ECS 222A: Assignment #1

Due on Tusday, January 13, 2015

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Problem 0

In the bit-model every bit-level operation must be counted. For example, to take the OR of two binary strings of length q each, takes q operations, and to use or set an index consisting of q bits, takes q operations. Show in detail that in the bit-model, then the 4-Russians method for bitmatrix multiplication only takes $\mathcal{O}(n^3/(\log n))$ operations.

Answer: The steps of 4-Russian's method for bitmatrix multiplication (BMM) are as follows

Algorithm 1 4-Russian's algorithm for BMM of $n \times n$ bit matrix A and B: C = AB

1: Define $k = \log_4 n$. Build a $2^k \times 2^k$ look up tables T. $T(u, v)$ in which $u = 0, \ldots, 2^k - 1$ and $v = 0, \ldots, 2^k - 1$ is the multiplication between the row bit vector representing i and the column vit vector representing j.
2: **for** i = 1 **to** n **do**
3: **for** j = 1 **to** n **do**
4: Divide A's i-th row $A_{i,:}$ into n/k row vectors of length k, denoted as $A^{(1)}_{i,:}, \ldots, A^{(n/k)}_{i,:}$. Similarly, divide B's j-th row $B_{:,j}$ into n/k column vectors of length k, denoted as $B^{(1)}_{:,j}, \ldots, B^{(n/k)}_{:,j}$.
5: Lookup the multiplication value $A^{(w)}_{i,:} B^{(w)}_{:,j}, w = 1, \ldots, n/k$, from table T.
6: Take the OR operation of all $A^{(w)}_{i,:} B^{(w)}_{:,j}, w = 1, \ldots, n/k$ as the result C_{ij}.
7: **end for**
8: **end for**

In line 1, to build the $2^k \times 2^k = \sqrt{n} \times \sqrt{n}$ lookup table, according to the matrix multiplication definition, it takes $\mathcal{O}(n^{3/2})$ operations
In line 5, given (i, j), for each w it takes $\mathcal{O}(k)$ to index the table T, so the total number of cost of index is $\mathcal{O}(\log_4 n)$.
In line 6, given (i, j), the sum-OR takes $\mathcal{O}(n/k) = \mathcal{O}(n/\log_4 n)$ operations.
Consequently, the total number of operations is

$$\mathcal{O}(n^{3/2}) + n^2(\mathcal{O}(\log_4 n) + \mathcal{O}(n/\log_4 n)) = \mathcal{O}(n^3/(\log(n)))$$

Problem 1

Prove that the edit distance is the same no matter which definition is used.

Answer: In order to change both S_1 and S_2 into a same string (no matter what exactly this string is), given a position p_1 in S_1 and position p_2 in S_2, we need to carry out some operation either at p_1 or p_2 (but not both) so that after the operation the charactors on this two positions are matched. In this sense the following pairs of operations are equivalent:

- Insertion in S_1 and deletion in S_2.
- Deletion in S_1 and insertion in S_2.
- Replacement in S_1 and replacement in S_2.

We can see that for each of the 3 allowed operation on 1 string, there is an equivalent operation on the other string that has the same effect (get a match in the current position). Therefore, the edit distance is the same no matter which definition is used.

Problem 2

From the mathematical standpoint, an alignment and an edit transcript are equivalent ways to describe a relationship between two strings. An alignment can be easily converted to the equivalent edit transcript and vice-versa. Completely explain and justify the above statement.
Answer:

Problem 3

Prove that any traceback path specifies an optimal edit transcript, and an optimal alignment. In the latter case, explain how the path specifies where the spaces should go in the two strings.
Answer:

Problem 4

Theorem Any path from (n, m) to $(0, 0)$ following pointers established during the computation of $D(i, j)$ specifies an edit transcript with the minimum number of edit operations. Conversely, any optimal edit transcript is specified by such a path. Moreover, since a path describes only one transcript, the correspondence between paths and optimal transcripts is one-one.
Prove this theorem.
Answer:

Problem 5

Since the traceback paths in a dynamic programming table correspond one-to-one with the optimal alignments, the number of distinct co-optimal alignments can be obtained by computing the number of distinct traceback paths. Give an algorithm to compute this number in $O(nm)$ time. Hint: use dynamic programming.
Answer: