Mixed integer programming for vehicle routing problem with time windows

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Abstract: Being a key element in logistics distribution, vehicle routing problem becomes an importance research topic in management and computation science. Vehicle routing problem (VRP) with time windows is a specialisation of VRP. In this paper, a brief description of VRP is presented. A mixed integer programming (MIP) is utilised to solve the vehicle routing problem with time windows (VRPTW). A novel mathematical model of MIP is formulated and implemented using IBM CPLEX. A novel constraint is designed to optimise the number of vehicle used. The proposed model is used to optimise both transportation cost and number of vehicle used simultaneously. The proposed model is tested on two well-known instances of Solomon's benchmark test problem. Experimental results illustrate that the proposed formulation provides promising solutions in reasonable computation time. The sensitivity analysis of customer nodes is also studied.

Keywords: VRP; vehicle routing problem; time windows; Solomon's instance; CPLEX; MIP; mixed integer programming.

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1 Introduction

Logistics distribution has a keen role in the present logistics management. For the last few decades, vehicle routing problem (VRP), due to its key role in logistics distribution, has become a topic of main concern for researchers working in the field of distribution and computation. It has been observed that distribution costs include half of the total system cost in most of the businesses such as food or drinks. These might sums up to 70% of the total cost (De Backer and Furnon, 1997; Golden and Wasil, 1987; Halse, 1992). Hence, the planning of vehicles and routes could help to minimise the input resources. Toth and Vigo (2002) reported in their work that computerised methods would decrease the transportation cost by 5–20%. The well-known examples of VRP are supermarket distribution, freight transportation and industrial gases delivery.

The simple routing problem is known as travelling salesman problem (TSP). It suggests that a salesman visiting number of cities should visit all the cities travelling the minimum distance and returning to the same city where started. The VRP is the extension of multiple travelling salesman problem (MTSP) in which each request is associated with each city and each vehicle has a specific capacity limit. When the time window is added to each customer location then the problem becomes vehicle routing problem with time windows (VRPTW). The VRPTW is an expansion of VRP. The aim of VRP is to promote an arrangement of customers with known requests while minimising the cost on given set of routes. VRPTW involves the delivery in specific time window provided by the customers.

The time window constraint can be further classified into two categories such as soft and hard time windows. The former one involves the violation of time constraint at the sake of cost. If the vehicle arrives before its earliest time period, then vehicle can violate time window constraints. A new penalty function is added to vehicle in case of arriving late or early. The former one does not take into consideration a vehicle to arrive after the latest time to begin service. Hard time window constraint restricts the service time to fall between the earliest and the latest time constraints. If the vehicle arrives before its earliest time, then it has to wait until the time window starts.

The focus of this paper is hard time window constraint. The mixed integer programming (MIP) has been used to solve the VRPTW. A novel constraint has been designed to minimise the number of vehicle used. This constraint has been utilised in the MIP. The proposed model has been tested on two instances of Solomon's benchmark problem. The remaining paper is structured as follows. Section 2 presents the basic concepts of VRP and VRPTW. Section 3 introduces the overview of related work done in the field of VRPTW. The problem formulation is described in Section 4. The computational results are presented in Section 5. Finally, the concluding remarks are drawn in Section 6.

2 Background

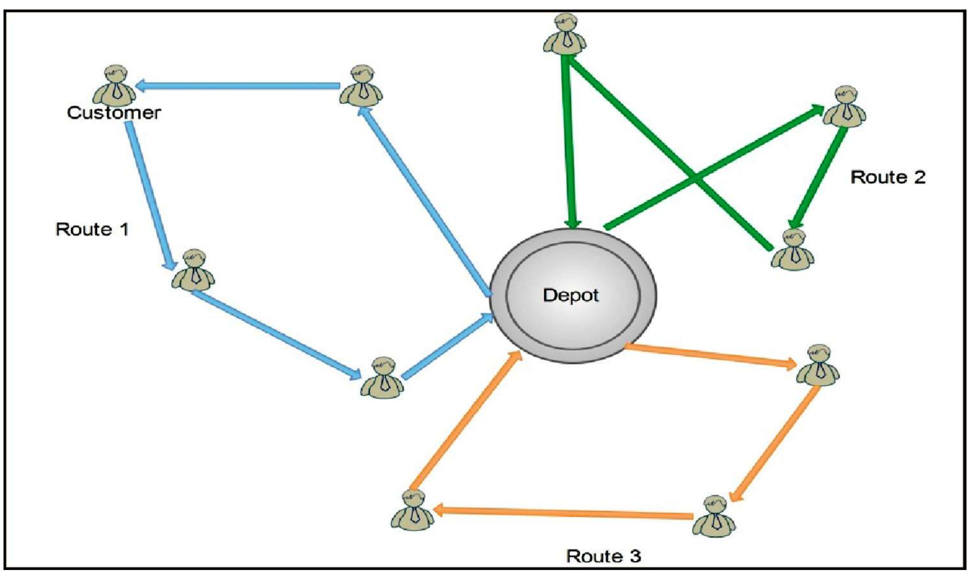
This section provides the basic concepts used in VRP and VRPTW.

2.1 Vehicle routing problem

The assignment of designing delivery or pickup service in the area of transportation and supply chain is termed as VRP. VRP was first introduced by Dantzig and Ramser (1959) as a generalisation of TSP. The main aim of VRP is to select the optimal routes for gasoline delivery trucks which provide service between single distribution terminal and number of service stations. Collection of household garbage, package delivery and delivery trucks are the main sources of distribution or collection in real world. VRP has a vital role in distributions and logistics.

VRP is the most studied problem in optimisation literature because of its practical applicability and its considerable difficulty. VRP refers to the situation where distinct routes can be determined for each vehicle whose starting and ending journey at one or more depots. In case of adequate supply of vehicles, it also aims to use a minimum number of vehicles. Figure 1 shows the illustration of classical VRP having 10 customers and 3 vehicles. The routes in VRP are discriminated through colour lines.

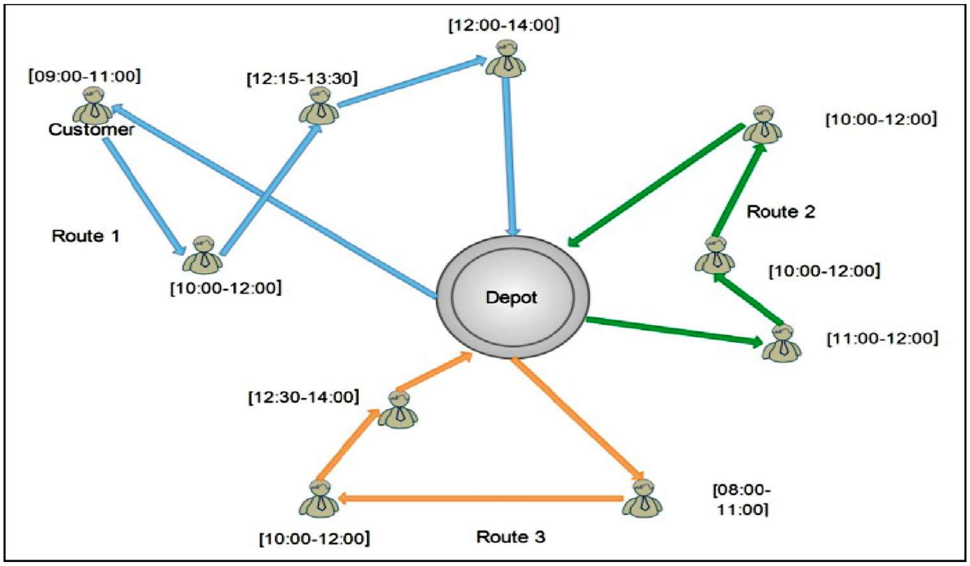
Figure 1 A classical vehicle routing problem (see online version for colours)



2.2 Vehicle routing problem with time windows

The VRPTW is the specialisation of classical VRP, being very applicable in real world problems. It involves a fleet of vehicles starting from a depot to serve a number of customers which are diversely located. Each customer has a fixed demand request. Owing to the applicability of VRPTW in real life situations, it has been widely investigated through various researchers. Figure 2 describes the illustration of VRPTW having hard time window constraint. The routes in VRPTW are discriminated through colour lines. Each customer has fixed time constraint window.

Figure 2 A typical vehicle routing problem with time windows (see online version for colours)



The main difference between VRPTW and VRP is that there are time windows imposed by customers. Each customer can only be visited within the time window. For VRPTW, the time window is the important factor during the selection of routes. The VRP is only routing problem. VRPTW is the combination of routing and scheduling problem. Besides these dissimilarities, both VRP and VRPTW are NP-hard optimisation problems.

3 Literature review

Various researches have been done in the field of VRPTW which can be used to specify many real world problems. The early works on the VRPTW include case studies such as Pullen and Webb (1967), Knight and Hofer (1968) and Madsen (1976). Earlier studies on VRPTW include both optimisation algorithms and heuristic approaches. However, the focus of current research is heuristic approaches. Various surveys have been done in the field of VRPTW such as Golden and Assad (1986, 1988), Desrochers et al. (1988) and Solomon and Desrosiers (1988). Many real-life applications have been proposed for VRPTW including bank deliveries (Lambert et al., 1993), postal deliveries (Mechti et al., 2001), industrial refuse collection (Golden et al., 2001), national franchise restaurant services (Russell, 1995) and industrial gases delivery (Campbell et al., 2001).

VRPTW has been solved using heuristic and exact optimisation approaches. Owing to the complexity of the VRP family problems, exact methods become useless especially when large-scale instances are tackled. Kolen et al. (1987) used branch and bound approach to solve a VRPTW having customer nodes varying between 6 and 15. When the number of nodes is set to 6, then computation took about 1 min. The branch and bound approach was unable to process for 12 customer nodes. Sam (1995) worked on reducing the delivery costs by considering only travel time taken by vehicle while traversing from one customer to another customer. Moreover, this work involves product delivery by violating the time windows and compensating it by adding a penalty in the objective function. Dondo et al. (2003) proposed a mixed integer linear programming (MILP) for solving multi-depot heterogeneous-fleet VRPTW problem. Dondo and Cerda (2007) offered three-phase heuristics for *m*-depot routing problem with time windows and heterogeneous vehicles. The clustering algorithm was applied on VRPTW which could solve at most 25 nodes by restructuring MILP.

Cetinkaya et al. (2013) proposed a hybrid approach that combines both MIP and memetic algorithm (MA) for solving two-stage VRP with arc time windows. He added time variant to each and every arcs. Suwansuksamran and Ongkunaruk (2013) presented a MIP for a case study company. In this problem, a binary variable is used to indicate whether a customer has time window constraint or not. Simbolon and Mawengkang (2014) suggested a mixed integer approach for VRP with window having a variant of avoiding routes. When pair of edges occurs frequently, then the sub-route is avoided to eliminate the heavy traffic. The experimental results revealed that MIP formulation gave good results for both small- and medium-size instances. However, it was concluded that MIP formulation is good enough for instances having up to 50 customers.

Besides these, various researchers have applied heuristics techniques for VRPTW. Holland (1975) proposed a genetic algorithm which initialises the random chromosome and generates the solution for VRPTW in bit strings. Thereafter, chromosomes were selected for crossover and mutation process to produce children. The process keeps on repeating until the evolution has been converged. El-Sherbeny (2001) solved VRPTW problem using a multi-objective simulated annealing (MOSA) method. Here, three objectives were defined such as minimising the vehicle used, total time and concerning the flexible duration of routes. Beatrice et al. (2006) presented a hybrid approach that comprises of genetic algorithm (GA) and Pareto ranking to solve VRPTW. This method was applicable on problems with more than 100 customer nodes.

Sabar et al. (2015) suggested an approach that consists of two phases, i.e., math phase and hyper heuristic phase. This approach is better than the simple hyper heuristics approach. Hokama et al. (2016) suggested branch-and-cut approach for solving VRP having loading constraints. It solves the problem in smaller computation time, but packing strategies and lower bounds could be improved. Ozbaygin et al. (2017) proposed a branch-and-price algorithm for VRP. This technique used the concept of roaming delivery locations. The proposed approach provided optimal solution. However, it requires the prior knowledge of travel time and itinerary of customer.

From the literature, it has been observed that there are two main research issues associated with VRPTW. First, the identification of route feasibility is an important issue for VRPTW. Second, the selection of time window during the design of routes is another research issue.

4 Proposed approach

In this section, we will describe motivation and problem formulation of VRPTW. Further, the algorithm of VRPTW is presented.

4.1 Motivation

After the thorough investigation of VRPTW, it has been observed that the high transportation cost and long scheduling time are the main two areas of concern. The time window constraint makes the VRP more applicable in real instances. Moreover, it has been noticed that there is scope to design VRPTW problem as a bi-objective problem that minimise both the transportation cost and used number of vehicles.

4.2 Problem statement

The problem is to determine the set of routes that minimise transportation cost and number of vehicles used while satisfying the following constraints:

- the load of a vehicle cannot increase its predefined capacity
- vehicle will not service the customer before the earliest time and not later than the latest time
- number of vehicle should be minimised
- each vehicle should visit customer only once.

The novel mathematical formulation of MIP proposed to satisfy service time constraint. Here, the main aim of proposed approach is to minimise the total transportation cost and number of vehicles used.

4.3 Problem formulation of VRPTW

From a graphical point of view, VRPTW is defined as a graph (C', E) which is to be traversed by a fleet of vehicles V. Here, the vehicles are considered homogeneous, i.e., every vehicle has same capacity.

Nomenclature indices

i, j	$(i, j) \in E$
i	customer i

Model parameters

C	Set of all customers $(1, 2, \ldots, n)$
C'	Set of all nodes $C' = C \cup 0 \cup n+1$
Q	Vehicle capacity
V	Number of vehicles
c_{ij}	Cost of travel from customer i to customer j
t_{ij}	Travel time from customer i to customer j
r_i	Demand to service customer i
$[e_i, l_i]$	Time window requested by customer i, where e_i indicates the earliest service starting time and l_i indicates the latest service starting time

Decision variables

The model includes two types of decision variables X and S. The decision variable X_{ijk} is defined as follows:

$$X_{ijk} = \begin{cases} 1, & \text{if edge from customer } i \text{ to customer } j \text{ is traversed by vehicle } k \\ 0, & \text{otherwise} \end{cases}$$

The another decision variable S_{ik} represents the time at which vehicle k service customer i.

Let us assume that $S_{0k} = 0$ and $S_{(n+1)k}$ denote the time at which vehicles arrives at returning depot. The objective of model is to design a set of minimal cost routes, one for each vehicle, such that all customers are serviced exactly once while minimising the number of vehicles. Therefore, the objective function is considered as bi-objective. The split deliveries are not allowed. The routes must be feasible with respect to the capacity of vehicles and time windows of the customers serviced.

The mathematical model of VRPTW is given below:

$$\text{Minimise} \quad \sum_{k \in C'} \sum_{(i,j) \in E} c_{ij} X_{ijk} \tag{1}$$

Subject to

$$\sum_{k \in V} \sum_{j \in C'} X_{ijk} = 1, \quad \forall i \in C \tag{2}$$

$$\sum_{i \in C} r_i \sum_{j \in C'} X_{ijk} \le Q, \quad \forall k \in V \tag{3}$$

$$\sum_{j \in C'} X_{ojk} = 1, \quad \forall k \in V \tag{4}$$

$$\sum_{i \in C'} X_{ipk} - \sum_{j \in C'} X_{hjk} = 0, \quad \forall p \in C, \forall k \in V \tag{5}$$

$$\sum_{i \in C'} X_{i,n+1,k} = 1, \quad \forall k \in V \tag{6}$$

$$X_{ijk}(S_{ik} + t_{ij} - S_{jk}) \le 0, \quad \forall (i,j) \in E, \forall k \in V \tag{7}$$

$$e_i \le S_{ik} \le l_i, \quad \forall i \in C', \forall k \in V \tag{8}$$

$$X_{ijk} \in \{0,1\} \quad \forall (i,j) \in E, \forall k \in V \tag{9}$$

The objective function (1) states that the total cost should be minimised. Constraint (2) suggests that only one vehicle should service one node while constraint (3) ensures that the capacity constraint is fulfilled. Here, no vehicle can serve the customer demands beyond its capacity permits. Constraints (4)–(6) fulfil the flow constraint for the path traversed by vehicle k. Constraint (7) makes sure that vehicle k, $k \in V$ should not arrive at customer j before $S_{ik} + t_{ij}$ if travelling from node i to j. Constraint (8) checks for the satisfaction of time constraint. An unused vehicle is represented by an empty route from node 0 to $n + 1$.

A novel constraint for minimisation of vehicles used is defined below:

$$\sum_{k \in V} \sum_{j \in C'} x_{0jk} \le |V|, \quad \forall k \in V, \forall j \in C' \tag{10}$$

Note that constraint (7) may results in non-convex optimisation because it involves quadratic terms. These terms are rewritten in linear form which is given below.

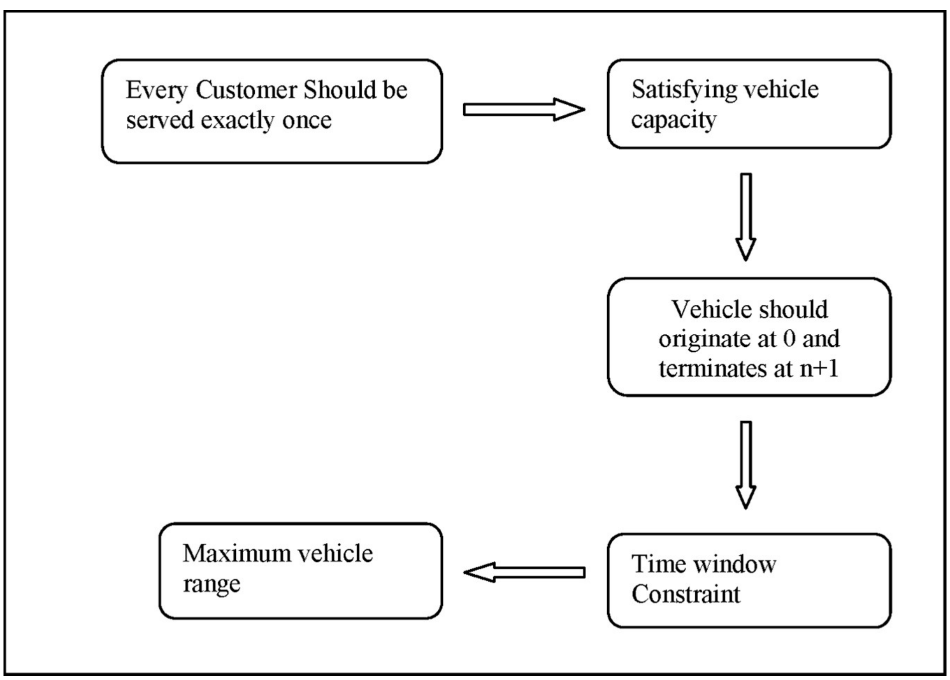
$$S_{ik} + t_{ij} - M_{ij}(1 - X_{ijk}) \le S_{jk} \quad \forall i, j \in C', \forall k \in V \tag{11}$$

Here, M_{ij} is described as a large constant that can be replaced by $\max(l_i + t_{ij} - e_i, 0)$.

4.4 Constraint workflow

Figure 3 shows the constraint workflow of VRPTW. The constraint workflow consists of five main stages. The first stage includes the integrity constraint of given problem. It is satisfied that each customer should be served exactly once by each vehicle. In the second stage, the capacity constraint of vehicle is maintained. It ensures that the vehicle should deliver the goods in their predefined capacity. The third stage ensures that each vehicle should have an edge between an intermediate node and the start node. The fourth stage is responsible for time window constraint. It ensures that no vehicle is allowed to serve before the earliest time and later than the latest time for a particular customer. The fifth stage is responsible for minimising the vehicle used.

Figure 3 Constraint workflow of VRPTW



4.4.1 Algorithm

Step 1. a) Obtain the initial linear relaxation of the mixed integer programming formulation.

b) Construct the initial feasible solution and designate this solution as upper bound.

c) Add the root node to tree and set the root node as current node.

Step 2. *If* termination criterion is satisfied, *then*

Return the upper bound and Exit

Else

Select a node that has the smallest objective function value

Endif

Step 3. Solve the corresponding linear relaxation and obtain the optimal solution

Step 4. *If* current node is infeasible or worse than upper bound, *then*

Remove the current node from the tree and *goto* Step 2.

Endif

Step 5. *If* Optimal solution is integer, *then*

Update the upper bound and remove it from tree and *goto* Step 2.

Endif

Step 6. *If* Optimal solution is not integer, *then*

Construct a feasible solution from existing solution and update the upper bound.

Endif

5 Experimental results and analysis

In this section, the experimentation is designed to evaluate the performance of proposed approach on two well-known benchmark instances of Solomon (1987). The algorithm was coded in IBM CPLEX and implemented on a Core i7 Intel Processor under Windows 7 with 1 GB RAM.

5.1 Solving tool and test instances

IBM CPLEX is one of the tools widely used to solve combinatorial optimisation problems. It has a concert technology that provides interfaces to C++, C# and Java languages. It is accessible through independent modelling systems such as a mathematical programming language (AMPL) and TOMLAB. It is also recognised as a constraint solving toolkit suitable for solving optimisation models. It uses inbuilt procedures to solve the MIP in short time. It can be used to solve a variety of different optimisation problems in a variety of computing environments. IBM CPLEX is an exact solver that uses MIP to search the desired solutions.

The proposed approach is tested on two test instances of VRPTW. The used test instances are taken from Solomon's VRPTW instances (Solomon, 1983). The test instances are R101 and C101. The first test instance namely R101 that belongs to class with randomly generated customers. This class is solvable up to 33 customer nodes. The second test instance namely C101 that belongs to class having clustered customer locations. This class is solvable up to 60 customer nodes. The objective of proposed approach is to minimise the number of vehicles and carrying capacity of each vehicle should be 200 units.

5.2 *Performance evaluation*

Table 1 shows the performance evaluation of proposed approach on test instances R101. The six different cases are generated from R101 instance. The number of customers is varying from 5 to 33. The computed distance, used number of vehicles and computational time are taken for performance comparison. As seen from Table 1, the computational time increases with increase in number of customer node. The optimum distance and vehicle used is also increase with increase in customer nodes.

The performance evaluation on test instances C101 is presented in Table 2. The seven different cases are generated for C101. The number of customer nodes is varying from 5 to 60. It has been observed from Table 2 that the number of vehicle used is one for number of customer nodes 5 and 10. For number of customer nodes 20 and 25, the optimum vehicles used are 3. The number of vehicle used, objective function value and computation time increase with increase in number of customer nodes.

Table 1 Performance evaluation of proposed approach on test instance R101

Instance	*Number of customer*	*Objective distance*	*Minimised number of vehicles*	*Average time (in seconds)*
R101	5	156.35	2	0.76
R101	10	269.54	4	1.92
R101	15	383.83	3	1.99
R101	20	511.26	6	2.45
R101	25	618.34	6	4.59
R101	33	713.58	11	11.59

Table 2 Performance evaluation of proposed approach on test instance C101

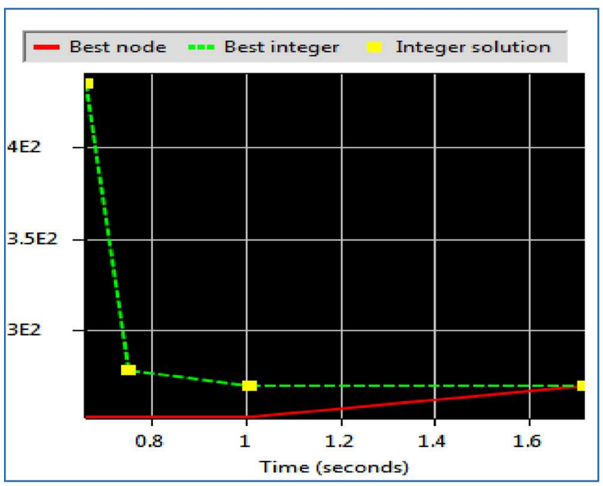
Instance	*Number of customer*	*Objective distance*	*Minimised number of vehicles*	*Average time (in seconds)*
C101	5	42.42	1	0.69
C101	10	58.34	1	1.20
C101	15	142.17	2	2.16
C101	20	175.39	3	2.89
C101	25	191.83	3	5.22
C101	50	363.28	5	19.49
C101	60	470.13	6	36.92

5.3 Convergence analysis

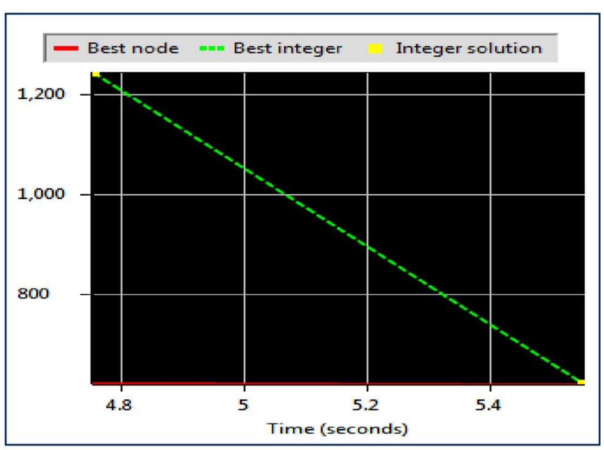
Figures 4 and 5 show the convergence curves obtained from proposed model using R101 and C101 test instances, respectively. In these figures, the yellow point indicates a node where an integer value has been found. The green line represents the evolution of best integer value computed. The red line gives a bound on the final solution. These figures are drawn between objective function value and time.

It has been observed from figures that the proposed model explore the promising regions for solutions. Thereafter, it converges towards the best solution. Figure 4(a) and (b) shows the convergence curve for 10 and 25 customer nodes. As seen from Figure 4(a), the slope of convergence curve is varying throughout the process whereas the slope of convergence curve is same throughout the process for 25 customer nodes. From Figure 5(a) and (b), it has also been observed that the slope of convergence curve is same throughout the process for both 10 and 25 customer nodes.

Figure 4 Convergence curve obtained from CPLEX (a) 10 customer nodes (b) 25 customer nodes for test instance R101 (see online version for colours)

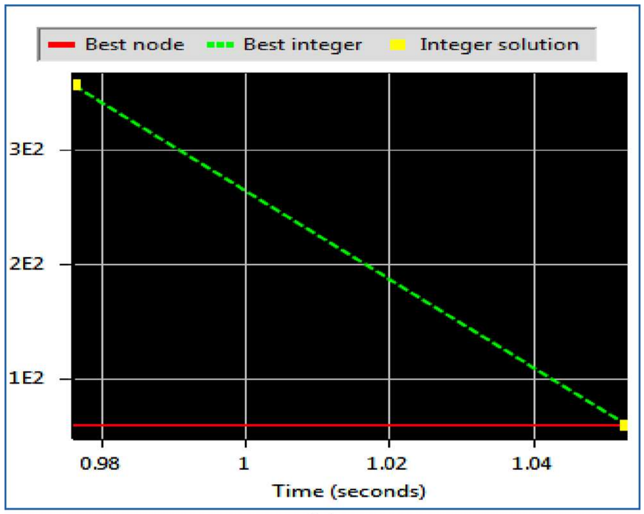


(a)

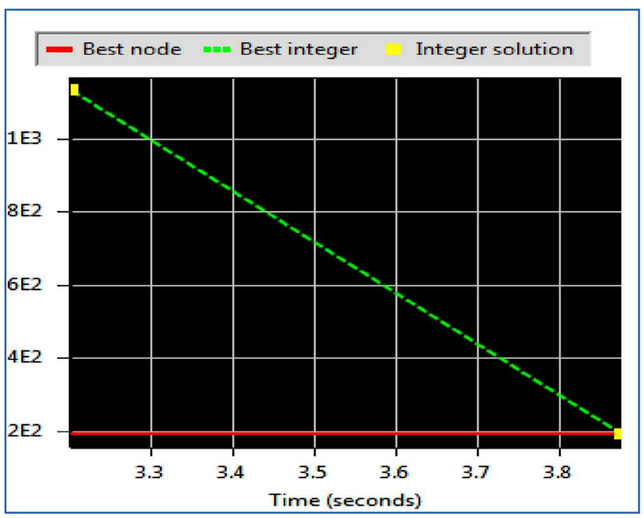


(b)

Figure 5 Convergence curve obtained from CPLEX (a) 10 customer nodes (b) 25 customer nodes for test instance C101 (see online version for colours)



(a)



(b)

Therefore, mentioned results depict that the proposed model provides better convergence towards the optimal solution. The performance of proposed model is least affected for above-mentioned customer nodes.

5.4 Sensitivity analysis

The effect of number of customer has been investigated on computational time and vehicle used is studied. Figures 6 and 7 show the performance of proposed approach over R101 and C101 instance, respectively. For R101 test instance, the computational time greatly increase with increase in number of customers. The number of vehicle used also increases with increase in the value of customer nodes. The result reveals that the number of vehicles used is six for both 20 and 25 customer nodes.

Figure 6 Comparison between customer nodes and vehicles used with average time on secondary axis for R101 instance (see online version for colours)

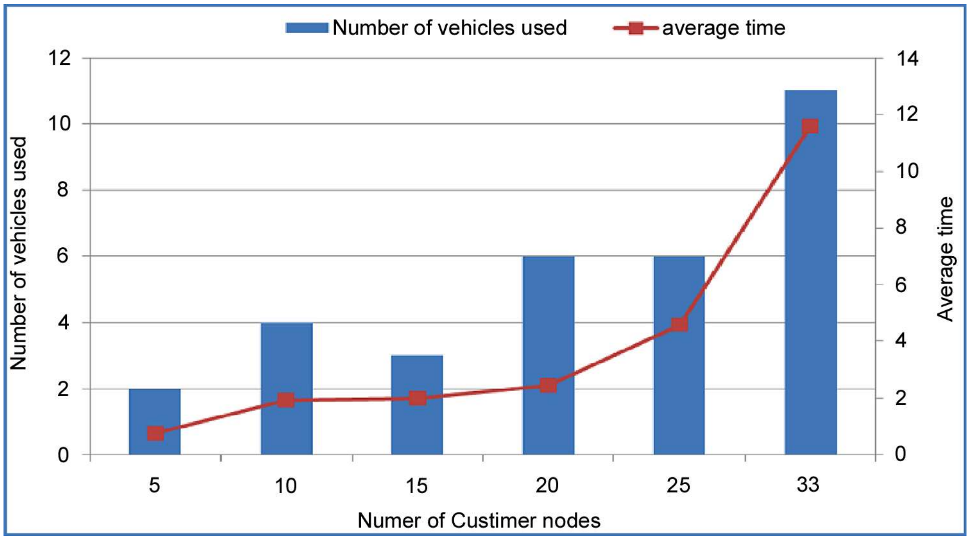
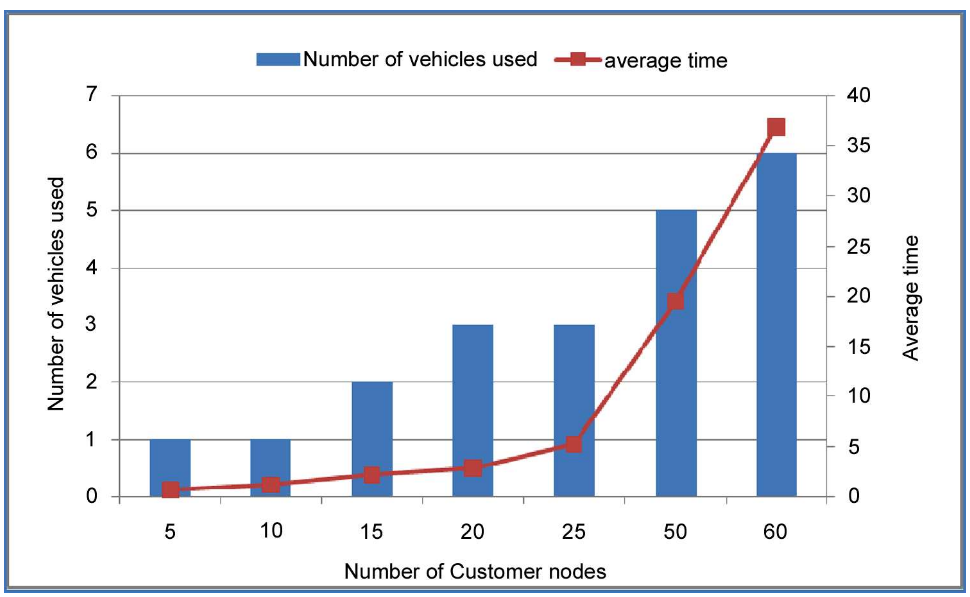


Figure 7 Comparison between customer nodes and vehicles used with average time on secondary axis for C101 instance (see online version for colours)



For C101 test instances, the performance of proposed approach is greatly reduced with increase in number of customers. The results obtained from Figure 6 depict that the number of vehicles used and average computation time increase with increase in customer nodes. The optimum number of vehicle used is one for 5 and 10 customer nodes. It can also be observed that the minimum number of vehicle used is three for 20 and 25 customer nodes.

6 Conclusion and future scope

This paper presents an approach for solving VRPTW using MIP. The proposed approach has been used to minimise transportation cost and number of vehicles used simultaneously. A new constraint has been designed to optimise the number of vehicles used. The proposed approach has been tested on two well-known test instances of Solomon's benchmark problem. The computational results reveal that both cost and number of vehicles are optimised at reasonable time. The sensitivity analysis for customer nodes has been investigated on proposed approach, where it was noticed that the computational time and number of vehicle used increase with increase in number of customer node.

This work opens several research directions for future studies. First, metaheuristic techniques can be applied to solve this problem. Second, metaheuristic can be integrated with MIP to improving their performance. Third, investigation of different constraint for VRPTW under different environment would be a valuable contribution. Also, it is worth to investigate and find the best constraint satisfaction technique among the available techniques.

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