ECS 222A: Assignment #4

Due on Tuesday,February 12, 2015

Daniel Gusfield TR 4:40pm-6:00pm

Wenhao Wu

Contents

Problem 1

(The bipartite node cover problem) Let G be an undirected graph with each node i given weight $w(i) > 0$. A set of nodes S is a *node cover* of G if every edge of G is incident to at least one node of S. The weight of a node cover S is the summation of the weights, denoted $w(S)$, of the nodes in S; the weighted node cover problem is to select a node cover with *minimum weight*.

There is no known polynomial time (in terms of worst case) algorithm for the node cover problem (even when all weights are one). But if G is bipartite, then the minimum weight node cover can be found in polynomial time by network flow. If you dont know what a bipartite graph is, look up a definition in the book.

Explain how to do this. Hint: Use maximum flow, but the minimun cut is the key, rather than the maximum flow.

Answer:

Problem 2

First, read Section 7.7 in the book to learn about circulations and maximum flow with lower bounds on edge flows. Then use what you have learned to solve the following problem. You must use network flow with lower bounds, even if you see another solution method.

(Table Rounding problem) You are given an n by m table of numbers between 0 and 1 along with row and column totals. Your objective is to round each entry to either 0 or 1 (you have complete freedom in this) so that each resulting row and column total is itself rounded to one of its *two* nearest integers, and so that the table total is rounded to one of its two nearest integers. However, any number which was originally an integer must not change. For example,

```
.2 .4  0 .2 |  0.8          0 0 0 0   0
.4  0 .2 .4 |  1            0 0 1 0   1
.8 .8 .8  0 |  2.4   ===>   1 1 1 0   3
.6  0 .2 .2 |  1            1 0 0 0   1
------------|
2 1.2 1.2 .8   5.2          2 1 2 0   5
```

Figure 1: A table rounding examples.

Give an efficient algorithm, using network flow, that always finds such a rounding. This also provides a proof that such a rounding is always possible.

Answer:

Problem 3

Suppose (X, Y) and (X', Y') are two distinct minimum-capacity s, t cuts in a directed network G. Prove that $(X \cup X', Y \cup Y')$ and $(X \cap X, Y \cap Y)$ are also a minimum-capacity s, t cuts in G. The key to this problem is to remember that an s, t cut is a partition of the nodes with s in one subset and t in the other, so there are four subsets of nodes when you examine X, Y, X', Y' together; then do a case analysis by looking closely at the capacities of the edges from one subset of nodes to another.

Problem 3 continued on next page...

Answer:

Problem 4

In some applications of numerical linear algebra you are given a sparse square matrix M (say n by n) and you want to permute the rows and columns of M so that the main diagonal has no 0, if possible. Show how to find such a permutation, if there is one, by using network flow. Hint: The key here is to use network flow to find a set of n non-zero entries in M such that no two are in the same row or column. For the network, start with a bipartite graph to represent the non-zero entries of M, and then add the s and t nodes.

Answer:

Problem 5

Show that if f is some non-maximum $s-t$ flow in a graph G, and G_f is the residual graph with respect to f, then flow f *superimposed* with a maximum $s-t$ flow g in G_f is a maximum flow in G. By superposition we mean the addition of the two flows; however if for an edge (i,j) there is flow from i to j in f and flow from j to i in g (recall that g is a flow in G_f so that this is possible, since (j,i) is a backward edge) then the superposition of these flows means the subtraction of $g(j,i)$ from $f(i,j)$. That is, forward flows in f and g are added, but a backward flow in g is subtracted from the corresponding forward flow in f.

Answer: