Tube-Based Robust Model Predictive Control Law for Uncertain Nonlinear Wheeled Inverted Pendulum Systems

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***Abstract* – This work presents the problem of tube-based robust model predictive controller for a class of continuous-time systems with input disturbance. It is able to obtain that the state trajectory of the uncertain system is maintained inside a sequence of tubes. An estimation of attraction region of the closed system is pointed out by using input state stability (ISS) theory. The theoretical analysis and simulation results demonstrate the performance of the proposed algorithm for a wheeled inverted pendulum system.**

Keywords - Tube-based robust MPC, continuous-time nonlinear systems, wheeled inverted pendulum.

INTRODUCTION

The dynamic model and several control methods are presented in [1]. This model is separated into two subsystems and authors in [2], [3] approached to control inverse pendulum based on energy function. Olfati-Saber [4] proposed coordinate transformation to change Cart-Pole model to strict forward system to design a control law based on nest saturation technique. It is hard to obtain a controller because some parameters are uncertainties or disturbances appear. Therefore, K.D.Do [5] propose a control scheme by using the similar transformation in [4] and combines with nest saturation method, disturbance observer. Researchers in [6] apply properties of nonholomic system and backstepping technique, but their drawback is assumption of satisfying state constraints. The instantaneous switching of control input has been proposed in [7], nevertheless, position of Cart is not able to stabilize at desired point. The papers in [8] researching into adaptive control have some weakness which is time-dependent inertia matrix, leading to the wrong of Lyapunov function control candidate. In [10], authors proposed a robust adaptive law with time-dependent inertia matrix and a new controller to control WIP system with uncertain parameters based on Backstepping and a separation technique. In [9], the proposed controller utilize the H – infinity method (Vanderschaft) for rotation motion and the additional controller is proposed to straight motion subsystem. In [11] – [15], Mayne *et al.* have been proposed a robust model predictive control law for nonlinear system with external disturbance that contains two components: a nominal controller create central path and an ancillary controller ensure that steer follows the trajectories of the uncertain system to the central path. Tube-based robust MPC has been proposed in [11]-[15] to deal with disturbance and ensure that state of the uncertain system differs from state of the nominal system. Therefore, the state trajectory of the uncertain system is maintained inside a sequence of tubes [11]-[15]. However, almost previous controllers have been implemented based on discrete-time systems and we develop these approaches for continuous time systems. This work propose a control law for uncertain wheeled inverted pendulum systems based on tube-based robust model predictive controller.

PROBLEM STATEMENTS

We consider a class of uncertain continuous-time nonlinear systems with the following equation:

$$\dot{x}(t) = f\left(x(t);u(t)\right) + B_{\omega}.\omega(x,t); x(0) = x_0 \quad (1)$$

Where $w(t) \in \mathbb{R}^p$ is the exogenous disturbances. The system (1) subject to the flowing set of constrains:

$$\left(x(t);u(t);w(x,t)\right) \in X \times U \times W \subseteq \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \quad (2)$$

Where $X;U$ are compact sets and both contain the origin as an interior point, $W = \left\{w \in \mathbb{R}^p : \|w\| \le w_{\max}\right\}; \forall t \ge 0$.

Definition: $\begin{array}{l} A \oplus B = \left\{x + y : x \in A; y \in B\right\}; \\ A \Theta B = \left\{x \in A : x \oplus B \subseteq A\right\} \end{array}$.

Assumption 1: $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a twice continuously differentiable function with $f(0;0) = 0$. $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n; B_w \in \mathbb{R}^{n \times p}$ are known function and known constant matrix, respectively.

Lemma 1: Let $V : \mathbb{R}^n \to \left[0;+\infty\right)$ be a continuously differentiable function and $\alpha_1\left(\|x\|\right) \le V(x) \le \alpha_2\left(\|x\|\right)$, where $\alpha_1;\alpha_2$ are class K_∞ functions. Suppose $u : \mathbb{R} \to \mathbb{R}^m$ is chosen, and there exist the positive numbers $\lambda;\mu$ such that:

$\frac{d}{dt}V\left(x(t)\right)+\lambda V\left(x(t)\right)-\mu w(x,t)^T w(x,t)\le 0;$ with $\forall w(x,t)\in W$ (3)

$x\in X$. Then, the system trajectory starting from $x(t_0)\in\Xi(x)$ will remain in $\Xi(x)$,

where: $\Xi(x)=\left\{x:V(x)\le\frac{\mu w_{\max}^2}{\lambda}\right\}$

Proof:
We have:

$$(3)\Leftrightarrow\frac{d}{dt}\left(e^{\lambda t}V\left(x(t)\right)\right)\le\mu e^{\lambda t}w(x,t)^T w(x,t)\Rightarrow$$
$$V\left(x(t)\right)\le$$
$$e^{-\lambda(t-t_0)}V\left(x(t_0)\right)+\mu e^{-\lambda t}\int_{t_0}^{t}e^{\lambda t}w(x,\tau)^T w(x,\tau)d\tau$$

Due to $w(x,\tau)^T w(x,\tau)\le w_{\max}^2$ then:

$$V\left(x(t)\right)\le e^{-\lambda(t-t_0)}V\left(x(t_0)\right)+\mu e^{-\lambda t}w_{\max}^2\int_{t_0}^{t}e^{\lambda t}d\tau$$
$$=e^{-\lambda(t-t_0)}\left[V\left(x(t_0)\right)-\frac{\mu w_{\max}^2}{\lambda}\right]+\frac{\mu w_{\max}^2}{\lambda}$$
$$\Rightarrow V\left(x(t)\right)\le\frac{\mu w_{\max}^2}{\lambda}$$

In order to design tube-based robust MPC, we consider a nominal system: $\dot{\bar{x}}=f(\bar{x};\bar{u})$ (4)

Define $\{t_k\}$ with k being nonnegative integers as the set of sampling instants with $t_{k+1}-t_k=\xi>0$. In this paper, we will design a tube-based RMPC consisting of a nominal input and a state feedback control:

$$u(t)=\bar{u}(t)+K\left(x(t)-\bar{x}\left(t;\bar{x}_0(t_k)\right)\right)\qquad(5)$$

where $\bar{x}\left(t_k;\bar{x}_0(t_k)\right)=\bar{x}_0(t_k);\bar{x}\left(t;\bar{x}_0(t_k)\right);t\in(t_k;t_{k+1})$ and $\bar{u}(t);t\in[t_k;t_{k+1})$ are the state and control of the nominal system with initial state $\bar{x}_0(t_k)$ to be designed, K is a matrix gain of linear feedback control to be designed.

Define $A=\left.\frac{\partial f(x;u)}{\partial x}\right|_{(0;0)};B=\left.\frac{\partial f(x;u)}{\partial u}\right|_{(0;0)}$, the system (1) can be rewritten:

$$\dot{x}(t)=Ax(t)+Bu(t)+g\left(x(t);u(t)\right)+B_w.w(x,t)\quad(6)$$
$$\dot{\bar{x}}(t)=A\bar{x}(t)+B\bar{u}(t)+g\left(\bar{x}(t);\bar{u}(t)\right)\qquad(7)$$

where $g\left(x(t);u(t)\right)=f\left(x(t);u(t)\right)-Ax(t)-Bu(t)$.

Define the error is $e(t)=x(t)-\bar{x}(t)$, we obtain:

$$\dot{e}(t)=(A+BK)e(t)+$$
$$\left[g\left(x(t);u(t)\right)-g\left(\bar{x}(t);\bar{u}(t)\right)\right]+B_w.w(x,t)\quad(8)$$

By using **assumption 1**, we have $g(x;u)$ being a twice continuously differentiable function, so there exists $p_1;p_2$ such that:

$$\left\|g(x;u)-g(\bar{x};\bar{u})\right\|\le p_1\|x-\bar{x}\|+p_2\|u-\bar{u}\|$$
$$\Leftrightarrow\left\|g(x;u)-g(\bar{x};\bar{u})\right\|\le\left(p_1+p_2\|K\|\right)\|e\|\quad(9)$$

Assumption 2: There exist positive define matrix $X\in\mathbb{R}^{n\times n}$, non-square matrix $Y\in\mathbb{R}^{m\times n}$, and $\lambda_0>0;\mu>0$ such that:

$$\begin{bmatrix}(AX+BY)^T+AX+BY+\lambda_0 X & B_w\\ B_w^{\ T} & -\mu I\end{bmatrix}\le 0\ (10)$$

With $X;Y$ is a solution of (10), define

$$P^*=X^{-1};K^*=YX^{-1}\qquad(11).$$

Assumption 3: There exist $\lambda>0$ such that :

$p_1+p_2\|K^*\|\le\frac{(\lambda_0-\lambda)\vartheta_{\min}(P^*)}{2\|P^*\|}$, where $\vartheta_{\min}(P^*)$ is the minimum eigenvalue of the matrix P^*.

Assumption 4: $A+BK^*$ is a Schur matrix.

This paper point out a robust invariant set of error modelling as described in the following lemma:

Lemma 2:

1. The set $\Omega=\left\{x\in\mathbb{R}^n:x^TP^*x\le\frac{\mu w_{\max}^2}{\lambda}\right\}$ is a robust invariant set for the error modelling: $\dot{e}(t)=(A+BK)e(t)+\left[g\left(x(t);u(t)\right)-g\left(\bar{x}(t);\bar{u}(t)\right)\right]+B_w.w(x,t)(8)$, if $K=K^*$.
2. For any matrices $R>0;Q>0$, there exists a constant $\varepsilon>0$ and a matrix $\bar{P}>0$ such that:

(i). The set $\Delta_\varepsilon=\left\{x\in\mathbb{R}^n:x^T\bar{P}x\le\varepsilon\right\}\subseteq X\Theta\Omega$ is an invariant set for the system: $\dot{x}=f(x;K^*x)$ such that: $K^*x\in U\Theta K^*\Omega;\forall x\in\Delta_\varepsilon$.

(ii). $\frac{d}{dt}\left(x^T\bar{P}x\right)\le-x^T\left(Q+K^{*T}RK^*\right)x$

Then $\frac{d}{dt}\left(x(t)^T\bar{P}x(t)\right)\le-x(t)^T\bar{Q}x(t)$. Since $\bar{P}>0;\bar{Q}>0$ then the set Δ_ε is an invariant set of the system: $\dot{x}=f(x;K^*x)$.

We denote $T = n\xi; n \in N^*$. For any $Q; R > 0$ in **lemma 2** , we obtain $\overline{P}$. Let the cost function $J(\overline{x}(k);\overline{u}(t))$ be defined by:

$$J(\overline{x}(t_k);\overline{u}(t)) = \frac{1}{2}\int_{t_k}^{t_k+T}\left[\overline{x}(\tau)^T Q\overline{x}(\tau) + \overline{u}(\tau)^T R\overline{u}(\tau)\right]d\tau + \overline{x}(t_k+T)^T \overline{P}\overline{x}(t_k+T)$$

The constraints of the optimization are defined by:

$$\Psi = \begin{Bmatrix} \overline{u}(\tau) \in U\Theta K^*\Omega; \overline{x}(\tau) \in X\Theta\Omega; \\ \forall \tau \in [t_k; t_{k+1}); x(t_k+T) \in \Delta_\varepsilon; x(t_k) \in \overline{x}(t_k) \oplus \Omega \end{Bmatrix}$$

Therefore, the resulting optimization problem is:

$$J^*\left(\overline{x}(t_k);\overline{u}(t;\overline{x}(t_k))\right) = \min_{(\overline{x}(t_k);\overline{u}(t))\in\Psi} J\left(\overline{x}(t_k);\overline{u}(t)\right)$$

$$\Leftrightarrow \left(\overline{x}^*(t_k);\overline{u}^*(t;\overline{x}^*(t_k))\right) = \arg\min\left\{J\left(\overline{x}(t_k);\overline{u}(t)\right) : \left(\overline{x}(t_k);\overline{u}(t)\right) \in \Psi\right\} \quad (15)$$

Lemma 3: The feasibility of the optimization problem (15) at sampling time t_k implies its feasibility for the sampling time t_{k+1}.

Proof:

Assume that the problem (15) is feasible at t_k and it's corresponding optimal solution is $\overline{x}^*(t_k);\overline{u}^*(t;\overline{x}^*(t_k))$. We choose the control input $\overline{u}(t)$ on $[t_{k+1}; t_{k+1}+T]$ as follows:

$$\overline{u}^*(t) = \begin{cases} \overline{u}^*(t;\overline{x}^*(t_k)); \text{ if } t \in [t_{k+1}; t_k+T] \\ K^*\overline{x}\left(t, \overline{x}^*(t_k+T)\right); \text{ if } t \in [t_k+T; t_{k+1}+T] \end{cases} \quad (16)$$

Since the problem (15) is feasible at the time t_k, we have:

$$\overline{u}(t) = \overline{u}^*(t;\overline{x}^*(t_k)) \in U\Theta K^*\Omega; \forall t \in [t_{k+1}; t_k+T]$$

