Constellation Rearrangement in Cooperative Relay-HARQ Network

Abstract—**We study the constellation rearrangement (CoRe) problem in a relay-HARQ network to achieve symbol mapping diversity for reliable communication. Specifically, we formulate the bit error rate (BER) maximization into a quadratic three-dimensional assignment problem (Q3AP) and solve it with an efficient modified Iterated Local Search (ILS) method to find the optimal CoRe solution. Performance gains on various channel settings are demonstrated with simulations.**

I. Introduction

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ combined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ systems can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a series of identical bits are mapped to different constellation points across different links.

There is a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate-optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER maximization in [7]. CoRe is also studied for the Nakagami-m channel [8] and in combination with power allocation [9]. Nevertheless, all the abovementioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on the same time slot or band, resulting in low bandwidth efficiency.

Historically, various CoRe problems for HARQ systems fall within the realm of the Quadratic Assignment Problem (QAP) or its extensions, such as the Quadratic 3-dimensional Assignment Problem (Q3AP) [10]. Since the CoRe problem is usually formulated into an NP-complete binary linear programming (BIP) problem, existing CoRe implementations are mostly based on a fixed rearrangement [4], [9], or by impractically dropping the binary constellation mapping constraints [11]. Other works apply heuristic/metaheuristic approaches such as simulated annealing [6] and genetic algorithms [7]. It has been reported that some heuristic QAP solvers [12], [13], [14], [15] provide very high-quality solutions over the cases in QAPLIB [16].

In this work, we study the CoRe for cooperative relay-HARQ channel based on BER maximization. The main contributions of this paper are as follows:

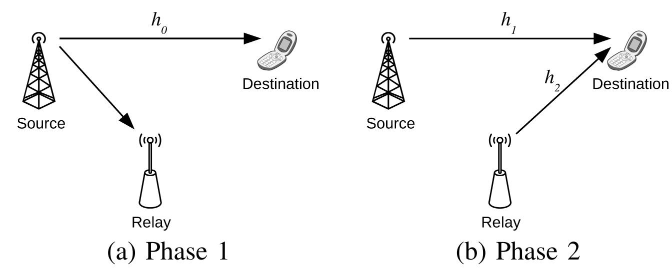


Fig. 1. Cooperative relay-HARQ networks.

- We propose to use CoRe for the relay-HARQ scheme similar to [17]. As depicted in Fig. 1, the source and the relay jointly perform the Chase Combining retransmission to the destination in a coordinated fashion, practically forming a 2-by-1 multiple-input single-output MISO system.
- In our cooperative relay-HARQ settings, we formulate the CoRe design into a Q3AP problem. Since Q3AP/QAP formulations are NP-complete, we adopt an efficient iterated local search (ILS) method to solve it numerically. Simulation results demonstrate that the optimized mapping rearrangement provides significant performance gain by exploring the signal-space diversity.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into a Q3AP solution and provides a brief description of the numerical algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model and Problem Formulation

We consider the cooperative relay-HARQ network depicted in Fig. 1. Denote $\mathcal{C}$ as the constellation used by this relay network whose size is $Q = |\mathcal{C}|$. In the first phase, the source converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \dots, Q-1\} \to \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \dots, Q-1\}$. Then the source transmits $\psi_0[p]$ to the destination via channel h_0 which is also overheard by the decode-and-forward (DF) relay. We assume that this relay is placed strategically so that it has negligible decoding error rate as in [8], [7]. Upon receiving a request for retransmission, the second phase is started. This time the source and the relay adopt a remap p into $\psi_1[p]$ and $\psi_2[p]$, respectively, where potentially $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are transmitted simultaneously on the same band to the destination via channel h_1 and h_2. In summary, the received signals at the destination during the 2 phases are

$$y_1 = h_0\psi_0[p] + v_1, \tag{1a}$$
$$y_2 = h_1\psi_1[p] + h_2\psi_2[p] + v_2, \tag{1b}$$

where $v_1 \sim \mathcal{CN}(0, \sigma_v^2)$ and $v_2 \sim \mathcal{CN}(0, \sigma_v^2)$ are the additive noise. Throughout this work, we assume the channels h_0, h_1 and h_2 follows independent Rician distribution.

