# Điều khiển Thích nghi bền vững Mô hình mẫu cho Hệ chuyển mạch phi tuyến

## Model Reference Robust Adaptive Control for Nonlinear Switched Systems

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#### **Abstract**

In this paper, a new model reference robust adaptive controller (MRRAC) for uncertain nonlinear switched systems is developed. The proposed control law with the adaptive law guarantees that the model errors converge to arbitrary attraction region. Finally, simulation results show the good behaviour of the model reference robust adaptive law.

#### **Keywords**

Robust Adaptive Controller, Nonlinear Switched Systems, Model Reference Control, Lyapunov Technique.

#### 1. Introduction

Switched systems are an important subclass belonging to hybrid systems. They appear in many applications, such as power electronics, smart energy systems,...[3].

Among many researches of this topic, model reference adaptive control have attracted the most attention [1],...,[5]. In [1], authors proposed the individual estimation for nonlinear switched systems and the asymptotic state tracking has been analyzed. However, the uncertain term of system was not depend on control input. Sinafar et al. proposed a model reference adaptive controller for uncertain switched systems in presence of uncertainty and unmodeled dynamics. However, the control law in [2] has been proposed without the efficiency of disturbance. In [3], the proposed control law has been mentioned to uncertain term depending on control input. In [4], a model reference adaptive control for switched LPV systems has been proposed by constructing special Lyapunov candidate function. Xie et al. [5] proposed the model reference adaptive controller and give out the global practical stability analysis without external disturbance. In this paper, we present an model reference robust adaptive control for nonlinear switched systems in presence of external disturbance and unknown parameters.

#### 2. Problem Statement

Consider a nonlinear switched system:  

$$\dot{x} = A_{\sigma}x + \theta_{\sigma}.f_{\sigma}(x) + B_{\sigma}.u + d_{\sigma}(t)$$
 (1)

 $\begin{array}{ll} \text{where } x \in \mathbb{R}^n \text{ is the state, } u \in \mathbb{R}^q \text{ is the control input.} \\ \text{The function } \sigma \colon & \left[0, +\infty\right) \to \Omega = \left\{1, 2, \ldots, N\right\} \text{ is a} \end{array}$ 

switching signal, which is a piecewise continuous function of time, and N is the number of subsystems.  $A_i \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{n\times q}, i \in \Omega \quad \text{are unknown constant}$  matrices.  $\theta_i \in \mathbb{R}^{n\times p}, i \in \Omega \quad \text{are the uncertain constant}$  parameter matrices, and  $f_i\left(x\right) \in \mathbb{R}^p, i \in \Omega \quad \text{are known}$  smooth vector functions with  $f_i\left(0\right) = 0 \ .$   $d_i\left(t\right) \in \mathbb{R}^n, i \in \Omega \quad \text{are disturbance and uncertain of systems.}$ 

**Assumption 1:** There exists a constant number D such that:  $\|\mathbf{d}_i(t)\| \le D, \forall i \in \Omega, t \ge 0$ .

In this paper, we propose the robust adaptive control to ensure the system tracks the performance of a group of switched reference models representing the desired behavior of each subsystem is described as follows:

$$\dot{x}_{m} = A_{m\sigma} x_{m} + B_{m\sigma} . r, \ \sigma \in \Omega(2)$$

where  $A_{mi} \in \mathbb{R}^{n \times n}$  satisfies:

$$A_{mi}^{T}.P_{i}+P_{i}.A_{mi}=-Q_{i}, \quad i \in \Omega(3)$$

for some positive symmetric matrices  $P_{i}\text{,}Q_{i}\text{,}B_{mi}\in\mathbb{R}^{n\times q}\text{ are the known constant matrices}$  and  $r\in\mathbb{R}^{q}$  is a bounded reference input signal.

#### 3. Control Design

We consider the state-feedback controllers being given as follows:

$$u(t) = K_{\sigma}^* x - L_{\sigma}^* f_{\sigma}(x) + M_{\sigma}^* r(4)$$

where  $K_{\sigma}^*, L_{\sigma}^*, M_{\sigma}^*$  are constant matrices satisfying:

$$B_{\sigma}K_{\sigma}^{*} = A_{m\sigma} - A_{\sigma}, B_{\sigma}L_{\sigma}^{*} = \theta_{\sigma}, B_{\sigma}M_{\sigma}^{*} = B_{m\sigma} \quad (5)$$

**Assumption 2:** We consider that  $M_{\sigma}^* = \text{diag}\left(m_{\sigma 1}^{-1}, m_{\sigma 2}^{-1}, ..., m_{\sigma q}^{-1}\right)$  with unknown

 $m_{\sigma i} > 0; i = 1, 2, ..., q.$ 

**Remark 1:** The necessary and sufficient condition for the existence of constant matrices  $K_{\sigma}^*, L_{\sigma}^*, M_{\sigma}^*$  have been discussed in [6].

Since  $A_{\sigma}$ ,  $B_{\sigma}$  and  $\theta_{\sigma}$  are unknown, we cannot obtain  $K_{\sigma}^*$ ,  $L_{\sigma}^*$ ,  $M_{\sigma}^*$  from (5). Then, we design the control as:  $u(t) = \hat{K}_{\sigma}(t)x - \hat{L}_{\sigma}(t)f_{\sigma}(x) + \hat{M}_{\sigma}(t)r$  (6)

where  $\hat{K}_{\sigma}(t), \hat{L}_{\sigma}(t), \hat{M}_{\sigma}(t)$  are the estimate of  $K_{-}^{*}, L_{-}^{*}, M_{-}^{*}$ , respectively.

The tracking error is defined as  $e(t) = x(t) - x_m(t)$ . We have:

$$\begin{split} \dot{e}\left(t\right) &= A_{m\sigma(t)}e\left(t\right) + \\ B_{\sigma(t)}\left(\tilde{K}_{\sigma}\left(t\right)x - \tilde{L}_{\sigma}\left(t\right)f_{\sigma}\left(x\right) + \tilde{M}_{\sigma}\left(t\right)r\right) + d_{\sigma}\left(t\right) \\ \dot{e}\left(t\right) &= A_{m\sigma(t)}e\left(t\right) + \\ B_{m\sigma}M_{\sigma}^{*-1}\left(\tilde{K}_{\sigma}\left(t\right)x - \tilde{L}_{\sigma}\left(t\right)f_{\sigma}\left(x\right) + \tilde{M}_{\sigma}\left(t\right)r\right) + d_{\sigma}\left(t\right)\left(7\right) \end{split}$$

where  $\tilde{K}_{g} = \hat{K}_{g} - K_{g}^{*}$ ,  $\tilde{L}_{g} = \hat{L}_{g} - L_{g}^{*}$ ,  $\tilde{M}_{g} = \hat{M}_{g} - M_{g}^{*}$ (8), which are the parameter errors.

**Lemma 1:** Let  $x \in \mathbb{R}^p$ ,  $y \in \mathbb{R}^q$ and M, N are appropriately dimensioned matrices, then for any positive number  $\theta$  and every appropriately dimensioned matrix X(t) satisfying  $X^{T}(t)X(t) \leq I$ , we have:

$$2x^TMXNy \leq \theta x^TMM^Tx + \theta^{-1}y^TN^TNy$$

**Assumption 3:** There exist known matrices  $S_{\sigma}$  such that  $R_{\sigma} = S_{\sigma} M_{\sigma}^{*T}$  are positive matrices. We defined:

$$\boldsymbol{R}^{-1} = \begin{bmatrix} \boldsymbol{R}_1^{-1} & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{R}_N^{-1} \end{bmatrix}, \boldsymbol{M}^* = \begin{bmatrix} \left(\boldsymbol{M}_1^{*-1}\right)^T & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \left(\boldsymbol{M}_q^{*-1}\right)^T \end{bmatrix}$$

$$\begin{split} & \Phi = \begin{bmatrix} \tilde{K}_1 \\ \tilde{K}_2 \\ \vdots \\ \tilde{K}_N \end{bmatrix}, \bar{\Phi} = \begin{bmatrix} \hat{K}_1 \\ \hat{K}_2 \\ \vdots \\ \hat{K}_N \end{bmatrix}, \ \Psi = \begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \\ \vdots \\ \tilde{L}_N \end{bmatrix}, \bar{\Psi} = \begin{bmatrix} \hat{L}_1 \\ \hat{L}_2 \\ \vdots \\ \hat{L}_N \end{bmatrix}, \\ & \Xi = \begin{bmatrix} \tilde{M}_1 \\ \tilde{M}_2 \\ \vdots \\ \tilde{M}_N \end{bmatrix}, \bar{\Xi} = \begin{bmatrix} \hat{M}_1 \\ \hat{M}_2 \\ \vdots \\ \hat{M}_N \end{bmatrix}, \end{split}$$

**Theorem 1:** By using the adaptive laws:

$$\begin{split} \dot{\hat{K}}_{\sigma} &= \begin{cases} -S_{i}B_{mi}^{T}P_{i}ex^{T} - \alpha S_{i} \ \hat{K}_{i}when \ \sigma(t) = i \\ -\alpha S_{i} \ \hat{K}_{i}when \ \sigma(t) \neq i \end{cases} \\ \dot{\hat{L}}_{\sigma} &= \begin{cases} S_{i}B_{mi}^{T}P_{i}ef_{i} \left(x\right)^{T} - \beta S_{i}\hat{L}_{i}when \ \sigma(t) = i \\ -\beta S_{i}\hat{L}_{i}when \ \sigma(t) \neq i \end{cases} \\ \dot{\hat{M}}_{\sigma} &= \begin{cases} -S_{i}B_{mi}^{T}P_{i}ex^{T} - \gamma S_{i} \ \hat{M}_{i}when \ \sigma(t) = i \\ -\gamma S_{i} \ \hat{M}_{i}when \ \sigma(t) \neq i \end{cases} \end{split}$$

and the  $% \frac{1}{2}$  are average dwell-time satisfies  $T>\frac{\mu _{2}\ln \left( \frac{\mu _{2}}{\mu _{1}}\right) }{..}$  with:

$$\mu_{1} = \frac{1}{2} min \Big( \lambda_{min} \left( P_{1} \right), \lambda_{min} \left( P_{2} \right), \ldots, \lambda_{min} \left( P_{N} \right), \lambda_{min} \left( R^{-1} \right) \Big)$$

$$\boldsymbol{\mu}_{2} = \frac{1}{2} \, max \Big( \boldsymbol{\lambda}_{max} \left( \boldsymbol{P}_{\!_{1}} \right), \boldsymbol{\lambda}_{max} \left( \boldsymbol{P}_{\!_{2}} \right), \ldots, \boldsymbol{\lambda}_{max} \left( \boldsymbol{P}_{\!_{N}} \right), \boldsymbol{\lambda}_{max} \left( \boldsymbol{R}^{-1} \right) \Big)$$

$$\mu_{0} = \frac{1}{2} min \begin{pmatrix} \lambda_{min}\left(Q_{1}\right) \left(1-\theta_{0}\right), \lambda_{min}\left(Q_{2}\right), \ldots, \\ \lambda_{min}\left(Q_{N}\right), \alpha\lambda_{min}\left(M\right), \beta\lambda_{min}\left(M\right), \gamma\lambda_{min}\left(M\right) \end{pmatrix}$$

we can appropriately choose design parameters such that the state tracking error satisfies:

 $\lim_{x \to \infty} \|e(t)\|^2 \le \kappa$  for any given constant  $\kappa > 0$ .

Consider the Lyapunov candidate function:

$$\begin{aligned} & \text{Assumption 3: There exist known matrices } S_{\sigma} \text{ such} \\ & \text{that } R_{\sigma} = S_{\sigma} M_{\sigma}^{\mathsf{T}} \text{ are positive matrices.} \\ & \text{We defined:} \\ & \text{We defined:} \\ & \text{R}^{-1} = \begin{bmatrix} R_{1}^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_{N}^{-1} \end{bmatrix}, M^{*} = \begin{bmatrix} (M_{1}^{*-1})^{\mathsf{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (M_{q}^{*-1})^{\mathsf{T}} \end{bmatrix}, \\ & \text{Description 3: There exist known matrices.} \\ & \text{We defined:} \\ & \text{V}_{\sigma} = V_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} P_{\sigma} e + \\ & \frac{1}{2} \operatorname{tr} (\Phi^{\mathsf{T}} R^{-1} \Psi) + \frac{1}{2} \operatorname{tr} (\Xi^{\mathsf{T}} R^{-1} \Xi) \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} P_{\sigma} + P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac{1}{2} e^{\mathsf{T}} (A_{m\sigma}^{\mathsf{T}} \cdot P_{\sigma} \cdot A_{m\sigma}) e \\ & \text{V}_{\sigma} (e, \Phi, \Psi, \Xi) = \frac$$

$$\begin{split} \dot{V}_{\sigma} &= -\frac{1}{2}e^{T}Q_{\sigma}e + e^{T}P_{\sigma}d_{\sigma} - \alpha.tr\Big(\Phi^{T}M\overline{\Phi}\Big) \\ -\beta.tr\Big(\Psi^{T}M\overline{\Psi}\Big) - \gamma.tr\Big(\Xi^{T}M\overline{\Xi}\Big) \\ \dot{V}_{\sigma} &= -\frac{1}{2}e^{T}Q_{\sigma}e + e^{T}P_{\sigma}d_{\sigma} - \alpha.tr\Big(M\Phi^{T}\overline{\Phi}\Big) \\ -\beta.tr\Big(M\Psi^{T}\overline{\Psi}\Big) - \gamma.tr\Big(M\Xi^{T}\overline{\Xi}\Big) \end{split}$$

From assumption 2 we have:

$$\begin{split} \dot{V}_{\sigma} & \leq -\frac{1}{2} e^{T} Q_{\sigma} e + e^{T} P_{\sigma} d_{\sigma} \\ -\lambda_{\min} \left( M \right) & \left[ \alpha. tr \left( \Phi^{T} \overline{\Phi} \right) + \beta. tr \left( \Psi^{T} \overline{\Psi} \right) + \gamma. tr \left( \Xi^{T} \overline{\Xi} \right) \right] \\ & \text{Applying:} \end{split}$$

$$2 \text{tr} \left( \boldsymbol{Y}^{T} \overline{\boldsymbol{Y}} \right) = \left\| \boldsymbol{Y} \right\|^{2} + \left\| \overline{\boldsymbol{Y}} \right\|^{2} - \left\| \boldsymbol{Y} - \overline{\boldsymbol{Y}} \right\|^{2} \geq \left\| \boldsymbol{Y} \right\|^{2} - \left\| \boldsymbol{Y} - \overline{\boldsymbol{Y}} \right\|^{2}$$

$$\begin{split} \dot{V}_{\sigma} &\leq -\frac{1}{2}e^{T}Q_{\sigma}e + e^{T}P_{\sigma}d_{\sigma} + \\ \lambda_{min}\left(M\right) & \left(\frac{\alpha}{2}\left\|\Phi - \overline{\Phi}\right\|^{2} - \frac{\alpha}{2}\left\|\Phi\right\|^{2} + \frac{\beta}{2}\left\|\Psi - \overline{\Psi}\right\|^{2} - \frac{\beta}{2}\left\|\Psi\right\|^{2} + \frac{\gamma}{2}\left\|\Xi - \overline{\Xi}\right\|^{2} - \frac{\gamma}{2}\left\|\Xi\right\|^{2} \right) \end{split}$$

Applying **lemma 1** with  $0 < \theta_0 < 1$  we have

$$e^{T}P_{\sigma}d_{\sigma} \leq \frac{1}{2} \left(\theta_{0}\lambda_{\min}\left(Q_{\sigma}\right)e^{T}e + \frac{1}{\theta_{0}\lambda_{\min}\left(Q_{\sigma}\right)}d_{\sigma}^{T}P_{\sigma}^{T}P_{\sigma}d_{\sigma}\right)$$

$$\begin{split} \dot{V}_{\sigma} \leq -\frac{1}{2} \left[ \frac{\lambda_{min} \left( Q_{\sigma} \right) \! \left( 1 \! - \! \boldsymbol{\theta}_{0} \right) \! \left\| \boldsymbol{e} \right\|^{2}}{+ \! \lambda_{min} \left( \boldsymbol{M} \right) \! \left( \boldsymbol{\alpha} \left\| \boldsymbol{\Phi} \right\|^{2} + \! \boldsymbol{\beta} \left\| \boldsymbol{\Psi} \right\|^{2} + \! \boldsymbol{\gamma} \left\| \boldsymbol{\Xi} \right\|^{2} \right)} \right] \! + \mathfrak{I} \end{split}$$

$$\begin{split} &\mathfrak{I} = \lambda_{\min}\left(M\right)\!\!\left(\frac{\alpha}{2}\!\left\|\Phi - \overline{\Phi}\right\|^2 + \frac{\beta}{2}\!\left\|\Psi - \overline{\Psi}\right\|^2 + \frac{\gamma}{2}\!\left\|\Xi - \overline{\Xi}\right\|^2\right) \\ &+ \frac{\left\|d_{\sigma}\right\|^2 \left\|P_{\sigma}\right\|^2}{\theta_{\sigma}\lambda_{\min}\left(O\right)} \end{split}$$

From **assumption 1** and (8) we imply that there exists a constant c such that  $\Im \leq c$ .

$$\begin{split} &\mu_{1}\left(\left\|\boldsymbol{e}\right\|^{2}+\left\|\boldsymbol{\Phi}\right\|^{2}+\left\|\boldsymbol{\Psi}\right\|^{2}+\left\|\boldsymbol{\Xi}\right\|^{2}\right)\leq V_{\sigma}\\ &\leq\mu_{2}\left(\left\|\boldsymbol{e}\right\|^{2}+\left\|\boldsymbol{\Phi}\right\|^{2}+\left\|\boldsymbol{\Psi}\right\|^{2}+\left\|\boldsymbol{\Xi}\right\|^{2}\right)\\ &\dot{V}_{\sigma}\leq-\mu_{0}\left(\left\|\boldsymbol{e}\right\|^{2}+\left\|\boldsymbol{\Phi}\right\|^{2}+\left\|\boldsymbol{\Psi}\right\|^{2}+\left\|\boldsymbol{\Xi}\right\|^{2}\right)+c \end{split}$$

$$\begin{aligned} & V_{i} \leq aV_{j}; \forall i, j \in \Omega \\ & \dot{V}_{i} \leq -bV_{i} + c; \forall i \in \Omega \end{aligned}$$

$$a = \frac{\mu_2}{\mu_1}$$
;  $b = \frac{\mu_0}{\mu_2}$ 

We define:

$$\begin{split} W_{\sigma(t)}\left(t\right) &= e^{bt} \left(V_{\sigma(t)}\left(t\right) - \frac{c}{b}\right) \\ \dot{W}_{\sigma(t)}\left(t\right) &= e^{bt} \left(\dot{V}_{\sigma(t)}\left(t\right) + bV_{\sigma(t)}\left(t\right) - c\right) \leq 0 \\ I_{et} \ t_{M} &\leq 9 < t_{M+1} \ then \ 9 = t_{M} + \Delta \end{split}$$

$$\begin{split} & W_{\sigma(t_{_{M}})}\left(t_{_{M}}+\Delta\right) \leq W_{\sigma(t_{_{M}})}\left(t_{_{M}}\right) \\ & \Rightarrow e^{b(t_{_{M}}+\Delta)}\bigg(V_{\sigma(t_{_{M}})}\left(t_{_{M}}+\Delta\right)-\frac{c}{b}\bigg) \leq e^{bt_{_{M}}}\bigg(V_{\sigma(t_{_{M}})}\left(t_{_{M}}\right)-\frac{c}{b}\bigg) \\ & \Rightarrow e^{b\Delta}V_{\sigma(t_{_{M}})}\left(t_{_{M}}+\Delta\right) \leq V_{\sigma(t_{_{M}})}\left(t_{_{M}}\right)+\frac{c}{b}\Big(e^{b\Delta}-1\Big) \\ & \Rightarrow V_{\sigma(t_{_{M}})}\Big(9\Big) \leq e^{-b\Delta}V_{\sigma(t_{_{M}})}\Big(t_{_{M}}\Big)+\frac{c}{b}\Big(1-e^{-b\Delta}\Big) \end{split}$$

Similar way, we have:

$$\begin{split} &V_{\sigma\left(t_{j}\right)}\left(t_{j+1}^{-}\right) \leq e^{-bT}V_{\sigma\left(t_{j}\right)}\left(t_{j}\right) + \frac{c}{b}\left(1 - e^{-bT}\right) \\ &V_{\sigma\left(t_{M}\right)}\left(t_{M}\right) \leq aV_{\sigma\left(t_{M-1}\right)}\left(t_{M}\right) \\ &\leq a\Bigg[e^{-bT}V_{\sigma\left(t_{M-1}\right)}\left(t_{M-1}\right) + \frac{c}{b}\left(1 - e^{-bT}\right)\Bigg] \\ &V_{\sigma\left(t_{M-1}\right)}\left(t_{M-1}\right) \leq aV_{\sigma\left(t_{M-2}\right)}\left(t_{M-1}\right) \\ &\leq a\Bigg[e^{-bT}V_{\sigma\left(t_{M-2}\right)}\left(t_{M-2}\right) + \frac{c}{b}\left(1 - e^{-bT}\right)\Bigg] \end{split}$$

$$V_{\sigma\!\left(t_{2}\right)}\!\left(t_{2}\right)\!\leq\!aV_{\sigma\!\left(t_{1}\right)}\!\left(t_{2}\right)\!\leq\!a\!\left[e^{-bT}V_{\sigma\!\left(t_{1}\right)}\!\left(t_{1}\right)\!+\!\frac{c}{b}\!\left(1\!-\!e^{-bT}\right)\right]$$

$$\begin{split} &V_{\sigma(t_1)}\left(t_1\right) \leq aV_{\sigma(t_0)}\left(t_1\right) \leq a \Bigg[ e^{-bT}V_{\sigma(t_0)}\left(t_0\right) + \frac{c}{b}\Big(1 - e^{-bT}\Big) \Bigg] \\ &V_{\sigma(t_M)}\left(t_M\right) \leq \Big(ae^{-bT}\Big)^MV_{\sigma(0)}(0) + \frac{1 - \Big(ae^{-bT}\Big)^M}{1 - ae^{-bT}}.\frac{ac}{b}.\Big(1 - e^{-bT}\Big) \\ &V_{\sigma(t_M)}\left(\vartheta\right) \leq e^{-b\Delta}.\Big(ae^{-bT}\Big)^MV_{\sigma(0)}\left(0\right) \\ &+ \frac{c}{b}.\Big(1 - e^{-bT}\Big).\Bigg[ e^{-b\Delta}.a.\frac{1 - \Big(ae^{-bT}\Big)^M}{1 - ae^{-bT}} + \frac{1 - e^{-b\Delta}}{1 - e^{-bT}} \Bigg] \end{split}$$

If 
$$T > \frac{\mu_2 \ln \left(\frac{\mu_2}{\mu_1}\right)}{\mu_0} = \frac{\ln a}{b}$$
 then  $0 < ae^{-bT} < 1$  so, when

 $9 \rightarrow +\infty$  or  $M \rightarrow +\infty$  then:

$$\Rightarrow \lim_{\theta \to \infty} \left\| x(\theta) - x_m(\theta) \right\|^2$$

$$\leq \frac{c}{\mu_1} \cdot \frac{1 - e^{-bT}}{b} \cdot \left[ a \cdot \frac{1}{1 - ae^{-bT}} + 1 \right] = \Theta_2$$

We can find the value b such that  $\Theta_2 \le \kappa$  with any given number  $\kappa > 0$ .

**Remark 2:** Unlike the previous works in [1]-[6], the proposed control law focus on the efficiency including nonlinear system, external disturbance and unknown parameters.

#### 4. Simulation Results

The proposed control law is applied for the switched system as follows:

Let N=2 and the subsystems of the switched system are :

$$\begin{split} \dot{x} &= \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} u + 4 \begin{bmatrix} x_1 x_2 \\ x_2^2 \end{bmatrix} + \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} \\ \dot{x} &= \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} x + \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} u + 5 \begin{bmatrix} x_1^2 \\ x_1 x_2 \end{bmatrix} + \begin{bmatrix} d_3(t) \\ d_4(t) \end{bmatrix} \end{split}$$

Where  $x = (x_1 \ x_2)^T$ ;  $u = (u_1 \ u_2)^T$ ;

 $\begin{aligned} &d_{_{1}}(t)\text{ ,}d_{_{2}}(t)\text{ ,}d_{_{3}}(t)\text{ ,}d_{_{4}}(t) &\text{are bounded disturbance}\\ &\text{and }\left\|d_{_{i}}(t)\right\|\leq0.1\text{ ,}t\geq0\text{ and initial condition is chosen}\\ &\text{as: }x_{_{0}}=\left(2\ 1\right)^{\text{T}} \end{aligned}$ 

The switched reference model is described as follows:

$$\dot{\mathbf{x}}_{\mathrm{m}} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{r}$$

$$\dot{\mathbf{x}}_{\mathrm{m}} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \mathbf{r}$$

where  $x_m = (x_{m1} \ x_{m2})^T$  is the desired state,  $r = (r_1 \ r_2)^T$  is the reference input. We choose  $r = \begin{cases} (1 \ 1)^T \ \text{if} \quad t \geq 0 \\ 0 \quad \text{if} \quad t < 0 \end{cases}$  and initial condition is chosen

as: 
$$x_{m0} = (0.5 \ 0.5)^T$$

From the Lyapunov equation:

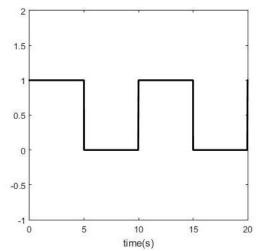
$$A_{mi}^{T}.P_{i} + P_{i}.A_{mi} = -Q_{i}$$

We obtain:

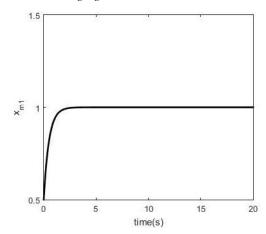
$$\begin{aligned} P_1 &= P_2 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}; Q_1 = \begin{bmatrix} 40 & 0 \\ 0 & 80 \end{bmatrix}; Q_2 = \begin{bmatrix} 80 & 0 \\ 0 & 120 \end{bmatrix} \\ S_1 &= S_2 = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} \end{aligned}$$

$$\alpha = \beta = \gamma = 0.5$$

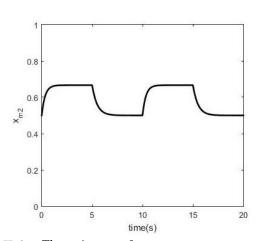
Simulation results are depicted in Fig. 1-6 and the good behavior are described



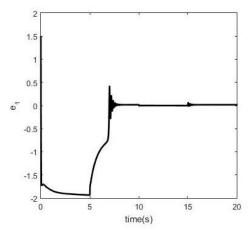
**H.1** Switching signal:



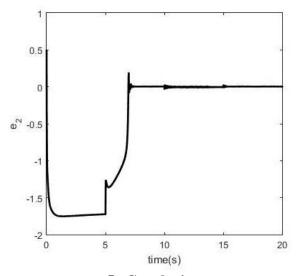
**H. 2** The trajectory of state  $x_{m1}$ :



**H.3** The trajectory of state  $x_{m2}$ :



### **H.4** The behavior of error 1



### 5. Conclusion

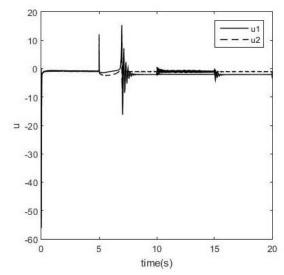
In this paper, robust adaptive controller have been implemented for the switched system in presence of nonlinear system, external disturbance and unknown parameters. The proposed control law ensure that closed system converges to reference model.

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#### **H. 5** The behavior of error 2



H. 6 The control input signals

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