Constellation Rearrangement in Cooperative Relay-HARQ Network

Abstract—**Constellation rearrangement (CoRe) is a technique to enhance diversity by using different mapping from bits to a constellation for retransmitted packets in HARQ system. In this paper, we study the CoRe problem in a cooperative relay-HARQ network where the source and relay carry out the retransmission in a coordinated fashion. By allowing the source and relay use two different remappings, the signal space diversity is exploited more thoroughly. Based on the minimum bit error rate (min-BER) criteria, the CoRe design problem is formulated into a quadratic three-dimensional assignment problem (Q3AP). To solve the NP-complete Q3AP, we adopted an efficient modified Iterated Local Search (ILS) method. Performance gains of this CoRe scheme are demonstrated and analyzed with numerical results under various channel settings.**

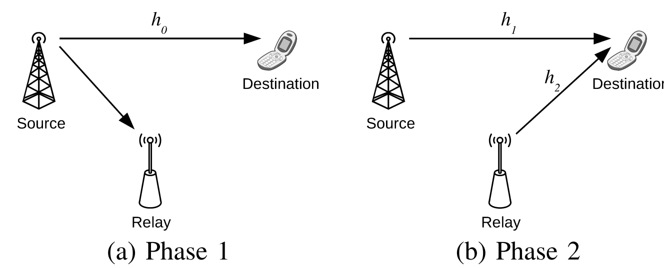


Fig. 1. Cooperative relay-HARQ networks.

I. Introduction

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ combined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ systems can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a specific sequence of bits are mapped to different constellation points across different links.

There is a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate-optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER minimization in [7]. CoRe is also studied for the Nakagami-m channel [8] and in combination with power allocation [9]. Note that the above-mentioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on the same time slot or band, resulting in low bandwidth efficiency. However, three-terminal relay networks where the S-D and R-D links are additive instead of orthogonal have long been proposed [10] and extensively studied [11], [12]. It would be interesting to extend CoRe for HARQ to this kind of relay networks.

One major difficulty for CoRe problems is rooted in the fact that they are hard to solve exactly. Historically, various CoRe problems for HARQ systems fall within the realm of the Quadratic Assignment Problem (QAP) [13], which finds a its applications in VLSI design, facility lay-out planning and analysis of chemical reactions, etc [14]. As a special case of binary programming problem, QAP is known to be NP-hard and finding an approximate solution is NP-hard, too [15]. Some of the existing CoRe implementations work around this difficulty by adopting a fixed remapping scheme [4], [9], or by impractically dropping the binary constellation mapping constraints [16]. Other works apply heuristic/metaheuristic approaches such as simulated annealing (SA) [6] and genetic algorithms [7]. It has been reported that some heuristic QAP solvers [17], [18], [19] provide very high-quality solutions over the cases in QAPLIB [20]. The joint optimization of the two remappings on the S-D and R-D link, respectively, results in an even harder Quadratic 3-dimensional Assignment Problem (Q3AP). Consequently, it is more challenging to find an efficient algorithm that gives high-quality solutions.

In this work, we study the CoRe for cooperative relay-HARQ networks based on BER minimization. The main contributions of this paper are as follows:

- We propose a novel CoRe scheme for the relay-HARQ network similar to [21]. As depicted in Fig. 1, the source and the relay jointly perform the Chase Combining retransmission to the destination in a coordinated fashion, practically forming a 2-by-1 multiple-input single-output MISO system.
- In our cooperative relay-HARQ settings, we formulate the CoRe design into a Q3AP problem, then solve it with an efficient iterated local search (ILS) method. The performance gains by fully exploiting the signal-space diversity with the optimized mapping rearrangement are demonstrated with simulation results under various channel settings.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into a Q3AP solution and provides an outline of the modified ILS algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model

We consider the cooperative relay-HARQ network shown in Fig. 1. Denote $\mathcal{C}$ as the constellation used by this relay network whose size is $Q = |\mathcal{C}|$. In the first phase, the source converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \to \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. Then the source transmits $\psi_0[p]$ to the destination via channel h_0 which is also overheard by the decode-and-forward (DF) relay. We assume that this relay is placed strategically so that it has negligible decoding error rate as in [8], [7]. Upon receiving a request for retransmission, the second phase is started. This time the source and the relay remap p into $\psi_1[p]$ and $\psi_2[p]$, respectively, where potentially $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are transmitted simultaneously on the same band to the destination via channel h_1 and h_2. In summary, the received signals at the destination during the two phases are

$$y_1 = h_0\psi_0[p] + v_1, \tag{1a}$$

$$y_2 = h_1\psi_1[p] + h_2\psi_2[p] + v_2, \tag{1b}$$

where $v_1 \sim \mathcal{CN}(0, \sigma_v^2)$ and $v_2 \sim \mathcal{CN}(0, \sigma_v^2)$ are the additive noise. Throughout this work, we assume the channels h_0, h_1 and h_2 follows independent Rician distribution.

Assuming that the destination has perfect channel state information (CSI). Based on the received symbols y_1 and y_2 from the two phases, it decides on the index p with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_1 - h_0\psi_0[\hat{p}]|^2 + |y_2 - h_1\psi_1[\hat{p}] - h_2\psi_2[\hat{p}]|^2. \tag{2}$$

III. Optimal Constellation Rearrangement

In this section we first formulate the min-BER CoRe design into a Q3AP problem, and explain the numerical approach to compute the input cost matrix of the Q3AP problem. Then we provide an efficient algorithm to determine the Q3AP solution.

A. BER Minimization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded and approximated using the pair-wise error probability (PEP) [13]

$$P_{BER} \approx \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}(q|p), \tag{3}$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q and $P_{PEP}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,q,\psi_1,\psi_2) < 0\}, \tag{4}$$

i.e. given indices p, q and the remapping scheme ψ_1, ψ_2, the probability of the random variable $\delta < 0$ being evaluated over the random variables h_0, h_1, h_2, v_1, v_2, and δ is defined as

$$\begin{aligned}\delta = {} & |h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 + \\ & |h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_2|^2 - |v_2|^2.\end{aligned} \tag{5}$$

In order to formulate the Q3AP problem, we introduce the binary variable $x_{pij} = 1$ if $\psi_1[p] = \psi_0[i]$ and $\psi_2[p] = \psi_0[j]$ and $x_{pij} = 0$ otherwise. Denote $\mathbf{x} = \{x_{pij} | p, i, j = 0, \ldots, Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\} \right\}, \tag{6a}$$

$$\mathcal{I} = \left\{ \mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\} \right\}, \tag{6b}$$

$$\mathcal{J} = \left\{ \mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\} \right\}. \tag{6c}$$

Then from (3)(4)(5), the BER-minimization CoRe scheme $\min_{\psi_1,\psi_2} P_{BER}$ can be reformulated as

$$\begin{aligned}&\min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl}x_{pij}x_{qkl}, \\ &\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}.\end{aligned} \tag{7}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}, \tag{8}$$

$$\begin{aligned}\delta = {} & |h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 + \\ & |h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_2|^2 - |v_2|^2.\end{aligned} \tag{9}$$

A Q3AP instance is fully defined by the 6-dimensional matrix $\{c_{pijqkl}\}$.

B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters $\{c_{pijqkl}\}$ of the Q3AP problem. According to (8), the key to compute c_{pijqkl} lies in the evaluation of $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l)$, i.e. the cumulative distribution function (CDF) of the random variable $\delta(p,i,j,q,k,l)$ as in (5). Under the general Rician channel assumption $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma_{h_m}^2), m = 0,1,2,$ we extend the method in [13], [22] to compute $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$:

$$\begin{aligned}& P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\} \\ & \approx \frac{1}{2v}\sum_{t=1}^{v} \Re\left\{\Phi_\delta(\xi + j\xi\tau_t)\right\} + \tau_t \Im\left\{\Phi_\delta(\xi + j\xi\tau_t)\right\},\end{aligned} \tag{10}$$

with the moment generating function (MGF) $\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)]$, $\tau_t = \tan((t-1/2)\pi/v)$ and $\Re\{\cdot\}$, $\Im\{\cdot\}$ denoting its real and image part, respectively. The parameter ξ is selected to ensure convergence of the integration and [22] suggests $\xi = 1/4$. The size v of the expansion (10) needs to be larger when $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$ is smaller in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote the Gaussian random vectors $\mathbf{z}_1 = [h_0, v_1]^T$, $\mathbf{z}_2 = [h_1, h_2, v_2]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1,2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T, \tag{11}$$

$$\boldsymbol{\Sigma}_1 = \text{diag}\left(\sigma_{h_0}^2, \sigma_v^2\right), \boldsymbol{\Sigma}_2 = \text{diag}\left(\sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_v^2\right). \tag{12}$$

Then (5) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix}, \tag{13a}$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix}, \tag{13b}$$

here $e_{ab} = \psi_0[a] - \psi_0[b]$. Then the MGF can be computed as [23]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \tag{14}$$

Note that for any instance of Q3AP a total number of Q^6 coefficients must be computed. Fortunately, the CoRe design is based on statistical CSI and does not require an online computation. In our simulation, we implement the above procedure for 16-QAM and 32-QAM with Armadillo library [24] on a workstation with 48 cores and finished the computation in a few days for a $Q = 32$ case and a few hours for a $16 - QAM$ case. For larger constellation such as 64-QAM, however, the time and spacial complexity would be too high. We will address in future works to reduce this complexity by adding a few rules to restrict the remapping schemes.

When the Rician-fading channel reduces to the Rayleigh-fading channel, i.e. when $\mu_{h_m} = 0, m = 0, 1, 2$ there is a simple upper bound of $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p, i, j, q, k, l) < 0\}$ similar to [4]. Note that

$$\begin{aligned} &P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p, i, j, q, k, l) < 0\} \\ &= \mathbb{E}_{h_0,h_1,h_2}\left\{ Q\left(\sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma_v^2}} \right) \right\}. \end{aligned} \tag{15}$$

Applying the same relaxation as in [4], we have

$$\begin{aligned} &P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p, i, j, q, k, l) < 0\} \\ &\le \frac{3\sigma_v^4}{\sigma_{h_0}^2 |e_{pq}|^2 (\sigma_{h_1}^2 |e_{ik}|^2 + \sigma_{h_2}^2 |e_{jl}|^2)}. \end{aligned} \tag{16}$$

which is tight in the high-SNR regime.

C. Q3AP Solution

For practical-sized constellation such as 16-QAM and 32-QAM, it is impractical to apply the exact branch-and-bound algorithm [25]. Also, our tests show that even the smaller 16-QAM does not have enough symmetry to exploit as does the 16-PSK constellation [26]. Consequently, the CoRe problem is solved with the ILS method [25] extended from its QAP version [27]. The basic procedure of this ILS method is outlined as follows:

- S1 Initialization: starting from a random mapping $\psi_1^{(0)}, \psi_2^{(0)}$.
- S2 Local search: find a locally optimum solution $\psi_1^{(0)*}, \psi_2^{(0)*}$. Set $n = 0$
- S3 Perturbation: from the last locally optimum solution $\psi_1^{(n)*}, \psi_2^{(n)*}$, generate a new mapping $\psi_1^{(n+1)}, \psi_2^{(n+1)}$.
- S4 Local search: from $\psi_1^{(n+1)}, \psi_2^{(n+1)}$, find a new locally optimum solution $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$.
- S5 Examination of the acceptance criterion: compare $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ with $\psi_1^{(n)*}, \psi_2^{(n)*}$. If the new mapping is not accepted, reset $\psi_1^{(n+1)*} = \psi_1^{(n)*}$ and $\psi_2^{(n+1)*} = \psi_2^{(n)*}$. Set $n \leftarrow n + 1$.
- S6 Return to S3 until the stopping criterion is satisfied.

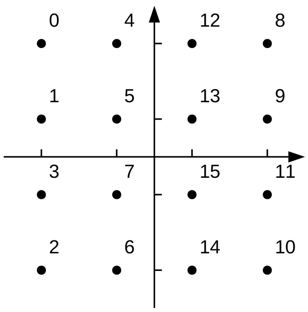
The local search in S2 and S4 attempts to exchange the mapping of exactly 2 indices in ψ_1 or ψ_2 in order to reduce the objective function, i.e. a 2-opt neighborhood search. A first-improvement rule is used, which means that whenever a reduction in the objective function is made the mapping is updated. The perturbation in S3 is performed by exchanging the mapping of k_p indices, where $k_p \in [k_{p,min}, k_{p,max}]$ is adaptively increased if the perturbation does not produce a better solution or is reset to $k_{p,min}$ otherwise. For the acceptance criterion examination in S5, $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ is accepted if the objective function $f^{(n+1)} < f^{(n)}$, or accepted with probability $\exp(-[f^{(n+1)} - f^{(n)}]/T)$, where T is the temperature parameter similar to that of simulated annealing.

IV. Numerical Results

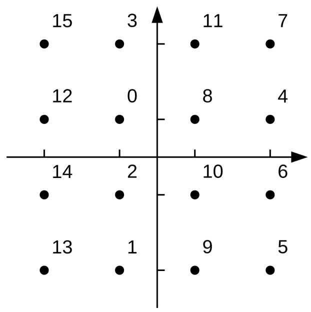
In this section we present the numerical results of CoRe for the relay-HARQ network under various channel settings. In our simulation, all Rician channels are assumed to have the same Rician parameter K. Also we assume that during the second phase, the phases of the line of sight (LOS) components of channels h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma_{h_0}^2 = \sigma_{h_1}^2 = 1/(K+1)$ and $\sigma_{h_2}^2 = |a|^2/(K+1)$, where a represents the ratio between the amplitude of the LOS component of the relay-to-destination and the S-D link. Throughout the simulation we consider 16-QAM constellation thus $Q = 16$. The noise power is parameterized with E_b/N_0 of the S-D link.

First, we provide an example of Q3AP optimized CoRe. When $E_b/N_0 = 2$dB, $K = 10$ and $a = 1$, the remapping scheme of ψ_0 (Gray-mapping), ψ_1 and ψ_2 is depicted in Fig. 2. We note that for the Chase combining HARQ scheme and the joint ML demodulator, the optimal ψ_1 and ψ_2 are no longer a Gray mapping. A general pattern of ψ_1 and ψ_2 is that the indices whose constellation points are close to each other during the first phase are remapped to constellation points far away from each other during the second phase, and vice versa. Both ψ_1 and ψ_2 are essentially the same as Seddik's heuristic result in [4], denoted as ψ_S. However, it is noted $\psi_1 \neq \psi_2$. In this special example or Rician channel with strong LOS component, an explanation is that the cooperative transmission is equivalent to transmission from a single transmitter with mapping $\psi_1 + \psi_2$. In this aspect, CoRe in effect results in a new constellation different from 16-QAM, while the individual constellations at the source and relay are kept unchanged.

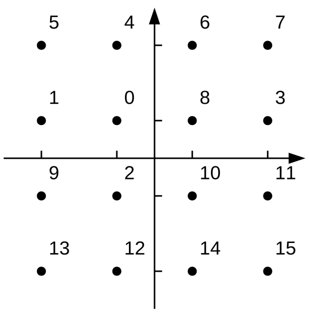
Next we compare the BER upper bound P_{BER}, which is also the Q3AP objective function, for three mapping schemes: the Q3AP optimized CoRe, the extension of Seddik's remapping $\psi_1 = \psi_2 = \psi_S$, and no CoRe at all, i.e. $\psi_1 = \psi_2 = \psi_0$. In Fig. 3, there is a considerable performance gain for CoRe versus no CoRe at all. Moreover, the gain of the Q3AP



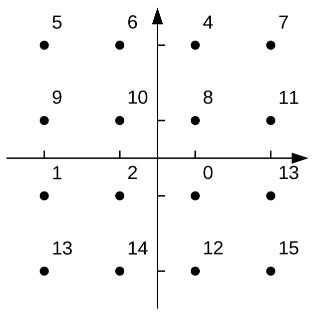
(a) ψ_0 (Gray-mapping)



(b) ψ_S

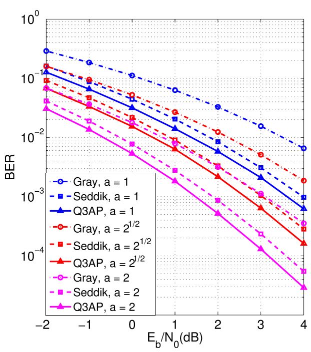


(c) ψ_1

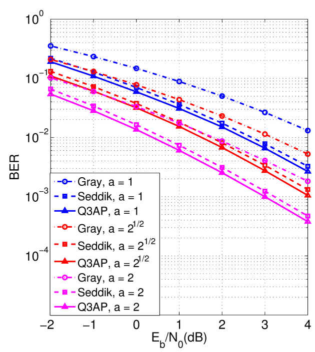


(d) ψ_2

Fig. 2. Q3AP optimized CoRe schemes.



(a) $K = 10$



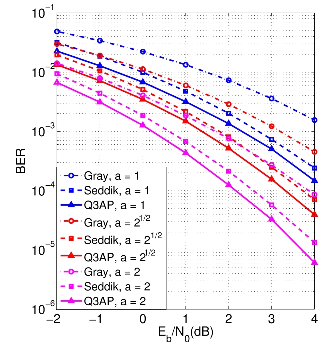
(b) $K = 5$

Fig. 3. BER upper bounds of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

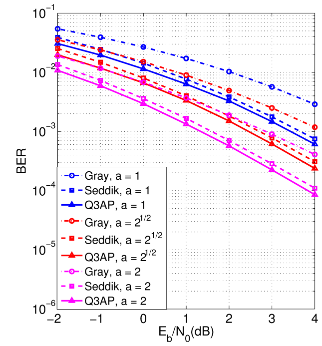
optimized CoRe over $\psi_1 = \psi_2 = \psi_S$ indicates that in cooperative relay-HARQ networks, the BER performance can be further improved by using different rearrangements at the source and relay during the cooperative transmission. This gain increases as K increases, i.e. when the channel becomes less random.

The performance gain of Q3AP optimized CoRe for the cooperative relay-HARQ system is further verified in Fig. 4, where we evaluate the actual BER by running a Monte-Carlo simulation. We implement the ML demodulator in (2) and evaluate the average BER of $M = 10^7$ randomly generated indices p. The actual BER curves demonstrate a similar pattern as the BER upper bound curves in Fig. 3.

Finally, we test the robustness of the Q3AP CoRe solution against mismatch in channel settings. The intuition is that in practical, the CSI may not be perfectly accurate and the phase



(a) $K = 10$



(b) $K = 5$

Fig. 4. Monte-Carlo simulated BER of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

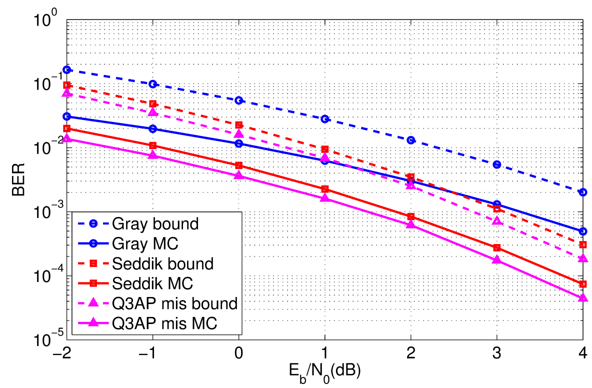


Fig. 5. BER performance under channel mismatch. Q3AP CoRe solution for $a = 1$, $K = 5$ in the actual channel settings $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$.

of the LOS component of the S-D and R-D links may not be perfectly aligned. Also, the Q3AP CoRe scheme can only be computed off-line for a finite number of channel settings. We deliberately test the BER performance of the Q3AP CoRe solution for $a = 1$, $K = 5$ in the actual channel settings $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$. The BER upper bound and Monte-Carlo simulated BER are plotted in Fig. 5. Apparently, the Q3AP CoRe solution is not sensitive to the relative amplitude and the phase alignment error between the S-D and R-D links.

V. Conclusion

In this work, we have investigated the constellation rearrangement (CoRe) problem in a three-terminal cooperative relay-HARQ systems featured by the coordinated retransmission from both the source and the relay. Based on the minimization of the bit error rate (BER) upper bound, the CoRe design was formulated into a quadratic three-dimensional assignment problem (Q3AP), then solved with an efficient modified iterated local search (ILS) method. Our numerical tests have demonstrated the performance gain of the Q3AP-based CoRe which allows the source and relay to use two different remapping schemes during the retransmission other than Gray mapping and verified its robustness against channel state information (CSI) imperfection.

References

[1] H. A. Ngo and L. Hanzo, "Hybrid Automatic-Repeat-reQuest systems for cooperative wireless communications," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 25–45, First Quarter 2014.

[2] G. Benelli, "A new method for the integration of modulation and channel coding in an ARQ protocol," *IEEE Trans. Commun.*, vol. 40, no. 10, pp. 1594–1606, Oct 1992.

[3] C. Wengerter, A. von Elbwart, E. Seidel, G. Velev, and M. Schmitt, "Advanced hybrid ARQ technique employing a signal constellation rearrangement," in *Proc. Vehic. Technol. Conf. (VTC)*, vol. 4, Sep 2002, pp. 2002–2006.

[4] K. Seddik, A. Ibrahim, and K. Liu, "Trans-modulation in wireless relay networks," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 170–172, Mar 2008.

[5] M. Khormuji and E. Larsson, "Improving collaborative transmit diversity by using constellation rearrangement," in *Proc. IEEE Wireless Commun. Networking Conf. (WCNC)*, Mar 2007, pp. 803–807.

[6] ——, "Rate-optimized constellation rearrangement for the relay channel," *IEEE Commun. Lett.*, vol. 12, no. 9, pp. 618–620, Sep 2008.

[7] J. W. Kim, H. Lee, J. Ahn, and C. Kang, "Design of signal constellation rearrangement (CoRe) for multiple relay links," in *Proc. IEEE GLOBECOM*, Nov 2009, pp. 1–6.

[8] H.-S. Ryu, J.-S. Lee, and C. Kang, "BER analysis of constellation rearrangement for cooperative relaying networks over Nakagami-m fading channel," in *Proc. IEEE Int. Commun. Conf. (ICC)*, Jun 2011, pp. 1–5.

[9] T.-W. Yu, W.-H. Sheen, and C.-H. Wang, "Power allocation for cooperative bit-interleaved coded modulation systems with decode-remap-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1712–1721, May 2012.

[10] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep 1979.

[11] R. Nabar, H. Bolcskei, and F. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug 2004.

[12] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec 2005.

[13] H. Samra, Z. Ding, and P. Hahn, "Symbol mapping diversity design for multiple packet transmissions," *IEEE Trans. Commun.*, vol. 53, no. 5, pp. 810–817, May 2005.

[14] G. Finke, R. E. Burkard, and F. Rendl, "Quadratic assignment problems," *North-Holland Mathematics Studies*, vol. 132, pp. 61–82, 1987.

[15] S. Sahni and T. Gonzalez, "P-complete approximation problems," *J. ACM*, vol. 23, no. 3, pp. 555–565, Jul. 1976.

[16] A. Bin Sediq, P. Djukic, H. Yanikomeroglu, and J. Zhang, "Optimized nonuniform constellation rearrangement for cooperative relaying," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2340–2347, Jun 2011.

[17] Z. Drezner, "Extensive experiments with hybrid genetic algorithms for the solution of the quadratic assignment problem," *Computers & Operations Research*, vol. 35, no. 3, pp. 717–736, 2008.

[18] T. James, C. Rego, and F. Glover, "Multistart tabu search and diversification strategies for the quadratic assignment problem," *IEEE Trans. Syst., Man, Cybern. A*, vol. 39, no. 3, pp. 579–596, May 2009.

[19] U. Benlic and J.-K. Hao, "Memetic search for the quadratic assignment problem," *Expert Systems with Applications*, vol. 42, no. 1, pp. 584–595, 2015.

[20] R. E. Burkard, S. E. Karisch, and F. Rendl, "QAPLIB–a quadratic assignment problem library," *Journal of Global optimization*, vol. 10, no. 4, pp. 391–403, 1997.

[21] Y. Qi, R. Hoshyar, M. Imran, and R. Tafazolli, "H2-ARQ-relaying: Spectrum and energy efficiency perspectives," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1547–1558, Sep 2011.

[22] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 510–513, Feb 2002.

[23] M. Schwartz, W. R. Bennett, and S. Stein, *Communication systems and techniques*. John Wiley & Sons, 1995.

[24] C. Sanderson, "Armadillo: An open source C++ linear algebra library for fast prototyping and computationally intensive experiments," 2010.

[25] P. M. Hahn, B.-J. Kim, T. Sttzle, S. Kanthak, W. L. Hightower, H. Samra, Z. Ding, and M. Guignard, "The quadratic three-dimensional assignment problem: Exact and approximate solution methods," *European Journal of Operational Research*, vol. 184, no. 2, pp. 416 – 428, 2008.

[26] H. D. Mittelmann and D. Salvagnin, "On solving a hard quadratic 3-dimensional assignment problem," *Mathematical Programming Computation*, pp. 1–16, 2015.

[27] T. Stützle, "Iterated local search for the quadratic assignment problem," *European Journal of Operational Research*, vol. 174, no. 3, pp. 1519 – 1539, 2006.