Assuming that the destination has perfect channel state information (CSI), it decides on the index p with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_1 - \hat{h}_0\psi_0[\hat{p}]|^2 + |y_2 - \hat{h}_1\psi_1[\hat{p}] - \hat{h}_2\psi_2[\hat{p}]|^2. \tag{2}$$

III. Optimal Constellation Rearrangement

In this section we first formulate the min-BER CoRe design into a Q3AP problem, and explain the numerical approach to compute the input cost matrix of the Q3AP problem. Then we provide an efficient algorithm to determine the Q3AP solution.

A. BER Maximization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded using pair-wise error probability (PEP) [10]

$$P_{BER} = \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}(q|p), \tag{3}$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q and $P_{PEP}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,q,\psi_1,\psi_2) < 0\}, \tag{4}$$

i.e. given indices p, q and the remapping scheme ψ_1, ψ_2, the probability of the random variable $\delta < 0$ being evaluated over the random variables h_0, h_1, h_2, v_0, v_1, and δ is defined as

$$\begin{aligned}\delta = &|h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + \\ &|h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_1|^2 - |v_1|^2.\end{aligned} \tag{5}$$

In order to formulate the Q3AP problem, we introduce the binary variable $x_{pij} = 1$ if $\psi_1[p] = \psi_0[i]$ and $\psi_2[p] = \psi_0[j]$ and $x_{pij} = 0$ otherwise. Denote $\mathbf{x} = \{x_{pij}|p, i, j = 0, \ldots, Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{\mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}, \tag{6a}$$

$$\mathcal{I} = \left\{\mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}, \tag{6b}$$

$$\mathcal{J} = \left\{\mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}. \tag{6c}$$

Then from (3)(4)(5), the BER-minimization CoRe scheme $\min_{\psi_1,\psi_2} P_{BER}$ can be reformulated as

$$\begin{aligned}&\min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl} x_{pij} x_{qkl}, \\ &\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}.\end{aligned} \tag{7}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}, \tag{8}$$

$$\begin{aligned}\delta = &|h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + \\ &|h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_1|^2 - |v_1|^2.\end{aligned} \tag{9}$$

B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters $\{c_{pijqkl}\}$ of the Q3AP problem. According to 8, the key to compute c_{pijqkl} lies in the evaluation of $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l)$, i.e. the CDF of the random variable $\delta(p,i,j,q,k,l)$ as in (5). Under the general Rician channel assumption $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma_{h_m}^2), m = 0,1,2$, we extend the method in [10], [18] to compute $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$:

$$\begin{aligned}&P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\ &\approx \frac{1}{2v}\sum_{t=1}^{v} \Re\{\Phi_\delta(\xi + j\xi\tau_t)\} + \tau_t \Im\{\Phi_\delta(\xi + j\xi\tau_t)\},\end{aligned} \tag{10}$$

with the moment generating function (MGF) $\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)]$, $\tau_t = \tan((t - 1/2)\pi/v)$ and $\Re\{\cdot\}$, $\Im\{\cdot\}$ denoting its real and image part, respectively. The parameter ξ is selected to ensure convergence of the integration and [18] it is suggested to use $\xi = 1/4$. The size u of the expansion (10) needs to be larger when $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$ is smaller in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote the Gaussian random vectors $\mathbf{z}_1 = [h_0, v_0]^T$, $\mathbf{z}_2 = [h_1, h_2, v_1]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1, 2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T, \tag{11}$$
$$\boldsymbol{\Sigma}_1 = \text{diag}\left(\sigma_{h_0}^2, \sigma_v^2\right), \boldsymbol{\Sigma}_2 = \text{diag}\left(\sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_v^2\right). \tag{12}$$

Then (5) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix}, \tag{13a}$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix}, \tag{13b}$$

here $e_{ab} = \psi_0[a] - \psi_0[b]$. Then the MGF can be computed as [19]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega\boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega\boldsymbol{\Sigma}_m\mathbf{A}_m)^{-1}\boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega\boldsymbol{\Sigma}_m\mathbf{A}_m)}. \tag{14}$$

