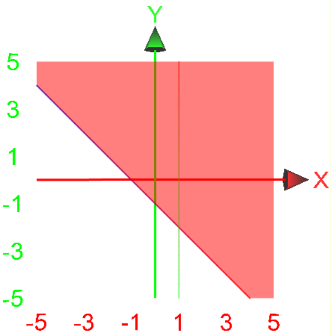
- Find and graph the domain of $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$.

  The domain of $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$ is specified by enforcing the following conditions:

  - $x+y+1 \geq 0$, giving $y \geq -x-1$;
  - $x-1 \neq 0$, giving $x \neq 1$.

  Thus, the domain is $\mathcal{D} = \{(x,y) : y \geq -x-1 \text{ and } x \neq 1\}$.

range |R



Another Example of a Domain

- Find and graph the domain of $f(x,y) = x\ln(y^2 - x)$.

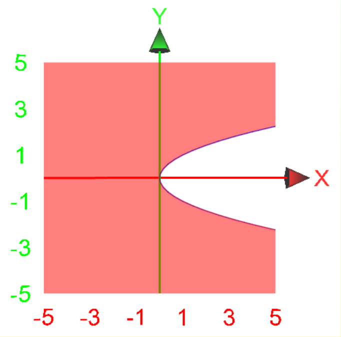
  The domain of $f(x,y) = x\ln(y^2 - x)$ is specified by enforcing the following condition:

  - $y^2 - x > 0$, giving $y^2 > x$.

  Thus, the domain is

  $$\mathcal{D} = \{(x,y) : y^2 > x\}.$$

  range |R



A Third Example of a Domain

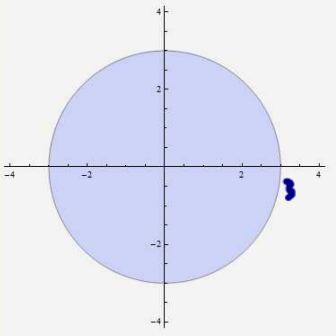
- Find and graph the domain of $f(x, y) = \sqrt{9 - x^2 - y^2}$.

  The domain of $f(x, y) = \sqrt{9 - x^2 - y^2}$ is specified by enforcing the following condition:

  - $9 - x^2 - y^2 \geq 0$, giving $x^2 + y^2 \leq 9$.

  Thus, the domain is

  $$\mathcal{D} = \{(x, y) : x^2 + y^2 \leq 9\}.$$



range [0,3]

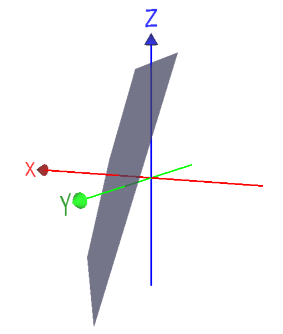
Graphs of Functions of Two Variables

- If $f(x, y)$ is a function of two variables, with domain $\mathcal{D}$, the **graph** of f is the set of points

$$G_f = \{(x, y, z) \in \mathbb{R}^3 : z = f(x, y), (x, y) \in \mathcal{D}\}.$$

- The graphs of functions of two variables are 3-dimensional surfaces.

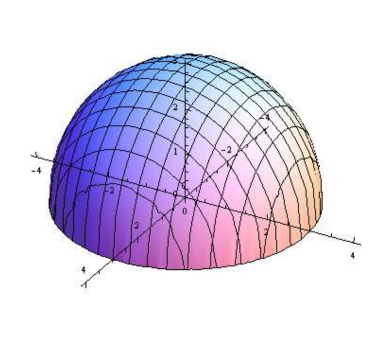
Example: Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.
$3x + 2y + z = 6$ is the equation of a plane in space. $\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$
It intersects the coordinate axes at the points $(2, 0, 0)$, $(0, 3, 0)$, $(0, 0, 6)$.



$y = f(x)$
$G_f = \{(x, y) : y = f(x),\ x \in \mathcal{D}_f\}$

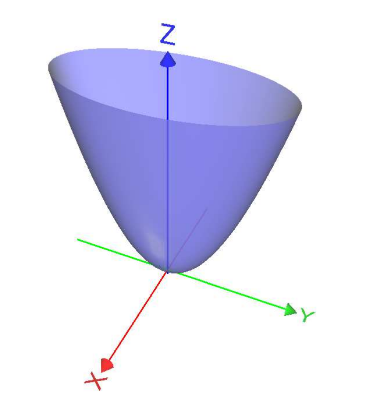
A Second Graph

- Sketch the graph of the function $f(x,y) = \sqrt{9 - x^2 - y^2}$. $z \geq 0$
  Rewriting $z = \sqrt{9 - x^2 - y^2}$ as $x^2 + y^2 + z^2 = 9$, we get the equation of a sphere with center at the origin and radius 3. But the positive square root allows only the upper hemisphere.



A Third Graph

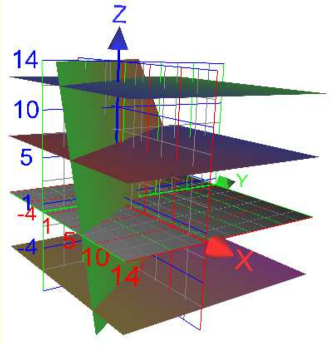
- Sketch the graph of the function $f(x, y) = 4x^2 + y^2$. $D_f = \mathbb{R}^2$ $E_f = \mathbb{R}_+$ range
  Calculating traces, we see that $z = 4x^2 + y^2$ is the equation of an elliptic paraboloid.

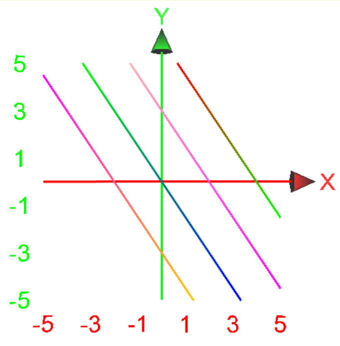


Level Curves

- The **level curves** of a function $f(x, y)$ of two variables are the curves with equations $f(x, y) = c$, where c is a constant in the range of f.

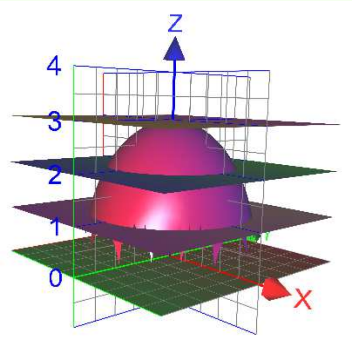
  Example: Sketch the level curves of the function $f(x, y) = 6 - 3x - 2y$ for $c = -6, 0, 6, 12$.

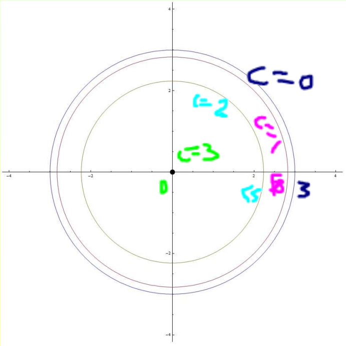




Level Curves: Second Example

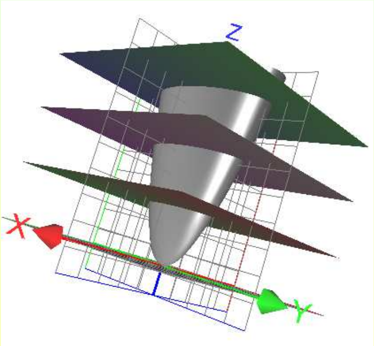
- Sketch the level curves of the function $f(x, y) = \sqrt{9 - x^2 - y^2}$ for $c = 0, 1, 2, 3$.

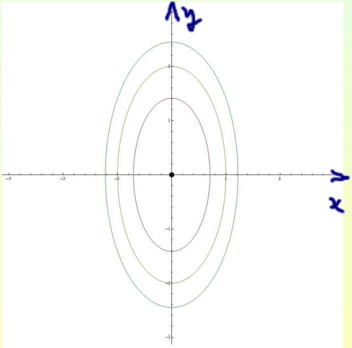




Level Curves: Third Example

- Sketch the level curves of the function $f(x, y) = 4x^2 + y^2$ for $c = 0, 2, 4, 6$.





Functions of Three Variables

$f: \mathbb{R}^3 \longrightarrow \mathbb{R}$

- A **function of three variables** $f(x, y, z)$ is a rule that assigns to each ordered triple (x, y, z) in a domain $\mathcal{D}$ a unique real number $f(x, y, z)$.

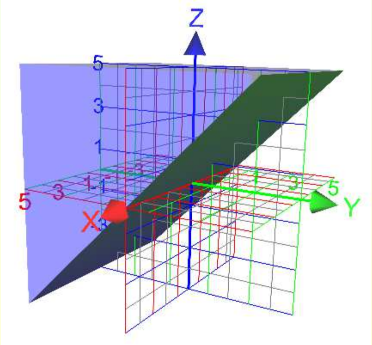
  Example: What is the domain $\mathcal{D}$ of the function

$$f(x, y, z) = \ln(z - y) + xy \sin z?$$

We must have $z - y > 0$, i.e., $z > y$. Thus, the domain of f is the following half-space

$$\mathcal{D} = \{(x, y, z) \in \mathbb{R}^3 : z > y\}$$

of $\mathbb{R}^3$:



$w = f(x,y,z)$

$G_f = \{ (x,y,z,w) :$ w=f(x,y,z), (x,y,z) $\in D_f \}$

hypersurface in 4-dimensional space

level surface, f(x,y,z)=c, c belongs to the range of f

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functions of n variables

$z = f(x_1, x_2, \dots, x_n)$

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$ hypersurface

$D_f \subseteq \mathbb{R}^n \quad , \quad E_f \subseteq \mathbb{R}$