ROBUST CONTROL FOR A DC – DC CONVERTER: A SWITCHED SYSTEMS APROACH

Dao Phuong Nam*, Nguyen Van Huong, Pham Van Dao,
Mai Xuan Sinh, Dương Ba Hai Dang

Abstract: *Power converters (Buck, Boost, Buck-Boost, multilevel converters) have many applications such as in variable speed DC motor drivers, solar cells, power supply for computers, TV, cell phones, cameras, etc. The converters are controlled by switches to transfer the energy from a power source to a load and these systems belong to switched system class. This work develops an optimization technique to design a robust control law for unstable autonomous switched systems. This method is then applied to the DC-DC buck converter under the consideration of the uncertainty of the parameters in the model. Based on this model, a robust control law is design to allow the output voltage track a reference voltage when the load is changed. The theory and simulation results point out the good abilities of the proposed robust technique control for the DC – DC converter.*

Keywords: DC – DC converter; Switched system; Robust control.

1. INTRODUCTION

Several methods have been used to model DC-DC converters such as State-Space Averaging model or frequency selective averaging model [1]-[4]. These converters are controlled by Pulse Width-Modulation (PWM) where the switching behaviour of the closed loop system is averaged by a nonlinear model. The state of the system can be described by averaged equations. However, the averaged model can not present the instant real-time values of the systems' state. The state of systems is modelled by averaged values, so it is required a very short sampling time. Alternatively, these converters can be modelled by switched systems in [5]-[7]. The state of system can be evaluated by the real time. The analysis and design of switched system is focused on stability, robustness, controllability and optimal control. The stability analysis and control design of a switched system can be solved by LMIs tool box in Matlab. The purpose of this paper is to develop a systematic state space approach so that the nonlinear dynamics of the buck converter can be more faithfully described. Based on switched system, the novel method to design the feedback control law for buck converter that consider the uncertainty parameter of the system is presented. The paper is organized as follow. Part II presents the methodology for the designing of the feedback control law. The optimal tracking problem is solved in part III and the result is validated by using the Simulink in Matlab. Part IV concludes the paper.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following uncertain switched linear sysem:

$$\begin{cases} \dfrac{dx}{dt} = \left(A_\sigma + \Delta_\sigma\right)x + \left(B_\sigma + \Gamma_\sigma\right)u, \quad x(0) = x_0 \\ z = \left(C_\sigma + \Upsilon_\sigma\right)x \end{cases} \tag{1}$$

where $x(.) \in R^n$ is the state, $u(.) \in R^m$ is the control, $z(.) \in R^s$ is the controlled output, $A_\sigma, B_\sigma, C_\sigma$ are the parameters and $\Delta_\sigma, \Gamma_\sigma, \Upsilon_\sigma$ are the uncertainties of this system.

The control objective is to find the switching strategy to obtain that $x(t)$ converge to the equilibrium x_e in the presence of uncertain parameters.

The notation used throughout is standard. Let $A_\lambda = \sum_{i=1}^{n} \lambda_i A_i$ denote the convex combination of a set of matrices $\{A_1, A_2, ..., A_n\}$, where λ belongs to the set Θ created by all non-negative numbers such that $\sum_{i=1}^{n} \lambda_i = n$.

3. ROBUST STATE FEEDBACK SWITCHING LAW BASED ON OPTIMIZATION

Theorem 1: (Attractor)

The uncertain linear switched systems

$$\begin{cases} \dfrac{dx}{dt} = (A_\sigma + \Delta_\sigma)x + (B_\sigma + \Gamma_\sigma)u, \quad x(0) = x_0 \\ z = (C_\sigma + \Upsilon_\sigma)x \end{cases}$$

with input $u(t) = u(const), \forall t \geq 0$ and let x_e (equilibrium point) be given. If there exist $\lambda \in \Theta$, and a symmetric positive definite matrix $P \in R^{n \times n}$ satisfy:

$$A_i^T P + PA_i + Q_i < 0 \quad \left(\forall i = \overline{1, n}\right) \tag{2}$$

$$A_\lambda x_e + B_\lambda u = 0 \tag{3}$$

Then switching strategy $\sigma(x) \in \{1, 2, ..., n\}$ minimizes the cost function $\varepsilon^T P(A_i x_e + B_i u)$, where $\varepsilon = x - x_e$ converges to the attractor as follows:

$$\Omega = \left\{\varepsilon \in R^n, -\varepsilon^T Q_\sigma \varepsilon + 2\varepsilon^T P(\Delta_\sigma x + \Gamma_\sigma u) \leq 0\right\}$$

Proof:

Consider the Lyapunov candidate function $V(\varepsilon) = \varepsilon^T P \varepsilon$. We obtain time derivative of $V(\varepsilon)$ along the trajectory of uncertain switched system (1) controlled by switching rule as above, described as follows:

$$\begin{aligned} \frac{d}{dt}V(\varepsilon) &= \dot{x}^T P \varepsilon + \varepsilon^T P \dot{x} = 2\varepsilon^T P((A_\sigma + \Delta_\sigma)x + (B_\sigma + \Gamma_\sigma)u) \\ &= 2\varepsilon^T P(A_\sigma x_e + B_\sigma u) + \varepsilon^T (A_\sigma{}^T P + PA_\sigma)\varepsilon + 2\varepsilon^T P(\Delta_\sigma x + \Gamma_\sigma u) \\ &= \min_{i \in \{1,2,...,n\}} (2\varepsilon^T P(A_i x_e + B_i u)) + \varepsilon^T (A_\sigma{}^T P + PA_\sigma)\varepsilon + 2\varepsilon^T P(\Delta_\sigma x + \Gamma_\sigma u) \\ &\leq 2\varepsilon^T P(A_\lambda x_e + B_\lambda u) - \varepsilon^T Q_\sigma \varepsilon + 2\varepsilon^T P(\Delta_\sigma x + \Gamma_\sigma u) \\ &\leq -\varepsilon^T Q_\sigma \varepsilon + 2\varepsilon^T P(\Delta_\sigma x + \Gamma_\sigma u) \end{aligned}$$

Therefore, if ε is out of the attractor Ω then ε converge to this attractor due to $\frac{d}{dt}V(\varepsilon) < 0$.

Remarks:

1. The novel of this control technique is applied in this study is the consideration to the uncertainty components, expressed by $\Delta_\sigma, \Gamma_\sigma, \Upsilon_\sigma$
2. The design problem is cast into an optimization problem, which can be effectively solved with linear matrix inequality (LMI) toolbox in Matlab.

4. SIMULATION RESULTS AND DISCUSSION

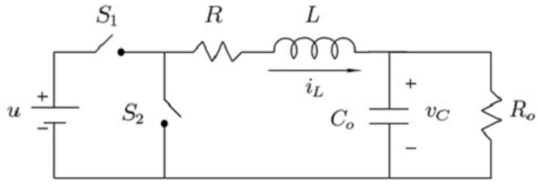

***Fig. 1**. Buck Converter.*

We constructed the Simulink model for the inverter using SimPower in Matlab. The parameters for the circuit in Fig. 1 are given as follows: $L = 500\mu H, R = 2\Omega, C_0 = 470\mu F$ The input voltage $u = 100V$. The load R_0 are changed from $R_0 = 100\Omega$ to $R_0 = 500\Omega$ to verify the stability of the output voltage. The output voltage also is set at different value $v_e = 45V$ and $v_e = 60V$. The average output current will be $i = \frac{v_e}{R_0}$. The model is included 2 MOSFET transistors working as switches S1 and S2, which are controlled by PWM signals.

The optimal solution $c_e = P(B_1 - B_2)u = \begin{bmatrix} 0,27 \\ 0,49 \end{bmatrix}$, $\sigma(x) = \begin{cases} 1 \text{ if SW} \le 0 \\ 2 \text{ if SW>0} \end{cases}$ in which $\text{SW}=\begin{bmatrix} 0,27 & 0,49 \end{bmatrix}\left(\begin{bmatrix} i_L \\ v \end{bmatrix} - \begin{bmatrix} i_0 \\ v_e \end{bmatrix}\right)$

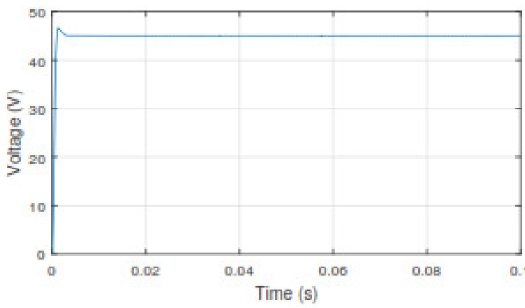

***Fig. 2.** Output Voltage with* $R_0 = 100\Omega$.

Figure 2 and 3 show the output voltage when the load R_0 is choosen $R_0 = 100\Omega$ and $R_0 = 500\Omega$, the output voltage is tracked by $v_e = 45V$

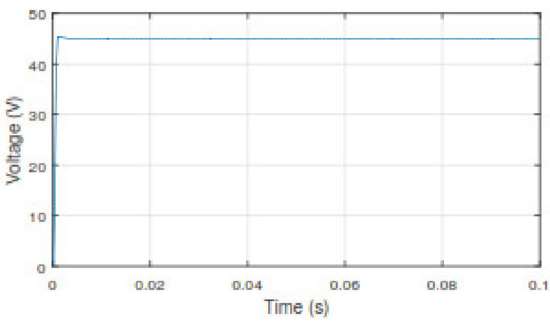

***Fig. 3.** Output Voltage with* $R_0 = 500\Omega$.

5. CONCLUSIONS

This paper presents the novel method to model the DC – DC buck converter as a switched system that considers the uncertainty parameter of the system, while most of the previous research has not mentioned. The analysis of the control law was obtained for controlling the buck converter and the simulation results confirm the design.

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TÓM TẮT

ĐIỀU KHIỂN BỀN VỮNG CHO BỘ BIẾN ĐỔI DC – DC: NHÌN NHẬN THEO HỆ CHUYỂN MẠCH

Bộ biến đổi điện tử công suất (Buck, Boost, Buck-Boost, biến tần đa mức) có nhiều ứng dụng trong truyền động động cơ 1 chiều, pin mặt trời, nguồn cho máy tính, TV, camera,... Chúng được điều khiển bằng cách đóng mở các van bán dẫn để chuyển năng lượng từ nguồn công suất đến tải và thuộc nhóm hệ chuyển mạch. Nhiệm vụ của chúng tôi phát triển kỹ thuật tối ưu hóa trong việc thiết kế bộ điều khiển bền vững cho hệ chuyển mạch không cân bằng. Phương pháp này vận dụng với bộ biến đổi giảm áp khi xem xét đến yếu tố bất định trong tham số của mô hình. Từ mô hình đó, 1 bộ điều khiển bền vững được thiết kế để đảm bảo điện áp đầu ra bám được theo lượng đặt khi tải thay đổi. Lý thuyết và kết quả mô phỏng đã chỉ ra khả năng của bộ điều khiển bền vững được đề xuất đối với bộ biến đổi DC – DC.

Từ khóa: Bộ biến đổi DC – DC, Hệ chuyển mạch, Điều khiển bền vững.

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Author affiliations: Hanoi University of Science and Technology.
*Corresponding author: nam.daophuong@hust.edu.vn