ECS 222A: Assignment #7

Due on Thursday,March 12, 2015

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Contents

Problem 1

In class on thursday, we talked about the factor of two approximation algorithm for minimum-size node cover problem in an undirected graph G: find a maximal Independent Set of edges of G, call it M , then form $A(G)$ by taking both ends of each edge in M. $A(G)$ is a node cover, and we proved that its size is at most twice the size of $O(G)$, where $O(G)$ is a minimum-size node cover of G. That is, $|A(G)|/|O(G)| \leq 2$. In class, I said that we can find a subset $A'(G)$ of $A(G)$ (possibly equal), which is also a node cover of G, such that $|A'(G)|/|O(G)| < 2$. More precisely, we have the following:

> Claim: Either some node x can be removed from $A(G)$ so that $A'(G) = A(G)\{x\}$ is a node cover of G, or $A(G)$ is a minimum-size node cover.

Prove the claim, and show how it establishes the better ratio. As a hint, examine two cases: either there is a node v in $A(G)$ such that for every edge (u, v) in G, u is also in $A(G)$; or there is no such node. To handle the latter case, extend the proof we gave in class that $|A(G)|/|O(G)| \leq 2$.

Answer:

Problem 2

In class we stated that the Satifiability problem is NP-complete, that the Independent Set problem is NP-complete and the Node-Cover Problem is NP-complete. So in this problem you may assume those problems, but only those problems, are known to be NP-complete.

Problem 2(a)

In a problem we call the ZZZ problem, the input is a number k, and bipartite graph G, where the two node sets on the two sides of G are denoted A and B. The answer to an instance of problem ZZZ is yes if and only if there is a subset S of size at most k of the nodes in A, such that every node in B is adjacent to at least one node in S. Prove that problem ZZZ is NP-complete.

Answer:

Problem 2(b)

In a problem we call the QQQ problem, the input is an undirected graph $G = (V, E)$ and an undirected graph G_1. There are no node or edge labels. The answer to an instance of problem QQQ is yes if and only if there is an "induced" subgraph $G' = (V', E')$ of G which is isomorphic (indentical in this context) to G_1. In an induced subgraph containing the set of nodes V', the edge set E' consists of *every* edge whose two endpoints are both in V'. Prove that Problem QQQ is NP-complete.

Answer:

Problem 3

In an undirected, connected graph G, a subset S of nodes of G is called a *Dominating Set* if every node in G is adjacent to at least one node in S. Note that a Dominating Set is not the same as a Node Cover. The Dominating Set Problem has input (G, k). The answer to an instance of the Dominating Set problem is yes, if and only if G has a Dominating Set of size at most k.

Problem 3 continued on next page. . .

The following idea shows how to reduce any instance of the Node Cover Problem, when the input graph is connected, to an Instance of the Dominating Set Problem. Note that the Node Cover problem is NP-complete even when the restricted to the case where the input graph is required to be connected.
Given an instance (H, t) of the Node Cover Problem (H is a connected, undirected graph, and t is the target), create a new graph G consisting of H plus one new node uv for each edge (u, v) in H. Node uv in G has an edge to node u in G and an edge to node v in G. So each edge (u, v) in H is associated with a triangle in G consisting of nodes u, v, uv. It helps to draw a picture. Then the input to the Dominating Set Problem is (G, k), where $k = t$.
Prove that H has a Node Cover of size at most t if and only if G has a Dominating Set of size at most t. Hint: establish first that a smallest Dominating Set of G can be found using only the original nodes in H.

Answer: