Bits, Bytes and Integers – Part 1

15-213/18-213/14-513/15-513: Introduction to Computer Systems
2nd Lecture, Aug. 30, 2018

Announcements

- **Recitations are on Mondays, but next Monday (9/3) is Labor Day, so recitations are cancelled**
- **Linux Boot Camp Monday evening 7pm, Rashid Auditorium**
- **Lab 0 is now available via course web page and Autolab.**
  - Due Thu Sept. 6, 11:59pm
  - No grace days
  - No late submissions
  - Just do it!

Logistics

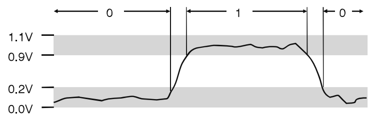
- **Waitlist**
  - 15-213: Mary Widom (marwidom@cs.cmu.edu)
  - 18-213: ECE Academic services
    ece-asc@andrew.cmu.edu
  - 15-513: Mary Widom (marwidom@cs.cmu.edu)
  - 14-513: INI Enrollment (ini-enrollment@andrew.cmu.edu)
  - Please don't contact the instructors with waitlist questions.
- **Autolab Accounts**
  - Check whether you have one
  - If not, refer to Piazza @68

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- **Representations in memory, pointers, strings**

Everything is bits

- **Each bit is 0 or 1**
- **By encoding/interpreting sets of bits in various ways**
  - Computers determine what to do (instructions)
  - … and represent and manipulate numbers, sets, strings, etc…
- **Why bits? Electronic Implementation**
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

- **Base 2 Number Representation**
  - Represent 15213_{10} as 11101101101101_2
  - Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
  - Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Encoding Byte Values

- **Byte = 8 bits**
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write $FA1D37B_{16}$ in C as
      - 0xFA1D37B
      - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

$$15213: \underbrace{0011}_{3}\ \underbrace{1011}_{B}\ \underbrace{0110}_{6}\ \underbrace{1101}_{D}$$

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

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Boolean Algebra

- **Developed by George Boole in 19th Century**
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- **Operate on Bit Vectors**
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- **All of the Properties of Boolean Algebra Apply**

Example: Representing & Manipulating Sets

- **Representation**
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if j ∈ A
    - 01101001 { 0, 3, 5, 6 }
    - *76543210*
    - 01010101 { 0, 2, 4, 6 }
    - *76543210*
- **Operations**
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - `~0x41` →
  - `~0x00` →
  - `0x69 & 0x55` →
  - `0x69 | 0x55` →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
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- **Examples (Char data type)**
  - `~0x41` → `0xBE`
    - `~0100 0001`$_2$ → `1011 1110`$_2$
  - `~0x00` → `0xFF`
    - `~0000 0000`$_2$ → `1111 1111`$_2$
  - `0x69 & 0x55` → `0x41`
    - `0110 1001`$_2$ `&` `0101 0101`$_2$ → `0100 0001`$_2$
  - `0x69 | 0x55` → `0x7D`
    - `0110 1001`$_2$ `|` `0101 0101`$_2$ → `0111 1101`$_2$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
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A	10	1010
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Contrast: Logic Operations in C

- **Contrast to Bit-Level Operators**
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- **Examples (char data type)**
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`
  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Watch out for && vs. & (and || vs. |)… one of the more common oopsies in C programming

Shift Operations

- **Left Shift: `x << y`**
  - Shift bit-vector `x` left `y` positions
    - Throw away extra bits on left
    - Fill with 0's on right
- **Right Shift: `x >> y`**
  - Shift bit-vector `x` right `y` positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on right
  - Arithmetic shift
    - Replicate most significant bit on left
- **Undefined Behavior**
  - Shift amount < 0 or ≥ word size

Argument `x`	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument `x`	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

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Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C does not mandate using two's complement**
  - But, most machines do, and we will assume so
- **C `short` 2 bytes long**

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Two-complement: Simple Example

```
          -16  8   4   2   1
   10 =    0   1   0   1   0      8+2 = 10

          -16  8   4   2   1
  -10 =    1   0   1   1   0      -16+4+2 = -10
```


Two-complement Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1
- **Two's Complement Values**
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
  - Minus 1
    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- **Observations**
  - \| *TMin* \| = *TMax* + 1
    - Asymmetric range
  - *UMax* = 2 * *TMax* + 1
- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **⇒ Can Invert Mappings**
  - $U2B(x) = B2U^{-1}(x)$
    - Bit pattern for unsigned integer
  - $T2B(x) = B2T^{-1}(x)$
    - Bit pattern for two's comp integer

Quiz Time!

Check out:

https://canvas.cmu.edu/courses/5835

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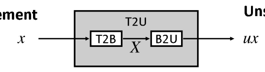
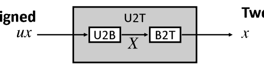
Mapping Between Signed & Unsigned

Two's Complement **Unsigned**



Maintain Same Bit Pattern

Unsigned **Two's Complement**



Maintain Same Bit Pattern

- **Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret**

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

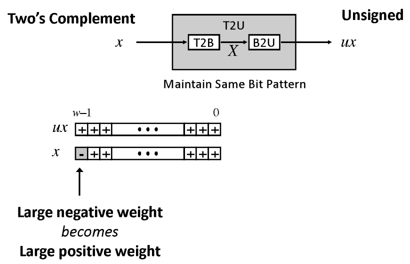
T2U →

← U2T

Mapping Signed ↔ Unsigned

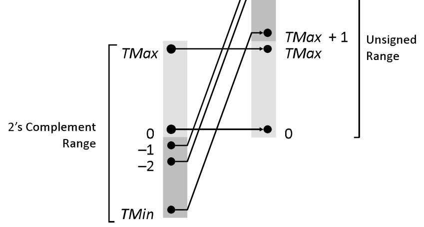
Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6	+/- 16	10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Relation between Signed & Unsigned



Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

    `0U, 4294967259U`
- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                    int fun(unsigned u);
uy = ty;                    uy = fun(tx);
```


Casting Surprises

- **Expression Evaluation**
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Unsigned vs. Signed: Easy to Make Mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

- Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```


Summary
Casting Signed ↔ Unsigned: Basic Rules

- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**

- **Expression containing signed and unsigned int**
  - `int` is cast to `unsigned`!!

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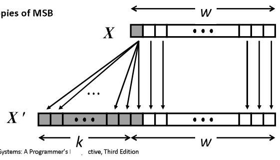
Sign Extension

- **Task:**
  - Given w-bit signed integer x
  - Convert it to w+k-bit integer with same value
- **Rule:**
  - Make k copies of sign bit:
  - $X' = \underbrace{x_{w-1}, \ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \ldots, x_0$



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Larger Sign Extension Example

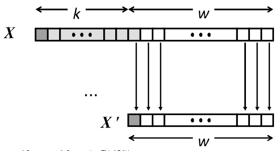
```
short int x =  15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- **Converting from smaller to larger integer data type**
- **C automatically performs sign extension**

Truncation

- **Task:**
  - Given k+w-bit signed or unsigned integer X
  - Convert it to w-bit integer X' with same value for "small enough" X
- **Rule:**
  - Drop top k bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

`2 mod 16 = 2`

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`-6 mod 16 = 26U mod 16 = 10U = -6`

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

`10 mod 16 = 10U mod 16 = 10U = -6`

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

`-10 mod 16 = 22U mod 16 = 6U = 6`

Summary: Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

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