ACM Template

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Last build at October 21, 2012

Contents

1 To Do List

所有带*的内容。。。

可以从原来的模板里面继承一些好东西过来。

set,map,multiset等的搞基用法，以及注意事项。

生成树计数

2 注意事项

10^6数量级慎用后缀数组

TLE的时候要冷静哟。。

思考的时候结合具体步骤来的话 会体会到一些不同的东西

C++与G++是很不一样的。。。

map套字符串是很慢的。。。

栈会被记录内存。。。

浮点数最短路要注意取≤来判断更新。。。

注意 long long

不要相信.size()

重复利用数组时 小心数组范围

先构思代码框架 每当实际拍马框架变化时 停手 重新思考

有时候四边形不等式也是帮得上忙的 dp 优化是可以水的

结构体里面带数组会非常慢,有时候 BFS 把数组压成数字会快很多。

```
1 void fun(int a[])
2 {
3   printf("%d\n",sizeof(a));
4 }
```

结果是 sizeof(a[0]),如果传数组指针然后要清空的话不要用 sizeof。

sqrt 某些时候会出现 sqrt(-0.00)的问题。

将code::blocks的默认终端改成gnome-terminal

```
1 gnome-terminal -t $TITLE -x
```

最小割割集找法 在残量网络中从源点出发能到的点集记为S原图中S到S'的边即是最小割集

double全局变量初始值可能不是0

3 字符串处理

3.1 *AC自动机

3.1.1 指针

```
const int CHAR=26;
const int TOTLEN=500000;
const int MAXLEN=1000000;
struct Vertex
{
  Vertex *fail,*next[CHAR];
  Vertex(){}
  Vertex(bool flag)//为什么要这样写?
  {
    fail=0;
    memset(next,0,sizeof(next));
  }
};
int size;
Vertex vertex[TOTLEN+1];
void init()
{
  vertex[0]=Vertex(0);
  size=1;
}
void add(Vertex *pos,int cha)
{
  vertex[size]=Vertex(0);
  pos->next[cha]=&vertex[size++];
}
void add(vector<int> s)
{
  int l=s.size();
  Vertex *pos=&vertex[0];
  for (int i=0; i<l; i++)
  {
    if (pos->next[s[i]]==NULL)
      add(pos,s[i]);
    pos=pos->next[s[i]];
  }
}
void bfs()
{
  queue<Vertex *> que;
  Vertex *u=&vertex[0];
  for (int i=0; i<CHAR; i++)
    if (u->next[i]!=NULL)
    {
      que.push(u->next[i]);
      u->next[i]->fail=u;
```

```
46      }
47      else
48        u->next[i]=u;
49    u->fail=NULL;
50    while (!que.empty())
51    {
52      u=que.front();
53      que.pop();
54      for (int i=0; i<CHAR; i++)
55        if (u->next[i]!=NULL)
56        {
57          que.push(u->next[i]);
58          u->next[i]->fail=u->fail->next[i];
59        }
60        else
61          u->next[i]=u->fail->next[i];
62    }
63  }
```

3.1.2 非指针

```
1   struct Trie
2   {
3     int next[50][10],fail[50];
4     bool end[50];
5     int L,root;
6
7     int newNode()
8     {
9       for (int i = 0;i < 10;i++)
10        next[L][i] = -1;
11      end[L] = false;
12      return L++;
13    }
14
15    void Init()
16    {
17      L = 0;
18      root = newNode();
19    }
20
21    void Insert(char s[])
22    {
23      int now = root;
24      for (int i = 0;s[i] != 0;i++)
25      {
26        if (next[now][s[i]-'0'] == -1)
27          next[now][s[i]-'0'] = newNode();
28        now = next[now][s[i]-'0'];
29      }
30      end[now] = true;
31    }
```

```
32
33    void Build()
34    {
35      queue<int> Q;
36      for (int i = 0;i < 10;i++)
37        if (next[root][i] == -1)
38          next[root][i] = root;
39        else
40        {
41          fail[next[root][i]] = root;
42          Q.push(next[root][i]);
43        }
44      while (!Q.empty())
45      {
46        int now = Q.front();
47        Q.pop();
48        end[now] |= end[fail[now]];
49        for (int i = 0;i < 10;i++)
50          if (next[now][i] == -1)
51            next[now][i] = next[fail[now]][i];
52          else
53          {
54            fail[next[now][i]] = next[fail[now]][i];
55            Q.push(next[now][i]);
56          }
57      }
58    }
59  };
```

3.2 后缀数组

3.2.1 DC3

所有下标都是0 n-1，height[0]无意义。

```
1  //所有相关数组都要开三倍
2  const int maxn = 300010;
3  # define F(x) ((x)/3+((x)%3==1?0:tb))
4  # define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)
5  int wa[maxn * 3], wb[maxn * 3], wv[maxn * 3], ws[maxn * 3];
6  int c0(int *r, int a, int b)
7  {
8    return r[a] == r[b] && r[a + 1] == r[b + 1] && r[a + 2] == r[b +
       2];
9  }
10 int c12(int k, int *r, int a, int b)
11 {
12   if (k == 2) return r[a] < r[b] || r[a] == r[b] && c12(1, r, a +
       1, b + 1);
13   else return r[a] < r[b] || r[a] == r[b] && wv[a + 1] < wv[b + 1];
14 }
15 void sort(int *r, int *a, int *b, int n, int m)
16 {
```

```
17    int i;
18    for (i = 0; i < n; i++) wv[i] = r[a[i]];
19    for (i = 0; i < m; i++) ws[i] = 0;
20    for (i = 0; i < n; i++) ws[wv[i]]++;
21    for (i = 1; i < m; i++) ws[i] += ws[i - 1];
22    for (i = n - 1; i >= 0; i--) b[--ws[wv[i]]] = a[i];
23    return;
24  }
25  void dc3(int *r, int *sa, int n, int m)
26  {
27    int i, j, *rn = r + n, *san = sa + n, ta = 0, tb = (n + 1) / 3,
          tbc = 0, p;
28    r[n] = r[n + 1] = 0;
29    for (i = 0; i < n; i++) if (i % 3 != 0) wa[tbc++] = i;
30    sort(r + 2, wa, wb, tbc, m);
31    sort(r + 1, wb, wa, tbc, m);
32    sort(r, wa, wb, tbc, m);
33    for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
34      rn[F(wb[i])] = c0(r, wb[i - 1], wb[i]) ? p - 1 : p++;
35    if (p < tbc) dc3(rn, san, tbc, p);
36    else for (i = 0; i < tbc; i++) san[rn[i]] = i;
37    for (i = 0; i < tbc; i++) if (san[i] < tb) wb[ta++] = san[i] * 3;
38    if (n % 3 == 1) wb[ta++] = n - 1;
39    sort(r, wb, wa, ta, m);
40    for (i = 0; i < tbc; i++) wv[wb[i] = G(san[i])] = i;
41    for (i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
42      sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
43    for (; i < ta; p++) sa[p] = wa[i++];
44    for (; j < tbc; p++) sa[p] = wb[j++];
45  }
46  //str和sa也要三倍
47  void da(int str[], int sa[], int rank[], int height[], int n, int m
        )
48  {
49    for (int i = n; i < n * 3; i++)
50      str[i] = 0;
51    dc3 (str , sa , n + 1 , m);
52    int i, j, k;
53    for (i = 0; i < n; i++)
54    {
55      sa[i] = sa[i + 1];
56      rank[sa[i]] = i;
57    }
58    for (i = 0, j = 0, k = 0; i < n; height[rank[i ++]] = k)
59      if (rank[i] > 0)
60        for (k ? k-- : 0 , j = sa[rank[i] - 1]; i + k < n && j + k <
              n &&
61             str[i + k] == str[j + k]; k ++) ;
62  }
```

3.2.2 DA

这份似乎就没啥要注意的了。

```
const int maxn = 200010;
int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];

bool cmp(int *r,int n,int a,int b,int l)
{
  return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
}
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
  int *s = str;
  int *x=wx,*y=wy,*t,p;
  int i,j;
  for(i=0; i<m; i++)wss[i]=0;
  for(i=0; i<n; i++)wss[x[i]=s[i]]++;
  for(i=1; i<m; i++)wss[i]+=wss[i-1];
  for(i=n-1; i>=0; i--)sa[--wss[x[i]]]=i;
  for(j=1,p=1; p<n && j<n; j*=2,m=p)
  {
    for(i=n-j,p=0; i<n; i++)y[p++]=i;
    for(i=0; i<n; i++)if(sa[i]-j>=0)y[p++]=sa[i]-j;
    for(i=0; i<n; i++)wv[i]=x[y[i]];
    for(i=0; i<m; i++)wss[i]=0;
    for(i=0; i<n; i++)wss[wv[i]]++;
    for(i=1; i<m; i++)wss[i]+=wss[i-1];
    for(i=n-1; i>=0; i--)sa[--wss[wv[i]]]=y[i];
    for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
      x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
  }
  for(int i=0; i<n; i++) rank[sa[i]]=i;
  for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
    if(rank[i]>0)
      for(k?k--:0,j=sa[rank[i]-1]; i+k < n && j+k < n && str[i+k]==
         str[j+k]; k++);
}
```

3.2.3 调用

注意几个数组的下标是不同的

```
char s[maxn];
int str[maxn],sa[maxn],rank[maxn],height[maxn];

int main()
{
  scanf("%s",s);
  int len = strlen(s);
  for (int i = 0;i <= len;i++)
    str[i] = s[i];
  da(str,sa,rank,height,len,128);
```

```
11
12    for (int i = 0;i < len;i++)
13    {
14      printf("sa␣=␣%d,␣height␣=␣%d,␣s␣=␣%s\n",sa[i],height[i],s+sa[i
          ]);
15    }
16    return 0;
17  }
```

3.2.4 最长公共前缀

记得不要忘记调用lcpinit！

```
1   int f[maxn][20];
2   int lent[maxn];
3   void lcpinit()
4   {
5     int i,j;
6     int n = len,k = 1,l = 0;
7     for (i = 0; i < n; i++)
8     {
9       f[i][0] = height[i];
10      if (i+1 > k*2)
11      {
12        k *= 2;
13        l++;
14      }
15      lent[i+1] = l;
16    }
17    for (j = 1; (1<<j)-1<n; j++)
18      for (i = 0; i+(1<<j)-1<n; i++)
19        f[i][j] = min(f[i][j-1],f[i+(1<<(j-1))][j-1]);
20  }
21  int lcp(int x,int y)
22  {
23    if (x > y)  swap(x,y);
24    if (x == y)
25      return x-sa[x];//自己和自己当然是自己的长度啦lcp
26    x++;
27    int k = lent[y-x+1];
28    return min(f[x][k],f[y-(1<<k)+1][k]);
29  }
```

3.2.5 最长公共前缀大于等于某个值的区间

```
1   void getinterv(int pos,int comlen,int& pl,int& pr)
2   {
3     int l,r,mid,cp;
4     l = 0;
5     r = pos;
6     while (l < r)
```

```
  {
    mid = l+r>>1;
    cp = lcp(mid,pos);
    if (cp < comlen)
      l = mid+1;
    else
      r = mid;
  }
  pl = l;

  l = pos;
  r = len-1;
  while (l < r)
  {
    mid = l+r+1>>1;
    cp = lcp(pos,mid);
    if (cp < comlen)
      r = mid-1;
    else
      l = mid;
  }
  pr = l;
}
```

3.3 后缀三兄弟

```
#include <cstdio>
#include <cstring>
#include <algorithm>
using namespace std;
const int CHAR = 26;
const int MAXN = 100000;
struct SAM_Node
{
  SAM_Node *fa,*next[CHAR];
  int len;
  int id,pos;
  SAM_Node() {}
  SAM_Node(int _len)
  {
    fa = 0;
    len = _len;
    memset(next,0,sizeof(next));
  }
};
SAM_Node SAM_node[MAXN * 2], *SAM_root, *SAM_last;
int SAM_size;
SAM_Node *newSAM_Node(int len)
{
  SAM_node[SAM_size] = SAM_Node(len);
  SAM_node[SAM_size].id=SAM_size;
  return &SAM_node[SAM_size++];
```

```
}
SAM_Node *newSAM_Node(SAM_Node *p)
{
  SAM_node[SAM_size] = *p;
  SAM_node[SAM_size].id=SAM_size;
  return &SAM_node[SAM_size++];
}
void SAM_init()
{
  SAM_size = 0;
  SAM_root = SAM_last = newSAM_Node(0);
  SAM_node[0].pos=0;
}
void SAM_add(int x,int len)
{
  SAM_Node *p = SAM_last, *np = newSAM_Node(p->len + 1);
  np->pos=len;
  SAM_last = np;
  for (; p && !p->next[x]; p = p->fa)
    p->next[x] = np;
  if (!p)
  {
    np->fa = SAM_root;
    return ;
  }
  SAM_Node *q = p->next[x];
  if (q->len == p->len + 1)
  {
    np->fa = q;
    return ;
  }
  SAM_Node *nq = newSAM_Node(q);
  nq->len = p->len + 1;
  q->fa = nq;
  np->fa = nq;
  for (; p && p->next[x] == q; p = p->fa)
    p->next[x] = nq;
}
void SAM_build(char *s)
{
  SAM_init();
  int l = strlen(s);
  for (int i = 0; i < l; i++)
    SAM_add(s[i] - 'a',i+1);
}

SAM_Node * SAM_add(SAM_Node *p, int x, int len)
{
  SAM_Node *np = newSAM_Node(p->len + 1);
  np->pos = len;
  SAM_last = np;
```

```
78    for (; p && !p->next[x]; p = p->fa)
79      p->next[x] = np;
80    if (!p)
81    {
82      np->fa = SAM_root;
83      return np;
84    }
85    SAM_Node *q = p->next[x];
86    if (q->len == p->len + 1)
87    {
88      np->fa = q;
89      return np;
90    }
91    SAM_Node *nq = newSAM_Node(q);
92    nq->len = p->len + 1;
93    q->fa = nq;
94    np->fa = nq;
95    for (; p && p->next[x] == q; p = p->fa)
96      p->next[x] = nq;
97    return np;
98  }
99  void SAM_build(char *s)//多串建立 注意SAM_init()的调用
100 {
101   int l = strlen(s);
102   SAM_Node *p = SAM_root;
103   for (int i = 0; i < l; i++)
104   {
105     if (!p->next[s[i] - 'a'] || !(p->next[s[i] - 'a']->len == i +
          1))
106       p=SAM_add(p,s[i] - 'a', i + 1);
107     else
108       p = p->next[s[i] - 'a'];
109   }
110 }
111
112 struct ST_Node
113 {
114   ST_Node *next[CHAR],*fa;
115   int len,pos;
116 }ST_node[MAXN*2],*ST_root;
117 int Sufpos[MAXN];
118 void ST_add(int u,int v,int chr,int len)
119 {
120   ST_node[u].next[chr]=&ST_node[v];
121   ST_node[v].len=len;
122 }
123 void init(int n)
124 {
125   for (int i=0;i<n;i++)
126   {
127     ST_node[i].pos=-1;
```

```
128     ST_node[i].fa=0;
129     memset(ST_node[i].next,0,sizeof(ST_node[i].next));
130   }
131   ST_node[0].pos=0;
132   ST_root=&ST_node[0];
133 }
134 void ST_build(char *s)
135 {
136   int n=strlen(s);
137   reverse(s,s+n);
138   SAM_build(s);
139   init(SAM_size);
140   for (int i=1;i<SAM_size;i++)
141   {
142     ST_add(SAM_node[i].fa->id,SAM_node[i].id,s[SAM_node[i].pos-
          SAM_node[i].fa->len-1]-'a',SAM_node[i].len-SAM_node[i].fa->
          len);
143     if (SAM_node[i].pos==SAM_node[i].len)
144     {
145       Sufpos[n-SAM_node[i].pos+1]=i;
146       ST_node[i].pos=n-SAM_node[i].pos+1;
147     }
148   }
149 }
150
151 int rank[MAXN],sa[MAXN+1];
152 int height[MAXN];
153 int L;
154 void ST_dfs(ST_Node *p)
155 {
156   if (p->pos!=-1)
157     sa[L++]=p->pos;
158   for (int i=0;i<CHAR;i++)
159     if (p->next[i])
160       ST_dfs(p->next[i]);
161 }
162 char s[MAXN+1];
163 int main()
164 {
165   gets(s);
166   ST_build(s);
167   L=0;
168   ST_dfs(ST_root);
169   int n=strlen(s);
170   for (int i=0; i<n; i++)
171     sa[i]=sa[i+1]-1;
172   for (int i=0; i<n; i++)
173     rank[sa[i]]=i;
174   reverse(s,s+n);
175   for (int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
176     if (rank[i])
```

```
177       for (k?k--:0,j=sa[rank[i]-1]; s[i+k]==s[j+k]; k++);
178 }
```

3.3.1 例题

```
1  #include <iostream>
2  #include <algorithm>
3  #include <cstdio>
4  #include <cstring>
5  using namespace std;
6
7  const int CHAR = 26;
8  const int MAXN = 100000;
9
10 struct SAM_Node
11 {
12   SAM_Node *fa,*next[CHAR];
13   int len;
14   int id;
15   int mat[9];
16   SAM_Node() {}
17   SAM_Node(int _len)
18   {
19     fa = 0;
20     len = _len;
21     memset(mat,0,sizeof(mat));
22     memset(next,0,sizeof(next));
23   }
24 };
25 SAM_Node SAM_node[MAXN*2],*SAM_root,*SAM_last;
26 int SAM_size;
27 SAM_Node *newSAM_Node(int len)
28 {
29   SAM_node[SAM_size] = SAM_Node(len);
30   SAM_node[SAM_size].id = SAM_size;
31   return &SAM_node[SAM_size++];
32 }
33 SAM_Node *newSAM_Node(SAM_Node *p)
34 {
35   SAM_node[SAM_size] = *p;
36   SAM_node[SAM_size].id = SAM_size;
37   return &SAM_node[SAM_size++];
38 }
39 void SAM_init()
40 {
41   SAM_size = 0;
42   SAM_root = SAM_last = newSAM_Node(0);
43 }
44 void SAM_add(int x,int len)
45 {
46   SAM_Node *p = SAM_last,*np = newSAM_Node(p->len+1);
47   SAM_last = np;
```

```
48    for (; p&&!p->next[x]; p=p->fa)
49      p->next[x] = np;
50    if (!p)
51    {
52      np->fa = SAM_root;
53      return;
54    }
55    SAM_Node *q = p->next[x];
56    if (q->len == p->len+1)
57    {
58      np->fa = q;
59      return;
60    }
61    SAM_Node *nq = newSAM_Node(q);
62    nq->len = p->len+1;
63    q->fa = nq;
64    np->fa = nq;
65    for (; p&&p->next[x] == q; p = p->fa)
66      p->next[x] = nq;
67  }
68  int getid(char ch)
69  {
70    return ch-'a';
71  }
72  void SAM_build(char *s)
73  {
74    SAM_init();
75    int l = strlen(s);
76    for (int i = 0; i < l; i++)
77      SAM_add(getid(s[i]),i+1);
78  }
79  char s[10][MAXN+1];
80  int ans;
81  int head[MAXN*2];
82  struct Edge
83  {
84    int to,next;
85  } edge[MAXN*2];
86  int M;
87  int n;
88  void add_edge(int u,int v)
89  {
90    edge[M].to=v;
91    edge[M].next=head[u];
92    head[u]=M++;
93  }
94  void dfs(int u)
95  {
96    for (int i=head[u]; i!=-1; i=edge[i].next)
97    {
98      int v=edge[i].to;
```

```
99      dfs(v);
100     for (int j=0; j<n-1; j++)
101       SAM_node[u].mat[j]=max(SAM_node[v].mat[j],SAM_node[u].mat[j])
            ;
102   }
103   int tmp=SAM_node[u].len;
104   for (int i=0; i<n-1; i++)
105     tmp=min(tmp,SAM_node[u].mat[i]);
106   ans=max(ans,tmp);
107 }
108 int main()
109 {
110
111   while (scanf("%s",s[n])!=EOF)
112     n++;
113   int L=strlen(s[0]);
114   ans=M=0;
115   SAM_build(s[0]);
116   for (int j=1; j<n; j++)
117   {
118     int l=strlen(s[j]),len=0;
119     SAM_Node *p=SAM_root;
120     for (int i=0; i<l; i++)
121     {
122       if (p->next[getid(s[j][i])])
123       {
124         p=p->next[getid(s[j][i])];
125         p->mat[j-1]=max(p->mat[j-1],++len);
126       }
127       else
128       {
129         while (p && !p->next[getid(s[j][i])])
130           p=p->fa;
131         if (!p)
132         {
133           p=SAM_root;
134           len=0;
135         }
136         else
137         {
138           len=p->len+1;
139           p=p->next[getid(s[j][i])];
140         }
141         p->mat[j-1]=max(p->mat[j-1],len);
142       }
143     }
144   }
145   memset(head,-1,4*SAM_size);
146   for (int i=1; i<SAM_size; i++)
147     add_edge(SAM_node[i].fa->id,i);
148   dfs(0);
```

```
149   printf("%d\n",ans);
150   return 0;
151 }
```

LCS2

```
 1 #include <iostream>
 2 #include <algorithm>
 3 #include <cstdio>
 4 #include <cstring>
 5 using namespace std;
 6
 7 const int CHAR = 26;
 8 const int MAXN = 100000;
 9
10 struct SAM_Node
11 {
12   SAM_Node *fa,*next[CHAR];
13   int len;
14   int id;
15   int mat[9];
16   SAM_Node() {}
17   SAM_Node(int _len)
18   {
19     fa = 0;
20     len = _len;
21     memset(mat,0,sizeof(mat));
22     memset(next,0,sizeof(next));
23   }
24 };
25 SAM_Node SAM_node[MAXN*2],*SAM_root,*SAM_last;
26 int SAM_size;
27 SAM_Node *newSAM_Node(int len)
28 {
29   SAM_node[SAM_size] = SAM_Node(len);
30   SAM_node[SAM_size].id = SAM_size;
31   return &SAM_node[SAM_size++];
32 }
33 SAM_Node *newSAM_Node(SAM_Node *p)
34 {
35   SAM_node[SAM_size] = *p;
36   SAM_node[SAM_size].id = SAM_size;
37   return &SAM_node[SAM_size++];
38 }
39 void SAM_init()
40 {
41   SAM_size = 0;
42   SAM_root = SAM_last = newSAM_Node(0);
43 }
44 void SAM_add(int x,int len)
45 {
46   SAM_Node *p = SAM_last,*np = newSAM_Node(p->len+1);
```

```
47    SAM_last = np;
48    for (; p&&!p->next[x]; p=p->fa)
49      p->next[x] = np;
50    if (!p)
51    {
52      np->fa = SAM_root;
53      return;
54    }
55    SAM_Node *q = p->next[x];
56    if (q->len == p->len+1)
57    {
58      np->fa = q;
59      return;
60    }
61    SAM_Node *nq = newSAM_Node(q);
62    nq->len = p->len+1;
63    q->fa = nq;
64    np->fa = nq;
65    for (; p&&p->next[x] == q; p = p->fa)
66      p->next[x] = nq;
67  }
68  int getid(char ch)
69  {
70    return ch-'a';
71  }
72  void SAM_build(char *s)
73  {
74    SAM_init();
75    int l = strlen(s);
76    for (int i = 0; i < l; i++)
77      SAM_add(getid(s[i]),i+1);
78  }
79  char s[MAXN+1];
80  int ans;
81  int head[MAXN*2];
82  struct Edge
83  {
84    int to,next;
85  } edge[MAXN*2];
86  int M;
87  int n;
88  void add_edge(int u,int v)
89  {
90    edge[M].to=v;
91    edge[M].next=head[u];
92    head[u]=M++;
93  }
94  void dfs(int u)
95  {
96    for (int i=head[u]; i!=-1; i=edge[i].next)
97    {
```

```
98      int v=edge[i].to;
99      /*for (int j=0; j<n; j++)
100       SAM_node[v].mat[j]=max(SAM_node[v].mat[j],SAM_node[u].mat[j])
            ;*/
101     dfs(v);
102     for (int j=0; j<n; j++)
103       SAM_node[u].mat[j]=max(SAM_node[v].mat[j],SAM_node[u].mat[j])
            ;
104   }
105   int tmp=SAM_node[u].len;
106   for (int i=0; i<n; i++)
107     tmp=min(tmp,SAM_node[u].mat[i]);
108   ans=max(ans,tmp);
109 }
110 int main()
111 {
112   //freopen("in.txt","r",stdin);
113   //freopen("out.txt","w",stdout);
114   n=0;
115   gets(s);
116   SAM_build(s);
117   while (gets(s))
118   {
119     int l=strlen(s),len=0;
120     SAM_Node *p=SAM_root;
121     for (int i=0; i<l; i++)
122     {
123       if (p->next[getid(s[i])])
124       {
125         p=p->next[getid(s[i])];
126         p->mat[n]=max(p->mat[n],++len);
127       }
128       else
129       {
130         while (p && !p->next[getid(s[i])])
131           p=p->fa;
132         if (!p)
133         {
134           p=SAM_root;
135           len=0;
136         }
137         else
138         {
139           len=p->len+1;
140           p=p->next[getid(s[i])];
141         }
142         p->mat[n]=max(p->mat[n],len);
143       }
144       //printf("%d %d %d\n",i,len,p->id);
145     }
146     n++;
```

```
147   }
148   memset(head,-1,4*SAM_size);
149   for (int i=1; i<SAM_size; i++)
150     add_edge(SAM_node[i].fa->id,i);
151   dfs(0);
152   printf("%d\n",ans);
153   return 0;
154 }
```

3.4 KMP

求A[0..i]的一个后缀最多能匹配B的前缀多长。 先对B进行自匹配然后与A匹配。 KMP[i]就是对应答案，p[i]+1是B[0..i]的一个后缀最多能匹配B的前缀多长。

```
1  //自匹配过程
2  int j;
3  p [0] = j = -1;
4  for ( int i = 1; i < lb; i++)
5  {
6    while (j >= 0 && b[j + 1] != b[i]) j = p[j];
7    if (b[j + 1] == b[i]) j ++;
8    p[i] = j;
9  }
10 //下面是匹配过程
11 j = -1;
12 for ( int i = 0; i < la; i++)
13 {
14   while (j >= 0 && b[j + 1] != a[i]) j = p[j];
15   if (b[j + 1] == a[i]) j ++;
16   KMP[i] = j + 1;
17 }
```

3.5 e-KMP

求A[i..len-1]和B的最长公共前缀有多长。 先对B进行自匹配然后与A匹配。 eKMP[i]就是对应答案。p[i]是B[i..len-1]和B的最长公共前缀有多长。

```
1  //自匹配过程
2  int j = 0;
3  while (j < lb && b[j] == b[j + 1])
4    j++;
5  p[0] = lb, p[1] = j;
6  int k = 1;
7  for (int i = 2; i < lb; i++)
8  {
9    int Len = k + p[k] - 1, L = p[i - k];
10   if (L < Len - i + 1)
11     p[i] = L;
12   else
13   {
14     j = max(0, Len - i + 1);
15     while (i + j < lb && b[i + j] == b[j])
16       j++;
```

```
    p[i] = j, k = i;
  }
}
//下面是匹配过程
j = 0;
while (j < la && j < lb && a[j] == b[j])
  j++;
eKMP[0] = j;
k = 0;
for (int i = 1; i < la; i++)
{
  int Len = k + eKMP[k] - 1, L = p[i - k];
  if (L < Len - i + 1)
    eKMP[i] = L;
  else
  {
    j = max(0, Len - i + 1);
    while (i + j < la && j < lb && a[i + j] == b[j])
      j++;
    eKMP[i] = j, k = i;
  }
}
```

3.6 *Manacher

待整理

```
char s[1000],a[3000];
int p[3000],len,l,pnow,pid,res,resid;

int main()
{
  while (scanf("%s",s) != EOF)
  {
    len = strlen(s);
    l = 0;
    a[l++] = '.';
    a[l++] = ',';
    for (int i = 0;i < len;i++)
    {
      a[l++] = s[i];
      a[l++] = ',';
    }
    pnow = 0;
    res = 0;
    for (int i = 1;i < l;i++)
    {
      if (pnow > i)
        p[i] = min(p[2*pid-i],pnow-i);
      else
        p[i] = 1;
      for (;a[i-p[i]] == a[i+p[i]];p[i]++);
```

```
26        if (i+p[i] > pnow)
27        {
28          pnow = i+p[i];
29          pid = i;
30        }
31        if (p[i] > res)
32        {
33          res = p[i];
34          resid = i;
35        }
36      }
37      for (int i = resid-res+2;i < resid+res-1;i += 2)
38        printf("%c",a[i]);
39      printf("\n");
40    }
41    return 0;
42  }
```

3.7 *字符串最小表示法

```
1  int Gao(char a[],int len)
2  {
3    int i = 0,j = 1,k = 0;
4    while (i < len && j < len && k < len)
5    {
6      int cmp = a[(j+k)%len]-a[(i+k)%len];
7      if (cmp == 0)
8        k++;
9      else
10     {
11       if (cmp > 0)
12         j += k+1;
13       else
14         i += k+1;
15       if (i == j) j++;
16       k = 0;
17     }
18   }
19   return min(i,j);
20 }
```

3.8 带*通配符的匹配

```
1  #include <iostream>
2  #include <algorithm>
3  #include <cstdio>
4  #include <cstring>
5  using namespace std;
6
7  char a[110],b[110],sp[110][110],tot,place[110];
8  int n,la,lb,ll;
```

```
9
10 bool check(int id,int pos)
11 {
12   for (int i = 0;sp[id][i] != 0;i++)
13     if (b[pos+i] != sp[id][i])
14       return false;
15   return true;
16 }
17
18 bool check()
19 {
20   lb = strlen(b);
21   int pre = 0;
22   for (int i = 0;i < tot;i++)
23   {
24     bool find = false;
25     for (int j = pre;j < lb;j++)
26       if (check(i,j) == true)
27       {
28         place[i] = j;
29         pre = place[i]+1;
30         find = true;
31         break;
32       }
33     if (find == false)  return false;
34   }
35   if (a[0] != '*')
36     if (place[0] != 0)
37       return false;
38   if (a[la-1] != '*')
39     if (check(tot-1,lb-ll) == false)
40       return false;
41   return true;
42 }
43
44 int main()
45 {
46   while (scanf("%s",a) != EOF)
47   {
48     tot = 0;
49     for (int i = 0;a[i] != 0;i++)
50       if (a[i] != '*')
51       {
52         int j;
53         for (j = i;a[j] != 0 && a[j] != '*';j++)
54           sp[tot][j-i] = a[j];
55         sp[tot++][j-i] = 0;
56         i = j;
57       }
58     la = strlen(a);
59     ll = strlen(sp[tot-1]);
```

```
60      scanf("%d",&n);
61      for (int i = 0;i < n;i++)
62      {
63        scanf("%s",b);
64        if (check() == true)
65          puts(b);
66      }
67    }
68    return 0;
69  }
70  /*
71  Sample Input 1
72  *.*
73  4
74  main.c
75  a.out
76  readme
77  yacc
78
79  Sample Input 2
80  *a*a*a
81  4
82  aaa
83  aaaaa
84  aaaaax
85  abababa
86
87  Sample Output 1
88  main.c
89  a.out
90
91  Sample Output 2
92  aaa
93  aaaaa
94  abababa
95  */
```

4 数学

4.1 扩展GCD

求ax+by=gcd(a,b)的一组解

```
long long ex_gcd(long long a,long long b,long long &x,long long &y)
{
  if (b)
  {
    long long ret = ex_gcd(b,a%b,x,y),tmp = x;
    x = y;
    y = tmp-(a/b)*y;
    return ret;
  }
  else
  {
    x = 1;
    y = 0;
    return a;
  }
}
```

4.2 模线性方程组

```
//有更新
int m[10],a[10];//模数m 余数a
bool solve(int &m0,int &a0,int m,int a)//模线性方程组
{
  int y,x;
  int g=ex_gcd(m0,m,x,y);
  if (abs(a-a0)%g) return 0;
  x*=(a-a0)/g;
  x%=m/g;
  a0=(x*m0+a0);
  m0*=m/g;
  a0%=m0;
  if (a0<0) a0+=m0;
  return 1;
}
int MLES()
{
  bool flag=1;
  int m0=1,a0=0;
  for (int i=0; i<n; i++)
    if (!solve(m0,a0,m[i],a[i]))
    {
      flag=0;
      break;
    }
  if (flag)
```

```
27      return a0;
28    else
29      return -1;
30  }
```

4.3 矩阵

乘法的时候将B数组转置一下然后$C[i][j] = \sum A[i][k] \times B[j][k]$会有奇效。

```
1  struct Matrix
2  {
3    int a[52][52];
4    Matrix operator * (const Matrix &b)const
5    {
6      Matrix res;
7      for (int i = 0; i < 52; i++)
8        for (int j = 0; j < 52; j++)
9        {
10         res.a[i][j] = 0;
11         for (int k = 0; k < 52; k++)
12           res.a[i][j] += a[i][k] * b.a[k][j];
13       }
14     return res;
15   }
16   Matrix operator ^ (int y)const
17   {
18     Matrix res, x;
19     for (int i = 0; i < 52; i++)
20     {
21       for (int j = 0; j < 52; j++)
22         res.a[i][j] = 0, x.a[i][j] = a[i][j];
23       res.a[i][i] = 1;
24     }
25     for (; y; y >>= 1, x = x * x)
26       if (y & 1)
27         res = res * x;
28     return res;
29   }
30 };
```

4.4 康拓展开

```
1  const int PermSize = 12;
2  int factory[PermSize] = {1, 1, 2, 6, 24, 120, 720, 5040, 40320,
     362880, 3628800, 39916800};
3  int Cantor(int a[])
4  {
5    int i, j, counted;
6    int result = 0;
7    for (i = 0; i < PermSize; ++i)
8    {
9      counted = 0;
```

```
    for (j = i + 1; j < PermSize; ++j)
      if (a[i] > a[j])
        ++counted;
    result = result + counted * factory[PermSize - i - 1];
  }
  return result;
}

bool h[13];

void UnCantor(int x, int res[])
{
  int i,j,l,t;
  for (i = 1;i <= 12;i++)
    h[i] = false;
  for (i = 1; i <= 12; i++)
  {
    t = x / factory[12 - i];
    x -= t * factory[12 - i];
    for (j = 1, l = 0; l <= t; j++)
      if (!h[j])l++;
    j--;
    h[j] = true;
    res[i - 1] = j;
  }
}
```

4.5 FFT

```
const double PI= acos(-1.0);
struct vir
{
  double re,im; //实部和虚部
  vir(double a=0,double b=0)
  {
    re=a;
    im=b;
  }
  vir operator +(const vir &b)
  {return vir(re+b.re,im+b.im);}
  vir operator -(const vir &b)
  {return vir(re-b.re, im-b.im);}
  vir operator *(const vir &b)
  {return vir(re*b.re-im*b.im , re*b.im+im*b.re);}
};
vir x1[200005],x2[200005];
void change(vir *x,int len,int loglen)
{
  int i,j,k,t;
  for(i=0;i<len;i++)
  {
    t=i;
```

```
24      for(j=k=0; j<loglen; j++,t>>=1)
25        k= (k<<1)|(t&1);
26      if(k<i)
27      {
28      //  printf("%d %d\n",k,i);
29        vir wt=x[k];
30        x[k]=x[i];
31        x[i]=wt;
32      }
33    }
34  }
35  void fft(vir *x,int len,int loglen)
36  {
37    int i,j,t,s,e;
38    change(x,len,loglen);
39    t=1;
40    for(i=0;i<loglen;i++,t<<=1)
41    {
42      s=0;
43      e=s+t;
44      while(s<len)
45      {
46        vir a,b,wo(cos(PI/t),sin(PI/t)),wn(1,0);
47        for(j=s;j<s+t;j++)
48        {
49          a=x[j];
50          b=x[j+t]*wn;
51          x[j]=a+b;
52          x[j+t]=a-b;
53          wn=wn*wo;
54        }
55        s=e+t;
56        e=s+t;
57      }
58    }
59  }
60  void dit_fft(vir *x,int len,int loglen)
61  {
62    int i,j,s,e,t=1<<loglen;
63    for(i=0;i<loglen;i++)
64    {
65      t>>=1;
66      s=0;
67      e=s+t;
68      while(s<len)
69      {
70        vir a,b,wn(1,0),wo(cos(PI/t),-sin(PI/t));
71        for(j=s;j<s+t;j++)
72        {
73          a=x[j]+x[j+t];
74          b=(x[j]-x[j+t])*wn;
```

```
75          x[j]=a;
76          x[j+t]=b;
77          wn=wn*wo;
78        }
79        s=e+t;
80        e=s+t;
81      }
82    }
83    change(x,len,loglen);
84    for(i=0;i<len;i++)
85      x[i].re/=len;
86  }
87  int main()
88  {
89    char a[100005],b[100005];
90    int i,len1,len2,len,loglen;
91    int t,over;
92    while(scanf("%s%s",a,b)!=EOF)
93    {
94      len1=strlen(a)<<1;
95      len2=strlen(b)<<1;
96      len=1;loglen=0;
97      while(len<len1)
98      {
99        len<<=1;  loglen++;
100     }
101     while(len<len2)
102     {
103       len<<=1;  loglen++;
104     }
105     for(i=0;a[i];i++)
106     {
107       x1[i].re=a[i]-'0';
108       x1[i].im=0;
109     }
110     for(;i<len;i++)
111       x1[i].re=x1[i].im=0;
112     for(i=0;b[i];i++)
113     {
114       x2[i].re=b[i]-'0';
115       x2[i].im=0;
116     }
117     for(;i<len;i++)
118       x2[i].re=x2[i].im=0;
119     fft(x1,len,loglen);
120     fft(x2,len,loglen);
121     for(i=0;i<len;i++)
122       x1[i] = x1[i]*x2[i];
123     dit_fft(x1,len,loglen);
124     for(i=(len1+len2)/2-2,over=len=0;i>=0;i--)
125     {
```

```
126       t=(int)(x1[i].re+over+0.5);
127       a[len++]= t%10;
128       over = t/10;
129     }
130     while(over)
131     {
132       a[len++]=over%10;
133       over/=10;
134     }
135     for(len--;len>=0&&!a[len];len--);
136       if(len<0)
137       putchar('0');
138       else
139         for(;len>=0;len--)
140           putchar(a[len]+'0');
141     putchar('\n');
142   }
143   return 0;
144 }
```

4.6 爬山法计算器

注意灵活运用。
双目运算符在calc()中，左结合单目运算符在P()中，右结合单目运算符在calc_exp中。（但是还没遇到过。。）

```
1  #include <iostream>
2  #include <cstdio>
3  #include <cstring>
4  #include <algorithm>
5  #include <string>
6  using namespace std;
7
8  char s[100000];
9  int n,cur;
10 const string OP = "+-*";
11
12 char next_char()
13 {
14   if (cur >= n) return EOF;
15   return s[cur];
16 }
17
18 int get_priority(char ch)
19 {
20   if (ch == '*')  return 2;
21   return 1;
22 }
23
24 int P();
25
26 int calc(int a,char op,int b)
```

```
27 {
28   if (op == '+')
29     return a+b;
30   if (op == '-')
31     return a-b;
32   if (op == '*')
33     return a*b;
34 }
35
36 int calc_exp(int p)
37 {
38   int a = P();
39   while ((OP.find(next_char()) != OP.npos) && (get_priority(
        next_char()) >= p))
40   {
41     char op = next_char();
42     cur++;
43     a = calc(a,op,calc_exp(get_priority(op)+1));
44   }
45   return a;
46 }
47
48 int totvar,m,var[26],varid[26];
49
50 int P()
51 {
52   if (next_char() == '-')
53   {
54     cur++;
55     return -P();
56   }
57   else if (next_char() == '+')
58   {
59     cur++;
60     return P();
61   }
62   else if (next_char() == '(')
63   {
64     cur++;
65     int res = calc_exp(0);
66     cur++;
67     return res;
68   }
69   else
70   {
71     cur++;
72     //cout << "getvar at " << cur << ' ' << var[varid[s[cur]-'a']]
          << endl;
73     return var[varid[s[cur-1]-'a']];
74   }
75 }
```

```
76
77  int id[26],minid;
78
79  int main()
80  {
81    while (true)
82    {
83      scanf("%d%d",&totvar,&var[0]);
84      if (totvar == 0 && var[0] == 0)  break;
85      for (int i = 1;i < totvar;i++)
86        scanf("%d",&var[i]);
87      scanf("%d",&m);
88      scanf("%s",s);
89      for (int i = 0;i < 26;i++)
90        id[i] = -1;
91      minid = 0;
92      n = strlen(s);
93      for (int i = 0;i < n;i++)
94        if (s[i] >= 'a' && s[i] <= 'z')
95        {
96          if (id[s[i]-'a'] == -1)
97          {
98            id[s[i]-'a'] = minid;
99            minid++;
100         }
101         s[i] = 'a'+id[s[i]-'a'];
102       }
103     for (int i = 0;i < totvar;i++)
104       varid[i] = i;
105     int res = 0;
106     do
107     {
108       cur = 0;
109       int tmp = calc_exp(0);
110       if (tmp == m)
111       {
112         res++;
113         break;
114       }
115     }
116     while (next_permutation(varid,varid+totvar));
117     //puts(s);
118     if (res > 0)
119       puts("YES");
120     else
121       puts("NO");
122   }
123   return 0;
124 }
```

4.7 线性筛

```
int N;
bool isPrime[10001];
int prime[10000];
void getPrime(int n)
{
  memset(isPrime,1,++n);
  N=0;
  isPrime[0]=isPrime[1]=0;
  for (int i=2;i<n;i++)
  {
    if (isPrime[i])
      prime[N++]=i;
    for (int j=0;j<N && prime[j]*i<n;j++)
    {
      isPrime[i*prime[j]]=0;
      if (i%prime[j]==0)
        break;
    }
  }
}
```

4.8 线性规划

```
#define MAXM 20  //max num of basic varibles
#define INF 1E200

double A[MAXM+5][MAXN+MAXM+5];
double b[MAXM+5],c[MAXN+MAXM+5];
int N[MAXN+5],B[MAXM+5];
double X[MAXN+MAXM+5],V;
int n,m,R,C,nCnt,bCnt;
int v1[MAXN],v2[MAXN];

int fcmp(double a,double b)
{
  if(fabs(a-b)<1E-7) return 0;
  if(a>b) return 1;
  return -1;
}

void Pivot(int l,int e)
{
  double t=A[l][e],p=c[e];
  b[l]=b[l]/t;
  for(int i=1;i<=C;i++)
    A[l][i]/=t;
  V=V-c[e]*b[l];
  for(int i=1;i<=R;i++)
  {
    if(i==l||fcmp(A[i][e],0.0)==0)
```

```
28        continue;
29      t=A[i][e];
30      b[i]=b[i]-t*b[l];
31      for(int j=1;j<=C;j++)
32        A[i][j]=A[i][j]-t*A[l][j];
33    }
34    for(int i=1;i<=C;i++)
35      c[i]=c[i]-p*A[l][i];
36    for(int i=1;i<=nCnt;i++)
37    {
38      if(N[i]==e)
39      {
40        N[i]=B[l];
41        break;
42      }
43    }
44    B[l]=e;
45  }
46
47  bool Process(double P[])
48  {
49    while(true)
50    {
51      int e=-1;
52      double mV=-INF;
53      for(int i=1;i<=nCnt;i++)
54        if(fcmp(P[N[i]],mV)==1)
55          mV=P[N[i]],e=N[i];
56
57      if(fcmp(mV,0.0)<=0) break;
58      int l=-1;
59      mV=INF;
60      for(int i=1;i<=bCnt;i++)
61      {
62        if(fcmp(A[i][e],0.0)==1)
63        {
64          double t=b[i]/A[i][e];
65          if(fcmp(mV,t)==1||(fcmp(mV,t)==0&&(l==-1||B[l]>B[i])))
66            mV=t,l=i;
67        }
68      }
69      if(l==-1) return false;
70      Pivot(l,e);
71    }
72    return true;
73  }
74
75  bool initSimplex()
76  {
77    nCnt=bCnt=0;
78    for(int i=1;i<=n;i++)
```

```
    N[++nCnt]=i;
  for(int i=1;i<=m;i++)
    B[++bCnt]=i+n,A[i][n+i]=1.0;
  R=bCnt,C=bCnt+nCnt;
  double minV=INF;
  int p=-1;
  for(int i=1;i<=m;i++)
    if(fcmp(minV,b[i])==1)
      minV=b[i],p=i;
  if(fcmp(minV,0.0)>=0)
    return true;
  N[++nCnt]=n+m+1;R++,C++;
  for(int i=0;i<=C;i++)
    A[R][i]=0.0;
  for(int i=1;i<=R;i++)
    A[i][n+m+1]=-1.0;
  Pivot(p,n+m+1);
  if(!Process(A[R])) return false;
  if(fcmp(b[R],0.0)!=0)
    return false;
  p=-1;
  for(int i=1;i<=bCnt&&p==-1;i++)
    if(B[i]==n+m+1) p=i;
  if(p!=-1)
  {
    for(int i=1;i<=nCnt;i++)
    {
      if(fcmp(A[p][N[i]],0.0)!=0)
      {
        Pivot(p,N[i]);
        break;
      }
    }
  }
  bool f=false;
  for(int i=1;i<=nCnt;i++)
  {
    if(N[i]==n+m+1) f=true;
    if(f&&i+1<=nCnt)
      N[i]=N[i+1];
  }
  nCnt--;
  R--,C--;
  return true;
}

//-1: no solution 1: no bound 0: has a solution -V
int Simplex()
{
  if(!initSimplex())
    return -1;
```

```
130   if(!Process(c))
131     return 1;
132   for(int i=1;i<=nCnt;i++)
133     X[N[i]]=0.0;
134   for(int i=1;i<=bCnt;i++)
135     X[B[i]]=b[i];
136   return 0;
137 }
138
139 int main()
140 {
141   //n = 1;m=1;
142   //V= 0.0;
143   //c[1] = 1.0;
144   //A[1][1] = 1.0;
145   //b[1] = 5.0;
146   //Simplex();
147   //printf("V = %.3f\n",V);
148
149   while(scanf("%d",&v1[1]) == 1)
150     {
151       for(int i = 2; i<=6;i++)
152         scanf("%d",&v1[i]);
153       n = 4; m = 6;
154       for(int i = 0 ; i<=m+1;i++)
155         for(int j=0;j<=n+m+2;j++)
156           A[i][j] = c[j] = 0;
157       memset(b,0,sizeof(b));
158       V = 0.0;
159       /*
160       n 为未知数个数
161       m 为约束个数
162       目标：siama(c[i]*xi)
163       约束：sigma(A[i][j]*xj) <=b[i]; j = 1 ...  n
164       解存在x里面
165       */
166       b[1] = v1[1] ; A[1][1] = 1;A[1][4] = 1;
167       b[2] = v1[2] ; A[2][1] = 1;A[2][3] = 1;
168       b[3] = v1[3] ; A[3][3] = 1;A[3][4] = 1;
169       b[4] = v1[4] ; A[4][2] = 1;A[4][3] = 1;
170       b[5] = v1[5] ; A[5][2] = 1;A[5][4] = 1;
171       b[6] = v1[6] ; A[6][1] = 1;A[6][2] = 1;
172       c[1] = 1;c[2] = 1;c[3] = 1;c[4] = 1;
173       Simplex();
174       //printf("V = %.3f\n",V);
175       printf("%.3f␣%.3f␣%.3f␣%.3f\n",X[1],X[2],X[3],X[4]);
176
177     }
178   return 0;
179 }
```

4.9 分解质因数

4.9.1 米勒拉宾+分解因数

```
#include<ctime>
#include<iostream>
#define bint long long
using namespace std;
const int TIME = 8;//测试次数，够了8~10
int factor[100],fac_top = -1;

//计算两个数的gcd
bint gcd(bint small,bint big)
{
  while(small)
  {
    swap(small,big);
    small%=big;
  }
  return abs(big);
}

//ret = (a*b)%n (n<2^62)
bint muti_mod(bint a,bint b,bint n)
{
  bint exp = a%n, res = 0;
  while(b)
  {
    if(b&1)
    {
      res += exp;
      if(res>n) res -= n;
    }
    exp <<= 1;
    if (exp>n) exp -= n;
    b>>=1;
  }
  return res;
}

// ret = (a^b)%n
bint mod_exp(bint a,bint p,bint m)
{
  bint exp=a%m, res=1; //
  while(p>1)
  {
    if(p&1)
      res=muti_mod(res,exp,m);
    exp = muti_mod(exp,exp,m);
    p>>=1;
  }
  return muti_mod(res,exp,m);
```

```
49 }
50
51 //miller-法测试素数rabin, time 测试次数
52 bool miller_rabin(bint n, int times)
53 {
54   if(n==2)return 1;
55   if(n<2||!(n&1))return 0;
56   bint a, u=n-1, x, y;
57   int t=0;
58   while(u%2==0)
59   {
60     t++;
61     u/=2;
62   }
63   srand(time(0));
64   for(int i=0; i<times; i++)
65   {
66     a = rand() % (n-1) + 1;
67     x = mod_exp(a, u, n);
68     for(int j=0; j<t; j++)
69     {
70       y = muti_mod(x, x, n);
71       if ( y == 1 && x != 1 && x != n-1 )
72         return false; //must not
73       x = y;
74     }
75     if( y!=1) return false;
76   }
77   return true;
78 }
79
80 bint pollard_rho(bint n,int c)//找出一个因子
81 {
82   bint x,y,d,i = 1,k = 2;
83   srand(time(0));
84   x = rand()%(n-1)+1;
85   y = x;
86   while(true)
87   {
88     i++;
89     x = (muti_mod(x,x,n) + c) % n;
90     d = gcd(y-x, n);
91     if (1 < d && d < n) return d;
92     if( y == x) return n;
93     if(i == k)
94     {
95       y = x;
96       k <<= 1;
97     }
98   }
99 }
```

```
100
101 void findFactor(bint n,int k)//二分找出所有质因子，存入factor
102 {
103   if(n==1)return;
104   if(miller_rabin(n, TIME))
105   {
106     factor[++fac_top] = n;
107     return;
108   }
109   bint p = n;
110   while(p >= n)
111     p = pollard_rho(p,k--);//值变化，防止死循环k
112   findFactor(p,k);
113   findFactor(n/p,k);
114 }
115
116 int main()
117 {
118   bint cs,n,min;
119   cin>>cs;
120   while (cs--)
121   {
122     cin>>n;
123     fac_top = min = -1;
124     if(miller_rabin(n,TIME)) cout<<"Prime"<<endl;
125     else
126     {
127       findFactor(n,107);
128       for(int i=0; i<=fac_top; i++)
129       {
130         if(min<0||factor[i]<min)
131           min = factor[i];
132       }
133       cout<<min<<endl;
134     }
135   }
136   return 0;
137 }
```

4.9.2 暴力版本

```
1  int N;
2  int num[30],fac[30];
3  void getFactor(int x)
4  {
5    N=0;
6    memset(num,0,sizeof(num));
7    for (int i=0; prime[i]*prime[i]<=x && i<L; i++)
8    {
9      if (x%prime[i]==0)
10     {
11       while (x%prime[i]==0)
```

```
12        {
13          x/=prime[i];
14          num[N]++;
15        }
16        fac[N++]=prime[i];
17      }
18    }
19    if (x>1)
20    {
21      num[N]=1;
22      fac[N++]=x;
23    }
24  }
```

4.10 baby step giant step

```
1  #define MOD 76543
2  int hs[MOD], head[MOD], next[MOD], id[MOD], top;
3  void insert(int x, int y)
4  {
5    int k = x%MOD;
6    hs[top] = x, id[top] = y, next[top] = head[k], head[k] = top++;
7  }
8  int find(int x)
9  {
10   int k = x%MOD;
11   for (int i = head[k]; i; i = next[i]) if (hs[i] == x) return id[i
       ];
12   return -1;
13 }
14 int BSGS(int a, int b, int n)
15 {
16   memset(head, 0, sizeof(head));
17   top = 1;
18   if (b==1) return 0;
19   int m = sqrt(n+.0), j;
20   long long x = 1, p = 1;
21   for (int i = 0; i < m; ++i, p = p*a%n) insert(p*b%n, i);
22   for (long long i = m; ; i += m)
23   {
24     if ((j = find(x=x*p%n)) != -1) return i-j;
25     if (i > n) break;
26   }
27   return -1;
28 }
```

4.11 原根

```
1  int getPriRoot(int p)
2  {
3    if (p==2) return 1;
4    int phi = p - 1;
```

```
5   getFactor(phi);
6   for (int g = 2; g < p; ++g)
7   {
8     bool flag=1;
9     for (int i = 0; flag && i < N; ++i)
10      if (power(g, phi/fac[i], p) == 1)
11        flag=0;
12    if (flag)
13      return g;
14  }
15 }
```

4.12 逆元

```
1  void getInv2(int x)
2  {
3    inv[1]=1;
4    for (int i=2; i<=x; i++)
5      inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
6  }
7  int getInv(int x)//为素数mod
8  {
9    return power(x,mod-2);
10 }
```

4.13 卢卡斯

卢卡斯，$num[i]$阶乘也

```
1  int comLucus(int n,int m,int p)
2  {
3    int ans=1;
4    for (; n && m && ans; n/=p,m/=p)
5    {
6      if (n%p>=m%p)
7        ans = ans*num[n%p]%p*getInv(num[m%p]%p)%p*getInv(num[n%p-m%p
             ])%p;
8      else
9        ans=0;
10   }
11   return ans;
12 }
```

4.14 欧拉函数

4.14.1 分解质因数

```
1  int getEuler(int x)
2  {
3    getFactor(x);
4    int ret=x;
5    for (int i=0; i<N; i++)
6      ret = ret/fac[i]*(fac[i]-1);
```

```
  return ret;
}
```

4.14.2 一次预处理

```
void getEuler2()
{
  memset(euler,0,sizeof(euler));
  euler[1] = 1;
  for (int i = 2; i <= 3000000; i++)
  {
    if (!euler[i])
    {
      for (int j = i; j <= 3000000; j += i)
      {
        if (!euler[j])
          euler[j] = j;
        euler[j] = euler[j]/i*(i-1);
      }
    }
  }
}
```

4.15 费马降阶法

分解素数p为x^2+y^2的费马降阶法，失败返回-1，主程序调用calcu(p,x,y)

```
#include <stdio.h>
#include <string.h>
#include <stdlib.h>
int p,expp,A,B,aa,ans,tt;
long long M;
long long exp(int a,int b,long long mod)
{
   long long ans=1,num=a;
   while (b!=0)
   {
       if (b&1)
       {
           ans=((ans%mod)*(num%mod))%mod;
       }
       num=((num%mod)*(num%mod))%mod;
       b>>=1;
   }
   return ans;
}
int calcu(int p,int &x,int &y)
{
      if (p%4!=1) return -1;
      else
      {
        expp=(p-1)/4;
        A,B;
```

```
27          while (1)
28          {
29            aa=rand()%p;
30            if (aa==0) continue;
31            A=exp(aa,expp,p);
32            ans=(((long long)A%p)*((long long)A%p))%p;
33            if (ans==p-1) break;
34          }
35          B=1;
36          M=((long long)A*(long long)A+(long long)B*(long long)B)/p;
37          if (M!=1) B=p;
38          while (M!=1)
39          {
40            if (B>A)
41            {tt=A; A=B; B=tt;}
42            tt=A;
43            A=B;
44            B=tt%B;
45            M=((long long)A*(long long)A+(long long)B*(long long)B)/p
                 ;
46          }
47          if (B<=A)
48          {
49               x=B;
50               y=A;
51          }
52          else
53          {
54           x=A;
55           y=B;
56          }
57        }
58  }
59  int main()
60  {
61    while (scanf("%d",&p)!=EOF)
62    {
63        int x,y;
64        if (calcu(p,x,y)!=-1)
65    }
66    return 0;
67  }
```

4.16 自适应simp

过了哈尔滨积分题，精度要求不高的时候可以考虑使用。
暂时我只能用这个做做类似于凸函数或者凹函数的函数。

```
1  double Simp(double l,double r)
2  {
3    double h = (r-l)/2.0;
4    return h*(calc(l)+4*calc((l+r)/2.0)+calc(r))/3.0;
```

```
}

double rSimp(double l,double r)
{
  double mid = (l+r)/2.0;
  if (abs((Simp(l,r)-Simp(l,mid)-Simp(mid,r)))/15 < eps)
    return Simp(l,r);
  else
    return rSimp(l,mid)+rSimp(mid,r);
}
```

4.17 组合数求模

模是质数

```
#include<cstdio>
#include<cstring>
#include<iostream>
using namespace std;
int mod;
long long num[100000];
int ni[100],mi[100];
int len;
void init(int p)
{
  mod=p;
  num[0]=1;
  for (int i=1; i<p; i++)
    num[i]=i*num[i-1]%p;
}
void get(int n,int ni[],int p)
{
  for (int i = 0; i < 100; i++)
    ni[i] = 0;
  int tlen = 0;
  while (n != 0)
  {
    ni[tlen++] = n%p;
    n /= p;
  }
  len = tlen;
}
long long power(long long x,long long y)
{
  long long ret=1;
  for (long long a=x%mod; y; y>>=1,a=a*a%mod)
    if (y&1)
      ret=ret*a%mod;
  return ret;
}
long long getInv(long long x)//mod为素数
{
```

```
38    return power(x,mod-2);
39  }
40  long long calc(int n,int m,int p)//C(n,m)%p
41  {
42    init(p);
43    long long ans=1;
44    for (; n && m && ans; n/=p,m/=p)
45    {
46      if (n%p>=m%p)
47        ans = ans*num[n%p]%p*getInv(num[m%p]%p)%p*getInv(num[n%p-m%p
            ])%p;
48      else
49        ans=0;
50    }
51    return ans;
52  }
53  int main()
54  {
55    int t;
56    scanf("%d",&t);
57    while (t--)
58    {
59      int n,m,p;
60      scanf("%d%d%d",&n,&m,&p);
61      printf("%I64d\n",calc(n+m,m,p));
62    }
63    return 0;
64  }
```

4.18 其它公式

4.18.1 Polya

设G是p个对象的一个置换群，用k种颜色去染这p个对象，若一种染色方案在群G的作用下变为另一种方案，则这两个方案当作是同一种方案，这样的不同染色方案数为：
$L = \frac{1}{|G|} \times \Sigma(k^{C(f)}), f \in G$
$C(f)$为循环节，$|G|$表示群的置换方法数

对于有n个位置的手镯，有n种旋转置换和n种翻转置换

对于旋转置换：
$C(f_i) = gcd(n,i)$,i表示一次转过i颗宝石，$i = 0$时$c = n$;

对于翻转置换：

如果n为偶数： 则有$\frac{n}{2}$个置换$C(f) = \frac{n}{2}$，有$\frac{n}{2}$个置换 $C(f) = \frac{n}{2} + 1$

如果n为奇数： $C(f) = \frac{n}{2} + 1$

4.18.2 拉格朗日插值法

已知$y = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$曲线上的$n$个点$(x_1, y_1), (x_2, y_2), (x_3, y_3) \cdots (x_n, y_n)$用拉格朗日插值法可以不求系数可知任意$x$对应的$y$值。

$$\begin{aligned} y = & y_1 \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} \\ & + y_2 \frac{(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} \\ & + \cdots \\ & + y_n \frac{(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} \end{aligned}$$

特别的，如果$x_1 \sim x_n$为 连续自然数，那么对于下一个自然数对应的y值为：

$$y_{n+1} = (-1)^{n-1}C_n^0 y_1 + (-1)^{n-2}C_n^1 y_2 + \cdots + (-1)^0 C_n^{n-1} y_n$$

这个组合系数可以通过高斯消元暴出来，前提是要猜到它满足递推关系。

4.18.3 正多面体顶点着色

正四面体：$N = \frac{(n^4+11\times n^2)}{12}$
正六面体：$N = \frac{(n^8+17\times n^4+6\times n^2)}{24}$
正八面体：$N = \frac{(n^6+3\times n^4+12\times n^3+8\times n^2)}{24}$
正十二面体：$N = \frac{(n^{20}+15\times n^{10}+20\times n^8+24\times n^4)}{60}$
正二十面体：$N = \frac{(n^{12}+15\times n^6+44\times n^4)}{60}$

4.18.4 求和公式

$\sum k = \frac{n\times(n+1)}{2}$
$\sum 2k-1 = n^2$
$\sum k^2 = \frac{n\times(n+1)\times(2n+1)}{6}$
$\sum (2k-1)^2 = \frac{n\times(4n^2-1)}{3}$
$\sum k^3 = (\frac{n\times(n+1)}{2})^2$
$\sum (2k-1)^3 = n^2 \times (2n^2-1)$
$\sum k^4 = \frac{n\times(n+1)\times(2n+1)\times(3n^2+3n-1)}{30}$
$\sum k^5 = \frac{n^2\times(n+1)^2\times(2n^2+2n-1)}{12}$
$\sum k\times(k+1) = \frac{n\times(n+1)\times(n+2)}{3}$
$\sum k\times(k+1)\times(k+2) = \frac{n\times(n+1)\times(n+2)\times(n+3)}{4}$
$\sum k\times(k+1)\times(k+2)\times(k+3) = \frac{n\times(n+1)\times(n+2)\times(n+3)\times(n+4)}{5}$

4.18.5 几何公式

球扇形：
全面积：$T = \pi r(2h + r_0)$，h为球冠高，r_0为球冠底面半径
体积：$V = \frac{2\pi r^2 h}{3}$

4.18.6 小公式

Pick 公式：$A = E \times 0.5 + I - 1$（A是多边形面积，E是边界上的整点，I是多边形内部的整点）

海伦公式：$S = \sqrt{p(p-a)(p-b)(p-c)}$，其中$p = \frac{(a+b+c)}{2}$，$abc$为三角形的三条边长

求$\binom{n}{k}$中素因子P的个数：

1. 把n转化为P进制，并记它每个位上的和为$S1$
2. 把$n-k$，k做同样的处理，得到$S2$，$S3$

则$\binom{n}{k}$中素因子P的个数：$\frac{S2+S3-S1}{P-1}$

部分错排公式：
$n+m$个数中m个数必须错排 求排列数

```
dp[i] = n*dp[i-1]+(i-1)*(dp[i-1]+dp[i-2]);
dp[0] = n!;
dp[1] = n*n!;
```

$dp[m]$为所求解

5 数据结构

5.1 *Splay

持续学习中。
注意节点的size值不一定是真实的值！如果有需要需要特别维护！

1. 旋转和Splay操作
2. rank操作
3. insert操作（。。很多题目都有）
4. del操作（郁闷的出纳员）
5. 由数组建立Splay
6. 前驱后继（营业额统计）
7. Pushdown Pushup的位置
8. *。。。暂时想不起了

节点定义。。

```
const int MaxN = 50003;

struct Node
{
  int size,key;

  Node *c[2];
  Node *p;
} mem[MaxN], *cur, *nil;
```

无内存池的几个初始化函数。

```
Node *newNode(int v, Node *p)
{
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;
  return cur++;
}

void Init()
{
  cur = mem;
  nil = newNode(0, cur);
  nil->size = 0;
}
```

带内存池的几个函数。

```
int emp[MaxN], totemp;

Node *newNode(int v, Node *p)
{
  cur = mem + emp[--totemp];
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;
  return cur;
}

void Init()
{
  for (int i = 0; i < MaxN; ++i)
    emp[i] = i;
  totemp = MaxN;
  cur = mem + emp[--totemp];
  nil = newNode(0, cur);
  nil->size = 0;
}

void Recycle(Node *p)
{
  if (p == nil)   return;
  Recycle(p->c[0]), Recycle(p->c[1]);
  emp[totemp++] = p - mem;
}
```

基本的Splay框架。维护序列用。
一切下标从0开始。

```
struct SplayTree
{
  Node *root;
  void Init()
  {
    root = nil;
  }
  void Pushup(Node *x)
  {
    if (x == nil)   return;
    Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
    x->size = x->c[0]->size + x->c[1]->size + 1;
  }
  void Pushdown(Node *x)
  {
    if (x == nil)   return;
    //do something
  }
  void Rotate(Node *x, int f)
  {
```

```
21    if (x == nil)   return;
22    Node *y = x->p;
23    y->c[f ^ 1] = x->c[f], x->p = y->p;
24    if (x->c[f] != nil)
25      x->c[f]->p = y;
26    if (y->p != nil)
27      y->p->c[y->p->c[1] == y] = x;
28    x->c[f] = y, y->p = x;
29    Pushup(y);
30  }
31  void Splay(Node *x, Node *f)
32  {
33    while (x->p != f)
34    {
35      Node *y = x->p;
36      if (y->p == f)
37        Rotate(x, x == y->c[0]);
38      else
39      {
40        int fd = y->p->c[0] == y;
41        if (y->c[fd] == x)
42          Rotate(x, fd ^ 1), Rotate(x, fd);
43        else
44          Rotate(y, fd), Rotate(x, fd);
45      }
46    }
47    Pushup(x);
48    if (f == nil)
49      root = x;
50  }
51  void Select(int k, Node *f)
52  {
53    Node *x = root;
54    Pushdown(x);
55    int tmp;
56    while ((tmp = x->c[0]->size) != k)
57    {
58      if (k < tmp)  x = x->c[0];
59      else
60        x = x->c[1], k -= tmp + 1;
61      Pushdown(x);
62    }
63    Splay(x, f);
64  }
65  void Select(int l, int r)
66  {
67    Select(l, nil), Select(r + 2, root);
68  }
69  Node *Make_tree(int a[], int l, int r, Node *p)
70  {
71    if (l > r)  return nil;
```

```
72      int mid = l + r >> 1;
73      Node *x = newNode(a[mid], p);
74      x->c[0] = Make_tree(a, l, mid - 1, x);
75      x->c[1] = Make_tree(a, mid + 1, r, x);
76      Pushup(x);
77      return x;
78    }
79    void Insert(int pos, int a[], int n)
80    {
81      Select(pos, nil), Select(pos + 1, root);
82      root->c[1]->c[0] = Make_tree(a, 0, n - 1, root->c[1]);
83      Splay(root->c[1]->c[0], nil);
84    }
85    void Insert(int v)
86    {
87      Node *x = root, *y = nil;
88      while (x != nil)
89      {
90        y = x;
91        y->size++;
92        x = x->c[v >= x->key];
93      }
94      y->c[v >= y->key] = x = newNode(v, y);
95      Splay(x, nil);
96    }
97    void Remove(int l, int r)
98    {
99      Select(l, r);
100     //Recycle(root->c[1]->c[0]);
101     root->c[1]->c[0] = nil;
102     Splay(root->c[1], nil);
103   }
104 };
```

例题：旋转区间赋值求和求最大子序列。
注意打上懒标记后立即Pushup。Pushup(root-c[1]-c[0]),Pushup(root-c[1]),Pushup(root);

```
1     void Pushup(Node *x)
2     {
3       if (x == nil) return;
4       Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
5       x->size = x->c[0]->size+x->c[1]->size+1;
6
7       x->sum = x->c[0]->sum+x->c[1]->sum+x->key;
8       x->lsum = max(x->c[0]->lsum,x->c[0]->sum+x->key+max(0,x->c[1]->
          lsum));
9       x->rsum = max(x->c[1]->rsum,x->c[1]->sum+x->key+max(0,x->c[0]->
          rsum));
10      x->maxsum = max(max(x->c[0]->maxsum,x->c[1]->maxsum),x->key+max
          (0,x->c[0]->rsum)+max(0,x->c[1]->lsum));
11    }
```

```
12    void Pushdown(Node *x)
13    {
14      if (x == nil) return;
15      if (x->rev)
16      {
17        x->rev = 0;
18        x->c[0]->rev ^= 1;
19        x->c[1]->rev ^= 1;
20        swap(x->c[0],x->c[1]);
21
22        swap(x->lsum,x->rsum);
23      }
24      if (x->same)
25      {
26        x->same = false;
27        x->key = x->lazy;
28        x->sum = x->key*x->size;
29        x->lsum = x->rsum = x->maxsum = max(x->key,x->sum);
30        x->c[0]->same = true, x->c[0]->lazy = x->key;
31        x->c[1]->same = true, x->c[1]->lazy = x->key;
32      }
33    }
34
35  int main()
36  {
37    int totcas;
38    scanf("%d",&totcas);
39    for (int cas = 1;cas <= totcas;cas++)
40    {
41      Init();
42      sp.Init();
43      nil->lsum = nil->rsum = nil->maxsum = -Inf;
44      sp.Insert(0);
45      sp.Insert(0);
46
47      int n,m;
48      scanf("%d%d",&n,&m);
49      for (int i = 0;i < n;i++)
50        scanf("%d",&a[i]);
51      sp.Insert(0,a,n);
52
53      for (int i = 0;i < m;i++)
54      {
55        int pos,tot,c;
56        scanf("%s",buf);
57        if (strcmp(buf,"MAKE-SAME") == 0)
58        {
59          scanf("%d%d%d",&pos,&tot,&c);
60          sp.Select(pos-1,pos+tot-2);
61          sp.root->c[1]->c[0]->same = true;
62          sp.root->c[1]->c[0]->lazy = c;
```

```
63          sp.Pushup(sp.root->c[1]), sp.Pushup(sp.root);
64        }
65        else if (strcmp(buf,"INSERT") == 0)
66        {
67          scanf("%d%d",&pos,&tot);
68          for (int i = 0;i < tot;i++)
69            scanf("%d",&a[i]);
70          sp.Insert(pos,a,tot);
71        }
72        else if (strcmp(buf,"DELETE") == 0)
73        {
74          scanf("%d%d",&pos,&tot);
75          sp.Remove(pos-1,pos+tot-2);
76        }
77        else if (strcmp(buf,"REVERSE") == 0)
78        {
79          scanf("%d%d",&pos,&tot);
80          sp.Select(pos-1,pos+tot-2);
81          sp.root->c[1]->c[0]->rev ^= 1;
82          sp.Pushup(sp.root->c[1]), sp.Pushup(sp.root);
83        }
84        else if (strcmp(buf,"GET-SUM") == 0)
85        {
86          scanf("%d%d",&pos,&tot);
87          sp.Select(pos-1,pos+tot-2);
88          printf("%d\n",sp.root->c[1]->c[0]->sum);
89        }
90        else if (strcmp(buf,"MAX-SUM") == 0)
91        {
92          sp.Select(0,sp.root->size-3);
93          printf("%d\n",sp.root->c[1]->c[0]->maxsum);
94        }
95      }
96    }
97    return 0;
98  }
```

维护多个序列的时候，不需要建立很多Splay。只需要记录某个点在内存池中的绝对位置就可以了。
需要操作它所在的序列时直接Splay到nil。此时Splay的root所在的Splay就是这个序列了。
新建序列的时候需要多加入两个额外节点。如果某个Splay只有两个节点了需要及时回收。
例题：Box（维护括号序列）

```
1   \\下面都是专用函数
2   \\判断x在不在f里面
3   bool Ancestor(Node *x,Node *f)
4   {
5     if (x == f) return true;
6     while (x->p != nil)
7     {
8       if (x->p == f)  return true;
```

```
9       x = x->p;
10    }
11    return false;
12  }
13  \\把Splay v插入到pos后面，pos=nil时新开一个序列
14  void Insert(Node *pos, Node *v)
15  {
16    int pl;
17    if (pos == nil)
18    {
19      Init();
20      Insert(0), Insert(0);
21      pl = 0;
22    }
23    else
24    {
25      Splay(pos, nil);
26      pl = root->c[0]->size;
27    }
28    Select(pl, nil), Select(pl + 1, root);
29    root->c[1]->c[0] = v;
30    v->p = root->c[1];
31    Splay(v, nil);
32  }
33  \\把[l,r]转出来（这里记录的是绝对位置）
34  void Select(Node *l, Node *r)
35  {
36    Splay(l, nil);
37    int pl = root->c[0]->size - 1;
38    Splay(r, nil);
39    int pr = root->c[0]->size - 1;
40    Select(pl, pr);
41  }
42  \\分离[l,r]
43  Node *Split(Node *l, Node *r)
44  {
45    Select(l, r);
46    Node *res = root->c[1]->c[0];
47    root->c[1]->c[0] = res->p = nil;
48    Splay(root->c[1], nil);
49    if (root->size == 2)
50    {
51      Recycle(root);
52      Init();
53    }
54    return res;
55  }
56
57 int main(int argc, char const *argv[])
58 {
59   freopen("P.in", "r", stdin);
```

```
60    bool first = true;
61    while (scanf("%d", &n) != EOF)
62    {
63      if (!first) puts("");
64      first = false;
65      Init();
66      for (int i = 0; i < n; i++)
67      {
68        \\建立独立的N个区间，记录绝对位置
69        sp.Init();
70        sp.Insert(0), sp.Insert(0);
71        sp.Insert(0,i+1),sp.Insert(1,i+1);
72        sp.Select(0, 0), l[i] = sp.root->c[1]->c[0];
73        sp.Select(1, 1), r[i] = sp.root->c[1]->c[0];
74      }
75      for (int i = 0; i < n; i++)
76      {
77        int f;
78        scanf("%d", &f);
79        if (f != 0)
80        {
81          \\把[l[i],r[i]]插入到l[f-1]后面
82          Node *pos = sp.Split(l[i], r[i]);
83          sp.Insert(l[f - 1], pos);
84        }
85      }
86      scanf("%d", &n);
87      for (int i = 0; i < n; i++)
88      {
89        scanf("%s", com);
90        if (com[0] == 'Q')
91        {
92          int pos;
93          scanf("%d", &pos);
94          \\求[l[pos-1],r[pos-1]]在哪个序列里面
95          sp.Splay(l[pos - 1], nil);
96          sp.Select(1, nil);
97          printf("%d\n", sp.root->key);
98        }
99        else
100       {
101         int u, v;
102         scanf("%d%d", &u, &v);
103         if (v == 0)
104           sp.Insert(nil, sp.Split(l[u-1], r[u-1]));
105         else
106         {
107           sp.Select(l[u-1],r[u-1]);
108           if (sp.Ancestor(l[v-1],sp.root->c[1]->c[0]) == false)
109             sp.Insert(l[v - 1], sp.Split(l[u-1], r[u-1]));
110         }
```

```
111       }
112     }
113   }
114   return 0;
115 }
```

5.2 动态树

懒标记是否及时Pushdown了?
修改之后有没有及时Pushup?

5.2.1 维护点权

查询链上的最长字段和
GetRoute是用换根写的

```
1  const int MaxN = 110000;
2
3  struct Node
4  {
5    int size, key;
6    bool rev;
7
8  //  bool same;
9  //  int lsum, rsum, sum, maxsum, sa;
10
11   Node *c[2];
12   Node *p;
13 } mem[MaxN], *cur, *nil, *pos[MaxN];
14
15 Node *newNode(int v, Node *p)
16 {
17   cur->c[0] = cur->c[1] = nil, cur->p = p;
18   cur->size = 1;
19   cur->key = v;
20   cur->rev = false;
21
22 //  cur->same = false;
23 //  cur->sa = 0;
24 //  cur->lsum = cur->rsum = cur->maxsum = 0;
25 //  cur->sum = v;
26
27   return cur++;
28 }
29
30 void Init()
31 {
32   cur = mem;
33   nil = newNode(0, cur);
34   nil->size = 0;
```

```
35 }
36
37 struct SplayTree
38 {
39   void Pushup(Node *x)
40   {
41     if (x == nil)   return;
42     Pushdown(x); Pushdown(x->c[0]); Pushdown(x->c[1]);
43     x->size = x->c[0]->size + x->c[1]->size + 1;
44
45 //    x->sum = x->c[0]->sum + x->c[1]->sum + x->key;
46 //    x->lsum = max(x->c[0]->lsum, x->c[0]->sum + x->key + max(0, x
       ->c[1]->lsum));
47 //    x->rsum = max(x->c[1]->rsum, x->c[1]->sum + x->key + max(0, x
       ->c[0]->rsum));
48 //    x->maxsum = max(max(x->c[0]->maxsum, x->c[1]->maxsum),
49 //      x->key + max(0, x->c[0]->rsum) + max(0, x->c[1]->lsum));
50
51   }
52   void Pushdown(Node *x)
53   {
54     if (x == nil)   return;
55     if (x->rev)
56     {
57       x->rev = 0;
58       x->c[0]->rev ^= 1;
59       x->c[1]->rev ^= 1;
60       swap(x->c[0], x->c[1]);
61 //注意修改与位置有关的量
62 //      swap(x->lsum,x->rsum);
63     }
64
65 //    if (x->same)
66 //    {
67 //      x->same = false;
68 //      x->key = x->sa;
69 //      x->sum = x->sa * x->size;
70 //      x->lsum = x->rsum = x->maxsum = max(0, x->sum);
71 //      if (x->c[0] != nil)
72 //        x->c[0]->same = true, x->c[0]->sa = x->sa;
73 //      if (x->c[1] != nil)
74 //        x->c[1]->same = true, x->c[1]->sa = x->sa;
75 //    }
76   }
77   bool isRoot(Node *x)
78   {
79     return (x == nil) || (x->p->c[0] != x && x->p->c[1] != x);
80   }
81   void Rotate(Node *x, int f)
82   {
83     if (isRoot(x))   return;
```

```
84      Node *y = x->p;
85      y->c[f ^ 1] = x->c[f], x->p = y->p;
86      if (x->c[f] != nil)
87        x->c[f]->p = y;
88      if (y != nil)
89      {
90        if (y == y->p->c[1])
91          y->p->c[1] = x;
92        else if (y == y->p->c[0])
93          y->p->c[0] = x;
94      }
95      x->c[f] = y, y->p = x;
96      Pushup(y);
97    }
98    void Splay(Node *x)
99    {
100     static Node *stack[MaxN];
101     int top = 0;
102     stack[top++] = x;
103     for (Node *y = x; !isRoot(y); y = y->p)
104       stack[top++] = y->p;
105     while (top)
106       Pushdown(stack[--top]);
107
108     while (!isRoot(x))
109     {
110       Node *y = x->p;
111       if (isRoot(y))
112         Rotate(x, x == y->c[0]);
113       else
114       {
115         int fd = y->p->c[0] == y;
116         if (y->c[fd] == x)
117           Rotate(x, fd ^ 1), Rotate(x, fd);
118         else
119           Rotate(y, fd), Rotate(x, fd);
120       }
121     }
122     Pushup(x);
123   }
124   Node *Access(Node *u)
125   {
126     Node *v = nil;
127     while (u != nil)
128     {
129       Splay(u);
130       v->p = u;
131       u->c[1] = v;
132       Pushup(u);
133       u = (v = u)->p;
134       if (u == nil)
```

```
135         return v;
136       }
137     }
138     Node *LCA(Node *u, Node *v)
139     {
140       Access(u);
141       return Access(v);
142     }
143     Node *Link(Node *u, Node *v)
144     {
145       Access(u);
146       Splay(u);
147       u->rev = true;
148       u->p = v;
149     }
150     void ChangeRoot(Node *u)
151     {
152       Access(u)->rev ^= 1;
153     }
154     Node *GetRoute(Node *u, Node *v)
155     {
156       ChangeRoot(u);
157       return Access(v);
158     }
159 };
160 
161 int n, m;
162 SplayTree sp;
163 
164 int main(int argc, char const *argv[])
165 {
166     while (scanf("%d", &n) != EOF)
167     {
168       Init();
169       for (int i = 0; i < n; i++)
170       {
171         int v;
172         scanf("%d", &v);
173         pos[i] = newNode(v, nil);
174       }
175       for (int i = 0; i < n - 1; i++)
176       {
177         int u, v;
178         scanf("%d%d", &u, &v);
179         u--, v--;
180         sp.Link(pos[u], pos[v]);
181       }
182 
183 //     scanf("%d", &m);
184 //     for (int i = 0; i < m; i++)
185 //     {
```

```
//      int typ, u, v, c;
//      scanf("%d%d%d", &typ, &u, &v);
//      u--, v--;
//      if (typ == 1)
//        printf("%d\n", sp.GetRoute(pos[u], pos[v])->maxsum);
//      else
//      {
//        scanf("%d", &c);
//        Node *p = sp.GetRoute(pos[u], pos[v]);
//        p->same = true;
//        p->sa = c;
//      }
//    }
  }
  return 0;
}
```

5.2.2 维护边权

刘汝佳的Happy Painting!
查询链上边的不同颜色数量
不能换根，但是可以Link和Cut

```
const int MaxN = 60000;

struct Node
{
  int size,key;

  int msk,lazy;

  Node *c[2];
  Node *p;
} mem[MaxN], *cur, *nil, *pos[MaxN];

Node *newNode(int v,Node *p)
{
  cur->c[0] = cur->c[1] = nil, cur->p = p;
  cur->size = 1;
  cur->key = v;

  cur->msk = 0;
  cur->lazy = -1;

  return cur++;
}

void Init()
{
  cur = mem;
  nil = newNode(0, cur);
```

```
29    nil->size = 0;
30  }
31
32  struct SplayTree
33  {
34    void Pushup(Node *x)
35    {
36      if (x == nil) return;
37      Pushdown(x);
38      Pushdown(x->c[0]);
39      Pushdown(x->c[1]);
40      x->size = x->c[0]->size + x->c[1]->size + 1;
41
42      x->msk = x->c[0]->msk | x->c[1]->msk | (1<<x->key);
43    }
44    void Pushdown(Node *x)
45    {
46      if (x == nil) return;
47
48      if (x->lazy != -1)
49      {
50        x->key = x->lazy;
51        x->msk = (1<<x->key);
52        x->c[0]->lazy = x->c[1]->lazy = x->lazy;
53        x->lazy = -1;
54      }
55    }
56    bool isRoot(Node *x)
57    {
58      return (x == nil) || (x->p->c[0] != x && x->p->c[1] != x);
59    }
60    void Rotate(Node *x, int f)
61    {
62      if (isRoot(x))  return;
63      Node *y = x->p;
64      y->c[f ^ 1] = x->c[f], x->p = y->p;
65      if (x->c[f] != nil)
66        x->c[f]->p = y;
67      if (y != nil)
68      {
69        if (y == y->p->c[1])
70          y->p->c[1] = x;
71        else if (y == y->p->c[0])
72          y->p->c[0] = x;
73      }
74      x->c[f] = y, y->p = x;
75      Pushup(y);
76    }
77    void Splay(Node *x)
78    {
79      static Node *stack[MaxN];
```

```
80      int top = 0;
81      stack[top++] = x;
82      for (Node *y = x; !isRoot(y); y = y->p)
83        stack[top++] = y->p;
84      while (top)
85        Pushdown(stack[--top]);
86
87      while (!isRoot(x))
88      {
89        Node *y = x->p;
90        if (isRoot(y))
91          Rotate(x, x == y->c[0]);
92        else
93        {
94          int fd = y->p->c[0] == y;
95          if (y->c[fd] == x)
96            Rotate(x, fd ^ 1), Rotate(x, fd);
97          else
98            Rotate(y, fd), Rotate(x, fd);
99        }
100     }
101     Pushup(x);
102   }
103   Node *Access(Node *u)
104   {
105     Node *v = nil;
106     while (u != nil)
107     {
108       Splay(u);
109       v->p = u;
110       u->c[1] = v;
111       Pushup(u);
112       u = (v = u)->p;
113       if (u == nil) return v;
114     }
115   }
116   Node *Root(Node *u)
117   {
118     Access(u);
119     Splay(u);
120     for (Pushdown(u); u->c[0] != nil; u = u->c[0])
121       Pushdown(u);
122     Splay(u);
123     return u;
124   }
125   Node *LCA(Node *u, Node *v)
126   {
127     if (Root(u) != Root(v))
128       return nil;
129     Access(u);
130     return Access(v);
```

```
131   }
132   void Cut(Node *u)
133   {
134     Access(u);
135     Splay(u);
136     u->c[0] = u->c[0]->p = nil;
137     Pushup(u);
138   }
139   void Link(Node *u, Node *v, int val)
140   {
141     Access(u);
142     Splay(u);
143     u->p = v;
144     u->key = val;
145     Pushup(u);
146   }
147 };
148 
149 int cntbit(int x)
150 {
151   x = (x & 0x55555555) + ((x >> 1) & 0x55555555);
152   x = (x & 0x33333333) + ((x >> 2) & 0x33333333);
153   x = (x & 0x0F0F0F0F) + ((x >> 4) & 0x0F0F0F0F);
154   x = (x & 0x00FF00FF) + ((x >> 8) & 0x00FF00FF);
155   x = (x & 0x0000FFFF) + ((x >> 16) & 0x0000FFFF);
156   return x;
157 }
158 
159 SplayTree sp;
160 int n,Q,f[MaxN];
161 
162 int main(int argc, char const *argv[])
163 {
164   while (scanf("%d%d",&n,&Q) != EOF)
165   {
166     Init();
167     for (int i = 0; i < n; i++)
168     {
169       scanf("%d",&f[i]);
170       pos[i] = newNode(0, nil);
171     }
172     for (int i = 0; i < n; i++)
173     {
174       int col;
175       scanf("%d",&col);
176       if (f[i] > 0)
177         sp.Link(pos[i],pos[f[i]-1],col-1);
178     }
179     for (int q = 0; q < Q; q++)
180     {
181       int typ,x,y,c;
```

```
182       scanf("%d%d%d",&typ,&x,&y);
183       x--,y--;
184       if (typ == 3)
185       {
186         Node *lca = sp.LCA(pos[x],pos[y]);
187         if (lca == nil || x == y)
188         {
189           printf("0 0\n");
190           continue;
191         }
192         int totedge = lca->c[1]->size;
193         int msk = lca->c[1]->msk;
194
195         if (pos[x] != lca)
196         {
197           sp.Splay(pos[x]);
198           totedge += pos[x]->size;
199           msk |= pos[x]->msk;
200         }
201
202         printf("%d %d\n",totedge,cntbit(msk));
203       }
204       else
205       {
206         scanf("%d",&c);
207         c--;
208         if (typ == 1)
209         {
210           if (x == y) continue;
211
212           Node *lca = sp.LCA(pos[x],pos[y]);
213           if (pos[x] == lca)  continue;
214
215           sp.Cut(pos[x]);
216           sp.Link(pos[x],pos[y],c);
217
218         }
219         else
220         {
221           Node *lca = sp.LCA(pos[x],pos[y]);
222
223           if (lca == nil || x == y)
224             continue;
225
226           lca->c[1]->lazy = c;
227           sp.Pushup(lca->c[1]);
228           sp.Pushup(lca);
229           if (pos[x] != lca)
230           {
231             sp.Splay(pos[x]);
232             pos[x]->lazy = c;
```

```
233             sp.Pushup(pos[x]);
234           }
235         }
236       }
237     }
238   }
239   return 0;
240 }
```

5.3 可持久化线段树

区间第k小数，内存压缩版，POJ2014。

```
1  #include <cstdio>
2  #include <algorithm>
3  using namespace std;
4
5  const int MAXN=100000,MAXM=100000;
6
7  struct node
8  {
9    node *l,*r;
10   int sum;
11 }tree[MAXN*4+MAXM*20];
12
13 int N;
14 node *newnode()
15 {
16   tree[N].l=tree[N].r=NULL;
17   tree[N].sum=0;
18   return &tree[N++];
19 }
20 node *newnode(node *x)
21 {
22   tree[N].l=x->l;
23   tree[N].r=x->r;
24   tree[N].sum=x->sum;
25   return &tree[N++];
26 }
27 node *build(int l,int r)
28 {
29   node *x=newnode();
30   if (l<r)
31   {
32     int mid=l+r>>1;
33     x->l=build(l,mid);
34     x->r=build(mid+1,r);
35     x->sum=x->l->sum+x->r->sum;
36   }
37   else
38     x->sum=0;
39   return x;
```

```
}
node *update(node *x,int l,int r,int p,int v)
{
  if (l<r)
  {
    int mid=l+r>>1;
    node *nx=newnode(x);
    if (p<=mid)
    {
      node *ret=update(x->l,l,mid,p,v);
      nx->l=ret;
    }
    else
    {
      node *ret=update(x->r,mid+1,r,p,v);
      nx->r=ret;
    }
    nx->sum=nx->l->sum+nx->r->sum;
    return nx;
  }
  else
  {
    node *nx=newnode(x);
    nx->sum+=v;
    return nx;
  }
}
int query(node *x1,node *x2,int l,int r,int k)
{
  if (l<r)
  {
    int mid=l+r>>1;
    int lsum=x2->l->sum-x1->l->sum;
    if (lsum>=k)
      return query(x1->l,x2->l,l,mid,k);
    else
      return query(x1->r,x2->r,mid+1,r,k-lsum);
  }
  else
    return l;
}
char s[10];
node *root[MAXM+1];
int a[MAXN],b[MAXN];
int init(int n)
{
  for (int i=0;i<n;i++)
    b[i]=a[i];
  sort(b,b+n);
  int tn=unique(b,b+n)-b;
  for (int i=0;i<n;i++)
```

```
91    {
92      int l=0,r=tn-1;
93      while (l<r)
94      {
95        int mid=l+r>>1;
96        if (b[mid]>=a[i])
97          r=mid;
98        else
99          l=mid+1;
100     }
101     a[i]=l;
102   }
103   return tn;
104 }
105 int main()
106 {
107   int cas=1,n;
108   while (scanf("%d",&n)!=EOF)
109   {
110     printf("Case␣%d:\n",cas++);
111     for (int i=0;i<n;i++)
112       scanf("%d",&a[i]);
113     int tn=init(n);
114     N=0;
115     root[0]=build(0,tn-1);
116     for (int i=1;i<=n;i++)
117       root[i]=update(root[i-1],0,tn-1,a[i-1],1);
118     int m;
119     scanf("%d",&m);
120     for (int i=0;i<m;i++)
121     {
122       int s,t;
123       scanf("%d%d",&s,&t);
124       printf("%d\n",b[query(root[s-1],root[t],0,tn-1,t-s+2>>1)]);
125     }
126   }
127   return 0;
128 }
```

5.4 treap正式版

支持翻转。

```
1 #include <cstdio>
2 #include <cstdlib>
3 #include <algorithm>
4 using namespace std;
5
6 const int MAXN = 100000;
7 const int MAXM = 100000;
8 const int inf = 0x7fffffff;
9 int a[MAXN];
```

```
10 struct Treap
11 {
12   int N;
13   Treap()
14   {
15     N = 0;
16     root = NULL;
17   }
18   void init()
19   {
20     N = 0;
21     root = NULL;
22   }
23   struct Treap_Node
24   {
25     Treap_Node *son[2];//left & right
26     int value, fix;
27     bool lazy;
28     int size;
29     Treap_Node() {}
30     Treap_Node(int _value)
31     {
32       son[0] = son[1] = NULL;
33       value = _value;
34       fix = rand() * rand();
35       lazy = 0;
36       size = 1;
37     }
38     int sonSize(bool flag)
39     {
40       if (son[flag] == NULL)
41         return 0;
42       else
43         return son[flag]->size;
44     }
45   } node[MAXN], *root, *pos[MAXN];
46   void up(Treap_Node *p)
47   {
48     p->size = p->sonSize(0) + p->sonSize(1) + 1;
49   }
50   void down(Treap_Node *p)
51   {
52     if (!p->lazy)
53       return ;
54     for (int i = 0; i < 2; i++)
55       if (p->son[i])
56         p->son[i]->lazy = !p->son[i]->lazy;
57     swap(p->son[0], p->son[1]);
58     p->lazy = 0;
59   }
60   Treap_Node *merge(Treap_Node *p, Treap_Node *q)
```

```
61  {
62    if (p == NULL)
63      return q;
64    else if (q == NULL)
65      return p;
66    if (p->fix <= q->fix)
67    {
68      down(p);
69      p->son[1] = merge(p->son[1], q);
70      up(p);
71      return p;
72    }
73    else
74    {
75      down(q);
76      q->son[0] = merge(p, q->son[0]);
77      up(q);
78      return q;
79    }
80  }
81  pair<Treap_Node *, Treap_Node *> split(Treap_Node *p, int n)
82  {
83    if (p == NULL)
84      return make_pair((Treap_Node *)NULL, (Treap_Node *)NULL);
85    if (!n)
86      return make_pair((Treap_Node *)NULL, p);
87    if (n == p->size)
88      return make_pair(p, (Treap_Node *)NULL);
89    down(p);
90    if (p->sonSize(0) >= n)
91    {
92      pair<Treap_Node *, Treap_Node *> ret = split(p->son[0], n);
93      p->son[0] = ret.second;
94      up(p);
95      return make_pair(ret.first, p);
96    }
97    else
98    {
99      pair<Treap_Node *, Treap_Node *> ret = split(p->son[1], n - p
           ->sonSize(0) - 1);
100     p->son[1] = ret.first;
101     up(p);
102     return make_pair(p, ret.second);
103   }
104 }
105 int smalls(Treap_Node *p,int value)
106 {
107   if (p==NULL)
108     return 0;
109   if (p->value<=value)
110     return 1+p->sonSize(0)+smalls(p->son[1],value);
```

```
111     else
112       return smalls(p->son[0],value);
113   }
114   void insert(int value)
115   {
116     Treap_Node *p = &node[N++];
117     *p = Treap_Node(value);
118     pair<Treap_Node *, Treap_Node *> ret = split(root, smalls(root,
          value));
119     root = merge(merge(ret.first, p), ret.second);
120   }
121   void remove(int value)
122   {
123     pair<Treap_Node *, Treap_Node *> ret = split(root, smalls(root,
          value) - 1);
124     root = merge(ret.first, split(ret.second, 1).second);
125   }
126   Treap_Node *build(int s, int t)
127   {
128     int idx = t + s >> 1;
129     Treap_Node *p = &node[N++];
130     *p = Treap_Node(a[idx]);
131     pos[a[idx]] = p;
132     if (idx > s)
133       p = merge(build(s, idx - 1), p);
134     if (idx < t)
135       p = merge(p, build(idx + 1, t));
136     up(p);
137     return p;
138   }
139   void build(int n)
140   {
141     root = build(0, n - 1);
142   }
143   void *reverse(int s, int t)
144   {
145     pair<Treap_Node *, Treap_Node *> tmp1, tmp2;
146     tmp1 = split(root, s - 1);
147     tmp2 = split(tmp1.second, t - s + 1);
148     tmp2.first->lazy = !tmp2.first->lazy;
149     root = merge(tmp1.first, merge(tmp2.first, tmp2.second));
150   }
151 };
152 Treap treap;
153 int main()
154 {
155   treap.init();
156   int n;
157   scanf("%d", &n);
158   for (int i = 0; i < n; i++)
159     scanf("%d", &a[i]);
```

```
160   treap.build(n);
161 }
```

5.5 树链剖分

5.5.1 点权

```
1  #include <cstdio>
2  #include <cstring>
3  #include <cstdlib>
4  #include <algorithm>
5  using namespace std;
6  const int MAX = 12000;
7  const int LOG = 15;
8  const int oo = 0x3f3f3f3f;
9  struct Edge
10 {
11     int to, w, id;
12     Edge* next;
13 } memo[MAX << 1], *cur, *g[MAX], *pree[MAX], *solid[MAX], *valid[
       MAX];
14 int dp[MAX][LOG], pos[MAX], lst[MAX], dep[MAX], cnt[MAX], h[MAX], K
       , n;
15 void init()
16 {
17   for (int i = 1; i <= n; i++)
18   {
19     g[i] = NULL;
20     valid[i] = NULL;
21     solid[i] = NULL;
22     pree[i] = NULL;
23   }
24   for (int i = 0; i < LOG; i++)
25   {
26     dp[1][i] = 1;
27   }
28   cur = memo;
29   K = 0;
30 }
31 void add(int u, int v, int w, int id)
32 {
33   cur->to = v;
34   cur->w = w;
35   cur->id = id;
36   cur->next = g[u];
37   g[u] = cur++;
38 }
39 void dfsLCA(int d, int u, int f)
40 {
41   dep[u] = d;
42   dp[u][0] = f;
```

```
43    cnt[u] = 1;
44    for (int i = 1; i < LOG; i++)
45    {
46      dp[u][i] = dp[dp[u][i - 1]][i - 1];
47    }
48    for (Edge* it = g[u]; it; it = it->next)
49    {
50      int v = it->to;
51      if (v != f)
52      {
53        pree[v] = it;
54        valid[it->id] = it;
55        dfsLCA(d + 1, v, u); //RE
56        cnt[u] += cnt[v];
57        if (solid[u] == NULL || cnt[solid[u]->to] < cnt[v])
58        {
59          solid[u] = it;
60        }
61      }
62    }
63  }
64  void dfsChain(int u, int head)
65  {
66    h[u] = head;
67    if (solid[u])
68    {
69      lst[pos[u] = K++] = u;
70      dfsChain(solid[u]->to, head);
71    }
72    else
73    for (Edge* it = g[u]; it; it = it->next)
74    {
75      int v = it->to;
76      if (it != solid[u] && v != dp[u][0])
77      {
78        dfsChain(v, v);
79      }
80    }
81  }
82  int getLCA(int u, int v)
83  {
84    if (dep[u] < dep[v])
85      swap(u, v);
86    for (int st = 1 << (LOG - 1), i = LOG - 1; i >= 0; i--, st >>= 1)
87    {
88      if (st <= dep[u] - dep[v])
89      {
90        u = dp[u][i];
91      }
92    }
93    if (u == v)
```

```
94      return u;
95    for (int i = LOG - 1; i >= 0; i--)
96    {
97      if (dp[u][i] != dp[v][i])
98      {
99        u = dp[u][i];
100       v = dp[v][i];
101     }
102   }
103   return dp[u][0];
104 }
105 struct Node
106 {
107     int l, r, ma, mi;
108     bool rev;
109 } seg[MAX << 2];
110 void reverse(int k)
111 {
112   seg[k].mi *= -1;
113   seg[k].ma *= -1;
114   seg[k].rev ^= 1;
115   swap(seg[k].mi, seg[k].ma);
116 }
117 void pushdown(int k)
118 {
119   if (seg[k].rev)
120   {
121     reverse(k << 1);
122     reverse(k << 1 | 1);
123     seg[k].rev = false;
124   }
125 }
126 void update(int k)
127 {
128   seg[k].mi = min(seg[k << 1].mi, seg[k << 1 | 1].mi);
129   seg[k].ma = max(seg[k << 1].ma, seg[k << 1 | 1].ma);
130 }
131 void init(int k, int l, int r)
132 {
133   seg[k].l = l;
134   seg[k].r = r;
135   seg[k].rev = false;
136   if (l == r)
137   {
138     seg[k].mi = seg[k].ma = solid[lst[l]]->w; //solid WA
139     return;
140   }
141   int mid = l + r >> 1;
142   init(k << 1, l, mid);
143   init(k << 1 | 1, mid + 1, r);
144   update(k);
```

```
}
void update(int k, int id, int v)
{
  if (seg[k].l == seg[k].r)
  {
    seg[k].mi = seg[k].ma = solid[lst[id]]->w = v;
    return;
  }
  pushdown(k);
  int mid = seg[k].l + seg[k].r >> 1;
  if (id <= mid)
    update(k << 1, id, v);
  else
    update(k << 1 | 1, id, v);
  update(k);
}
void reverse(int k, int l, int r)
{
  if (seg[k].l > r || seg[k].r < l)
    return;
  if (seg[k].l >= l && seg[k].r <= r)
  {
    reverse(k);
    return;
  }
  pushdown(k);
  reverse(k << 1, l, r);
  reverse(k << 1 | 1, l, r);
  update(k);
}
int read(int k, int l, int r)
{
  if (seg[k].l > r || seg[k].r < l)
    return -oo;
  if (seg[k].l >= l && seg[k].r <= r)
    return seg[k].ma;
  pushdown(k);
  return max(read(k << 1, l, r), read(k << 1 | 1, l, r));
}
void setEdge(int id, int v)
{
  Edge* it = valid[id];
  if (h[it->to] != it->to)
  {
    update(1, pos[dp[it->to][0]], v);
  }
  else
  {
    it->w = v;
  }
}
```

```
196 void negateLCA(int t, int u)
197 {
198   while (t != u)
199   {
200     int tmp = h[u];
201     if (dep[tmp] < dep[t])
202       tmp = t;
203     if (h[u] == u)
204     {
205       pree[u]->w *= -1;
206       u = dp[u][0];
207     }
208     else
209     {
210       reverse(1, pos[tmp], pos[dp[u][0]]);
211       u = tmp;
212     }
213   }
214 }
215 void negate(int u, int v)
216 {
217   int t = getLCA(u, v);
218   negateLCA(t, u);
219   negateLCA(t, v);
220 }
221 int maxLCA(int t, int u)
222 {
223   int ret = -oo;
224   while (t != u)
225   {
226     int tmp = h[u];
227     if (dep[tmp] < dep[t])
228       tmp = t;
229     if (h[u] == u)
230     {
231       ret = max(ret, pree[u]->w);
232       u = dp[u][0];
233     }
234     else
235     {
236       ret = max(ret, read(1, pos[tmp], pos[dp[u][0]]));
237       u = tmp;
238     }
239   }
240   return ret;
241 }
242 int query(int u, int v)
243 {
244   int t = getLCA(u, v);
245   return max(maxLCA(t, u), maxLCA(t, v));
246 }
```

```
247 int main()
248 {
249   int T;
250   int u, v, w;
251   char op[15];
252   scanf("%d", &T);
253   while (T--)
254   {
255     scanf("%d", &n);
256     init();
257     for (int i = 1; i < n; i++)
258     {
259       scanf("%d%d%d", &u, &v, &w);
260       add(u, v, w, i);
261       add(v, u, w, i);
262     }
263     dfsLCA(0, 1, 1);
264     dfsChain(1, 1);
265     init(1, 0, K - 1);
266     while (scanf("%s", op), op[0] != 'D')
267     {
268       scanf("%d%d", &u, &v);
269       if (op[0] == 'C')
270       {
271         setEdge(u, v);
272       }
273       else if (op[0] == 'N')
274       {
275         negate(u, v);
276       }
277       else
278       {
279         printf("%d\n", query(u, v));
280       }
281     }
282   }
283   return 0;
284 }
```

5.5.2 边权

```
1  #include <cstdio>
2  #include <iostream>
3  #include <cstdlib>
4  #include <algorithm>
5  #include <cmath>
6  #include <cstring>
7  using namespace std;
8  int n,m,sum,pos;
9  int head[50005],e;
10 int s[50005],from[50005];
11 int fa[50005][20],deep[50005],num[50005];
```

```
int solid[50005],p[50005],fp[50005];
struct N
{
  int l,r,mid;
  int add,w;
}nod[50005*4];
struct M
{
  int v,next;
}edge[100005];
void addedge(int u,int v)
{
  edge[e].v=v;
  edge[e].next=head[u];
  head[u]=e++;

  edge[e].v=u;
  edge[e].next=head[v];
  head[v]=e++;
}
void LCA(int st,int f,int d)
{
  deep[st]=d;
  fa[st][0]=f;
  num[st]=1;
  int i,v;
  for(i=1;i<20;i++)
    fa[st][i]=fa[fa[st][i-1]][i-1];
  for(i=head[st];i!=-1;i=edge[i].next)
  {
    v=edge[i].v;
    if(v!=f)
    {
      LCA(v,st,d+1);
      num[st]+=num[v];
      if(solid[st]==-1||num[v]>num[solid[st]])
        solid[st]=v;
    }
  }
}
void getpos(int st,int sp)
{
  from[st]=sp;
  if(solid[st]!=-1)
  {
    p[st]=pos++;
    fp[p[st]]=st;
    getpos(solid[st],sp);
  }
  else
  {
```

```
63      p[st]=pos++;
64      fp[p[st]]=st;
65      return;
66    }
67    int i,v;
68    for(i=head[st];i!=-1;i=edge[i].next)
69    {
70      v=edge[i].v;
71      if(v!=solid[st]&&v!=fa[st][0])
72        getpos(v,v);
73    }
74  }
75  int getLCA(int u,int v)
76  {
77    if(deep[u]<deep[v])
78      swap(u,v);
79    int d=1<<19,i;
80    for(i=19;i>=0;i--)
81    {
82      if(d<=deep[u]-deep[v])
83        u=fa[u][i];
84      d>>=1;
85    }
86    if(u==v)
87      return u;
88    for(i=19;i>=0;i--)
89      if(fa[u][i]!=fa[v][i])
90      {
91        u=fa[u][i];
92        v=fa[v][i];
93      }
94    return fa[u][0];
95  }
96  void init(int p,int l,int r)
97  {
98    nod[p].l=l;
99    nod[p].r=r;
100   nod[p].mid=(l+r)>>1;
101   nod[p].add=0;
102   if(l==r)
103     nod[p].w=s[fp[l]];
104   else
105   {
106     init(p<<1,l,nod[p].mid);
107     init(p<<1|1,nod[p].mid+1,r);
108   }
109 }
110 void lazy(int p)
111 {
112   if(nod[p].add!=0)
113   {
```

```
114     nod[p<<1].add+=nod[p].add;
115     nod[p<<1|1].add+=nod[p].add;
116     nod[p].add=0;
117   }
118 }
119 void update(int p,int l,int r,int v)
120 {
121   if(nod[p].l==l&&nod[p].r==r)
122   {
123     nod[p].add+=v;
124     return;
125   }
126   lazy(p);
127   if(nod[p].mid<l)
128     update(p<<1|1,l,r,v);
129   else if(nod[p].mid>=r)
130     update(p<<1,l,r,v);
131   else
132   {
133     update(p<<1,l,nod[p].mid,v);
134     update(p<<1|1,nod[p].mid+1,r,v);
135   }
136 }
137 int read(int p,int l,int r)
138 {
139   if(nod[p].l==l&&nod[p].r==r)
140     return nod[p].w+nod[p].add;
141   lazy(p);
142   if(nod[p].mid<l)
143     return read(p<<1|1,l,r);
144   else if(nod[p].mid>=r)
145     return read(p<<1,l,r);
146 }
147 void jump(int st,int ed,int val)
148 {
149   while(deep[st]>=deep[ed])
150   {
151     int tmp=from[st];
152     if(deep[tmp]<deep[ed])
153       tmp=ed;
154     update(1,p[tmp],p[st],val);
155     st=fa[tmp][0];
156   }
157 }
158 void change(int st,int ed,int val)
159 {
160   int lca=getLCA(st,ed);
161   jump(st,lca,val);
162   jump(ed,lca,val);
163   jump(lca,lca,-val);
164 }
```

```
int main()
{
  while(scanf("%d%d%d",&n,&m,&sum)==3)
  {
    int i;
    s[0]=0;pos=0;deep[0]=-1;
    memset(fa,0,sizeof(fa));
    for(i=1;i<=n;i++)
    {
      solid[i]=-1;
      scanf("%d",&s[i]);
    }
    memset(head,-1,sizeof(head));
    e=0;
    for(i=0;i<m;i++)
    {
      int a,b;
      scanf("%d%d",&a,&b);
      addedge(a,b);
    }
    LCA(1,0,0);
    getpos(1,1);
    init(1,0,pos-1);
    for(i=0;i<sum;i++)
    {
      char que[5];
      scanf("%s",que);
      if(que[0]!='Q')
      {
        int a,b,c;
        scanf("%d%d%d",&a,&b,&c);
        if(que[0]=='D')
          c=-c;
        change(a,b,c);
      }
      else
      {
        int a;
        scanf("%d",&a);
        printf("%d\n",read(1,p[a],p[a]));
      }
    }
  }
  return 0;
}
```

5.6 划分树

```
int n,m;
struct elem
{
  int v,index;
```

```
}a[120000];
int d[30][120000];
int s[30][120000];

bool cmp(elem a,elem b)
{
  if (a.v == b.v)
    return a.index <= b.index;
  return a.v < b.v;
}

void build(int depth,int l,int r)
{
  if (l == r)
    return;
  int mid = (l+r)/2;
  int tl,tr;
  tl = tr = 0;
  for (int i = l;i <= r;i++)
  {
    if (cmp(a[d[depth][i]],a[mid]))
    {
      d[depth+1][l+tl] = d[depth][i];
      tl++;
    }
    else
    {
      d[depth+1][mid+1+tr] = d[depth][i];
      tr++;
    }
    s[depth][i] = tl;
  }
  build(depth+1,l,mid);
  build(depth+1,mid+1,r);
}

int find(int depth,int dl,int dr,int fl,int fr,int k)
{
  if (fl == fr)
    return a[d[depth][fl]].v;
  int ls,rs;
  int mid = (dl+dr)/2;
  ls = (fl == dl)? 0 : s[depth][fl-1];
  rs = s[depth][fr];
  return (rs-ls < k)? find(depth+1,mid+1,dr,mid+fl-dl-ls+1,mid+fr-
     dl-rs+1,k-(rs-ls)) : find(depth+1,dl,mid,dl+ls,dl+rs-1,k);
}

int main()
{
  while (scanf("%d%d",&n,&m) != EOF)
```

```
55    {
56      for (int i = 1;i <= n;i++)
57      {
58        scanf("%d",&a[i].v);
59        a[i].index = i;
60      }
61      sort(a+1,a+n+1,cmp);
62      for (int i = 1;i <= n;i++)
63        d[0][a[i].index] = i;
64      build(0,1,n);
65      int l,r,k;
66      for (int i = 1;i <= m;i++)
67      {
68        scanf("%d%d%d",&l,&r,&k);
69        printf("%d\n",find(0,1,n,l,r,k));
70      }
71    }
72    return 0;
73  }
```

5.7 树状数组

```
1  int read(int k)
2  {
3    int sum = 0;
4    for (; k; k^=k&-k)
5      sum+=tree[k];
6    return sum;
7  }
8  void update(int k, int v)
9  {
10   for (; k<=MaxN; k+=k&-k)
11     tree[k]+=v;
12 }
13 int find_Kth(int k)
14 {
15   int idx = 0;
16   for(int i=20; i>=0; i--)
17   {
18     idx |= 1 << i;
19     if(idx <= MaxN && tree[idx] < k)
20       k -= tree[idx];
21     else  idx ^= 1 << i;
22   }
23   return idx + 1;
24 }
```

6 图论

6.1 优先队列优化的dijkstra

```
#include<cstdio>
#include<cstring>
#include<iostream>
#include<algorithm>
#include<queue>
#include<vector>
using namespace std;
const int MAXN=100;
const int MAXM=1000;
int N,L;
int head[MAXN];
struct edges
{
  int to,next,cost;
} edge[MAXM];
int dist[MAXN];
class states
{
public:
  int cost,id;
};
class cmp
{
public:
  bool operator ()(const states &i,const states &j)
  {
    return i.cost>j.cost;
  }
};
void init(int n)
{
  N=n;
  L=0;
  for (int i=0; i<n; i++)
    head[i]=-1;
}
void add_edge(int x,int y,int cost)
{
  edge[L].to=y;
  edge[L].cost=cost;
  edge[L].next=head[x];
  head[x]=L++;
}
int dijkstra(int s,int t)
{
  memset(dist,63,sizeof(dist));
  states u;
```

```
48    u.id=s;
49    u.cost=0;
50    dist[s]=0;
51    priority_queue<states,vector<states>,cmp> q;
52    q.push(u);
53    while (!q.empty())
54    {
55      u=q.top();
56      q.pop();
57      if (u.id==t) return dist[t];
58      if (u.cost!=dist[u.id]) continue;
59      for (int i=head[u.id]; i!=-1; i=edge[i].next)
60      {
61        states v=u;
62        v.id=edge[i].to;
63        if (dist[v.id]>dist[u.id]+edge[i].cost)
64        {
65          v.cost=dist[v.id]=dist[u.id]+edge[i].cost;
66          q.push(v);
67        }
68      }
69    }
70    return -1;
71  }
72  int main()
73  {
74    int n,m;
75    scanf("%d%d",&n,&m);
76    init(n);
77    for (int i=0; i<m; i++)
78    {
79      int x,y,z;
80      scanf("%d%d%d",&x,&y,&z);
81      add_edge(x,y,z);
82      add_edge(y,x,z);
83    }
84    int s,t;
85    scanf("%d%d",&s,&t);
86    printf("%d\n",dijkstra(s,t));
87    return 0;
88  }
```

6.2 SAP四版

```
1  const int MAXEDGE=20400;
2  const int MAXN=400;
3  const int inf=0x3fffffff;
4  struct edges
5  {
6    int cap,to,next,flow;
7  } edge[MAXEDGE+100];
8  struct nodes
```

```
{
  int head,label,pre,cur;
} node[MAXN+100];
int L,N;
int gap[MAXN+100];
void init(int n)
{
  L=0;
  N=n;
  for (int i=0; i<N; i++)
    node[i].head=-1;
}
void add_edge(int x,int y,int z,int w)
{
  edge[L].cap=z;
  edge[L].flow=0;
  edge[L].to=y;
  edge[L].next=node[x].head;
  node[x].head=L++;
  edge[L].cap=w;
  edge[L].flow=0;
  edge[L].to=x;
  edge[L].next=node[y].head;
  node[y].head=L++;
}
int maxflow(int s,int t)
{
  memset(gap,0,sizeof(gap));
  gap[0]=N;
  int u,ans=0;
  for (int i=0; i<N; i++)
  {
    node[i].cur=node[i].head;
    node[i].label=0;
  }
  u=s;
  node[u].pre=-1;
  while (node[s].label<N)
  {
    if (u==t)
    {
      int min=inf;
      for (int i=node[u].pre; i!=-1; i=node[edge[i^1].to].pre)
        if (min>edge[i].cap-edge[i].flow)
          min=edge[i].cap-edge[i].flow;
      for (int i=node[u].pre; i!=-1; i=node[edge[i^1].to].pre)
      {
        edge[i].flow+=min;
        edge[i^1].flow-=min;
      }
      u=s;
```

```
60        ans+=min;
61        continue;
62      }
63      bool flag=false;
64      int v;
65      for (int i=node[u].cur; i!=-1; i=edge[i].next)
66      {
67        v=edge[i].to;
68        if (edge[i].cap-edge[i].flow && node[v].label+1==node[u].
             label)
69        {
70          flag=true;
71          node[u].cur=node[v].pre=i;
72          break;
73        }
74      }
75      if (flag)
76      {
77        u=v;
78        continue;
79      }
80      node[u].cur=node[u].head;
81      int min=N;
82      for (int i=node[u].head; i!=-1; i=edge[i].next)
83        if (edge[i].cap-edge[i].flow && node[edge[i].to].label<min)
84          min=node[edge[i].to].label;
85      gap[node[u].label]--;
86      if (!gap[node[u].label]) return ans;
87      node[u].label=min+1;
88      gap[node[u].label]++;
89      if (u!=s) u=edge[node[u].pre^1].to;
90    }
91    return ans;
92  }
```

6.3 费用流三版

T了可以改成栈。

```
1  const int MAXM=60000;
2  const int MAXN=400;
3  const int inf=0x3fffffff;
4  int L,N;
5  int K;
6  struct edges
7  {
8    int to,next,cap,flow,cost;
9  } edge[MAXM];
10 struct nodes
11 {
12   int dis,pre,head;
13   bool visit;
```

```
} node[MAXN];
void init(int n)
{
  N=n;
  L=0;
  for (int i=0; i<N; i++)
    node[i].head=-1;
}
void add_edge(int x,int y,int cap,int cost)
{
  edge[L].to=y;
  edge[L].cap=cap;
  edge[L].cost=cost;
  edge[L].flow=0;
  edge[L].next=node[x].head;
  node[x].head=L++;
  edge[L].to=x;
  edge[L].cap=0;
  edge[L].cost=-cost;
  edge[L].flow=0;
  edge[L].next=node[y].head;
  node[y].head=L++;
}
bool spfa(int s,int t)
{
  queue <int> q;
  for (int i=0; i<N; i++)
  {
    node[i].dis=0x3fffffff;
    node[i].pre=-1;
    node[i].visit=0;
  }
  node[s].dis=0;
  node[s].visit=1;
  q.push(s);
  while (!q.empty())
  {
    int u=q.front();
    node[u].visit=0;
    for (int i=node[u].head; i!=-1; i=edge[i].next)
    {
      int v=edge[i].to;
      if (edge[i].cap>edge[i].flow &&
          node[v].dis>node[u].dis+edge[i].cost)
      {
        node[v].dis=node[u].dis+edge[i].cost;
        node[v].pre=i;
        if (!node[v].visit)
        {
          node[v].visit=1;
          q.push(v);
```

```
65          }
66        }
67      }
68      q.pop();
69    }
70    if (node[t].pre==-1)
71      return 0;
72    else
73      return 1;
74  }
75  int mcmf(int s,int t,int &cost)
76  {
77    int flow=0;
78    while (spfa(s,t))
79    {
80      int max=inf;
81      for (int i=node[t].pre; i!=-1; i=node[edge[i^1].to].pre)
82      {
83        if (max>edge[i].cap-edge[i].flow)
84          max=edge[i].cap-edge[i].flow;
85      }
86      for (int i=node[t].pre; i!=-1; i=node[edge[i^1].to].pre)
87      {
88        edge[i].flow+=max;
89        edge[i^1].flow-=max;
90        cost+=edge[i].cost*max;
91      }
92      flow+=max;
93    }
94    return flow;
95  }
```

6.4 匈牙利

6.4.1 新版,隐式图可解

```
1  bool check(int u)
2  {
3    for (int i=head[u]; i!=-1; i=edge[i].next)
4    {
5      int v=edge[i].to;
6      if (matc[v]==u) continue;
7      if (!use[v])
8      {
9        use[v]=1;
10       if (matc[v]==-1 || check(matc[v]))
11       {
12         matc[v]=u;
13         matc[u]=v;
14         return 1;
15       }
```

```
    }
  }
  return 0;
}
int match()
{
  int ret=0;
  memset(matc,-1,sizeof(matc));
  for (int u=0; u<N; u++)
  {
    if (matc[u]!=-1) continue;
    memset(use,0,sizeof(use));
    if (check(u))
      ret++;
  }
  return ret;
}
```

6.4.2 邻接矩阵

```
bool check(int u)
{
  for (int v=0; v<N; v++)
    if (am[u][v] && !use[v])
    {
      use[v]=1;
      if (pre[v]==-1 || check(pre[v]))
      {
        pre[v]=u;
        return 1;
      }
    }
  return 0;
}
int match()
{
  int ret=0;
  memset(pre,-1,sizeof(pre));
  for (int u=0; u<N; u++)
  {
    memset(use,0,sizeof(use));
    if (check(u))
      ret++;
  }
  return ret;
}
```

6.4.3 邻接表

```
bool check(int u)
{
  for (int i=head[u]; i!=-1; i=edge[i].next)
  {
```

```
    int v=edge[i].to;
    if (!use[v])
    {
      use[v]=1;
      if (pre[v]==-1 || check(pre[v]))
      {
        pre[v]=u;
        return 1;
      }
    }
  }
  return 0;
}
int match()
{
  int ret=0;
  memset(pre,-1,sizeof(pre));
  for (int u=1; u<=N; u++)
  {
    memset(use,0,sizeof(use));
    if (check(u))
      ret++;
  }
  return ret;
}
```

6.5 一般图匹配带花树

```
const int MaxN = 222;
int N;
bool Graph[MaxN+1][MaxN+1];
int Match[MaxN+1];
bool InQueue[MaxN+1],InPath[MaxN+1],InBlossom[MaxN+1];
int Head,Tail;
int Queue[MaxN+1];
int Start,Finish;
int NewBase;
int Father[MaxN+1],Base[MaxN+1];
int Count;
void CreateGraph()
{
  int u,v;
  memset(Graph,false,sizeof(Graph));
  scanf("%d",&N);
  while (scanf("%d%d",&u,&v) != EOF)
    Graph[u][v] = Graph[v][u] = true;
}
void Push(int u)
{
  Queue[Tail] = u;
  Tail++;
  InQueue[u] = true;
```

```
}
int Pop()
{
  int res = Queue[Head];
  Head++;
  return res;
}
int FindCommonAncestor(int u,int v)
{
  memset(InPath,false,sizeof(InPath));
  while (true)
  {
    u = Base[u];
    InPath[u] = true;
    if (u == Start) break;
    u = Father[Match[u]];
  }
  while (true)
  {
    v = Base[v];
    if (InPath[v]) break;
    v = Father[Match[v]];
  }
  return v;
}
void ResetTrace(int u)
{
  int v;
  while (Base[u] != NewBase)
  {
    v = Match[u];
    InBlossom[Base[u]] = InBlossom[Base[v]] = true;
    u = Father[v];
    if (Base[u] != NewBase) Father[u] = v;
  }
}
void BlossomContract(int u,int v)
{
  NewBase = FindCommonAncestor(u,v);
  memset(InBlossom,false,sizeof(InBlossom));
  ResetTrace(u);
  ResetTrace(v);
  if (Base[u] != NewBase) Father[u] = v;
  if (Base[v] != NewBase) Father[v] = u;
  for (int tu = 1; tu <= N; tu++)
    if (InBlossom[Base[tu]])
    {
      Base[tu] = NewBase;
      if (!InQueue[tu]) Push(tu);
    }
}
```

```
76  void FindAugmentingPath()
77  {
78    memset(InQueue,false,sizeof(InQueue));
79    memset(Father,0,sizeof(Father));
80    for (int i = 1; i <= N; i++)
81      Base[i] = i;
82    Head = Tail = 1;
83    Push(Start);
84    Finish = 0;
85    while (Head < Tail)
86    {
87      int u = Pop();
88      for (int v = 1; v <= N; v++)
89        if (Graph[u][v] && (Base[u] != Base[v]) && (Match[u] != v))
90        {
91          if ((v == Start) || ((Match[v] > 0) && (Father[Match[v]] >
               0)))
92            BlossomContract(u,v);
93          else if (Father[v] == 0)
94          {
95            Father[v] = u;
96            if (Match[v] > 0)
97              Push(Match[v]);
98            else
99            {
100             Finish = v;
101             return;
102           }
103         }
104       }
105   }
106 }
107 void AugmentPath()
108 {
109   int u,v,w;
110   u = Finish;
111   while (u > 0)
112   {
113     v = Father[u];
114     w = Match[v];
115     Match[v] = u;
116     Match[u] = v;
117     u = w;
118   }
119 }
120 void Edmonds()
121 {
122   memset(Match,0,sizeof(Match));
123   for (int u = 1; u <= N; u++)
124     if (Match[u] == 0)
125     {
```

```
126       Start = u;
127       FindAugmentingPath();
128       if (Finish > 0) AugmentPath();
129     }
130 }
131 void PrintMatch()
132 {
133   for (int u = 1; u <= N; u++)
134     if (Match[u] > 0)
135       Count++;
136   printf("%d\n",Count);
137   for (int u = 1; u <= N; u++)
138     if (u < Match[u])
139       printf("%d␣%d\n",u,Match[u]);
140 }
141 int main()
142 {
143   CreateGraph();
144   Edmonds();
145   PrintMatch();
146 }
```

6.6 KM

6.6.1 最大加权匹配

```
1  bool visx[N],visy[N];//x,y中的点是否被访问
2  int lx[N],ly[N];//x,y中的点的标号
3  int matchy[N];//y中各点匹配状态
4  int map[N][N];//二分图描述[x][y]
5  bool find(int x)
6  {
7    visx[x]=true;
8    int t;
9    for (int y=0;y<ycnt;y++)
10   {
11     if (!visy[y])
12     {
13       t=lx[x]+ly[y]-map[x][y];
14       if (t==0)
15       {
16         visy[y]=true;
17         if (matchy[y]==-1 || find(matchy[y]))
18         {
19           matchy[y]=x;
20           return true;
21         }
22       }
23       else if (lack>t) lack=t;
24     }
25   }
26   return false;
```

```
}
void KM()
{
  memset(lx,0,sizeof(lx));
  memset(ly,0,sizeof(ly));
  memset(matchy,-1,sizeof(matchy));
  for (int i=0;i<xcnt;i++)
    for (int j=0;j<ycnt;j++)
      if (map[i][j]>lx[i])
        lx[i]=map[i][j];
  for (int x=0;x<xcnt;x++)
  {
    while (true)
    {
      memset(visx,false,sizeof(visx));
      memset(visy,false,sizeof(visy));
      lack=INFI;
      if (find(x)) break;
      for (int i=0;i<xcnt;i++)
      {
        if (visx[i]) lx[i]-=lack;
        if (visy[i]) ly[i]+=lack;
      }
    }
  }
  int cost=0;
  for (int i=0;i<ycnt;i++)
    cost+=map[matchy[i]][i];
}
```

6.6.2 自认为正确的Kuhn_Munkras

未验证

```
#include<cstdio>
#include<cstring>
#include<algorithm>
using namespace std;
const int MAXN=100;
const int inf=0x3f3f3f3f;
bool visitx[MAXN],visity[MAXN];
int labx[MAXN],laby[MAXN],matx[MAXN],maty[MAXN],slack[MAXN];
int ma[MAXN][MAXN];
bool check(int x,int n)
{
  visitx[x]=1;
  for (int i=0; i<n; i++)
    if (!visity[i])
      if (labx[x]+laby[i]==ma[x][i])
      {
        visity[i]=1;
        if (maty[i]==-1 || check(maty[i],n))
        {
```

```
20            matx[x]=i;
21            maty[i]=x;
22            return 1;
23          }
24        }
25        else
26          slack[i]=min(slack[i],labx[x]+laby[i]-ma[x][i]);
27
28    return 0;
29  }
30  void maintain(int n)
31  {
32    int diff=inf;
33    for (int i=0; i<n; i++)
34      if (!visity[i])
35        diff=min(diff,slack[i]);
36    for (int i=0; i<n; i++)
37    {
38      if (visitx[i])
39        labx[i]-=diff;
40      if (visity[i])
41        laby[i]+=diff;
42      else
43        slack[i]-=diff;
44    }
45  }
46  int Kuhn_Munkras(int n)
47  {
48    for (int i=0; i<n; i++)
49    {
50      labx[i]=-inf;
51      for (int j=0; j<n; j++)
52        labx[i]=max(labx[i],ma[i][j]);
53    }
54    memset(laby,0,4*n);
55    memset(matx,-1,4*n);
56    memset(maty,-1,4*n);
57    for (int i=0; i<n; i++)
58    {
59      memset(visitx,0,n);
60      memset(visity,0,n);
61      memset(slack,63,4*n);
62      while (!check(i,n))
63      {
64        maintain(n);
65        memset(visitx,0,n);
66        memset(visity,0,n);
67      }
68    }
69    int ret=0;
70    for (int i=0;i<n;i++)
```

```
71      ret+=labx[i]+laby[i];
72    return ret;
73  }
74  int main()
75  {
76    int n,m;
77    scanf("%d%d",&m,&n);
78    for (int i=m; i<n; i++)
79      for (int j=0; j<n; j++)
80        ma[i][j]=0;
81    for (int i=0; i<m; i++)
82      for (int j=0; j<n; j++)
83        scanf("%d",&ma[i][j]);
84    printf("%d\n",Kuhn_Munkras(n));
85    printf("%d",matx[0]+1);
86    for (int i=1;i<m;i++)
87      printf("␣%d",matx[i]+1);
88    puts("");
89    return 0;
90  }
```

6.7 *二维平面图的最大流

待整理

```
1   #include <iostream>
2   #include <algorithm>
3   #include <cstdio>
4   #include <cstring>
5   #include <vector>
6   #include <cmath>
7   #include <map>
8   #include <queue>
9   using namespace std;
10
11  const int maxn = 100100;
12  const int inf = 0x3f3f3f3f;
13  struct Point
14  {
15    int x,y,id;
16    double theta;
17    Point() {}
18    Point(int _x,int _y)
19    {
20      x = _x;
21      y = _y;
22    }
23    Point(Point _s,Point _e,int _id)
24    {
25      id = _id;
26      x = _s.x-_e.x;
```

```
27      y = _s.y-_e.y;
28      theta = atan2(y,x);
29    }
30    bool operator < (const Point &b)const
31    {
32      return theta < b.theta;
33    }
34  };
35
36  map<pair<int,int>,int > idmap;
37  struct Edge
38  {
39    int from,to,next,cap,near,mark;
40  };
41  Edge edge[maxn*2];
42  int head[maxn],L;
43  int cntd[maxn];
44  void addedge(int u,int v,int cap)
45  {
46    cntd[u]++;
47    cntd[v]++;
48    idmap[make_pair(u,v)] = L;
49    edge[L].from = u;
50    edge[L].to = v;
51    edge[L].cap = cap;
52    edge[L].next = head[u];
53    edge[L].mark = -1;
54    head[u] = L++;
55  }
56
57  int rtp[maxn];
58  Point p[maxn],tp[maxn];
59  int n,m,S,T;
60  int vid;
61
62  struct Edge2
63  {
64    int to,next,dis;
65  } edge2[maxn*2];
66  int head2[maxn],L2;
67
68  void addedge2(int u,int v,int dis)
69  {
70    edge2[L2].to = v;
71    edge2[L2].dis = dis;
72    edge2[L2].next = head2[u];
73    head2[u] = L2++;
74  }
75
76  int dist[maxn];
77  bool inq[maxn];
```

```
78  int SPFA(int s,int t)
79  {
80    queue<int> Q;
81    memset(inq,false,sizeof(inq));
82    memset(dist,63,sizeof(dist));
83    Q.push(s);
84    dist[s] = 0;
85    while (!Q.empty())
86    {
87      int now = Q.front();
88      Q.pop();
89      for (int i = head2[now]; i != -1; i = edge2[i].next)
90        if (dist[edge2[i].to] > dist[now]+edge2[i].dis)
91        {
92          dist[edge2[i].to] = dist[now]+edge2[i].dis;
93          if (inq[edge2[i].to] == false)
94          {
95            inq[edge2[i].to] = true;
96            Q.push(edge2[i].to);
97          }
98        }
99      inq[now] = false;
100   }
101   return dist[t];
102 }
103
104 int main()
105 {
106   int totcas;
107   scanf("%d",&totcas);
108   for (int cas = 1; cas <= totcas; cas++)
109   {
110     idmap.clear();
111     L = 0;
112     scanf("%d%d",&n,&m);
113     S = T = 0;
114     for (int i = 0; i < n; i++)
115     {
116       head[i] = -1;
117       scanf("%d%d",&p[i].x,&p[i].y);
118       if (p[S].x > p[i].x)
119         S = i;
120       if (p[T].x < p[i].x)
121         T = i;
122       cntd[i] = 0;
123     }
124     //源汇中间加入一个特殊节点
125     head[n] = -1;
126     n ++;
127     addedge(S,n-1,inf);
128     addedge(n-1,S,inf);
```

```
129     addedge(T,n-1,inf);
130     addedge(n-1,T,inf);
131
132     for (int i = 0; i < m; i++)
133     {
134       int u,v,cap;
135       scanf("%d%d%d",&u,&v,&cap);
136       u--;
137       v--;
138       addedge(u,v,cap);
139       addedge(v,u,cap);
140     }
141
142     for (int i = 0; i < n; i++)
143     {
144       int tot = 0;
145       //源点汇点连到特殊点的方向需要特别考虑一下
146       if (i == S)
147         tp[tot++] = Point(Point(0,0),Point(-1,0),n-1);
148       else if (i == T)
149         tp[tot++] = Point(Point(0,0),Point(1,0),n-1);
150       else if (i == n-1)
151       {
152         tp[tot++] = Point(Point(0,0),Point(1,0),S);
153         tp[tot++] = Point(Point(0,0),Point(-1,0),T);
154       }
155       if (i < n-1)
156       {
157         for (int j = head[i]; j != -1; j = edge[j].next)
158         {
159           if (i == S && edge[j].to == n-1)  continue;
160           if (i == T && edge[j].to == n-1)  continue;
161           tp[tot++] = Point(p[i],p[edge[j].to],edge[j].to);
162         }
163       }
164       sort(tp,tp+tot);
165       for (int j = 0; j < tot; j++)
166         rtp[tp[j].id] = j;
167       for (int j = head[i]; j != -1; j = edge[j].next)
168         edge[j].near = tp[(rtp[edge[j].to]+1)%tot].id;
169     }
170
171     vid = 0;
172     for (int i = 0;i < L;i++)
173       if (edge[i].mark == -1)
174       {
175         int now = edge[i].from;
176         int eid = i;
177         int to  = edge[i].to;
178         while (true)
179         {
```

```
180             edge[eid].mark = vid;
181             eid ^= 1;
182             now = to;
183             to = edge[eid].near;
184             eid = idmap[make_pair(now,to)];
185
186             if (now == edge[i].from)  break;
187           }
188           vid++;
189         }
190
191       L2 = 0;
192       for (int i = 0; i < vid; i++)
193         head2[i] = -1;
194       for (int i = 0; i < L; i++)
195         addedge2(edge[i].mark,edge[i^1].mark,edge[i].cap);
196       printf("%d\n",SPFA(edge[0].mark,edge[1].mark));
197     }
198     return 0;
199 }
```

6.8 强联通

```
1  int dfsnum[2000];
2  int low[2000];
3  int stack[2000];
4  int top;
5  int ans;
6  int an;
7  int be[2000];
8  int flag[2000];
9  void dfs(int x)
10 {
11   dfsnum[x] = low[x] = ans++;
12   stack[++top] = x;
13   flag[x] = 1;
14   for (int i = head[x];i != -1;i = edge[i].next)
15   {
16     int y = edge[i].to;
17     if (dfsnum[y] == -1)
18     {
19       dfs(y);
20       low[x] = min(low[x],low[y]);
21     }
22     else if (flag[y] == 1)
23       low[x] = min(low[x],dfsnum[y]);
24   }
25   if (dfsnum[x] == low[x])
26   {
27     while (stack[top] != x)
28     {
```

```
29        flag[stack[top]] = 0;
30        be[stack[top]] = an;
31        top--;
32      }
33      flag[x] = 0;
34      be[x] = an++;
35      top--;
36    }
37  }
```

调用:

```
1  void SC()
2  {
3    memset(dfsnum,-1,sizeof(dfsnum));
4    memset(flag,0,sizeof(flag));
5    top = 0;
6    an = 0;
7    ans = 0;
8    for (int i = 0;i < n;i++)
9      if (dfsnum[i] == -1)
10       dfs(i);
11 }
```

6.9 最大团以及相关知识

独立集： 独立集是指图的顶点集的一个子集,该子集的导出子图不含边.如果一个独立集不是任何一个独立集的子集, 那么称这个独立集是一个极大独立集.一个图中包含顶点数目最多的独立集称为最大独立集。最大独立集 一定是极大独立集，但是极大独立集不一定是最大的独立集。

支配集： 与独立集相对应的就是支配集，支配集也是图顶点集的一个子集，设S是图G的一个支配集，则对于图中的任意一个顶点u，要么属于集合s, 要么与s中的顶点相邻。在s中除去任何元素后s不再是支配集，则支配集s是极小支配集。称G的所有支配集中顶点个数最 少的支配集为最小支配集，最小支配集中的顶点个数成为支配数。

最小点的覆盖： 最小点的覆盖也是图的顶点集的一个子集，如果我们选中一个点，则称这个点将以他为端点的所有边都覆盖了。将图中所有的边都覆盖所用顶点数最少，这个集合就是最小的点的覆盖。

最大团： 图G的顶点的子集，设D是最大团，则D中任意两点相邻。若u，v是最大团，则u,v有边相连，其补图u,v没有边相连，所以图G的最大团=其补图的最大独立集。给定无向图$G = (V, E)$，如果U属于V，并且对于任意u,v包含于U 有$< u, v >$包含于E，则称U是G的完全子图，G的完全子图U是G的团，当且仅当U不包含在G的更大的完全子图中，G的最大团是指G中所含顶点数目最多的团。如果U属于V，并且对于任意u, v包含于U有$< u, v >$不包含于E，则称U是G的空子图，G的空子图U是G的独立集，当且仅当U不包含在G的更大的独立集，G的最大团是指G中所含顶点数目最多的独立集。

一些性质： 最大独立集+最小覆盖集$= V$，最大团=补图的最大独立集，最小覆盖集=最大匹配

```cpp
#include <cstdio>
bool am[100][100];
int ans;
int c[100];
int U[100][100];
int n;
bool dfs(int rest,int num)
{
  if (!rest)
  {
    if (num>=ans)
      return 1;
    else
      return 0;
  }
  int pre=-1;
  for (int i=0;i<rest && rest-i+num>=ans;i++)
  {
    int idx=U[num][i];
    if (num+c[idx]<ans)
      return 0;
    int nrest=0;
    for (int j=i+1; j<rest; j++)
      if (am[idx][U[num][j]])
        U[num+1][nrest++]=U[num][j];
    if (dfs(nrest,num+1))
      return 1;
  }
  return 0;
}
int main()
{
  while (scanf("%d",&n),n)
  {
    for (int i=0;i<n;i++)
      for (int j=0;j<n;j++)
        scanf("%d",&am[i][j]);
    ans=0;
    for (int i=n-1; i>=0; i--)
    {
      int rest=0;
      for (int j=i+1; j<n; j++)
        if (am[i][j])
          U[0][rest++]=j;
      ans+=dfs(rest,0);
      c[i]=ans;
    }
    printf("%d\n",ans);
  }
  return 0;
}
```

6.10 双连通分量

标号从0起

```
#include<cstdio>
#include<cstring>
#include<stack>
#include<queue>
#include<algorithm>
using namespace std;
const int MAXN=100000*2;
const int MAXM=200000;
struct edges
{
  int to,next;
  bool cut,visit;
} edge[MAXM<<1];
int head[MAXN],low[MAXN],dpt[MAXN],L;
bool visit[MAXN],cut[MAXN];
void init(int n)
{
  L=0;
  memset(head,-1,4*n);
  memset(visit,0,n);
}
void add_edge(int u,int v)
{
  edge[L].cut=edge[L].visit=0;
  edge[L].to=v;
  edge[L].next=head[u];
  head[u]=L++;
}
int idx;
stack<int> st;
int bcc[MAXM];
void dfs(int u,int fu,int deg)
{
  cut[u]=0;
  visit[u]=1;
  low[u]=dpt[u]=deg;
  int tot=0;
  for (int i=head[u]; i!=-1; i=edge[i].next)
  {
    int v=edge[i].to;
    if (edge[i].visit)
      continue;
    st.push(i/2);
    edge[i].visit=edge[i^1].visit=1;
    if (visit[v])
    {
      low[u]=dpt[v]>low[u]?low[u]:dpt[v];
      continue;
```

```
49      }
50      dfs(v,u,deg+1);
51      edge[i].cut=edge[i^1].cut=(low[v]>dpt[u] || edge[i].cut);
52      if (u!=fu) cut[u]=low[v]>=dpt[u]?1:cut[u];
53      if (low[v]>=dpt[u] || u==fu)
54      {
55        while (st.top()!=i/2)
56        {
57          int x=st.top()*2,y=st.top()*2+1;
58          bcc[st.top()]=idx;
59          st.pop();
60        }
61        bcc[i/2]=idx++;
62        st.pop();
63      }
64      low[u]=low[v]>low[u]?low[u]:low[v];
65      tot++;
66    }
67    if (u==fu && tot>1) cut[u]=1;
68  }
69  int main()
70  {
71    int n,m;
72    while (scanf("%d%d",&n,&m)!=EOF)
73    {
74      init(n);
75      for (int i=0; i<m; i++)
76      {
77        int u,v;
78        scanf("%d%d",&u,&v);
79        add_edge(u,v);
80        add_edge(v,u);
81      }
82      idx=0;
83      for (int i=0; i<n; i++)
84        if (!visit[i])
85          dfs(i,i,0);
86    }
87    return 0;
88  }
```

6.11 割点与桥

```
1  #include<cstdio>
2  #include<cstring>
3  const int MAXN=10000;
4  struct edges
5  {
6    int to,next;
7    bool cut,visit;
8    int from;
```

```
} edge[MAXN-1<<1];
int head[MAXN],low[MAXN],dfn[MAXN],L;
bool visit[MAXN],cut[MAXN];
void init(int n)
{
  L=0;
  memset(head,-1,4*n);
  memset(cut,0,4*n);
  memset(visit,0,4*n);
}
void add_edge(int u,int v)
{
  edge[L].from=u;
  edge[L].cut=edge[L].visit=0;
  edge[L].to=v;
  edge[L].next=head[u];
  head[u]=L++;
}
int idx;
void dfs(int u,int fu)
{
  visit[u]=1;
  low[u]=dfn[u]=idx++;
  int tot=0;
  for (int i=head[u]; i!=-1; i=edge[i].next)
  {
    int v=edge[i].to;
    if (edge[i].visit)
      continue;
    edge[i].visit=edge[i^1].visit=1;
    if (visit[v])
    {
      low[u]=dfn[v]>low[u]?low[u]:dfn[v];
      continue;
    }
    dfs(v,u);
    edge[i].cut=edge[i^1].cut=low[v]>dfn[u] || edge[i].cut;
    if (u!=fu) cut[u]=low[v]>=dfn[u]?1:cut[u];
    low[u]=low[v]>low[u]?low[u]:low[v];
    tot++;
  }
  if (u==fu && tot>1) cut[u]=1;
}
int main()
{
  int t;
  scanf("%d",&t);
  while (t--)
  {
    int n,m;
    scanf("%d%d",&n,&m);
```

```
60      init(n);
61      for (int i=0; i<m; i++)
62      {
63        int u,v;
64        scanf("%d%d",&u,&v);
65        add_edge(--u,--v);
66        add_edge(v,u);
67      }
68      for (int i=0; i<n; i++)
69        if (!visit[i])
70        {
71          idx=0;
72          dfs(i,i);
73        }
74    }
75    return 0;
76  }
```

6.12 LCA

在线LCA，bfs

```
1  #include<cstdio>
2  #include<cstring>
3  #include<queue>
4  using namespace std;
5  const int NSIZE = 50000;
6  const int DEG = 20;
7  struct trees
8  {
9
10   int fa[DEG];
11   int head,deg;
12 } tree[NSIZE];
13 struct edges
14 {
15   int to , next;
16 } edge[NSIZE];
17 struct states
18 {
19   int u,fu,deg;
20 };
21 int L;
22 void add_edge(int x, int y)
23 {
24   edge[L].to = y;
25   edge[L].next = tree[x].head;
26   tree[x].head = L++;
27 }
28 int Root;
29 void BFS(int s)
30 {
31   queue<states> que;
```

```
32    states st;
33    st.deg=0;
34    st.fu=st.u=s;
35    que.push(st);
36    while(!que.empty())
37    {
38      states st=que.front();
39      que.pop();
40      tree[st.u].deg = st.deg;
41      tree[st.u].fa[0] = st.fu;
42      for (int i=1;i<DEG;i++)
43        tree[st.u].fa[i]=s;
44      for (int tmp=st.fu,num=1;tree[tmp].deg;tmp=tree[st.u].fa[num
          ++])
45        tree[st.u].fa[num]=tree[tmp].fa[num-1];
46      for(int i = tree[st.u].head ; i != -1; i = edge[i].next)
47      {
48        int v = edge[i].to;
49        if (v == st.fu) continue;
50        states nst;
51        nst.u=v;
52        nst.fu=st.u;
53        nst.deg=st.deg+1;
54        que.push(nst);
55      }
56    }
57  }
58  int LCA(int x, int y)
59  {
60    if(tree[x].deg > tree[y].deg) swap(x,y);
61    int hx=tree[x].deg,hy=tree[y].deg;
62    int tx=x,ty=y;
63    for (int det=hy-hx,i=0; det; det>>=1,i++)
64      if (det&1)
65        ty=tree[ty].fa[i];
66    if(tx == ty) return tx;
67    for (int i=DEG-1; i>=0; i--)
68    {
69      if(tree[tx].fa[i] == tree[ty].fa[i])
70        continue;
71      tx = tree[tx].fa[i];
72      ty = tree[ty].fa[i];
73    }
74    return tree[tx].fa[0];
75  }
76  int main()
77  {
78    int t;
79    scanf("%d",&t);
80    while(t--)
81    {
```

```
82      int n;
83      scanf("%d",&n);
84      L = 0;
85      for(int i = 0 ; i < n ; i++)
86        tree[i].head = -1;
87      for(int i = 0 ; i < n-1 ; i++)
88      {
89        int a,b;
90        scanf("%d%d",&a ,&b);
91        add_edge(a-1,b-1);
92        add_edge(b-1,a-1);
93      }
94      Root=0;
95      BFS(Root);
96      int a,b;
97      scanf("%d%d",&a,&b);
98      int lca=LCA(a-1,b-1)+1;
99      printf("%d\n",lca);
100   }
101   return 0;
102 }
```

6.13 最优比例生成树

```
1  #include<stdio.h>
2  #include<string.h>
3  #include<math.h>
4  struct
5  {
6    int x,y;
7    double z;
8  } node[1100];
9  struct
10 {
11   double l,c;
12 } map[1100][1100];
13 int n,l,f[1100],pre[1100];
14 double dis[1100];
15 double mst(double x)
16 {
17   int i,j,tmp;
18   double min,s=0,t=0;
19   memset(f,0,sizeof(f));
20   f[1]=1;
21   for (i=2; i<=n; i++)
22   {
23     dis[i]=map[1][i].c-map[1][i].l*x;
24     pre[i]=1;
25   }
26   for (i=1; i<n; i++)
27   {
```

```
28      min=1e10;
29      for (j=1; j<=n; j++)
30        if (!f[j] && min>dis[j])
31        {
32          min=dis[j];
33          tmp=j;
34        }
35      f[tmp]=1;
36      t+=map[pre[tmp]][tmp].l;
37      s+=map[pre[tmp]][tmp].c;
38      for (j=1; j<=n; j++)
39        if (!f[j] && map[tmp][j].c-map[tmp][j].l*x<dis[j])
40        {
41          dis[j]=map[tmp][j].c-map[tmp][j].l*x;
42          pre[j]=tmp;
43        }
44    }
45    return s/t;
46  }
47  int main()
48  {
49    int i,j;
50    double a,b;
51    scanf("%d",&n);
52    while (n)
53    {
54      for (i=1; i<=n; i++)
55        scanf("%d%d%lf",&node[i].x,&node[i].y,&node[i].z);
56      for (i=1; i<=n; i++)
57        for (j=i+1; j<=n; j++)
58        {
59          map[j][i].l=map[i][j].l=sqrt(1.0*(node[i].x-node[j].x)*(
              node[i].x-node[j].x)+(node[i].y-node[j].y)*(node[i].y-
              node[j].y));
60          map[j][i].c=map[i][j].c=fabs(node[i].z-node[j].z);
61        }
62      a=0,b=mst(a);
63      while (fabs(b-a)>1e-8)
64      {
65        a=b;
66        b=mst(a);
67      }
68      printf("%.3f\n",b);
69      scanf("%d",&n);
70    }
71  }
```

6.14 全局最小割

```
1  #include <iostream>
2  using namespace std;
3  const int maxn=510;
```

```
4  int map[maxn][maxn];
5  int n;
6  void contract(int x,int y)
7  {
8    int i,j;
9    for (i=0; i<n; i++)
10     if (i!=x) map[x][i]+=map[y][i],map[i][x]+=map[i][y];
11   for (i=y+1; i<n; i++) for (j=0; j<n; j++)
12     {
13       map[i-1][j]=map[i][j];
14       map[j][i-1]=map[j][i];
15     }
16   n--;
17 }
18 int w[maxn],c[maxn];
19 int sx,tx;
20 int mincut()
21 {
22   int i,j,k,t;
23   memset(c,0,sizeof(c));
24   c[0]=1;
25   for (i=0; i<n; i++) w[i]=map[0][i];
26   for (i=1; i+1<n; i++)
27   {
28     t=k=-1;
29     for (j=0; j<n; j++) if (c[j]==0&&w[j]>k)
30         k=w[t=j];
31     c[sx=t]=1;
32     for (j=0; j<n; j++) w[j]+=map[t][j];
33   }
34   for (i=0; i<n; i++) if (c[i]==0) return w[tx=i];
35 }
36 int main()
37 {
38   int i,j,k,m;
39   while (scanf("%d%d",&n,&m)!=EOF)
40   {
41     memset(map,0,sizeof(map));
42     while (m--)
43     {
44       scanf("%d%d%d",&i,&j,&k);
45       map[i][j]+=k;
46       map[j][i]+=k;
47     }
48     int mint=999999999;
49     while (n>1)
50     {
51       k=mincut();
52       if (k<mint) mint=k;
53       contract(sx,tx);
54     }
```

```
55      printf("%d\n",mint);
56    }
57    return 0;
58  }
```

6.15 欧拉路

6.15.1 有向图

```
1  void solve(int x)
2  {
3    int i;
4    if (!match[x])
5    {
6      path[++l]=x;
7      return ;
8    }
9    for (i=1; i<=n; i++)
10     if (b[x][i])
11     {
12       b[x][i]--;
13       match[x]--;
14       solve(i);
15     }
16   path[++l]=x;
17 }
```

6.15.2 无向图

```
1  void solve(int x)
2  {
3    int i;
4    if (!match[x])
5    {
6      path[++l]=x;
7      return ;
8    }
9    for (i=1; i<=n; i++)
10     if (b[x][i])
11     {
12       b[x][i]--;
13       b[i][x]--;
14       match[x]--;
15       match[i]--;
16       solve(i);
17     }
18   path[++l]=x;
19 }
```

6.15.3 混合图

zju1992

```
1  int in[MAXN+100],out[MAXN+100];
2  int main()
```

```
3  {
4    int t;
5    scanf("%d",&t);
6    while (t--)
7    {
8      int n,m;
9      scanf("%d%d",&n,&m);
10     N=n+2;L=-1;
11     for (int i=0;i<N;i++)
12       head[i]=-1;
13     memset(in,0,sizeof(in));
14     memset(out,0,sizeof(out));
15
16     for (int i=0;i<m;i++)
17     {
18       int x,y,z;
19       scanf("%d%d%d",&x,&y,&z);
20       in[y]++;out[x]++;
21       if (!z)
22         add_edge(x,y,1);
23     }
24     int flag=1;
25     for (int i=1;i<=n;i++)
26     {
27       if (in[i]-out[i]>0)
28         add_edge(i,n+1,(in[i]-out[i])/2);
29       else
30       if (out[i]-in[i]>0)
31         add_edge(0,i,(out[i]-in[i])/2);
32       //printf("%d %d %d\n",i,out[i],in[i]);
33       if ((in[i]+out[i])&1)
34       {
35         flag=0;
36         break;
37       }
38     }
39     maxflow(0,n+1);
40     for (int i=head[0];i!=-1;i=edge[i].next)
41       if (edge[i].cap>0 && edge[i].cap>edge[i].flow)
42       {
43         flag=0;
44         break;
45       }
46     if (flag)
47       puts("possible");
48     else
49       puts("impossible");
50   }
51   return 0;
52 }
```

6.16 K短路

```
#include<cstdio>
#include<cstring>
#include<queue>
using namespace std;
int K;
class states
{
public:
  int cost,id;
};
int dist[1000];
class cmp
{
public:
  bool operator ()(const states &i,const states &j)
  {
    return i.cost>j.cost;
  }
};
class cmp2
{
public:
  bool operator ()(const states &i,const states &j)
  {
    return i.cost+dist[i.id]>j.cost+dist[j.id];
  }
};
struct edges
{
  int to,next,cost;
} edger[100000],edge[100000];
int headr[1000],head[1000],Lr,L;
void dijkstra(int s)
{
  states u;
  u.id=s;
  u.cost=0;
  dist[s]=0;
  priority_queue<states,vector<states>,cmp> q;
  q.push(u);
  while (!q.empty())
  {
    u=q.top();
    q.pop();
    if (u.cost!=dist[u.id]) continue;
    for (int i=headr[u.id]; i!=-1; i=edger[i].next)
    {
      states v=u;
      v.id=edger[i].to;
```

```
50        if (dist[v.id]>dist[u.id]+edger[i].cost)
51        {
52          v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
53          q.push(v);
54        }
55      }
56    }
57  }
58  int num[1000];
59  void init(int n)
60  {
61    Lr=L=0;
62    memset(head,-1,4*n);
63    memset(headr,-1,4*n);
64    memset(dist,63,4*n);
65    memset(num,0,4*n);
66  }
67  void add_edge(int u,int v,int x)
68  {
69    edge[L].to=v;
70    edge[L].cost=x;
71    edge[L].next=head[u];
72    head[u]=L++;
73    edger[Lr].to=u;
74    edger[Lr].cost=x;
75    edger[Lr].next=headr[v];
76    headr[v]=Lr++;
77  }
78  int a_star(int s,int t)
79  {
80    if (dist[s]==0x3f3f3f3f)
81      return -1;
82    priority_queue<states,vector<states>,cmp2> q;
83    states tmp;
84    tmp.id=s;
85    tmp.cost=0;
86    q.push(tmp);
87    while (!q.empty())
88    {
89      states u=q.top();
90      q.pop();
91      num[u.id]++;
92      if (num[t]==K)
93        return u.cost;
94      for (int i=head[u.id]; i!=-1; i=edge[i].next)
95      {
96        int v=edge[i].to;
97        tmp.id=v;
98        tmp.cost=u.cost+edge[i].cost;
99        q.push(tmp);
100     }
```

```
  }
  return -1;
}
int main()
{
  int n,m;
  scanf("%d%d",&n,&m);
  init(n);
  for (int i=0; i<m; i++)
  {
    int u,v,x;
    scanf("%d%d%d",&u,&v,&x);
    add_edge(u-1,v-1,x);
  }
  int s,t;
  scanf("%d%d%d",&s,&t,&K);
  if (s==t)
    K++;
  dijkstra(t-1);
  printf("%d\n",a_star(s-1,t-1));
}
```

6.17 稳定婚姻

假定有n个男生和M个女生，理想的拍拖状态就是对于每对情侣(a,b)，找不到另一对情侣(c,d)使得c更喜欢b,b也更喜欢c，同理，对a来说也没有(e,f)使得a更喜欢e而e更喜欢a,当然最后会有一些人落单。这样子一个状态可以称为理想拍拖状态，它也有一个专业的名词叫稳定婚姻。

求解这个问题可以用一个专有的算法，延迟认可算法，其核心就是让每个男生按自己喜欢的顺序逐个向女生表白，例如leokan向一个女生求爱，这个过程中，若这个女生没有男朋友，那么这个女生就暂时成为leokan的女朋友，或这个女生喜欢她现有男朋友的程度没有喜欢leokan高，这个女生也暂时成为leokan的女朋友，而她原有的男朋友则再将就找下一个次喜欢的女生来当女朋友。

```
#include<string.h>
#include<stdio.h>
#define N 1050
int boy[N][N];
int girl[N][N];
int ans[N];
int cur[N];
int n;
void getMarry(int g)
{
  for (int i=ans[g]+1;i<n;i++)
  {
    int b=girl[g][i]-1;
    if (cur[b]<0)
    {
      ans[g]=i;
      cur[b]=g;
      return;
    }
```

```
20      int og=cur[b];
21      if (boy[b][og] > boy[b][g])
22      {
23        cur[b]=g;
24        ans[g]=i;
25        getMarry(og);
26        return;
27      }
28    }
29  };
30  int main()
31  {
32    int t,a;
33    scanf("%d",&t);
34    while(t--)
35    {
36      memset(girl,0,sizeof(girl));
37      memset(boy,0,sizeof(boy));
38      scanf("%d",&n);
39      for (int i=0;i<n;i++)
40        for (int j=0;j<n;j++)
41          scanf("%d",&girl[i][j]);
42      for (int i=0;i<n;i++)
43        for (int j=0;j<n;j++)
44        {
45          scanf("%d",&a);
46          boy[i][a-1]=j;
47        }
48      memset(cur,0xff,sizeof(cur));
49      memset(ans,0xff,sizeof(ans));
50      for (int i=0;i<n;i++)
51        getMarry(i);
52      for (int i=0;i<n;i++)
53        printf("%d\n",girl[i][ans[i]]);
54    }
55    return 0;
56  }
```

6.18 最小树形图

```
1  const int inf = 19921005;
2  int n,m,u,v,cost,dis[1001][1001],L;
3
4  void init(int n)
5  {
6    L = 0;
7    for (int i = 0; i < n; i++)
8      for (int j = 0; j < n; j++)
9        dis[i][j] = inf;
10 }
11
```

```
12 struct Edge
13 {
14   int u,v,cost;
15 };
16
17 Edge e[1001*1001];
18
19 int pre[1001],id[1001],visit[1001],in[1001];
20
21 int zhuliu(int root,int n,int m,Edge e[])
22 {
23   int res = 0,u,v;
24   while (true)
25   {
26     for (int i = 0; i < n; i++)
27       in[i] = inf;
28     for (int i = 0; i < m; i++)
29       if (e[i].u != e[i].v && e[i].cost < in[e[i].v])
30       {
31         pre[e[i].v] = e[i].u;
32         in[e[i].v] = e[i].cost;
33       }
34     for (int i = 0; i < n; i++)
35       if (i != root)
36         if (in[i] == inf)   return -1;
37     int tn = 0;
38     memset(id,-1,sizeof(id));
39     memset(visit,-1,sizeof(visit));
40     in[root] = 0;
41     for (int i = 0; i < n; i++)
42     {
43       res += in[i];
44       v = i;
45       while (visit[v] != i && id[v] == -1 && v != root)
46       {
47         visit[v] = i;
48         v = pre[v];
49       }
50       if(v != root && id[v] == -1)
51       {
52         for(int u = pre[v] ; u != v ; u = pre[u])
53           id[u] = tn;
54         id[v] = tn++;
55       }
56     }
57     if(tn == 0) break;
58     for (int i = 0; i < n; i++)
59       if (id[i] == -1)
60         id[i] = tn++;
61     for (int i = 0; i < m;)
62     {
```

```
63        int v = e[i].v;
64        e[i].u = id[e[i].u];
65        e[i].v = id[e[i].v];
66        if (e[i].u != e[i].v)
67          e[i++].cost -= in[v];
68        else
69          swap(e[i],e[--m]);
70      }
71      n = tn;
72      root = id[root];
73    }
74    return res;
75  }
76
77  int main()
78  {
79    freopen("in.txt","r",stdin);
80    while (scanf("%d%d",&n,&m) != EOF)
81    {
82      init(n);
83      for (int i = 0; i < m; i++)
84      {
85        scanf("%d%d%d",&u,&v,&cost);
86        if (u == v) continue;
87        dis[u][v] = min(dis[u][v],cost);
88      }
89      L = 0;
90      for (int i = 0; i < n; i++)
91        for (int j = 0; j < n; j++)
92          if (dis[i][j] != inf)
93          {
94            e[L].u = i;
95            e[L].v = j;
96            e[L++].cost = dis[i][j];
97          }
98      printf("%d\n",zhuliu(0,n,L,e));
99    }
100   return 0;
101 }
```

7 计算几何

7.1 注意事项

如果用整数小心越界（多次乘法？）
如果用浮点数判断的时候一定要用eps！

7.2 基本函数

7.2.1 Point定义

```
struct Point
{
  double x, y;
  Point() {}
  Point(double _x, double _y)
  {
    x = _x, y = _y;
  }
  Point operator -(const Point &b)const
  {
    return Point(x-b.x, y-b.y);
  }
  double operator *(const Point &b)const
  {
    return x*b.y-y*b.x;
  }
  double operator &(const Point &b)const
  {
    return x*b.x+y*b.y;
  }
  void transXY(double B)
  {
    double tx = x,ty = y;
    x = tx*cos(B)-ty*sin(B);
    y = tx*sin(B)+ty*cos(B);
  }
};
```

7.2.2 Line定义

```
struct Line
{
  Point s, e;
  double k;
  Line() {}
  Line(Point _s, Point _e)
  {
    s = _s, e = _e;
    k = atan2(e.y-s.y, e.x-s.x);
  }
```

```
  Point operator &(const Line &b)const
  {
    Point res = s;
    //注意：有些题目可能会有直线相交或者重合情况
    //可以把返回值改成pair<Point,int>来返回两直线的状态。
    double t = ((s-b.s)*(b.s-b.e))/((s-e)*(b.s-b.e));
    res.x += (e.x-s.x)*t;
    res.y += (e.y-s.y)*t;
    return res;
  }
};
```

7.2.3 距离：点到直线距离

result:点到直线最近点

```
Point NPT(Point P, Line L)
{
  Point result;
  double a, b, t;

  a = L.e.x-L.s.x;
  b = L.e.y-L.s.y;
  t = ((P.x-L.s.x)*a+(P.y-L.s.y)*b)/(a*a+b*b);

  result.x = L.s.x+a*t;
  result.y = L.s.y+b*t;
  return dist(P, result);
}
```

7.2.4 距离：点到线段距离

res：点到线段最近点

```
Point NearestPointToLineSeg(Point P, Line L)
{
  Point result;
  double a, b, t;

  a = L.e.x-L.s.x;
  b = L.e.y-L.s.y;
  t = ( (P.x-L.s.x)*a+(P.y-L.s.y)*b )/(a*a+b*b);

  if (t >= 0 && t <= 1)
  {
    result.x = L.s.x+a*t;
    result.y = L.s.y+b*t;
  }
  else
  {
    if (dist(P,L.s) < dist(P,L.e))
      result = L.s;
    else
```

```
20        result = L.e;
21    }
22    return result;
23  }
```

旧版

```
1  double CalcDis(Point a,Point s,Point e) //点到线段距离
2  {
3    if (sgn((e-s)*(a-s)) < 0 || sgn((s-e)*(a-e)) < 0)
4      return min(dist(a,s),dist(a,e));
5    return abs((s-a)*(e-a))/dist(s-e));
6  }
```

7.2.5 面积：多边形

点按逆时针排序。

```
1  double CalcArea(Point p[], int n)
2  {
3    double res = 0;
4    for (int i = 0; i < n; i++)
5      res += (p[i]*p[(i+1) % n])/2;
6    return res;
7  }
```

7.2.6 判断：线段相交

```
1  bool inter(Line l1,Line l2)
2  {
3    return
4    max(l1.s.x,l1.e.x) >= min(l2.s.x,l2.e.x) &&
5    max(l2.s.x,l2.e.x) >= min(l1.s.x,l1.e.x) &&
6    max(l1.s.y,l1.e.y) >= min(l2.s.y,l2.e.y) &&
7    max(l2.s.y,l2.e.y) >= min(l1.s.y,l1.e.y) &&
8    sgn((l2.s-l1.s)*(l1.e-l1.s))*sgn((l2.e-l1.s)*(l1.e-l1.s)) <= 0 &&
9    sgn((l1.s-l2.s)*(l2.e-l2.s))*sgn((l1.e-l2.s)*(l2.e-l2.s)) <= 0;
10 }
```

7.2.7 判断：点在线段上

```
1  bool OnSeg(Line a,Point b)
2  {
3    return ((a.s-b)*(a.e-b) == 0 &&
4        (b.x-a.s.x)*(b.x-a.e.x) <= 0 &&
5        (b.y-a.s.y)*(b.y-a.e.y) <= 0);
6  }
```

7.2.8 判断：点在多边形内

凸包且按逆时针排序

```
1  bool inPoly(Point a,Point p[],int n)
2  {
3    for (int i = 0;i < n;i++)
```

```
    if ((p[i]-a)*(p[(i+1)%n]-a) < 0)
      return false;
  return true;
}
```

射线法, 多边形可以是凸的或凹的
poly的顶点数目要大于等于3
返回值为:
0 – 点在poly内
1 – 点在poly边界上
2 – 点在poly外

```
int inPoly(Point p,Point poly[], int n)
{
  int i, count;
  Line ray, side;

  count = 0;
  ray.s = p;
  ray.e.y  = p.y;
  ray.e.x  = -1;//-INF, 注意取值防止越界!

  for (i = 0; i < n; i++)
  {
    side.s = poly[i];
    side.e = poly[(i+1)%n];

    if(OnSeg(p, side))
      return 1;

    // 如果平行轴则不作考虑sidex
    if (side.s.y == side.e.y)
      continue;

    if (OnSeg(side.s, ray))
    {
      if (side.s.y > side.e.y) count++;
    }
    else if (OnSeg(side.e, ray))
    {
      if (side.e.y > side.s.y) count++;
    }
    else if (inter(ray, side))
    {
      count++;
    }
  }
  return ((count % 2 == 1) ? 0 : 2);
}
```

7.2.9 判断：两凸包相交

需要考虑这几个：一个凸包的点在另外一个图包内（包括边界）；一个凸包的某条边与另一个凸包某条边相交；如果凸包可能退化成点线还需要判断点在线段上和点和点重合。

7.2.10 排序：叉积极角排序

```
bool cmp(const Point& a,const Point& b)
{
  if (a.y*b.y <= 0)
  {
    if (a.y > 0 || b.y > 0) return a.y < b.y;
    if (a.y == 0 && b.y == 0) return a.x < b.x;
  }
  return a*b > 0;
}
```

7.3 三维几何

运算符和二维一样

7.3.1 叉积

```
Point3D operator*(Point3D s, Point3D e)
{
  return Point3D(s.y*e.z-s.z*e.y,
    s.z*e.x-s.x*e.z,
    s.x*e.y-s.y*e.x );
}
```

7.3.2 判断：直线相交

```
bool LineIntersect(Line3D L1, Line3D L2)
{
  Point3D s = L1.s-L1.e;
  Point3D e = L2.s-L2.e;
  Point3D p  = s*e;
  if (ZERO(p)) return false;     //是否平行
  p = (L2.s-L1.e)*(L1.s-L1.e);
  return ZERO(p&L2.e);      //是否共面
}
```

7.3.3 判断：线段相交

需要先判断是否在一个平面上：

```
bool inter(Point a,Point b,Point c,Point d)
{
  Point ret = (a-b)*(c-d);
  Point t1 = (b-a)*(c-a);
  Point t2 = (b-a)*(d-a);
  Point t3 = (d-c)*(a-c);
  Point t4 = (d-c)*(b-c);
  return sgn(t1&ret)*sgn(t2&ret) < 0 &&
         sgn(t3&ret)*sgn(t4&ret) < 0;
```

```
10 }
```

7.3.4 判断：三维向量是否为0

```
1 inline bool ZERO(Point3D p)
2 {
3   return (ZERO(p.x) && ZERO(p.y) && ZERO(p.z));
4 }
```

7.3.5 判断：点在直线上

```
1 bool OnLine(Point3D p, Line3D L)
2 {
3   return ZERO((p-L.s)*(L.e-L.s));
4 }
```

7.3.6 判断：点在线段上

```
1 bool OnSeg(Point3D p, Line3D L)
2 {
3   return (ZERO((L.s-p)*(L.e-p)) &&
4     EQ(Norm(p-L.s)+Norm(p-L.e),Norm(L.e-L.s)));
5 }
```

7.3.7 距离：点到直线

```
1 double Distance(Point3D p, Line3D L)
2 {
3   return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
4 }
```

7.3.8 夹角

返回值是$[0,\pi]$之间的弧度

```
1 double Inclination(Line3D L1, Line3D L2)
2 {
3   Point3D u = L1.e - L1.s;
4   Point3D v = L2.e - L2.s;
5   return acos( (u & v) / (Norm(u)*Norm(v)) );
6 }
```

7.4 圆

7.4.1 面积：两圆相交

圆不可包含

```
1 double dis(int x,int y)
2 {
3   return sqrt((double)(x*x+y*y));
4 }
5 double area(int x1,int y1,int x2,int y2,double r1,double r2)
6 {
7   double s=dis(x2-x1,y2-y1);
```

```
  if(r1+r2<s) return 0;
  else if(r2-r1>s) return PI*r1*r1;
  else if(r1-r2>s) return PI*r2*r2;
  double q1=acos((r1*r1+s*s-r2*r2)/(2*r1*s));
  double q2=acos((r2*r2+s*s-r1*r1)/(2*r2*s));
  return (r1*r1*q1+r2*r2*q2-r1*s*sin(q1));
}
```

7.4.2 三角形外接圆

```
void CircumscribedCircle()
{
  for (int i = 0; i < 3; i++)
    scanf("%lf%lf",&p[i].x,&p[i].y);
  tp = Point((p[0].x+p[1].x)/2,(p[0].y+p[1].y)/2);
  l[0] = Line(tp,Point(tp.x-(p[1].y-p[0].y),tp.y+(p[1].x-p[0].x)));
  tp = Point((p[0].x+p[2].x)/2,(p[0].y+p[2].y)/2);
  l[1] = Line(tp,Point(tp.x-(p[2].y-p[0].y),tp.y+(p[2].x-p[0].x)));
  tp = LineToLine(l[0],l[1]);
  r = Point(tp,p[0]).Length();
  printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
}
```

7.4.3 三角形内切圆

```
void InscribedCircle()
{
  for (int i = 0; i < 3; i++)
    scanf("%lf%lf",&p[i].x,&p[i].y);
  if (xmult(Point(p[0],p[1]),Point(p[0],p[2])) < 0)
    swap(p[1],p[2]);
  for (int i = 0; i < 3; i++)
    len[i] = Point(p[i],p[(i+1)%3]).Length();
  tr = (len[0]+len[1]+len[2])/2;
  r = sqrt((tr-len[0])*(tr-len[1])*(tr-len[2])/tr);
  for (int i = 0; i < 2; i++)
  {
    v = Point(p[i],p[i+1]);
    tv = Point(-v.y,v.x);
    tr = tv.Length();
    tv = Point(tv.x*r/tr,tv.y*r/tr);
    tp = Point(p[i].x+tv.x,p[i].y+tv.y);
    l[i].s = tp;
    tp = Point(p[i+1].x+tv.x,p[i+1].y+tv.y);
    l[i].e = tp;
  }
  tp = LineToLine(l[0],l[1]);
  printf("(%.6f,%.6f,%.6f)\n",tp.x,tp.y,r);
}
```

7.4.4 点对圆的两个切点

```
void calc_qie(Point poi,Point o,double r,Point &result1,Point &
   result2)
{
  double line = sqrt((poi.x-o.x)*(poi.x-o.x)+(poi.y-o.y)*(poi.y-o.y
     ));
  double angle = acos(r/line);
  Point unitvector,lin;
  lin.x = poi.x-o.x;
  lin.y = poi.y-o.y;
  unitvector.x = lin.x/sqrt(lin.x*lin.x+lin.y*lin.y)*r;
  unitvector.y = lin.y/sqrt(lin.x*lin.x+lin.y*lin.y)*r;
  result1 = unitvector.Rotate(-angle);
  result2 = unitvector.Rotate(angle);
  result1.x += o.x;
  result1.y += o.y;
  result2.x += o.x;
  result2.y += o.y;
}
```

7.4.5 两圆公切点

```
void Gao()
{
  tn = 0;
  Point a,b,vab;
  double tab,tt,dis,theta;
  for (int i = 0; i < tc; i++)
    for (int j = 0; j < tc; j++)
      if (i != j)
      {
        a = c[i];
        b = c[j];
        vab = Point(a,b);
        tab = atan2(vab.y,vab.x);
        dis = sqrt(vab.x*vab.x+vab.y*vab.y);
        if (b.r > a.r)
          tt = asin((b.r-a.r)/dis);
        else
          tt = -asin((a.r-b.r)/dis);
        theta = tab+pi/2+tt;
        tp[tn++] = Point(a.x+a.r*cos(theta),a.y+a.r*sin(theta));
        tp[tn++] = Point(b.x+b.r*cos(theta),b.y+b.r*sin(theta));
      }
}
```

7.5 矩阵

7.5.1 基本矩阵

按向量$\overrightarrow{(x,y,z)}$平移:

$$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

按比例(x,y,z)缩放:

$$\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

绕单位向量$\overrightarrow{(x,y,z)}$旋转$angle$角度:

$$\begin{pmatrix} x^2\times(1-c)+c & x\times y\times(1-c)-z\times s & x\times z\times(1-c)+y\times s & 0 \\ y\times x\times(1-c)+z\times s & y^2\times(1-c)+c & y\times z\times(1-c)-x\times s & 0 \\ x\times z\times(1-c)-y\times s & y\times z\times(1-c)+x\times s & z^2\times(1-c)+c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{cases} s=sin(angle) \\ c=cos(angle) \end{cases}$$

以上矩阵变换都把点当作列向量，旋转角度的正负由右手定则决定

7.5.2 刘汝佳的几何教室

```
const double pi = acos(-1.0);

int n,m,q;
struct Point
{
  double a,b,c,d;
};
Point p[50000],f[50000];

double a,b,c,theta,mt[4][4],tmp[4][4],tmt[4][4],rmt[4][8];
char com[20];

void TRANSLATE()
{
  memset(tmt,0,sizeof(tmt));
  tmt[0][0] = tmt[1][1] = tmt[2][2] = tmt[3][3] = 1;
  tmt[3][0] = a;
  tmt[3][1] = b;
  tmt[3][2] = c;
  memset(tmp,0,sizeof(tmp));
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      for (int k = 0; k < 4; k++)
        tmp[i][j] += mt[i][k]*tmt[k][j];
  for (int i = 0; i < 4; i++)
    for (int j = 0; j < 4; j++)
      mt[i][j] = tmp[i][j];
}

void ROTATE()
{
```

```
32    theta = -theta*pi/180;
33    memset(tmt,0,sizeof(tmt));
34    tmt[3][3] = 1;
35    tmt[0][0] = cos(theta)+(1-cos(theta))*a*a;
36    tmt[1][0] = (1-cos(theta))*a*b+c*sin(theta);
37    tmt[2][0] = (1-cos(theta))*a*c-b*sin(theta);
38    tmt[0][1] = (1-cos(theta))*a*b-c*sin(theta);
39    tmt[1][1] = cos(theta)+(1-cos(theta))*b*b;
40    tmt[2][1] = (1-cos(theta))*b*c+a*sin(theta);
41    tmt[0][2] = (1-cos(theta))*a*c+b*sin(theta);
42    tmt[1][2] = (1-cos(theta))*b*c-a*sin(theta);
43    tmt[2][2] = cos(theta)+(1-cos(theta))*c*c;
44    memset(tmp,0,sizeof(tmp));
45    for (int i = 0; i < 4; i++)
46      for (int j = 0; j < 4; j++)
47        for (int k = 0; k < 4; k++)
48          tmp[i][j] += mt[i][k]*tmt[k][j];
49    for (int i = 0; i < 4; i++)
50      for (int j = 0; j < 4; j++)
51        mt[i][j] = tmp[i][j];
52  }
53
54  void SCALE()
55  {
56    memset(tmt,0,sizeof(tmt));
57    tmt[0][0] = a;
58    tmt[1][1] = b;
59    tmt[2][2] = c;
60    tmt[3][3] = 1;
61    memset(tmp,0,sizeof(tmp));
62    for (int i = 0; i < 4; i++)
63      for (int j = 0; j < 4; j++)
64        for (int k = 0; k < 4; k++)
65          tmp[i][j] += mt[i][k]*tmt[k][j];
66    for (int i = 0; i < 4; i++)
67      for (int j = 0; j < 4; j++)
68        mt[i][j] = tmp[i][j];
69  }
70
71  void solvep(Point p)
72  {
73    memset(tmt,0,sizeof(tmt));
74    tmt[0][0] = p.a;
75    tmt[0][1] = p.b;
76    tmt[0][2] = p.c;
77    tmt[0][3] = 1;
78    memset(tmp,0,sizeof(tmp));
79    for (int i = 0; i < 1; i++)
80      for (int j = 0; j < 4; j++)
81        for (int k = 0; k < 4; k++)
82          tmp[i][j] += tmt[i][k]*mt[k][j];
```

```
83    printf("%.2f␣%.2f␣%.2f\n",tmp[0][0],tmp[0][1],tmp[0][2]);
84  }
85
86  void solvef(Point f)
87  {
88    memset(tmt,0,sizeof(tmt));
89    tmt[0][0] = f.a;
90    tmt[1][0] = f.b;
91    tmt[2][0] = f.c;
92    tmt[3][0] = 0;
93    memset(tmp,0,sizeof(tmp));
94    for (int i = 0;i < 4;i++)
95      for (int j = 0;j < 1;j++)
96        for (int k = 0;k < 4;k++)
97          tmp[i][j] += mt[i][k]*tmt[k][j];
98    tmp[3][0] += f.d;
99    double kk = tmp[0][0]*tmp[0][0]+tmp[1][0]*tmp[1][0]+tmp[2][0]*tmp
        [2][0];
100   kk = sqrt(1/kk);
101   for (int i = 0;i < 4;i++)
102     printf("%.2f␣",tmp[i][0]*kk);
103   printf("\n");
104 }
105
106 void solvermt()
107 {
108   memset(rmt,0,sizeof(rmt));
109   for (int i = 0;i < 4;i++)
110     for (int j = 0;j < 4;j++)
111       rmt[i][j] = mt[i][j];
112   rmt[0][4] = rmt[1][5] = rmt[2][6] = rmt[3][7] = 1;
113   for (int i = 0;i < 4;i++)
114   {
115     for (int j = i;j < 4;j++)
116       if (fabs(rmt[j][i]) > 1e-8)
117       {
118         for (int k = i;k < 8;k++)
119           swap(rmt[i][k],rmt[j][k]);
120         break;
121       }
122     double tt = rmt[i][i];
123     for (int j = i;j < 8;j++)
124       rmt[i][j] /= tt;
125     for (int j = 0;j < 4;j++)
126       if (i != j)
127       {
128         tt = rmt[j][i];
129         for (int k = i;k < 8;k++)
130           rmt[j][k] -= rmt[i][k]*tt;
131       }
132   }
```

```
133   for (int i = 0;i < 4;i++)
134     for (int j = 0;j < 4;j++)
135       mt[i][j] = rmt[i][4+j];
136 }
137
138 int main()
139 {
140   scanf("%d%d%d",&n,&m,&q);
141   for (int i = 0; i < n; i++)
142     scanf("%lf%lf%lf",&p[i].a,&p[i].b,&p[i].c);
143   for (int i = 0; i < m; i++)
144     scanf("%lf%lf%lf%lf",&f[i].a,&f[i].b,&f[i].c,&f[i].d);
145   memset(mt,0,sizeof(mt));
146   mt[0][0] = mt[1][1] = mt[2][2] = mt[3][3] = 1;
147   for (int i = 0; i < q; i++)
148   {
149     scanf("%s",com);
150     if (strcmp(com,"TRANSLATE") == 0)
151     {
152       scanf("%lf%lf%lf",&a,&b,&c);
153       TRANSLATE();
154     }
155     else if (strcmp(com,"ROTATE") == 0)
156     {
157       scanf("%lf%lf%lf%lf",&a,&b,&c,&theta);
158       ROTATE();
159     }
160     else if (strcmp(com,"SCALE") == 0)
161     {
162       scanf("%lf%lf%lf",&a,&b,&c);
163       SCALE();
164     }
165   }
166   //处理点
167   for (int i = 0; i < n; i++)
168     solvep(p[i]);
169   //处理面
170   solvermt();
171   for (int i = 0; i < m; i++)
172     solvef(f[i]);
173   return 0;
174 }
```

7.6 重心

```
1  Point CenterOfPolygon(Point poly[],int n)
2  {
3    Point p, p0, p1, p2, p3;
4    double m, m0;
5    p1 = poly[0];
6    p2 = poly[1];
7    p.x = p.y = m = 0;
```

```
  for (int i = 2; i < n; i++)
  {
    p3 = poly[i];
    p0.x = (p1.x + p2.x + p3.x) / 3.0;
    p0.y = (p1.y + p2.y + p3.y) / 3.0;
    m0 = p1.x*p2.y+p2.x*p3.y+p3.x*p1.y-p1.y*p2.x-p2.y*p3.x-p3.y*p1.
       x;
    if (cmp(m + m0,0.0) == 0)
    m0 += eps;
    p.x = (m * p.x + m0 * p0.x) / (m + m0);
    p.y = (m * p.y + m0 * p0.y) / (m + m0);
    m = m + m0;
    p2 = p3;
  }
  return p;
}
```

7.7 KD树

查找某个点距离最近的点，基本思想是每次分治把点分成两部分，建议按照坐标规模决定是垂直划分还是水平划分，查找时先往分到的那一部分查找，然后根据当前最优答案决定是否去另一个区间查找。

```
bool Div[MaxN];
void BuildKD(int deep,int l, int r, Point p[])\\记得备份一下P
{
  if (l > r) return;
  int mid = l + r >> 1;
  int minX, minY, maxX, maxY;
  minX = min_element(p + l, p + r + 1, cmpX)->x;
  minY = min_element(p + l, p + r + 1, cmpY)->y;
  maxX = max_element(p + l, p + r + 1, cmpX)->x;
  maxY = max_element(p + l, p + r + 1, cmpY)->y;
  Div[mid] = (maxX - minX >= maxY - minY);
  nth_element(p + l, p + mid, p + r + 1, Div[mid] ? cmpX : cmpY);
  BuildKD(l, mid - 1, p);
  BuildKD(mid + 1, r, p);
}

long long res;
void Find(int l, int r, Point a, Point p[])\\查找
{
  if (l > r)  return;
  int mid = l + r >> 1;
  long long dist = dist2(a, p[mid]);
  if (dist > 0)//如果有重点不能这样判断
    res = min(res, dist);
  long long d = Div[mid] ? (a.x - p[mid].x) : (a.y - p[mid].y);
  int l1, l2, r1, r2;
  l1 = l, l2 = mid + 1;
  r1 = mid - 1, r2 = r;
  if (d > 0)
    swap(l1, l2), swap(r1, r2);
```

```
  Find(l1, r1, a, p);
  if (d * d < res)
    Find(l2, r2, a, p);
}
```

7.7.1 例题

查询一个点为中心的给定正方形内所有点并删除（2012金华网赛A）

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <algorithm>
#include <cmath>
#include <queue>
using namespace std;

const int MaxN = 100000;
struct Point
{
  int x,y,r;
  int id;
  bool del;
};

int cmpTyp;
bool cmp(const Point& a,const Point& b)
{
  if (cmpTyp == 0)
    return a.x < b.x;
  else
    return a.y < b.y;
}

int cnt[MaxN];
bool Div[MaxN];
int minX[MaxN],minY[MaxN],maxX[MaxN],maxY[MaxN];
void BuildKD(int l,int r,Point p[])
{
  if (l > r)  return;
  int mid = l+r>>1;
  cmpTyp = 0;
  minX[mid] = min_element(p+l,p+r+1,cmp)->x;
  maxX[mid] = max_element(p+l,p+r+1,cmp)->x;
  cmpTyp = 1;
  minY[mid] = min_element(p+l,p+r+1,cmp)->y;
  maxY[mid] = max_element(p+l,p+r+1,cmp)->y;

  cnt[mid] = r-l+1;
  cmpTyp = Div[mid] = (maxX[mid]-minX[mid] < maxY[mid]-minY[mid]);
  nth_element(p+l,p+mid,p+r+1,cmp);
```

```
43    BuildKD(l,mid-1,p);
44    BuildKD(mid+1,r,p);
45  }
46
47  queue<int> Q;
48  int Find(int l,int r,Point a,Point p[])
49  {
50    if (l > r)  return 0;
51    int mid = l+r>>1;
52    if (cnt[mid] == 0)  return 0;
53
54    if (maxX[mid] < a.x-a.r ||
55        minX[mid] > a.x+a.r ||
56        maxY[mid] < a.y-a.r ||
57        minY[mid] > a.y+a.r)
58      return 0;
59
60    int totdel = 0;
61
62    if (p[mid].del == false)
63      if (abs(p[mid].x-a.x) <= a.r && abs(p[mid].y-a.y) <= a.r)
64      {
65        p[mid].del = true;
66        Q.push(p[mid].id);
67        totdel++;
68      }
69
70    totdel += Find(l,mid-1,a,p);
71    totdel += Find(mid+1,r,a,p);
72
73    cnt[mid] -= totdel;
74
75    return totdel;
76  }
77
78  Point p[MaxN],tp[MaxN];
79  int n;
80
81  int main()
82  {
83    int cas = 1;
84    while (true)
85    {
86      scanf("%d",&n);
87      if (n == 0) break;
88
89      for (int i = 0;i < n;i++)
90      {
91        p[i].id = i;
92        int tx,ty;
93        scanf("%d%d%d",&tx,&ty,&p[i].r);
```

```
94        p[i].x = tx-ty;
95        p[i].y = tx+ty;
96        p[i].del = false;
97        tp[i] = p[i];
98      }
99      BuildKD(0,n-1,tp);
100
101     printf("Case␣#%d:\n",cas++);
102     int q;
103     scanf("%d",&q);
104     for (int i = 0;i < q;i++)
105     {
106       int id;
107       scanf("%d",&id);
108       int res = 0;
109       id--;
110       Q.push(id);
111       while (!Q.empty())
112       {
113         int now = Q.front();
114         Q.pop();
115         if (p[now].del == true) continue;
116         p[now].del = true;
117         res += Find(0,n-1,p[now],tp);
118       }
119       printf("%d\n",res);
120     }
121   }
122   return 0;
123 }
```

7.8 半平面交

直线左边代表有效区域。

```
1  bool HPIcmp(Line a, Line b)
2  {
3    if (fabs(a.k - b.k) > eps)  return a.k < b.k;
4    return ((a.s - b.s) * (b.e-b.s)) < 0;
5  }
6
7  Line Q[100];
8  void HPI(Line line[], int n, Point res[], int &resn)
9  {
10   int tot = n;
11   sort(line, line + n, HPIcmp);
12   tot = 1;
13   for (int i = 1; i < n; i++)
14     if (fabs(line[i].k - line[i - 1].k) > eps)
15       line[tot++] = line[i];
16   int head = 0, tail = 1;
17   Q[0] = line[0];
```

```
18    Q[1] = line[1];
19    resn = 0;
20    for (int i = 2; i < tot; i++)
21    {
22      if (fabs((Q[tail].e-Q[tail].s) * (Q[tail - 1].e-Q[tail - 1].s))
          < eps ||
23          fabs((Q[head].e-Q[head].s) * (Q[head + 1].e-Q[head + 1].s))
              < eps)
24        return;
25      while (head < tail && (((Q[tail]&Q[tail - 1]) - line[i].s) * (
          line[i].e-line[i].s)) > eps)
26        tail--;
27      while (head < tail && (((Q[head]&Q[head + 1]) - line[i].s) * (
          line[i].e-line[i].s)) > eps)
28        head++;
29      Q[++tail] = line[i];
30    }
31    while (head < tail && (((Q[tail]&Q[tail - 1]) - Q[head].s) * (Q[
        head].e-Q[head].s)) > eps)
32      tail--;
33    while (head < tail && (((Q[head]&Q[head + 1]) - Q[tail].s) * (Q[
        tail].e-Q[tail].s)) > eps)
34      head++;
35    if (tail <= head + 1) return;
36    for (int i = head; i < tail; i++)
37      res[resn++] = Q[i] & Q[i + 1];
38    if (head < tail + 1)
39      res[resn++] = Q[head] & Q[tail];
40  }
```

7.9 凸包

得到的凸包按照逆时针方向排序。

```
1  //判断是否是共点或者共线用
2  bool conPoint(Point p[],int n)
3  {
4    for (int i = 1;i < n;i++)
5      if (p[i].x != p[0].x || p[i].y != p[0].y)
6        return false;
7    return true;
8  }
9  bool conLine(Point p[],int n)
10 {
11   for (int i = 2;i < n;i++)
12     if ((p[i]-p[0])*(p[1]-p[0]) != 0)
13       return false;
14   return true;
15 }
16
17 bool GScmp(Point a, Point b)
18 {
```

```
19    if (fabs(a.x - b.x) < eps)
20      return a.y < b.y - eps;
21    return a.x < b.x - eps;
22  }
23
24  void GS(Point p[],int n,Point res[],int &resn)
25  {
26    resn = 0;
27    int top = 0;
28    sort(p,p+n,GScmp);
29
30    if (conPoint(p,n))
31    {
32      res[resn++] = p[0];
33      return;
34    }
35    if (conLine(p,n))
36    {
37      res[resn++] = p[0];
38      res[resn++] = p[n-1];
39      return;
40    }
41
42    for (int i = 0;i < n;)
43      if (resn < 2 || (res[resn-1]-res[resn-2])*(p[i]-res[resn-1]) >
           0)
44        res[resn++] = p[i++];
45      else
46        --resn;
47    top = resn-1;
48    for (int i = n-2;i >= 0;)
49      if (resn < top+2 || (res[resn-1]-res[resn-2])*(p[i]-res[resn
           -1]) > 0)
50        res[resn++] = p[i--];
51      else
52        --resn;
53    resn--;
54  }
```

7.10 直线与凸包求交点

复杂度$O(\log n)$。
需要先预处理几个东西。

```
1  //二分[la,lb]这段区间那条边与line相交
2  int Gao(int la,int lb,Line line)
3  {
4    if (la > lb)
5      lb += n;
6    int l = la,r = lb,mid;
7    while (l < r)
```

```
8   {
9     mid = l+r+1>>1;
10    if (cmp((line.e-line.s)*(p[la]-line.s),0)*cmp((line.e-line.s)*(
          p[mid]-line.s),0) >= 0)
11      l = mid;
12    else
13      r = mid-1;
14  }
15  return l%n;
16 }
17 //求l与凸包的交点
18
19 //先调用Gettheta预处理出凸包每条边的斜率，然后处理成升序排列
20 double theta[maxn];
21
22 void Gettheta()
23 {
24   for (int i = 0;i < n;i++)
25   {
26     Point v = p[(i+1)%n]-p[i];
27     theta[i] = atan2(v.y,v.x);
28   }
29   for (int i = 1;i < n;i++)
30     if (theta[i-1] > theta[i]+eps)
31       theta[i] += 2*pi;
32 }
33
34 double Calc(Line l)
35 {
36   double tnow;
37   Point v = l.e-l.s;
38   tnow = atan2(v.y,v.x);
39   if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
40   int pl = lower_bound(theta,theta+n,tnow)-theta;
41   tnow = atan2(-v.y,-v.x);
42   if (cmp(tnow,theta[0]) < 0) tnow += 2*pi;
43   int pr = lower_bound(theta,theta+n,tnow)-theta;
44   //pl和pr是在l方向上距离最远的点对
45   pl = pl%n;
46   pr = pr%n;
47
48   if (cmp(v*(p[pl]-l.s),0)*cmp(v*(p[pr]-l.s),0) >= 0)
49     return 0.0;
50
51   int xa = Gao(pl,pr,l);
52   int xb = Gao(pr,pl,l);
53
54   if (xa > xb)  swap(xa,xb);
55   //与[xa,xa+1]和[xb,xb+1]这两条线段相交
56
57   if (cmp(v*(p[xa+1]-p[xa]),0) == 0)  return 0.0;
```

```
58    if (cmp(v*(p[xb+1]-p[xb]),0) == 0)  return 0.0;
59
60    Point pa,pb;
61    pa = Line(p[xa],p[xa+1])&l;
62    pb = Line(p[xb],p[xb+1])&l;
63    //题目：求直线切凸包得到的两部分的面积
64    double area0 = sum[xb]-sum[xa+1]+(pa*p[xa+1])/2.0+(p[xb]*pb)
         /2.0+(pb*pa)/2.0;
65    double area1 = sum[xa+n]-sum[xb+1]+(pb*p[xb+1])/2.0+(p[xa]*pa)
         /2.0+(pa*pb)/2.0;
66
67    return min(area0,area1);
68  }
```

7.11 三维凸包

暴力写法

```
1   #define eps 1e-7
2   #define MAXV 505
3
4   struct pt
5   {
6     double x, y, z;
7     pt() {}
8     pt(double _x, double _y, double _z): x(_x), y(_y), z(_z) {}
9     pt operator - (const pt p1)
10    {
11      return pt(x - p1.x, y - p1.y, z - p1.z);
12    }
13    pt operator * (pt p)
14    {
15      return pt(y*p.z-z*p.y, z*p.x-x*p.z, x*p.y-y*p.x);
16    }
17    double operator ^ (pt p)
18    {
19      return x*p.x+y*p.y+z*p.z;
20    }
21  };
22  struct _3DCH
23  {
24    struct fac
25    {
26      int a, b, c;
27      bool ok;
28    };
29    int n;
30    pt P[MAXV];
31    int cnt;
32    fac F[MAXV*8];
33    int to[MAXV][MAXV];
34    double vlen(pt a)
```

```
35  {
36    return sqrt(a.x*a.x+a.y*a.y+a.z*a.z);
37  }
38  double area(pt a, pt b, pt c)
39  {
40    return vlen((b-a)*(c-a));
41  }
42  double volume(pt a, pt b, pt c, pt d)
43  {
44    return (b-a)*(c-a)^(d-a);
45  }
46  double ptof(pt &p, fac &f)
47  {
48    pt m = P[f.b]-P[f.a], n = P[f.c]-P[f.a], t = p-P[f.a];
49    return (m * n) ^ t;
50  }
51  void deal(int p, int a, int b)
52  {
53    int f = to[a][b];
54    fac add;
55    if (F[f].ok)
56    {
57      if (ptof(P[p], F[f]) > eps)
58        dfs(p, f);
59      else
60      {
61        add.a = b, add.b = a, add.c = p, add.ok = 1;
62        to[p][b] = to[a][p] = to[b][a] = cnt;
63        F[cnt++] = add;
64      }
65    }
66  }
67  void dfs(int p, int cur)
68  {
69    F[cur].ok = 0;
70    deal(p, F[cur].b, F[cur].a);
71    deal(p, F[cur].c, F[cur].b);
72    deal(p, F[cur].a, F[cur].c);
73  }
74  bool same(int s, int t)
75  {
76    pt &a = P[F[s].a], &b = P[F[s].b], &c = P[F[s].c];
77    return fabs(volume(a, b, c, P[F[t].a])) < eps && fabs(volume(a,
        b, c,
78        P[F[t].b])) < eps && fabs(volume(a, b, c, P[F[t].c])) < eps
           ;
79  }
80  void construct()
81  {
82    cnt = 0;
83    if (n < 4)
```

```
84        return;
85      bool sb = 1;
86      for (int i = 1; i < n; i++)
87      {
88        if (vlen(P[0] - P[i]) > eps)
89        {
90          swap(P[1], P[i]);
91          sb = 0;
92          break;
93        }
94      }
95      if (sb)return;
96      sb = 1;
97      for (int i = 2; i < n; i++)
98      {
99        if (vlen((P[0] - P[1]) * (P[1] - P[i])) > eps)
100       {
101         swap(P[2], P[i]);
102         sb = 0;
103         break;
104       }
105     }
106     if (sb)return;
107     sb = 1;
108     for (int i = 3; i < n; i++)
109     {
110       if (fabs((P[0] - P[1]) * (P[1] - P[2]) ^ (P[0] - P[i])) > eps
          )
111       {
112         swap(P[3], P[i]);
113         sb = 0;
114         break;
115       }
116     }
117     if (sb)return;
118     fac add;
119     for (int i = 0; i < 4; i++)
120     {
121       add.a = (i+1)%4, add.b = (i+2)%4, add.c = (i+3)%4, add.ok =
            1;
122       if (ptof(P[i], add) > 0)
123         swap(add.b, add.c);
124       to[add.a][add.b] = to[add.b][add.c] = to[add.c][add.a] = cnt;
125       F[cnt++] = add;
126     }
127     for (int i = 4; i < n; i++)
128     {
129       for (int j = 0; j < cnt; j++)
130       {
131         if (F[j].ok && ptof(P[i], F[j]) > eps)
132         {
```

```
133             dfs(i, j);
134             break;
135           }
136         }
137       }
138       int tmp = cnt;
139       cnt = 0;
140       for (int i = 0; i < tmp; i++)
141       {
142         if (F[i].ok)
143         {
144           F[cnt++] = F[i];
145         }
146       }
147     }
148 //表面积
149     double area()
150     {
151       double ret = 0.0;
152       for (int i = 0; i < cnt; i++)
153       {
154         ret += area(P[F[i].a], P[F[i].b], P[F[i].c]);
155       }
156       return ret / 2.0;
157     }
158 //体积
159     double volume()
160     {
161       pt O(0, 0, 0);
162       double ret = 0.0;
163       for (int i = 0; i < cnt; i++)
164       {
165         ret += volume(O, P[F[i].a], P[F[i].b], P[F[i].c]);
166       }
167       return fabs(ret / 6.0);
168     }
169 //表面三角形数
170     int facetCnt_tri()
171     {
172       return cnt;
173     }
174 //表面多边形数
175     int facetCnt()
176     {
177       int ans = 0;
178       for (int i = 0; i < cnt; i++)
179       {
180         bool nb = 1;
181         for (int j = 0; j < i; j++)
182         {
183           if (same(i, j))
```

```
        {
          nb = 0;
          break;
        }
      }
      ans += nb;
    }
    return ans;
  }

  pt Fc[MAXV*8];
  double V[MAXV*8];
  pt Center()//重心
  {
    pt O(0,0,0);
    for (int i = 0; i < cnt; i++)
    {
      Fc[i].x = (O.x+P[F[i].a].x+P[F[i].b].x+P[F[i].c].x)/4.0;
      Fc[i].y = (O.y+P[F[i].a].y+P[F[i].b].y+P[F[i].c].y)/4.0;
      Fc[i].z = (O.z+P[F[i].a].z+P[F[i].b].z+P[F[i].c].z)/4.0;
      V[i] = volume(O,P[F[i].a],P[F[i].b],P[F[i].c]);
    }
    pt res = Fc[0],tmp;
    double m = V[0];
    for (int i = 1; i < cnt; i++)
    {
      if (fabs(m+V[i]) < eps)
        V[i] += eps;
      tmp.x = (m*res.x+V[i]*Fc[i].x)/(m+V[i]);
      tmp.y = (m*res.y+V[i]*Fc[i].y)/(m+V[i]);
      tmp.z = (m*res.z+V[i]*Fc[i].z)/(m+V[i]);
      m += V[i];
      res = tmp;
    }
    return res;
  }
};

_3DCH hull;

int main()
{
  while (scanf("%d",&hull.n) != EOF)
  {
    for (int i = 0; i < hull.n; i++)
      scanf("%lf%lf%lf",&hull.P[i].x,&hull.P[i].y,&hull.P[i].z);
    hull.construct();
  }
  return 0;
}
```

7.12 旋转卡壳

“对踵”

7.12.1 单个凸包

```
void solve(Point p[],int n)
{
  Point v;
  int cur = 1;
  for (int i = 0;i < n;i++)
  {
    v = p[i]-p[(i+1)%n];
    while (v*(p[(cur+1)%n]-p[cur]) < 0)
      cur = (cur+1)%n;
    //p[cur] -> p[i]
    //p[cur] -> p[i+1]
    //p[cur] -> (p[i],p[i+1])
  }
}
```

7.12.2 两个凸包

注意初始点的选取，代码只是个示例。
有时候答案需要取solve(p0,n,p1,m)和solve(p1,m,p0,n)的最优值。
何老鱼说我的是错的。。

```
void solve(Point p0[],int n,Point p1[],int m)
{
  Point v;
  int cur = 0;
  for (int i = 0;i < n;i++)
  {
    v = p0[i]-p0[(i+1)%n];
    while (v*(p1[(cur+1)%m]-p1[cur]) < 0)
      cur = (cur+1)%m;
    //p1[cur] -> p0[i]
    //p1[cur] -> p0[i+1]
    //p1[cur] -> (p0[i],p0[i+1])
  }
}
```

7.12.3 外接矩形

```
void solve()
{
  resa = resb = 1e100;
  double dis1,dis2;
  Point xp[4];
  Line l[4];
  int a,b,c,d;
```

```
  int sa,sb,sc,sd;
  a = b = c = d = 0;
  sa = sb = sc = sd = 0;
  Point va,vb,vc,vd;
  for (a = 0; a < n; a++)
  {
    va = Point(p[a],p[(a+1)%n]);
    vc = Point(-va.x,-va.y);
    vb = Point(-va.y,va.x);
    vd = Point(-vb.x,-vb.y);
    if (sb < sa)
    {
      b = a;
      sb = sa;
    }
    while (xmult(vb,Point(p[b],p[(b+1)%n])) < 0)
    {
      b = (b+1)%n;
      sb++;
    }
    if (sc < sb)
    {
      c = b;
      sc = sb;
    }
    while (xmult(vc,Point(p[c],p[(c+1)%n])) < 0)
    {
      c = (c+1)%n;
      sc++;
    }
    if (sd < sc)
    {
      d = c;
      sd = sc;
    }
    while (xmult(vd,Point(p[d],p[(d+1)%n])) < 0)
    {
      d = (d+1)%n;
      sd++;
    }

    //卡在p[a],p[b],p[c],p[d]上
    sa++;
  }
}
```

7.13 三角形内点个数

7.13.1 无三点共线

```
Point p[1000], tp[2000], base;

```

```
3  bool cmp(const Point &a, const Point &b)
4  {
5    return a.theta < b.theta;
6  }
7
8  int cnt[1000][1000];
9  int cntleft[1000][1000];
10 int n, m;
11
12 int calc(int a, int b, int c)
13 {
14   Point p1 = p[b] - p[a], p2 = p[c] - p[a];
15   if (atan2(p1.y, p1.x) > atan2(p2.y, p2.x))
16     swap(b, c);
17   if ((p[b] - p[a]) * (p[c] - p[a]) > 0)
18     return cnt[a][c] - cnt[a][b] - 1;
19   else
20     return n - 3 - (cnt[a][c] - cnt[a][b] - 1);
21 }
22
23 int main(int argc, char const *argv[])
24 {
25   int totcas;
26   scanf("%d", &totcas);
27   for (int cas = 1; cas <= totcas; ++cas)
28   {
29     scanf("%d", &n);
30     for (int i = 0; i < n; ++i)
31     {
32       scanf("%lld%lld", &p[i].x, &p[i].y);
33       p[i].id = i;
34     }
35     for (int i = 0; i < n; ++i)
36     {
37       m = 0;
38       base = p[i];
39       for (int j = 0; j < n; ++j)
40         if (i != j)
41         {
42           tp[m] = p[j];
43           Point v = tp[m]-base;
44           tp[m++].theta = atan2(v.y,v.x);
45         }
46
47       sort(tp, tp + m, cmp);
48       for (int j = 0; j < m; ++j)
49         tp[m + j] = tp[j];
50
51       //calc cnt
52       for (int j = 0; j < m; ++j)
53         cnt[i][tp[j].id] = j;
```

```
54
55        //calc cntleft
56        for (int j = 0, k = 0, tot = 0; j < m; ++j)
57        {
58          while (k == j || (k < j + m && (tp[j] - base) * (tp[k] -
               base) > 0))
59            k++, tot++;
60          cntleft[i][tp[j].id] = --tot;
61        }
62      }
63
64      printf("Case␣%d:\n", cas);
65      int q;
66      scanf("%d", &q);
67      for (int i = 0; i < q; ++i)
68      {
69        int x, y, z;
70        scanf("%d%d%d", &x, &y, &z);
71        if ((p[z] - p[x]) * (p[y] - p[x]) > 0)
72          swap(y, z);
73        int res = cntleft[x][z] + cntleft[z][y] + cntleft[y][x];
74        res += calc(x, y, z) + calc(y, z, x) + calc(z, x, y);
75        res -= 2 * (n - 3);
76        printf("%d\n", res);
77      }
78    }
79    return 0;
80  }
```

7.13.2 有三点共线且点有类别之分

```
1  int n,n0,n1,m;
2  Point p[3000], tp[3000], base;
3
4  bool cmp(const Point &a, const Point &b)
5  {
6    if ((a-base)*(b-base) == 0)
7    {
8      return (a-base).getMol() < (b-base).getMol();
9    }
10   return a.theta < b.theta;
11 }
12
13 int cnt[100][100];
14 int cntleft[100][100];
15
16 int calc(int a,int b,int c)
17 {
18   Point p1 = p[b]-p[a],p2 = p[c]-p[a];
19   if (atan2(1.0*p1.y,1.0*p1.x) > atan2(1.0*p2.y,1.0*p2.x))
20     swap(b,c);
21   int res = cnt[a][c]-cnt[a][b];
```

```
22    if ((p[b]-p[a])*(p[c]-p[a]) > 0)
23      return res;
24    else
25      return n1-res;
26  }
27
28  int main()
29  {
30    int cas = 0;
31    while (scanf("%d%d",&n0,&n1) != EOF)
32    {
33      n = n1+n0;
34      for (int i = 0; i < n; i++)
35      {
36        scanf("%I64d%I64d",&p[i].x,&p[i].y);
37        p[i].id = i;
38      }
39      for (int i = 0; i < n0; ++i)
40      {
41        m = 0;
42        base = p[i];
43        for (int j = 0; j < n; ++j)
44          if (i != j)
45          {
46            tp[m] = p[j];
47            Point v = tp[m]-base;
48            tp[m++].theta = atan2(1.0*v.y,1.0*v.x);
49          }
50
51        sort(tp, tp + m, cmp);
52        for (int j = 0; j < m; ++j)
53          tp[m + j] = tp[j];
54
55        for (int j = 0,tot = 0; j < m; ++j)
56        {
57          if (tp[j].id < n0)
58            cnt[i][tp[j].id] = tot;
59          else
60            tot++;
61        }
62
63        for (int j = 0, k = 0, tot = 0; j < m; ++j)
64        {
65          while (k == j || (k < j + m && (tp[j] - base) * (tp[k] -
               base) > 0))
66          {
67            if (tp[k].id >= n0)
68              tot++;
69            k++;
70          }
71          if (tp[j].id >= n0)
```

```
72            tot--;
73          else
74            cntleft[i][tp[j].id] = tot;
75        }
76      }
77
78      int ans = 0;
79      for (int i = 0; i < n0; i++)
80        for (int j = i+1; j < n0; j++)
81          for (int k = j+1; k < n0; k++)
82          {
83            int x = i,y = j,z = k;
84
85            if ((p[z] - p[x]) * (p[y] - p[x]) > 0)
86              swap(y, z);
87            int res = cntleft[x][z] + cntleft[z][y] + cntleft[y][x];
88
89            res += calc(x, y, z) + calc(y, z, x) + calc(z, x, y);
90
91            res -= 2 * n1;
92
93            //printf("%d %d %d %d\n",x,y,z,res);
94
95            if (res%2 == 1)
96              ans++;
97          }
98      printf("Case␣%d:␣%d\n",++cas,ans);
99    }
100   return 0;
101 }
```

7.14 最近点对

7.14.1 类快排算法

```
1  double calc_dis(Point &a ,Point &b) {
2    return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
3  }
4  //别忘了排序
5  bool operator<(const Point &a ,const Point &b) {
6    if(a.y != b.y) return a.x < b.x;
7    return a.x < b.x;
8  }
9  double Gao(int l ,int r ,Point pnts[]) {
10   double ret = inf;
11   if(l == r) return ret;
12   if(l+1 ==r) {
13     ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
14     return ret;
15   }
16   if(l+2 ==r) {
17     ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
```

```
    ret = min(calc_dis(pnts[l],pnts[l+2]) ,ret);
    ret = min(calc_dis(pnts[l+1],pnts[l+2]) ,ret);
    return ret;
  }

  int mid = l+r>>1;
  ret = min (ret ,Gao(l ,mid,pnts));
  ret = min (ret , Gao(mid+1, r,pnts));

  for(int c = l ; c<=r; c++)
    for(int d = c+1; d <=c+7 && d<=r; d++) {
      ret = min(ret , calc_dis(pnts[c],pnts[d]));
    }
  return ret;
}
```

7.14.2 随机增量法

```
#include <iostream>
#include <cstdio>
#include <cstring>
#include <map>
#include <vector>
#include <cmath>
#include <algorithm>
#define Point pair<double,double>
using namespace std;

const int step[9][2] =
   {{-1,-1},{-1,0},{-1,1},{0,-1},{0,0},{0,1},{1,-1},{1,0},{1,1}};
int n,x,y,nx,ny;
map<pair<int,int>,vector<Point > > g;
vector<Point > tmp;
Point p[20000];
double tx,ty,ans,nowans;
vector<Point >::iterator it,op,ed;
pair<int,int> gird;
bool flag;

double Dis(Point p0,Point p1)
{
  return sqrt((p0.first-p1.first)*(p0.first-p1.first)+
        (p0.second-p1.second)*(p0.second-p1.second));
}

double CalcDis(Point p0,Point p1,Point p2)
{
  return Dis(p0,p1)+Dis(p0,p2)+Dis(p1,p2);
}

void build(int n,double w)
{
```

```
34    g.clear();
35    for (int i = 0;i < n;i++)
36      g[make_pair((int)floor(p[i].first/w),(int)floor(p[i].second/w))
          ].push_back(p[i]);
37  }
38
39  int main()
40  {
41    int t;
42    scanf("%d",&t);
43    for (int ft = 1;ft <= t;ft++)
44    {
45      scanf("%d",&n);
46      for (int i = 0;i < n;i++)
47      {
48        scanf("%lf%lf",&tx,&ty);
49        p[i] = make_pair(tx,ty);
50      }
51      random_shuffle(p,p+n);
52      ans = CalcDis(p[0],p[1],p[2]);
53      build(3,ans/2.0);
54      for (int i = 3;i < n;i++)
55      {
56        x = (int)floor(2.0*p[i].first/ans);
57        y = (int)floor(2.0*p[i].second/ans);
58        tmp.clear();
59        for (int k = 0;k < 9;k++)
60        {
61          nx = x+step[k][0];
62          ny = y+step[k][1];
63          gird = make_pair(nx,ny);
64          if (g.find(gird) != g.end())
65          {
66            op = g[gird].begin();
67            ed = g[gird].end();
68            for (it = op;it != ed;it++)
69              tmp.push_back(*it);
70          }
71        }
72        flag = false;
73        for (int j = 0;j < tmp.size();j++)
74          for (int k = j+1;k < tmp.size();k++)
75          {
76            nowans = CalcDis(p[i],tmp[j],tmp[k]);
77            if (nowans < ans)
78            {
79              ans = nowans;
80              flag = true;
81            }
82          }
83        if (flag == true)
```

```
84          build(i+1,ans/2.0);
85        else
86          g[make_pair((int)floor(2.0*p[i].first/ans),(int)floor(2.0*p
               [i].second/ans))].push_back(p[i]);
87      }
88      printf("%.3f\n",ans);
89    }
90  }
```

7.15 多圆面积并

7.15.1 去重

有时候可能需要去掉不需要的圆

```
1  for (int i = 0; i < n; i++)
2  {
3    scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
4    del[i] = false;
5  }
6  for (int i = 0; i < n; i++)
7    if (del[i] == false)
8    {
9      if (c[i].r == 0.0)  del[i] = true;
10     for (int j = 0; j < n; j++)
11       if (i != j)
12         if (del[j] == false)
13           if (cmp(Point(c[i].c,c[j].c).Len()+c[i].r,c[j].r) <= 0)
14             del[i] = true;
15   }
16 tn = n;
17 n = 0;
18 for (int i = 0; i < tn; i++)
19   if (del[i] == false)
20     c[n++] = c[i];
```

7.15.2 圆并

$ans[i]$表示被覆盖i次的面积

```
1  const double pi = acos(-1.0);
2  const double eps = 1e-8;
3  struct Point
4  {
5    double x,y;
6    Point(){}
7    Point(double _x,double _y)
8      {
9        x = _x;
10       y = _y;
11     }
12   double Length()
13     {
14       return sqrt(x*x+y*y);
```

```
15      }
16  };
17  struct Circle
18  {
19    Point c;
20    double r;
21  };
22  struct Event
23  {
24    double tim;
25    int typ;
26    Event(){}
27    Event(double _tim,int _typ)
28      {
29        tim = _tim;
30        typ = _typ;
31      }
32  };
33
34  int cmp(const double& a,const double& b)
35  {
36    if (fabs(a-b) < eps)  return 0;
37    if (a < b)  return -1;
38    return 1;
39  }
40
41  bool Eventcmp(const Event& a,const Event& b)
42  {
43    return cmp(a.tim,b.tim) < 0;
44  }
45
46  double Area(double theta,double r)
47  {
48    return 0.5*r*r*(theta-sin(theta));
49  }
50
51  double xmult(Point a,Point b)
52  {
53    return a.x*b.y-a.y*b.x;
54  }
55
56  int n,cur,tote;
57  Circle c[1000];
58  double ans[1001],pre[1001],AB,AC,BC,theta,fai,a0,a1;
59  Event e[4000];
60  Point lab;
61
62  int main()
63  {
64    while (scanf("%d",&n) != EOF)
65    {
```

```
66      for (int i = 0;i < n;i++)
67        scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
68      for (int i = 1;i <= n;i++)
69        ans[i] = 0.0;
70      for (int i = 0;i < n;i++)
71      {
72        tote = 0;
73        e[tote++] = Event(-pi,1);
74        e[tote++] = Event(pi,-1);
75        for (int j = 0;j < n;j++)
76          if (j != i)
77          {
78            lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i].c.y);
79            AB = lab.Length();
80            AC = c[i].r;
81            BC = c[j].r;
82            if (cmp(AB+AC,BC) <= 0)
83            {
84              e[tote++] = Event(-pi,1);
85              e[tote++] = Event(pi,-1);
86              continue;
87            }
88            if (cmp(AB+BC,AC) <= 0) continue;
89            if (cmp(AB,AC+BC) > 0)  continue;
90            theta = atan2(lab.y,lab.x);
91            fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
92            a0 = theta-fai;
93            if (cmp(a0,-pi) < 0)  a0 += 2*pi;
94            a1 = theta+fai;
95            if (cmp(a1,pi) > 0)  a1 -= 2*pi;
96            if (cmp(a0,a1) > 0)
97            {
98              e[tote++] = Event(a0,1);
99              e[tote++] = Event(pi,-1);
100             e[tote++] = Event(-pi,1);
101             e[tote++] = Event(a1,-1);
102           }
103           else
104           {
105             e[tote++] = Event(a0,1);
106             e[tote++] = Event(a1,-1);
107           }
108         }
109       sort(e,e+tote,Eventcmp);
110       cur = 0;
111       for (int j = 0;j < tote;j++)
112       {
113         if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0)
114         {
115           ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
```

```
116             ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos(pre[cur]),c[i
                    ].c.y+c[i].r*sin(pre[cur])),
117                           Point(c[i].c.x+c[i].r*cos(e[j].tim),c[i].c.y+c[
                                i].r*sin(e[j].tim)))/2.0;
118           }
119           cur += e[j].typ;
120           pre[cur] = e[j].tim;
121         }
122       }
123       for (int i = 1;i < n;i++)
124         ans[i] -= ans[i+1];
125       for (int i = 1;i <= n;i++)
126         printf("[%d] = %.3f\n",i,ans[i]);
127     }
128     return 0;
129 }
```

7.16 一个圆与多边形面积交

```
1  bool InCircle(Point a,double r)
2  {
3    return cmp(a.x*a.x+a.y*a.y,r*r) <= 0; //这里判断的时候EPS一定不要太
       小！！
4  }
5
6  double CalcArea(Point a,Point b,double r)
7  {
8    Point p[4];
9    int tot = 0;
10   p[tot++] = a;
11
12   Point tv = Point(a,b);
13   Line tmp = Line(Point(0,0),Point(tv.y,-tv.x));
14   Point near = LineToLine(Line(a,b),tmp);
15   if (cmp(near.x*near.x+near.y*near.y,r*r) <= 0)
16   {
17     double A,B,C;
18     A = near.x*near.x+near.y*near.y;
19     C = r;
20     B = C*C-A;
21     double tvl = tv.x*tv.x+tv.y*tv.y;
22     double tmp = sqrt(B/tvl); //这样做只用一次开根
23     p[tot] = Point(near.x+tmp*tv.x,near.y+tmp*tv.y);
24     if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
25     p[tot] = Point(near.x-tmp*tv.x,near.y-tmp*tv.y);
26     if (OnSeg(Line(a,b),p[tot]) == true)  tot++;
27   }
28   if (tot == 3)
29   {
30     if (cmp(Point(p[0],p[1]).Length(),Point(p[0],p[2]).Length()) >
         0)
```

```
      swap(p[1],p[2]);
  }
  p[tot++] = b;

  double res = 0.0,theta,a0,a1,sgn;
  for (int i = 0;i < tot-1;i++)
  {
    if (InCircle(p[i],r) == true && InCircle(p[i+1],r) == true)
    {
      res += 0.5*xmult(p[i],p[i+1]);
    }
    else
    {
      a0 = atan2(p[i+1].y,p[i+1].x);
      a1 = atan2(p[i].y,p[i].x);
      if (a0 < a1)  a0 += 2*pi;
      theta = a0-a1;
      if (cmp(theta,pi) >= 0) theta = 2*pi-theta;
      sgn = xmult(p[i],p[i+1])/2.0;
      if (cmp(sgn,0) < 0) theta = -theta;
      res += 0.5*r*r*theta;
    }
  }
  return res;
}
```

调用

```
area2 = 0.0;
for (int i = 0;i < resn;i++) //遍历每条边，按照逆时针
  area2 += CalcArea(p[i],p[(i+1)%resn],r);
```

7.17 精度问题

7.17.1 浮点数为啥会有精度问题

浮点数(以C/C++为准)，一般用的较多的是float、double。

	占字节数	数值范围	十进制精度位数
float	4	$-3.4e-38 \sim 3.4e38$	$6 \sim 7$
double	8	$-1.7e-308 \sim 1.7e308$	$14 \sim 15$

如果内存不是很紧张或者精度要求不是很低，一般选用double。14位的精度(是有效数字位，不是小数点后的位数)通常够用了。注意，问题来了，数据精度位数达到了14位，但有些浮点运算的结果精度并达不到这么高，可能准确的结果只有$10 \sim 12$位左右。那低几位呢？自然就是不可预料的数字了。这给我们带来这样的问题：即使是理论上相同的值，由于是经过不同的运算过程得到的，他们在低几位有可能(一般来说都是)是不同的。这种现象看似没太大的影响，却会一种运算产生致命的影响：==。恩，就是判断相等。注意，C/C++中浮点数的==需要完全一样才能返回true。

7.17.2 eps

eps缩写自epsilon，表示一个小量，但这个小量又要确保远大于浮点运算结果的不确定量。eps最常见的取值是$1e-8$左右。引入eps后，我们判断两浮点数a、b相等的方式如下:

```
1 int sgn(double a){return a < -eps ? -1 : a < eps ? 0 : 1;}
```

这样，我们才能把相差非常近的浮点数判为相等；同时把确实相差较大(差值大于eps)的数判为不相等。
养成好习惯，尽量不要再对浮点数做==判断。

7.17.3 eps带来的函数越界

如果sqrt(a), asin(a), acos(a) 中的a是你自己算出来并传进来的，那就得小心了。
如果a本来应该是0的，由于浮点误差，可能实际是一个绝对值很小的负数（比如$-1e-12$），这样sqrt(a)应得0的，直接因a不在定义域而出错。
类似地，如果a本来应该是±1,则asin(a)、acos(a)也有可能出错。
因此，对于此种函数，必需事先对a进行校正。

7.17.4 输出陷阱I

现在考虑一种情况，题目要求输出保留两位小数。有个case的正确答案的精确值是0.005,按理应该输出0.01，但你的结果可能是0.005000000001(恭喜)，也有可能是0.004999999999(悲剧)，如果按照printf("%.2lf", a)输出，那你的遭遇将和括号里的字相同。
解决办法是，如果a为正，则输出$a+eps$, 否则输出$a-eps$

7.17.5 输出陷阱II

ICPC题目输出有个不成文的规定(有时也成文)，不要输出：-0.000
那我们首先要弄清，什么时候按printf("%.3lf", a)输出会出现这个结果。
直接给出结果好了：$a \in (-0.000499999\cdots, -0.000\cdots 1)$
所以，如果你发现a落在这个范围内，请直接输出0.000。更保险的做法是用sprintf直接判断输出结果是不是-0.000再予处理。

7.17.6 范围越界

请注意，虽然double可以表示的数的范围很大，却不是不穷大，上面说过最大是$1e308$。所以有些时候你得小心了，比如做连乘的时候，必要的时候要换成对数的和。

7.17.7 关于set

经观察,set不是通过==来判断相等的，是通过<来进行的，具体说来，只要$a<b$ 和$b<a$都不成立，就认为a和b相等，可以发现，如果将小于定义成:

```
1 bool operator < (const Dat dat)const{return val < dat.val - eps;}
```

就可以解决问题了。(基本类型不能重载运算符，所以封装了下)

7.17.8 输入值波动过大

这种情况不常见，不过可以帮助你更熟悉eps。假如一道题输入说，给一个浮点数a, $1e-20 < a < 1e20$。那你还敢用$1e-8$做eps么？合理的做法是把eps按照输入规模缩放到合适大小。

7.17.9 一些建议

容易产生较大浮点误差的函数有asin、 acos。欢迎尽量使用atan2。
另外，如果数据明确说明是整数，而且范围不大的话，使用int或者long long代替double都是极佳选择，因为就不存在浮点误差了

8 搜索

8.1 Dancing Links

8.1.1 估价函数

```
int h()
{
  bool vis[100];
  memset(vis,false,sizeof(vis));
  int i,j,k,res=0,mi,col;
  while(1)
  {
    mi=inf;
    for(i=R[head]; i!=head&&i<=2*n; i=R[i])
      if(mi>nk[i]&&!vis[i])
      {
        mi=nk[i];
        col=i;
      }
    if(mi==inf)
      break;
    res++;
    vis[col]=true;
    for(j=D[col]; j!=col; j=D[j])
      for(k=R[j]; k!=j; k=R[k])
      {
        if(C[k]>2*n)
          continue;
        vis[C[k]]=true;
      }
  }
  return res;
}
```

8.1.2 DLX

```
void remove1(int col)
{
  int i,j;
  L[R[col]]=L[col];
  R[L[col]]=R[col];
  for(i=D[col];i!=col;i=D[i])
  {
    L[R[i]]=L[i];
    R[L[i]]=R[i];
  }
}
void remove2(int col)
{
  int i,j;
  L[R[col]]=L[col];
```

```
16    R[L[col]]=R[col];
17    for(i=D[col];i!=col;i=D[i])
18    {
19      for(j=R[i];j!=i;j=R[j])
20      {
21        U[D[j]]=U[j];
22        D[U[j]]=D[j];
23        --nk[C[j]];
24      }
25    }
26  }
27  void resume1(int col)
28  {
29    int i,j;
30    for(i=U[col];i!=col;i=U[i])
31    {
32      L[R[i]]=i;
33      R[L[i]]=i;
34    }
35    L[R[col]]=col;
36    R[L[col]]=col;
37  }
38  void resume2(int col)
39  {
40    int i,j;
41    for(i=U[col];i!=col;i=U[i])
42    {
43      for(j=L[i];j!=i;j=L[j])
44      {
45        ++nk[C[j]];
46        U[D[j]]=j;
47        D[U[j]]=j;
48      }
49    }
50    L[R[col]]=col;
51    R[L[col]]=col;
52  }
53  int h()
54  {
55    bool vis[100];
56    memset(vis,false,sizeof(vis));
57    int i,j,k,res=0,mi,col;
58    while(1)
59    {
60      mi=inf;
61      for(i=R[head];i!=head&&i<=2*n;i=R[i])
62        if(mi>nk[i]&&!vis[i])
63        {
64          mi=nk[i];
65          col=i;
66        }
```

```
67      if(mi==inf)
68        break;
69      res++;vis[col]=true;
70      for(j=D[col];j!=col;j=D[j])
71        for(k=R[j];k!=j;k=R[k])
72        {
73          if(C[k]>2*n)
74            continue;
75          vis[C[k]]=true;
76        }
77    }
78    return res;
79  }
80  bool DLX(int d,int deep)
81  {
82    if(d+h()>deep) return false;
83    if(R[head]==head||R[head]>2*n)
84      return true;
85    if(d>=deep)
86      return false;
87    int col,ma=inf;
88    int i,j;
89    for(i=R[head];i!=head&&i<=2*n;i=R[i])
90      if(nk[i]<ma)
91      {
92        col=i;
93        ma=nk[i];
94      }
95    remove1(col);
96    for(i=D[col];i!=col;i=D[i])
97    {
98      int flag=1;
99      for(j=R[i];;j=R[j])
100     {
101       if(j==R[i]&&!flag)
102         break;
103       U[D[j]]=U[j];
104       D[U[j]]=D[j];
105       if(C[j]>2*n)
106         remove2(C[j]);
107       else
108         remove1(C[j]);
109       flag=0;
110     }
111     if(DLX(d+1,deep))
112       return true;
113     flag=1;
114     for(j=L[i];;j=L[j])
115     {
116       if(j==L[i]&&!flag)
117         break;
```

```
118         if(C[j]>2*n)
119           resume2(C[j]);
120         else
121           resume1(C[j]);
122         U[D[j]]=j;
123         D[U[j]]=j;
124         flag=0;
125       }
126     }
127     resume1(col);
128     return false;
129 }
```

9 动态规划

9.1 斜率优化

```
1  #include<cstdio>
2  #include<algorithm>
3  using namespace std;
4  int a[1000],sum[1001],dp[1000][1000];
5  int deque[1000];
6  const int inf=0x7fffffff;
7  int N,s,t;
8  int calc(int i,int l,int j)//决策值计算
9  {
10   return dp[j][l-1]-(sum[i]-sum[j])*(sum[N]-sum[i]);
11 }
12 bool check(int i,int l)//尾端判断
13 {
14   int k1=deque[t-1],k2=deque[t-2];
15   return (long long)(dp[k1][l]-dp[k2][l])*(sum[i]-sum[k1])>(long
       long)(dp[i][l]-dp[k1][l])*(sum[k1]-sum[k2]);
16 }
17 int main()
18 {
19   int n,m;
20   while (scanf("%d%d",&n,&m),n)
21   {
22     for (int i=0; i<n; i++)
23       scanf("%d",&a[i]);
24     N=n;
25     sum[0]=0;
26     for (int i=0; i<n; i++)
27       sum[i+1]=sum[i]+a[i];
28     dp[0][0]=0;
29     for (int i=0; i<n; i++)
30       for (int j=i+1; j<n; j++)
31         dp[0][0]+=a[i]*a[j];
32     for (int i=1; i<n; i++)
33       dp[i][0]=inf;
34     for (int i=1; i<n; i++)
35     {
36       dp[i][1]=inf;
37       for (int j=0; j<i; j++)
38         dp[i][1]=min(dp[i][1],calc(i,1,j));
39     }
40     for (int l=2; l<=m; l++)
41     {
42       s=t=0;//双端队列清空
43       for (int i=l; i<n; i++)
44       {
45         while (t-s>1 && check(i-1,l-1)) t--;
46         deque[t++]=i-1;//决策加入
```

```
47          while (t-s>1 && calc(i,l,deque[s])>calc(i,l,deque[s+1])) s
                ++;
48          dp[i][l]=calc(i,l,deque[s]);
49        }
50      }
51      int ans=0x7fffffff;
52      for (int i=m; i<n; i++)
53        ans=min(ans,dp[i][m]);
54      printf("%d\n",ans);
55    }
56    return 0;
57  }
```

9.2 RMQ二版

```
1  void init()
2  {
3    int i,j;
4    int n=N,k=1,l=0;
5    for (i=0; i<n; i++)
6    {
7      f[i][0]=ele[i].num;
8      if (i+1>k*2)
9      {
10       k*=2;
11       l++;
12     }
13     lent[i+1]=l;
14   }
15   for (j=1; (1<<j)-1<n; j++)
16     for (i=0; i+(1<<j)-1<n; i++)
17       f[i][j]=max(f[i][j-1],f[i+(1<<(j-1))][j-1]);
18 }
19 int fint(int x,int y)
20 {
21   int k=lent[y-x+1];
22   return max(f[x][k],f[y-(1<<k)+1][k]);
23 }
```

9.3 二维LIS

```
1  #include<cstdio>
2  #include<map>
3  using namespace std;
4  map<int,int> mp[100001];
5  bool check(int idx,int x,int y)
6  {
7    if (!idx) return 1;
8    if (mp[idx].begin()->first>=x) return 0;
9    map<int,int> ::iterator it=mp[idx].lower_bound(x);
10   it--;
11   if (it->second<y) return 1;
```

```
12    else return 0;
13  }
14  int main()
15  {
16    int n;
17    scanf("%d",&n);
18    int l=0,r=0;
19    for (int i=0;i<n;i++)
20    {
21      int x,y;
22      scanf("%d%d",&x,&y);
23      int tl=l,tr=r;
24      while (tl<tr)
25      {
26        int mid=(tl+tr+1)/2;
27        if (check(mid,x,y))
28          tl=mid;
29        else
30          tr=mid-1;
31      }
32      if (tl==r) r++;
33      int idx=tl+1;
34      map<int,int> ::iterator itl=mp[idx].lower_bound(x),itr=itl;
35      while (itr!=mp[idx].end() && itr->second>y) itr++;
36      if (mp[idx].find(x)!=mp[idx].end())
37        y=min(y,mp[idx][x]);
38      if (itl!=itr) mp[idx].erase(itl,itr);
39      if (mp[idx].find(x)==mp[idx].end() || mp[idx][x]>y)
40        mp[idx][x]=y;
41    }
42    printf("%d\n",r);
43    return 0;
44  }
```

9.4 插头DP

Tower Defence独立插头+构造解
构造解的时候保存的是在hash_map的ele数组的下标位置
没想清楚千万别去写

```
1   int bit[12];
2
3   inline int getbit(long long sta,int pos)
4   {
5     return sta/bit[pos]%bit[1];
6   }
7
8   inline long long setbit(long long sta,int pos,int val)
9   {
10    return sta/bit[pos+1]*bit[pos+1]+val*bit[pos]+sta%bit[pos];
11  }
12
```

```
13 int n,m,mp[30][10];
14 char buf[30][10];
15 hash_map dp[2];
16 bool flag;
17 int key,val,upd,l,u,res,msk,cov,now,pr,resnow,resmsk,pru;
18 int w[15],s[15],top;
19 int pre[210][10007],preuse[210][10007];
20
21 void decode(int msk,int& key,int& cov)
22 {
23   int tmp;
24   key = cov = 0;
25   for (int i = 0; i < m+1; i++)
26   {
27     tmp = getbit(msk,i);
28     if (tmp > 0)
29     {
30       key = setbit(key,i,tmp-1);
31       cov = setbit(cov,i,1);
32     }
33   }
34 }
35
36 int encode(int key,int cov)
37 {
38   int res = 0,tmp;
39   for (int i = 0; i < m+1; i++)
40   {
41     tmp = getbit(cov,i);
42     if (tmp > 0)
43     {
44       tmp = getbit(key,i);
45       res = setbit(res,i,tmp+1);
46     }
47   }
48   return res;
49 }
50
51 void update(int a,int key,int cov,int val)
52 {
53   int msk = encode(key,cov);
54   int pos;
55   if (dp[a][msk] < val)
56   {
57     dp[a][msk] = val;
58     pos = dp[a].fint(msk);
59     pre[now][pos] = pr;
60     preuse[now][pos] = pru;
61   }
62 }
63
```

```
int count3(int sta)
{
  int res = 0;
  for (int i = 0; i < m+1; i++)
    if (getbit(sta,i) == 3)
      res++;
  return res;
}

void expand(int sta)
{
  top = 0;
  for (int i = 0; i < m+1; i++)
    if (getbit(sta,i) == 1)
      s[top++] = i;
    else if (getbit(sta,i) == 2)
    {
      w[s[top-1]] = i;
      w[i] = s[top-1];
      top--;
    }
}

int main()
{
  //freopen("TD.in","r",stdin);
  //freopen("TDM.out","w",stdout);
  bit[0] = 1;
  for (int i = 1; i < 12; i++)  bit[i] = bit[i-1]*5;
  int t;
  scanf("%d",&t);
  dp[0].init();
  dp[1].init();
  for (int ft = 1; ft <= t; ft++)
  {
    scanf("%d%d",&n,&m);
    res = 0;
    memset(mp,0,sizeof(mp));
    memset(pre,0,sizeof(pre));
    memset(preuse,0,sizeof(preuse));
    for (int i = 0; i < n; i++)
    {
      scanf("%s",buf[i]);
      for (int j = 0; j < m; j++)
        if (buf[i][j] == '.')
          mp[i][j] = 1;
        else if (buf[i][j] != 'B')
          mp[i][j] = 2;
    }
    dp[0].clear();
    dp[1].clear();
```

```
115     flag = 0;
116     dp[flag][0] = 0;
117     int res = 0;
118     now = 0;
119     for (int i = 0; i < n; i++)
120     {
121       for (int j = 0; j < m; j++)
122       {
123         dp[!flag].clear();
124         for (int k = 0; k < dp[flag].N; k++)
125         {
126           msk = dp[flag].ele[k].key;
127           pr = k;
128           val = dp[flag].ele[k].val;
129           decode(msk,key,cov);
130           l = getbit(key,j);
131           u = getbit(key,j+1);
132           if (mp[i][j] == 0)//是障碍
133           {
134             if (l == 0 && u == 0)
135             {
136               pru = 0;
137               update(!flag,key,setbit(setbit(cov,j,0),j+1,0),val);
138             }
139           }
140           else
141           {
142             if (mp[i][j] == 1 && l == 0 && u == 0)//不要插头
143             {
144               pru = 1;
145               update(!flag,key,setbit(setbit(cov,j,0),j+1,0),val);
146             }
147             if (getbit(cov,j) == 1 && l == 0)   continue;//不可以在
                  这里搞插头
148             if (getbit(cov,j+1) == 1 && u == 0) continue;
149             cov = setbit(setbit(cov,j,1),j+1,1);//更新覆盖情况
150             upd = setbit(setbit(key,j,0),j+1,0);
151             pru = 2;
152             if (mp[i][j] == 2)
153             {
154               if (l == 0 && u == 0)
155               {
156                 if (count3(key) < 2)//可以新建独立插头
157                 {
158                   if (mp[i][j+1] != 0)
159                     update(!flag,setbit(setbit(key,j,0),j+1,3),cov,
                          val+1);
160                   if (mp[i+1][j] != 0)
161                     update(!flag,setbit(setbit(key,j,3),j+1,0),cov,
                          val+1);
162                 }
```

```
163                 }
164                 else if (l == 0 || u == 0)
165                 {
166                   if (l+u < 3 && count3(key) < 2)//可以用一个独立插头来
                        结束这条路径
167                   {
168                     expand(key);
169                     if (l > 0)
170                       update(!flag,setbit(upd,w[j],3),cov,val+1);
171                     else
172                       update(!flag,setbit(upd,w[j+1],3),cov,val+1);
173                   }
174                   else if (l+u == 3 && upd == 0)//路径的一端
175                   {
176                     if (res < val+1)
177                     {
178                       res = val+1;
179                       resnow = now-1;
180                       resmsk = k;
181                     }
182                   }
183                 }
184               }
185               else if (l == 0 && u == 0)
186               {
187                 if (mp[i][j+1] != 0 && mp[i+1][j] != 0)//可以新建插头
188                   update(!flag,setbit(setbit(key,j,1),j+1,2),cov,val
                        +1);
189               }
190               else if (l == 0 || u == 0)
191               {
192                 if (mp[i][j+1] != 0)//可以延续插头
193                   update(!flag,setbit(upd,j+1,l+u),cov,val+1);
194                 if (mp[i+1][j] != 0)//可以延续插头
195                   update(!flag,setbit(upd,j,l+u),cov,val+1);
196               }
197               else if (l == u)
198               {
199                 if (l < 3) //合并两个相同的括号
200                 {
201                   expand(key);
202                   if (l == 1)
203                     update(!flag,setbit(upd,w[j+1],1),cov,val+1);
204                   else
205                     update(!flag,setbit(upd,w[j],2),cov,val+1);
206                 }
207                 else if (upd == 0)//合并两个独立插头
208                 {
209                   if (res < val+1)
210                   {
211                     res = val+1;
```

```
                    resnow = now-1;
                    resmsk = k;
                  }
                }
              }
              else if (l == 3 || u == 3)//合并独立插头与括号
              {
                expand(key);
                if (l == 3)
                  update(!flag,setbit(upd,w[j+1],3),cov,val+1);
                else
                  update(!flag,setbit(upd,w[j],3),cov,val+1);
              }
              else if (l == 2 || u == 1) //合并)(
                update(!flag,upd,cov,val+1);
            }
          }
          flag = !flag;
          now++;
        }
        if (i+1 == n)   break;

        dp[!flag].clear();
        for (int k = 0; k < dp[flag].N; k++)
        {
          msk = dp[flag].ele[k].key;
          pr = k;
          val = dp[flag].ele[k].val;
          pru = 0;
          decode(msk,key,cov);
          update(!flag,key*bit[1],cov*bit[1],val);
        }
        now++;
        flag = !flag;
      }

      printf("Case␣%d:␣%d\n",ft,res);
      for (int i = resnow; i >= 0; i--)
      {
        if (preuse[i][resmsk] == 1)
          buf[i/(m+1)][i%(m+1)] = 'W';
        resmsk = pre[i][resmsk];
      }
      for (int i = 0; i < n; i++)
        printf("%s\n",buf[i]);
      printf("\n");
    }
    return 0;
}
```

10 杂物

10.1 高精度数

支持乘以整数和加法。

```
struct BigInt
{
  const static int mod = 100000000;
  int a[600],len;
  BigInt (){}
  BigInt (int v)
  {
    len = 0;
    do
    {
      a[len++] = v%mod;
      v /= mod;
    }while(v);
  }
  BigInt operator *(const int& b) const
  {
    BigInt res;
    res.len = len;
    for (int i = 0; i <= len; ++i)
      res.a[i] = 0;
    for (int i = 0; i < len; ++i)
    {
      res.a[i] += a[i]*b;
      res.a[i+1] += res.a[i]/mod;
      res.a[i] %= mod;
    }
    if (res.a[len] > 0)  res.len++;
    return res;
  }
  BigInt operator +(const BigInt& b) const
  {
    BigInt res;
    res.len = max(len,b.len);
    for (int i = 0; i <= res.len; ++i)
      res.a[i] = 0;
    for (int i = 0; i < res.len; ++i)
    {
      res.a[i] += ((i < len)?a[i]:0)+((i < b.len)?b.a[i]:0);
      res.a[i+1] += res.a[i]/mod;
      res.a[i] %= mod;
    }
    if (res.a[res.len] > 0)  res.len++;
    return res;
  }
  void output()
```

```
46    {
47      printf("%d",a[len-1]);
48      for (int i = len-2; i >= 0; --i)
49        printf("%08d",a[i]);
50      printf("\n");
51    }
52  };
```

10.2 整数外挂

```
1   int wg;
2   char ch;
3   bool ng;
4
5   inline int readint()
6   {
7     ch = getchar();
8     while (ch != '-' && (ch < '0' || ch > '9')) ch = getchar();
9     if (ch == '-')
10    {
11      ng = true;
12      ch = getchar();
13    }
14    else
15      ng = false;
16    wg = ch-'0';
17    ch = getchar();
18    while (ch >= '0' && ch <= '9')
19    {
20      wg = wg*10+ch-'0';
21      ch = getchar();
22    }
23    if (ng == true) wg = -wg;
24    return wg;
25  }
```

10.3 Java

10.3.1 文件操作

```
1   import java.io.*;
2   import java.util.*;
3   import java.math.*;
4   import java.text.*;
5
6   public class Main
7   {
8
9     public static void main(String args[]) throws
          FileNotFoundException, IOException
10    {
11      Scanner sc = new Scanner(new FileReader("a.in"));
```

```
12      PrintWriter pw = new PrintWriter(new FileWriter("a.out"));
13      int n,m;
14      n=sc.nextInt();//读入下一个INT
15      m=sc.nextInt();
16
17      for(ci=1; ci<=c; ++ci)
18      {
19        pw.println("Case␣#"+ci+":␣easy␣for␣output");
20      }
21
22      pw.close();//关闭流并释放，这个很重要，否则是没有输出的
23      sc.close();//关闭流并释放
24    }
25  }
```

10.3.2 优先队列

```
1  PriorityQueue queue = new PriorityQueue( 1, new Comparator()
2  {
3    public int compare( Point a, Point b )
4    {
5    if( a.x < b.x || a.x == b.x && a.y < b.y )
6      return -1;
7    else if( a.x == b.x && a.y == b.y )
8      return 0;
9    else
10     return 1;
11   }
12 });
```

10.3.3 Map

```
1  Map map = new HashMap();
2  map.put("sa","dd");
3  String str = map.get("sa").toString;
4
5  for(Object obj : map.keySet()){
6    Object value = map.get(obj );
7  }
```

10.3.4 sort

```
1  static class cmp implements Comparator
2  {
3    public int compare(Object o1,Object o2)
4    {
5    BigInteger b1=(BigInteger)o1;
6    BigInteger b2=(BigInteger)o2;
7    return b1.compareTo(b2);
8    }
9  }
10 public static void main(String[] args) throws IOException
11 {
```

```
12    Scanner cin = new Scanner(System.in);
13    int n;
14    n=cin.nextInt();
15    BigInteger[] seg = new BigInteger[n];
16    for (int i=0;i<n;i++)
17    seg[i]=cin.nextBigInteger();
18    Arrays.sort(seg,new cmp());
19  }
```

10.4 hashmap

```
1   struct hash_map
2   {
3     const static int mod=10007;
4     int head[mod];
5     struct hash_tables
6     {
7       int key;
8       int val;
9       int next;
10    } ele[10007];
11    int N;
12    int getHash(int x)
13    {
14      return x%mod;
15    }
16    void init()
17    {
18      memset(head,255,sizeof(head));
19      N=0;
20    }
21    void clear()
22    {
23      for (int i = 0; i < N; i++)
24        head[getHash(ele[i].key)] = -1;
25      N = 0;
26    }
27    int fint(int x)
28    {
29      for (int i=head[getHash(x)]; i!=-1; i=ele[i].next)
30        if (ele[i].key==x) return i;
31      return -1;
32    }
33    void insert(int x)
34    {
35      int tmp=getHash(x);
36      ele[N].key=x;
37      ele[N].val=0;
38      ele[N].next=head[tmp];
39      head[tmp]=N++;
40    }
41    int& operator [](int x)
```

```
42    {
43      int tmp=fint(x);
44      if (tmp==-1)
45      {
46        insert(x);
47        return ele[N-1].val;
48      }
49      else
50        return ele[tmp].val;
51    }
52  };
```

10.5 C++&STL常用函数

10.5.1 lower_bound/upper_bound

不解释

```
1  iterator lower_bound(const key_type &key )
2  \\返回一个迭代器，指向键值>= key的第一个元素。
3  iterator upper_bound(const key_type &key )
4  \\返回一个迭代器，指向键值> key的第一个元素。
5
6  #include <iostream>
7  #include <algorithm>
8  #include <vector>
9  using namespace std;
10
11 int main () {
12   int myints[] = {10,20,30,30,20,10,10,20};
13   vector<int> v(myints,myints+8);
14   // 10 20 30 30 20 10 10 20
15   vector<int>::iterator low,up;
16
17   sort (v.begin(), v.end());
18   // 10 10 10 20 20 20 30 30
19
20   low=lower_bound (v.begin(), v.end(), 20);
21   // 10 10 10 20 20 20 30 30
22   //          ^
23   up= upper_bound (v.begin(), v.end(), 20);
24   // 10 10 10 20 20 20 30 30
25   //                ^
26
27   cout << "lower_bound␣at␣position␣" << int(low- v.begin()) << endl
       ;
28   cout << "upper_bound␣at␣position␣" << int(up - v.begin()) << endl
       ;
29
30   return 0;
31 }
```

Output:

```
lower_bound at position 3
upper_bound at position 6
```

10.5.2 rotate

把数组后一半搬到前面

```
template <class ForwardIterator>
  void rotate ( ForwardIterator first, ForwardIterator middle,
                ForwardIterator last );
```

10.5.3 nth_element

```
template <class RandomAccessIterator>
  void nth_element ( RandomAccessIterator first,
     RandomAccessIterator nth,
                     RandomAccessIterator last );

template <class RandomAccessIterator, class Comapre>
  void nth_element ( RandomAccessIterator first,
     RandomAccessIterator nth,
                     RandomAccessIterator last, Compare comp );
```

10.5.4 bitset

取用

```
bitset<4> mybits;

mybits[1]=1;              // 0010
mybits[2]=mybits[1];      // 0110
```

翻转

```
bitset<4> mybits (string("0001"));

cout << mybits.flip(2) << endl;      // 0101
cout << mybits.flip() << endl;       // 1010
```

运算

```
bitset<4> first (string("1001"));
bitset<4> second (string("0011"));

cout << (first^=second) << endl;          // 1010 (XOR,assign)
cout << (first&=second) << endl;          // 0010 (AND,assign)
cout << (first|=second) << endl;          // 0011 (OR,assign)

cout << (first<<=2) << endl;              // 1100 (SHL,assign)
```

```
cout << (first>>=1) << endl;              // 0110 (SHR,assign)

cout << (~second) << endl;                // 1100 (NOT)
cout << (second<<1) << endl;              // 0110 (SHL)
cout << (second>>1) << endl;              // 0001 (SHR)

cout << (first==second) << endl;          // false (0110==0011)
cout << (first!=second) << endl;          // true  (0110!=0011)

cout << (first&second) << endl;           // 0010
cout << (first|second) << endl;           // 0111
cout << (first^second) << endl;           // 0101
```

10.5.5 multimap

遍历

```
multimap<char,int> mymm;
multimap<char,int>::iterator it;
char c;

mymm.insert(pair<char,int>('x',50));
mymm.insert(pair<char,int>('y',100));
mymm.insert(pair<char,int>('y',150));
mymm.insert(pair<char,int>('y',200));
mymm.insert(pair<char,int>('z',250));
mymm.insert(pair<char,int>('z',300));

for (c='x'; c<='z'; c++)
{
  cout << "There are " << (int)mymm.count(c);
  cout << " elements with key " << c << ":";
  for (it=mymm.equal_range(c).first; it!=mymm.equal_range(c).second
     ; ++it)
    cout << " " << (*it).second;
  cout << endl;
}
/*
Output:

There are 1 elements with key x: 50
There are 3 elements with key y: 100 150 200
There are 2 elements with key z: 250 300
*/
```

二分查找

```
multimap<char,int> mymultimap;
multimap<char,int>::iterator it,itlow,itup;

mymultimap.insert(pair<char,int>('a',10));
```

```
mymultimap.insert(pair<char,int>('b',121));
mymultimap.insert(pair<char,int>('c',1001));
mymultimap.insert(pair<char,int>('c',2002));
mymultimap.insert(pair<char,int>('d',11011));
mymultimap.insert(pair<char,int>('e',44));

itlow=mymultimap.lower_bound ('b');  // itlow points to b
itup=mymultimap.upper_bound ('d');   // itup points to e (not d)

// print range [itlow,itup):
for ( it=itlow ; it != itup; it++ )
  cout << (*it).first << " => " << (*it).second << endl;

/*
Output:

b => 121
c => 1001
c => 2002
d => 11011
*/
```

删除

```
multimap<char,int> mymultimap;
multimap<char,int>::iterator it;

// insert some values:
mymultimap.insert(pair<char,int>('a',10));
mymultimap.insert(pair<char,int>('b',20));
mymultimap.insert(pair<char,int>('b',30));
mymultimap.insert(pair<char,int>('c',40));
mymultimap.insert(pair<char,int>('d',50));
mymultimap.insert(pair<char,int>('d',60));
mymultimap.insert(pair<char,int>('e',70));
mymultimap.insert(pair<char,int>('f',80));

it=mymultimap.find('b');
mymultimap.erase (it);
// erasing by iterator (1 element)

mymultimap.erase ('d');
// erasing by key (2 elements)

it=mymultimap.find ('e');
mymultimap.erase ( it, mymultimap.end() );
// erasing by range

// show content:
for ( it=mymultimap.begin() ; it != mymultimap.end(); it++ )
  cout << (*it).first << " => " << (*it).second << endl;

```

```
/*
Output:

a => 10
b => 30
c => 40
*/
```

10.6 位运算

10.6.1 基本操作

注意括号

功能	示例	位运算
去掉最后一位	(101101 → 10110)	x shr 1
在最后加一个0	(101101 → 1011010)	x shl 1
在最后加一个1	(101101 → 1011011)	x shl 1+1
把最后一位变成1	(101100 → 101101)	x or 1
把最后一位变成0	(101101 → 101100)	x or 1-1
最后一位取反	(101101 → 101100)	x xor 1
把右数第k位变成1	(101001 → 101101, $k=3$)	x or (1 shl (k-1))
把右数第k位变成0	(101101 → 101001, $k=3$)	x and not (1 shl (k-1))
右数第k位取反	(101001 → 101101, $k=3$)	x xor (1 shl (k-1))
取末三位	(1101101 → 101)	x and 7
取末k位	(1101101 → 1101, $k=5$)	x and (1 shl k-1)
取右数第k位	(1101101 → 1, $k=4$)	x shr (k-1) and 1
把末k位变成1	(101001 → 101111, $k=4$)	x or (1 shl k-1)
末k位取反	(101001 → 100110, $k=4$)	x xor (1 shl k-1)
把右边连续的1变成0	(100101111 → 100100000)	x and (x+1)
把右起第一个0变成1	(100101111 → 100111111)	x or (x+1)
把右边连续的0变成1	(11011000 → 11011111)	x or (x-1)
取右边连续的1	(100101111 → 1111)	(x xor (x+1)) shr 1
去掉右起第一个1的左边	(100101000 → 1000)	x and (x xor (x-1))

10.6.2 枚举长为n含k个1的01串

```
int n = 5,k = 3;
for (int s = (1 << k)-1,u = 1 << n; s < u;)
{
  for (int i = 0;i < n;i++)
    printf("%d",(((s>>(n-1-i))&1) == 1));
  printf("\n");

  int b = s & -s;
  s = (s+b)|(((s^(s+b))>>2)/b);
}
```

10.7 其它

10.7.1 对跑脚本

```
while true; do
  ./gen > input
  ./sol < input > output.sol
  ./bf < input > output.bf

  diff output.sol output.bf
  if [ $? -ne 0 ] ; then break; fi
done
```