Since $\overline{x}^*(t_k+T) \in \Delta_\varepsilon$ then by using **lemma 2**, we obtain:

$$\overline{u}(t) = K^*\overline{x}\left(t, \overline{x}^*(t_k+T)\right) \in U\Theta K^*\Omega; \forall t \in [t_k+T; t_{k+1}+T]$$

From the set Δ_ε is an invariant set for the system: $\dot{x} = f(x; K^*x)$, we point out the following result:

$$\overline{x}(\tau) \in \Delta_\varepsilon \subset X\Theta\Omega; \forall t \in [t_k+T; t_{k+1}+T]$$

Lemma 4: The feasibility of the optimization problem (15) at sampling time t_k implies that:

$$J^*\left(\overline{x}(t_{k+1});\overline{u}(t)\right) \le J^*\left(\overline{x}(t_k);\overline{u}(t)\right) - \int_{t_k}^{t_{k+1}}\left[\overline{x}^*(\tau)^T Q\overline{x}^*(\tau) + \overline{u}^*(\tau)^T R\overline{u}^*(\tau)\right]d\tau$$

Proof:

Using (16) and **lemma 2**, we have:

$$\frac{d}{dt}\left(\overline{x}^*(\tau)^T \overline{P}\overline{x}^*(\tau)\right) \le -\overline{x}^*(\tau)^T\left(Q + K^{*T}RK^*\right)\overline{x}^*(\tau);$$

$$\forall \tau \in [t_k+T; t_{k+1}+T]$$

$$\Rightarrow \overline{x}^*(t_{k+1}+T)^T \overline{P}\overline{x}^*(t_{k+1}+T) - \overline{x}^*(t_k+T)^T \overline{P}\overline{x}^*(t_k+T) \le -\int_{t_k+T}^{t_{k+1}+T}\left[\overline{x}^*(\tau)^T Q\overline{x}^*(\tau) + \overline{u}^*(\tau)^T R\overline{u}^*(\tau)\right]d\tau$$

Therefore:

$$J^*\left(\overline{x}(t_{k+1});\overline{u}(t)\right) \le J^*\left(\overline{x}(t_k);\overline{u}(t)\right) - \int_{t_k}^{t_{k+1}}\left[\overline{x}^*(\tau)^T Q\overline{x}^*(\tau) + \overline{u}^*(\tau)^T R\overline{u}^*(\tau)\right]d\tau$$

Theorem 1: By using the control $u(t) = \overline{u}^*(t) + K^*\left(x(t) - \overline{x}^*(t)\right)$, the system (1) has the region of attractor Ω.

Proof:

From the definition of Ψ, we have: $x(t_0) \in \overline{x}^*(t_0) \oplus \Omega$. Because Ω is a robust invariant set for the error system (8), we imply: $x(t) \in \overline{x}^*(t) \oplus \Omega; \forall t \ge t_0$.

We have:

$$J\left(\overline{x}(t_k);\overline{u}(t)\right) \ge 0; \forall t \ge t_0 \Rightarrow$$

$$J^*\left(\overline{x}(t_k);\overline{u}(t;\overline{x}(t_k))\right) = \min_{(\overline{x}(t_k);\overline{u}(t))\in\Psi} J\left(\overline{x}(t_k);\overline{u}(t)\right) \ge 0$$

So by **lemma 4**, the sequence $J^*\left(\overline{x}(t_{k+1});\overline{u}(t)\right)$ is an unordered sequence and is bounded by 0. It is clear that, $\lim_{k\to\infty} J^*\left(\overline{x}(t_{k+1});\overline{u}(t)\right) = \phi < \infty$. We have:

$$0 \le \lim_{k\to\infty}\int_{t_k}^{t_{k+1}}\left[\overline{x}^*(\tau)^T Q\overline{x}^*(\tau) + \overline{u}^*(\tau)^T R\overline{u}^*(\tau)\right]d\tau$$

$$\le \lim_{k\to\infty}\left[J^*\left(\overline{x}(t_k);\overline{u}(t)\right) - J^*\left(\overline{x}(t_{k+1});\overline{u}(t)\right)\right]$$

$$\Rightarrow 0 \le \lim_{k\to\infty}\int_{t_k}^{t_{k+1}}\overline{x}^*(\tau)^T Q\overline{x}^*(\tau)d\tau \le \phi - \phi = 0$$

$$\Rightarrow \lim_{\tau\to\infty}\left\|\overline{x}^*(\tau)\right\| = 0$$

Since $x(t) \in \overline{x}^*(t) \oplus \Omega; \forall t \ge t_0$, Ω is the region of attraction.

Remark 1: It is different from [10]-[15], we obtain the tube-based robust model predictive control law for continuous time systems affected by external disturbances.

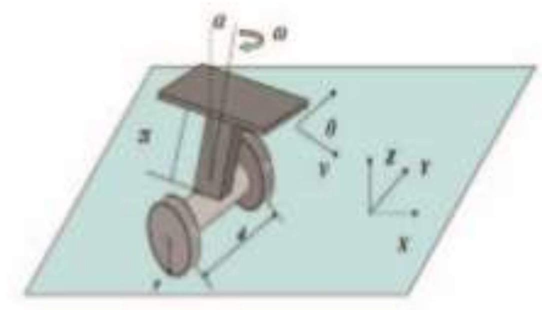

Fig.1 Model of WIP [16]

Parameter	Symbol	Variable	Symbol
Distance between two wheels	D	Heading angle of pendulum	φ
Radius of wheel	R	Tilt of pendulum	α
Moment of inertia of the wheel about y – axis	I_ω	Torque control in left and right wheel	τ_L, τ_R
Moment of inertia of heading angle pendulum about z – axis	I_m	Position of chassis	x, y
Mass of load and chassis	m	Disturbances impacting on two wheels	d_L, d_R
Mass of wheel	M_ω		
Gravity acceleration	g		
Distance between central point load and chassis	l		

Table 1. Parameters and Variables

TUBE-BASED ROBUST MODEL PREDICTIVE FOR UNCERTAIN NONLIEAR WIP SYSTEMS

In [16], the dynamic equation of WIP systems has been described as follows (Table. 1, Fig. 1):

$$\begin{bmatrix} \dot{\alpha} \\ \ddot{\alpha} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ t_1 & 0 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ t_3 & 0 \\ t_4 & 0 \\ 0 & 0 \\ 0 & t_5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + g(x,u) \quad 17$$

$$= Ax + Bu + g(x,u) + B_\omega.\omega(x,t)$$

Where

$$t_1 = \frac{M_x mgl}{M_x I_\alpha - m^2 l^2}, t_2 = \frac{-m^2 l^2 g}{M_x I_\alpha - m^2 l^2},$$
$$t_3 = \frac{-ml}{(M_x I_\alpha - m^2 l^2) r}, t_4 = \frac{I_\alpha}{(M_x I_\alpha - m^2 l^2) r}, \quad 18$$
$$t_5 = \frac{d}{2rI_\theta}, u_1 = (\tau_l + \tau_r), u_2 = (\tau_l - \tau_r)$$

And

$$g(x,u) = f(x,u) - Ax - Bu \ ,$$

$$\omega(x,t) = \begin{bmatrix} \frac{d}{2I_\theta}(d_l - d_r) \\ \frac{I_\alpha}{\Omega_0}(d_l + d_r) \\ -\frac{ml\cos(x_1)(d_l + d_r)}{\Omega_0} \end{bmatrix}, B_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x,u) = \begin{bmatrix} x_2 \\ \frac{1}{\Omega_0}[M_x mgl\sin(x_1) - m^2 l^2 \cos(x_1)\sin(x_1).(x_2)^2] - \frac{ml\cos(x_1)u_1}{\Omega_0 r} \\ \frac{1}{\Omega_0}[I_\alpha ml\sin(x_1).(x_2)^2 - m^2 l^2 g\sin(x_1)\cos(x_1)] + \frac{I_\alpha}{\Omega_0 r}u_1 \\ x_5 \\ \frac{d}{2rI_\theta}u_2 \end{bmatrix}$$

Remark 2: The proposed control law (5) is absolutely applied for uncertain nonlinear wheeled inverted pendulum systems (17).

SIMULATION RESULTS

In this section, offline simulation is implemented to demonstrate the proposed control law for the studied WIP as follows:

$$M = 9(kg), m = 4(kg), I_\omega = 0.2kgm^2, r = 0.065(m),$$
$$M_w = 2(kg), I_p = 0.34(kgm^2), d = 0.33(m),$$
$$l = 0.14(m), I_M = 0.12(kgm^2)$$

The Controller is designed by using above contents as the following steps:

- From (10, 11) , we choose $\lambda_0 = 10,\ \mu = 1$ and we obtain:

$$K^* = YX^{-1} =$$
$$\begin{bmatrix} 182.57 & 19.78 & 13.05 & 0 & 0 \\ 0 & 0 & 0 & -3.37 & -2.64 \end{bmatrix},$$
$$\|K^*\| = 184$$

$$P^* = \begin{bmatrix} 2.8 & 0.49 & 0.41 & 0 & 0 \\ 0.59 & 0.17 & 0.12 & 0 & 0 \\ 0.41 & 0.12 & 0.15 & 0 & 0 \\ 0 & 0 & 0 & 0.22 & 0.1 \\ 0 & 0 & 0 & 0.1 & 0.12 \end{bmatrix}, \left\| P^* \right\| = 2.9,$$

$$\vartheta_{\min}\left(P^*\right) = 0.0357$$

Selecting $R = 0.01 I_2$; $Q = 0.1 I_5$; $\chi = 1$, from (13) we have :

$$\overline{P} = \begin{bmatrix} 75.74 & 11.42 & 10.22 & 0 & 0 \\ 11.42 & 2.671 & 1.715 & 0 & 0 \\ 10.22 & 1.715 & 2.807 & 0 & 0 \\ 0 & 0 & 0 & 2.164 & 0.8053 \\ 0 & 0 & 0 & 0.805 & 0.7419 \end{bmatrix}$$

We continue to choose the sample time: T_s=0.1(s), Initial state: $x_1 = 0.1; x_2 = 0; x_3 = 0; x_4 = 0.1; x_5 = 0$

$-20 \le u_1, u_2 \le 20$ (N.m)

$-0.2 \le x_1, x_2, x_3, x_4, x_5 \le 0.2$

(9)=> $p_1 = 10^{-3}; p_2 = 2.10^{-5}$

then there exist $\lambda > 0$ such that :

$$p_1 + p_2 \left\| K^* \right\| \le \frac{\left(\lambda_0 - \lambda\right) \vartheta_{\min}\left(P^*\right)}{2\left\| P^* \right\|}.$$

External forces acting on the left and right wheels d_l, d_r are random function with amplitude being 0.1(N).

The simulation results (fig. 2,3) pointed out the good tracking behavior of tilt and heading angle.

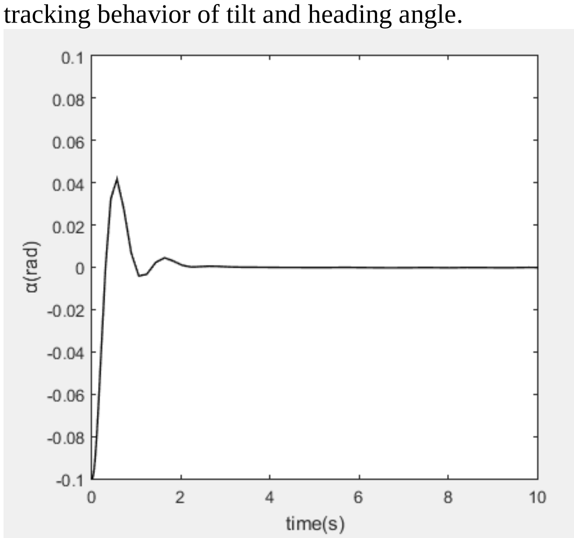

Fig.2 The behaviour of tilt angle

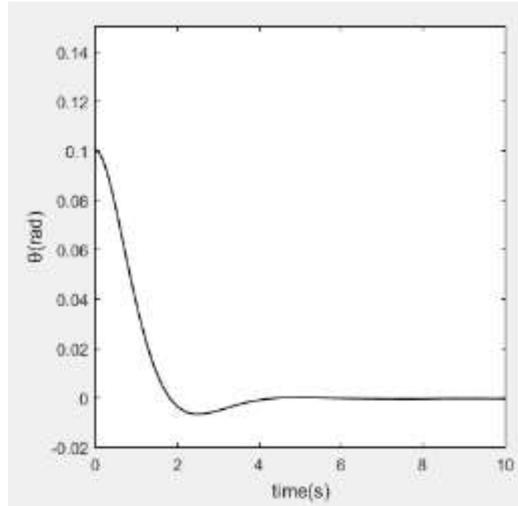

Fig.3 The behaviour of heading angle

CONCULUSION

This paper presents a tube-based robust MPC of continuous-time systems with external disturbance. The proposed algorithm pointed out the region of attractor. The theory analysis and simulation results illustrate the effectiveness of proposed algorithm.

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