Constellation Rearrangement in Cooperative Relay-HARQ Network

Abstract—**We study the constellation rearrangement (CoRe) problem in a multi-relay-HARQ network to achieve symbol mapping diversity for reliable communication. Specifically, we propose a framework of three relay-HARQ protocol to be chosen from according to the channel settings and the capability of cooperation of the relay network. The corresponding bit error rate (BER) based CoRe can be formulated into a quadratic three-dimensional assignment problem (Q3AP) or traditional quadratic assignment problem (QAP) and can be numerically solved with an efficient modified Iteratied Local Search (ILS) method. Performance gains on various channel settings are demonstrated with comprehensive simulations.**

I. Introduction

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ comined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ system can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a same series of bits are mapped to different constellation points across different links.

There are a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER maximization in [7]. CoRe is also studied in Nakagami-m channel [8] and in combination with power allocation [9]. Nevertheless, all the abovementioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on a same time slot or band, resulting in low bandwidth efficiency.

Historically, various CoRe problem for HARQ system fall within the realm of Quadratic Assignment Problem (QAP) or its extensions like Quadratic 3-dimensional Assignment Problem (Q3AP) [10]. Since the CoRe problem is usually formulated into a NP-complete binary linear programming (BIP) problem, existing CoRe implementation are mostly based on fixed rearrangement [4], [9], or by impractically dropping the binary constellation mapping constraints [11]. Other works apply heurisitc/metaheuristic approaches such as simulated annealing [6] and genetic algorithm [7]. It has been reported that some heuristic QAP solvers [12], [13], [14], [15] provides very high-quality solutions over the cases in QAPLIB [16].

In this work, we study the CoRe for cooperative relay-HARQ channel based on BER maximization. The main contributions of this paper are as follows:

- We propose a framework of three relay-HARQ protocol to be chosen from according to the channel settings and the capability of cooperation of the relay network. Similar to the relay-HARQ scheme in [17], multiple-input single-output (MISO) retransmission to the destination is allowed. As depicted in Fig. 1, in a multi-relay networks, when the source and the relay have joint transmission capability, different relays and the source rearrange the constellation mapping differently for the Chase Combining retransmission in order to provide additional signal-space gain. The CoRe design based on BER maximization can be formulated into a Q3AP problem. A simpler scheme where the relays and the source adopt a same constellation rearrangement is also allowed when the performance loss compared with the previous scheme is not significant. When the source and the relay does not have joint transmission capability, a single relay or the source with the best channel can be seleted for the retransmission. The latter 2 schemes have a conventional QAP formulation [10].
- Since Q3AP/QAP formulation is NP-complete, we adopt an efficient iterated local search (ILS) method to solve it numerically. Simulation results demonstrate that the optimized mapping rearrangement provides significant performance gain by exploring the signal-space diversity.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into Q3AP/QAP problem and provides a brief description of the numerical algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model and Problem Formulation

We consider the cooperative relay-HARQ network depicted in Fig. 1. Denote $\mathcal{C}$ as the constellation used by this relay network whose size is $Q = |\mathcal{C}|$. In the first phase, the source convert a bit sequence of length $\log_2 |\mathcal{C}|$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \to \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. Then the source transmit $\psi_0[p]$ to the destination via channel g which is also received by N decode-and-forward (DF) relays, as shown in Fig. 1(a). We assume that the relays are placed strategically so that it has negligible decoding error rate as in [8].

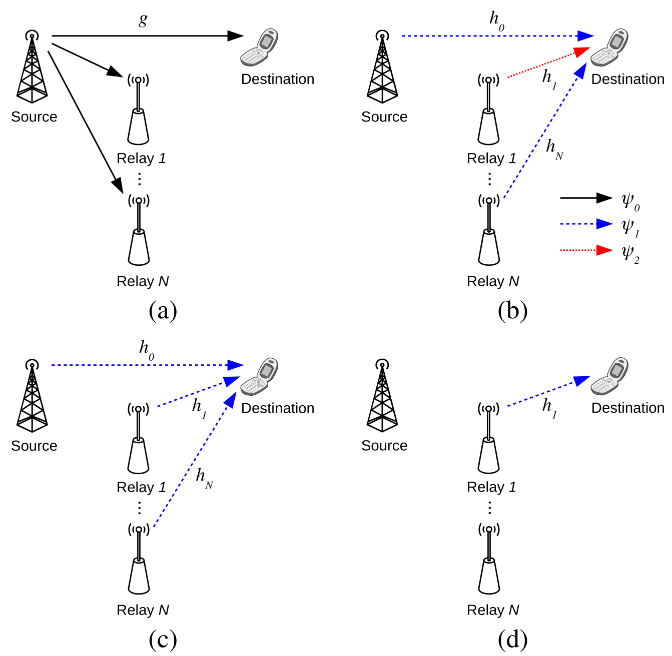

Fig. 1. Cooperative relay-HARQ networks. (a) Transmission in the first phase. (b) Joint retransmission using 2 different remappings. (c) Joint retransmission using 1 different remapping. (d) Selective retransmission using 1 different remapping and 1 node only,

Upon receiving a request for retransmission, the second phase is started. We assume that source and the N relays are capable of joint transmission and are considered a set $\mathcal{T} = \{t_0, t_1, \ldots, t_N\}$ as a whole, where t_0 represents the source and t_n represents the n-th relay, $1 \leq n \leq N$. To better explore the signal-space diversity during the retransmission, $\mathcal{T}$ is further partitioned into two subsets $\mathcal{T}_1$, $\mathcal{T}_2$, and nodes in $\mathcal{T}_1$, $\mathcal{T}_2$ remap p into $\psi_1[p]$ and $\psi_2[p]$, reszpectively. Potentially $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are then transmitted jointly on the same band to the destination via channel $h_0, h_1, \ldots, h_N$, as shown in Fig. 1(b). In summary, the received signal at the destination during the 2 phases are

$$y_t = g\psi_0[p] + v_t \tag{1a}$$

$$\begin{aligned} y_r &= \left(\sum_{n:t_n \in \mathcal{T}_1} h_n\right)\psi_1[p] + \left(\sum_{n:t_n \in \mathcal{T}_2} h_n\right)\psi_2[p] + v_r \\ &= g_1\psi_1[p] + g_2\psi_2[p] + v_r, \end{aligned} \tag{1b}$$

respectively. where $v_t, v_r \sim \mathcal{CN}(0, \sigma_v^2)$ are the additive noise. Thoughout this work, we assume the channels g and h_n, $n = 0, \ldots, N$ follow independent Rician distribution, thus g_1 and g_2 are also Rician distributed.Assuming that the destination has perfect channel state information (CSI), it decides the index p with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_t - g\psi_0[\hat{p}]|^2 + |y_r - g_1\psi_1[\hat{p}] - g_2\psi_2[\hat{p}]|^2. \tag{2}$$

Note that theoretically $\mathcal{T}$ can be partitioned into K subsets where $K > 2$ and the nodes in each individual subset uses a specific mapping. However, the resulting formulation would be a Q$(K+1)$AP problem which is intractable for medium size constellation like 16-QAM. On the other hand, as we will show in Section III, by setting $K = 1$, i.e. $\psi_1 = \psi_2$ as in Fig. 1(c), the CoRe design problem can be simplified from Q3AP to QAP, and our numerical results in Section IV will demonstrate that under certain channel conditions the performance loss is negligible compared to the case where $K = 2$. When the multi-relay network is not capable of joint transmission, we can select a single node with the best channel for retransmission, the formulation of which is essentially the same as [8] (Fig. 1(d)).

III. Optimal Constellation Rearrangement

In this section we first present the the Q3AP/QAP formulations for the min-BER CoRe design, and explain the numerical approach to compute the Q3AP/QAP cost matrices. Then we outline an efficient ILS algorithm which provides very high quality solutions.

A. BER Maximization via Q3AP solution

Firstly we consider the joint retransmission using 2 different remappings as in Fig. 1(b). Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded using pair-wise error probability (PEP) [10]

$$P_{BER} = \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}(q|p) \tag{3}$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q and $P_{PEP}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{g,g_1,g_2,v_t,v_r}\{\delta(p,q,\psi_1,\psi_2) < 0\} \tag{4}$$

i.e. given indices p, q and the remapping scheme ψ_1, ψ_2, the probability of random variable $\delta < 0$ evaluated over the random variables g, g_1, g_2, v_t, v_r, and δ is defined as

$$\begin{aligned} \delta &= |g(\psi_0[p] - \psi_0[q]) + v_t|^2 - |v_t|^2 + \\ &\quad |g_1(\psi_1[p] - \psi_1[q]) + g_2(\psi_2[p] - \psi_2[q]) + v_r|^2 - |v_r|^2. \end{aligned} \tag{5}$$

In order to formulate the Q3AP problem, we denote binary variable $x_{pij} = 1$ if $\psi_1[p] = \psi_0[i]$ and $\psi_2[p] = \psi_0[j]$ and $x_{pij} = 0$ otherwise. Denote $\mathbf{x} = \{x_{pij}|p,i,j = 0,\ldots,Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{\mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\} \tag{6a}$$

$$\mathcal{I} = \left\{\mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\} \tag{6b}$$

$$\mathcal{J} = \left\{\mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}. \tag{6c}$$

Then from (3)(4)(5), the BER-minimization CoRe scheme $\min_{\psi_1,\psi_2} P_{BER}$ can be reformulated as

$$\begin{aligned} &\min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl}x_{pij}x_{qkl} \\ &\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J} \end{aligned} \tag{7}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\} \quad (8)$$

$$\delta = |g(\psi_0[p] - \psi_0[q]) + v_t|^2 - |v_t|^2 + \\ |g_1(\psi_0[i] - \psi_0[k]) + g_2(\psi_0[j] - \psi_0[l]) + v_r|^2 - |v_r|^2. \quad (9)$$

The joint retransmission using 1 different remapping as in Fig. 1(c) has a QAP formulation exactly the same as in [10]: Denote $\mathbf{x} = \{x_{pi}|p, i = 0, \ldots, Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0,1\} \right\} \quad (10a)$$

$$\mathcal{I} = \left\{ \mathbf{x} : \sum_{i=0}^{Q-1} x_{pi} = 1, x_{pi} \in \{0,1\} \right\}. \quad (10b)$$

the BER-minimization $\min_{\psi_1} P_{BER}$ is

$$\min_{\mathbf{x}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} c_{piqk} x_{pi} x_{qk} \quad (11)$$
$$\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I}$$

where c_{piqk} is defined exactly as in (8) by dropping the indeices j, l and the random channel g_2, and δ is redefined as in (5) by setting $g_2 = 0$ and $g_1 = \sum_{n=0}^{N} h_n$.

B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters c_{pijqkl} of the Q3AP problem formulated from the joint retransmission using 2 different remappings. According to (8), the key to compute c_{pijqkl} lies in the evaluation of $P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l)$, i.e. the CDF of random variable $\delta(p,i,j,q,k,l)$ as in (5). For the general Rician channel assumption $g \sim \mathcal{CN}(\mu_g, \sigma_g^2)$, $g_m \sim \mathcal{CN}(\mu_{g_m}, \sigma_{g_m}^2)$, $m = 1,2$, we extend the method in [10], [18] to compute $P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\}$:

$$P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\} \\ \approx \frac{1}{2v} \sum_{t=1}^{v} \Re\{\Phi_\delta(\xi + j\xi\tau_t)\} + \tau_t \Im\{\Phi_\delta(\xi + j\xi\tau_t)\} \quad (12)$$

where the moment generating function (MGF) $\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)]$, $\tau_t = \tan((t-1/2)\pi/v)$ and $\Re\{\cdot\}$, $\Im\{\cdot\}$ denotes the real and image part, respectively. Parameter ξ is selected to ensure convergence of the integration and [18] suggested to use $\xi = 1/4$. The size u of the expansion (12) needs to be larger when $P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\}$ is smaller in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote Gaussian random vectors $\mathbf{z}_1 = [g, v_t]^T$, $\mathbf{z}_2 = [g_1, g_2, v_r]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1,2$, where

$$\boldsymbol{\mu}_1 = [\mu_g, 0]^T,\ \boldsymbol{\mu}_2 = [\mu_{g_1}, \mu_{g_2}, 0]^T \quad (13)$$

$$\boldsymbol{\Sigma}_1 = \text{diag}\left(\sigma_g^2, \sigma_v^2\right),\ \boldsymbol{\Sigma}_2 = \text{diag}\left(\sigma_{g_1}^2, \sigma_{g_2}^2, \sigma_v^2\right) \quad (14)$$

Then (5) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix} \quad (15a)$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix} \quad (15b)$$

in which where $e_{ab} = \psi_0[a] - \psi_0[b]$. Then the MGF can be computed as [19]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \quad (16)$$

When the Rician-fading channel reduces to Rayleigh-fading channel, i.e. when $\mu_g = \mu_{g_1} = \mu_{g_2}$ there is a simple upperbound of $P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\}$ similar to [4]. Note that

$$P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\} \\ = \mathbb{E}_{g,g_1,g_2}\left\{ Q\left(\sqrt{\frac{|ge_{pq}|^2 + |g_1 e_{ik} + g_2 e_{jl}|^2}{2\sigma_v^2}} \right) \right\} \quad (17)$$

Applying the same relaxation as in [4], we have

$$P_{g,g_1,g_2,v_t,v_r}\{\delta(p,i,j,q,k,l) < 0\} \\ \leq \frac{3\sigma_v^4}{\sigma_g^2 |e_{pq}|^2 (\sigma_{g_1}^2 |e_{ik}|^2 + \sigma_{g_2}^2 |e_{jl}|^2)} \quad (18)$$

which is tight in high-SNR regime.

The computation of the cost matrix c_{piqk} for the joint retransmission using 1 different remapping is very similar to the above procedure and is thus omitted.

C. Q3AP Solution

IV. Numerical Results

V. Conclusion

In this work we investigated the constellation rearrangement (CoRe) problem in cooperative relay-HARQ system. Based on the capacity of coordination of the relay network and the channel settings, three retransmission schemes can be chosen from. The corresponding maximimum bit error rate (BER) based CoRe design problems can formulated into either quadratic three-dimensional assignment problem (Q3AP) or quadratic assignment problem (QAP), then solved with an efficient modified iterative local search (ILS) method. Our numerical tests demonstrated the performance gain of the Q3AP/QAP-based CoRe which allows the source and relay to use different remapping schemes during the retransmission other than Gray mapping and verified its robustness against channel state information (CSI) imperfection.

References

[1] H. A. Ngo and L. Hanzo, "Hybrid automatic-repeat-request systems for cooperative wireless communications," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 25–45, First Quarter 2014.

[2] G. Benelli, "A new method for the integration of modulation and channel coding in an ARQ protocol," *IEEE Trans. Commun.*, vol. 40, no. 10, pp. 1594–1606, Oct 1992.

[3] C. Wengerter, A. von Elbwart, E. Seidel, G. Velev, and M. Schmitt, "Advanced hybrid ARQ technique employing a signal constellation rearrangement," in *Proc. Vehic. Technol. Conf. (VTC)*, vol. 4, Sep 2002, pp. 2002–2006.

[4] K. Seddik, A. Ibrahim, and K. Liu, "Trans-modulation in wireless relay networks," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 170–172, Mar 2008.

[5] M. Khormuji and E. Larsson, "Improving collaborative transmit diversity by using constellation rearrangement," in *Proc. IEEE Wireless Commun. Networking Conf. (WCNC)*, Mar 2007, pp. 803–807.

[6] ——, "Rate-optimized constellation rearrangement for the relay channel," *IEEE Commun. Lett.*, vol. 12, no. 9, pp. 618–620, Sep 2008.

[7] J. W. Kim, H. Lee, J. Ahn, and C. Kang, "Design of signal constellation rearrangement (CoRe) for multiple relay links," in *Proc. IEEE GLOBECOM*, Nov 2009, pp. 1–6.

[8] H.-S. Ryu, J.-S. Lee, and C. Kang, "BER analysis of constellation rearrangement for cooperative relaying networks over Nakagami-m fading channel," in *Proc. IEEE Int. Commun. Conf. (ICC)*, Jun 2011, pp. 1–5.

[9] T.-W. Yu, W.-H. Sheen, and C.-H. Wang, "Power allocation for cooperative bit-interleaved coded modulation systems with decode-remap-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1712–1721, May 2012.

[10] H. Samra, Z. Ding, and P. Hahn, "Symbol mapping diversity design for multiple packet transmissions," *IEEE Trans. Commun.*, vol. 53, no. 5, pp. 810–817, May 2005.

[11] A. Bin Sediq, P. Djukic, H. Yanikomeroglu, and J. Zhang, "Optimized nonuniform constellation rearrangement for cooperative relaying," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2340–2347, Jun 2011.

[12] T. Stützle, "Iterated local search for the quadratic assignment problem," *European Journal of Operational Research*, vol. 174, no. 3, pp. 1519 – 1539, 2006. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0377221705002651

[13] Z. Drezner, "Extensive experiments with hybrid genetic algorithms for the solution of the quadratic assignment problem," *Computers & Operations Research*, vol. 35, no. 3, pp. 717–736, 2008.

[14] T. James, C. Rego, and F. Glover, "Multistart tabu search and diversification strategies for the quadratic assignment problem," *IEEE Trans. Syst., Man, Cybern. A*, vol. 39, no. 3, pp. 579–596, May 2009.

[15] U. Benlic and J.-K. Hao, "Memetic search for the quadratic assignment problem," *Expert Systems with Applications*, vol. 42, no. 1, pp. 584–595, 2015.

[16] R. E. Burkard, S. E. Karisch, and F. Rendl, "Qaplib–a quadratic assignment problem library," *Journal of Global optimization*, vol. 10, no. 4, pp. 391–403, 1997.

[17] Y. Qi, R. Hoshyar, M. Imran, and R. Tafazolli, "H2-ARQ-relaying: Spectrum and energy efficiency perspectives," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1547–1558, Sep 2011.

[18] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 510–513, Feb 2002.

[19] M. Schwartz, W. R. Bennett, and S. Stein, *Communication systems and techniques*. John Wiley & Sons, 1995.