1) © $f(x) = \sin(\pi x) \longrightarrow f'(x) = \pi \cdot \cos(\pi x)$

$x_0 = 1 \longrightarrow f(x_0) = \sin(\pi) = 0$ $\qquad f''(x) = -\pi^2 \cdot \sin(\pi x)$

$x_1 = 1.25 \longrightarrow f(x_1) = \sin(1.25\pi) \approx -0.707106$ $\qquad f'''(x) = -\pi^3 \cos(\pi x)$

$x_2 = 1.6 \longrightarrow f(x_2) = \sin(1.6\pi) \approx -0.9510565$ $\qquad f''''(x) = \pi^4 \sin(\pi x)$

$x_3 = 1.8 \longrightarrow f(x_3) = \sin(1.8\pi) \approx -0.587785$ $\qquad f'''''(x) = \pi^5 \cos(\pi x)$

$P_{1,2}(x)$: $\left| \frac{f^2(\xi(x))}{2!} (x-x_1)(x-x_2) \right|$

$= \underbrace{\pi^2 \cdot \sin(\pi \xi(x))} \cdot 1/2 \cdot (x-1.25)(x-1.6)$

$= \pi^2 \cdot 1/2 \cdot (0.15)(0.2)$

$= 0.148044066$

$P_{0,1,2}(x)$: $\left| \frac{f^3(\xi(x))}{3!} (x-x_0)(x-x_1)(x-x_2) \right|$

$= \underbrace{\pi^3 \cos(\pi \xi(x))} \cdot 1/6 \cdot (x-1)(x-1.25)(x-1.6)$

$= \pi^3 \cos(1.8\pi) \cdot 1/6 \cdot (0.4)(0.15)(0.2)$

$= 0.050169209$

$P_{1,2,3}(x)$: $\left| \frac{f^3(\xi(x))}{3!} (x-x_0)(x-x_1)(x-x_2) \right|$

$= \underbrace{\pi^3 \cos(\pi \xi(x))} \cdot 1/6 \cdot (x-1.25)(x-1.6)(x-1.8)$

$= \pi^3 \cos(1.8\pi) \cdot 1/6 \cdot (0.15)(0.2)(0.4)$

$= 0.050169209$

$P_{0,1,2,3}(x)$: $\left| \frac{f^4(\xi(x))}{4!} (x-x_0)(x-x_1)(x-x_2)(x-x_3) \right|$

$= \underbrace{\pi^4 \sin(\pi \xi(x))} \cdot 1/24 \cdot (x-1)(x-1.25)(x-1.6)(x-1.8)$

$= \pi^4 \cdot 1/24 \cdot (0.4)(0.15)(0.2)(0.4)$

$= 0.0194818182$

The error bounds for the interpolating polynomials support the result from the previous part because the error bounds are greater than the actual error.