When the Rician-fading channel reduces to the Rayleigh-fading channel, i.e. when $\mu_{h_m} = 0, m = 0, 1, 2$ there is a simple upper bound of $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$ similar to [4]. Note that

$$
\begin{aligned}
&P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\
&= \mathbb{E}_{h_0,h_1,h_2}\left\{ Q\left(\sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma_v^2}} \right) \right\}.
\end{aligned} \tag{15}
$$

Applying the same relaxation as in [4], we have

$$
\begin{aligned}
&P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\
&\leq \frac{3\sigma_v^4}{\sigma_{h_0}^2 |e_{pq}|^2 (\sigma_{h_1}^2 |e_{ik}|^2 + \sigma_{h_2}^2 |e_{jl}|^2)}.
\end{aligned} \tag{16}
$$

which is tight in the high-SNR regime.

C. Q3AP Solution

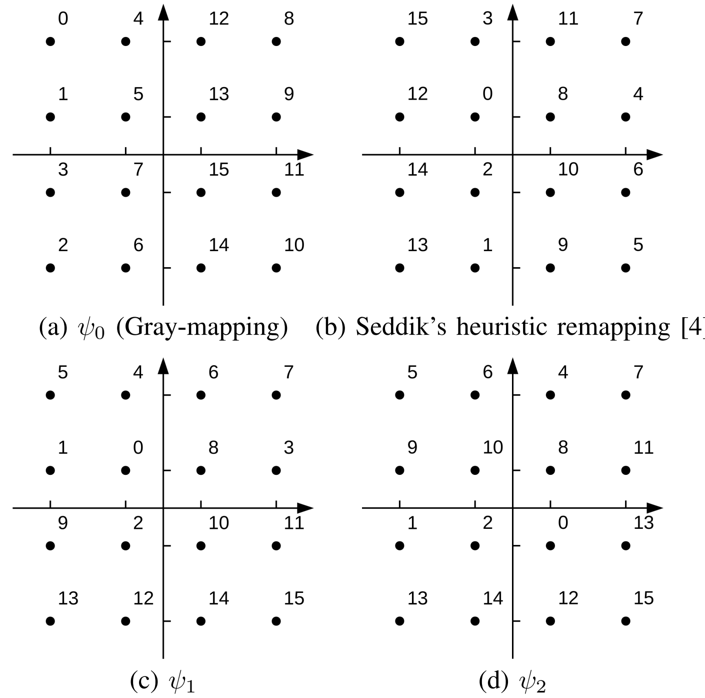
For Q3AP problems of size $Q = 16$ it is impractical to apply the exact branch-and-bound algorithm [20]. Also, our tests show that 16-QAM does not have enough symmetry to exploit as does the 16-PSK constellation [21]. Consequently, the CoRe problem is solved with the ILS method [20] extended from its QAP version [12]. The basic procedure of this ILS method is outlined as follows:

- S1 Initialization: starting from a random mapping $\psi_1^{(0)}, \psi_2^{(0)}$.
- S2 Local search: find a locally optimum solution $\psi_1^{(0)*}, \psi_2^{(0)*}$. Set $n = 0$
- S3 Perturbation: from the last locally optimum solution $\psi_1^{(n)*}, \psi_2^{(n)*}$, generate a new mapping $\psi_1^{(n+1)}, \psi_2^{(n+1)}$.
- S4 Local search: from $\psi_1^{(n+1)}, \psi_2^{(n+1)}$, find a new locally optimum solution $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$.
- S5 Examination of the acceptance criterion: compare $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ with $\psi_1^{(n)*}, \psi_2^{(n)*}$. If the new mapping is not accepted, reset $\psi_1^{(n+1)*} = \psi_1^{(n)*}$ and $\psi_2^{(n+1)*} = \psi_2^{(n)*}$. Set $n \leftarrow n + 1$.
- S6 Return to S3 until the stopping criterion is satisfied.

The local search in S2 and S4 attempts to exchange the mapping of exactly 2 indices in ψ_1 or ψ_2 in order to reduce the objective function, i.e. a 2-opt neighborhood search. A first-improvement rule is used, which means that whenever a reduction in the objective function is made the mapping is updated. The perturbation in S3 is performed by exchanging the mapping of k_p indices, where $k_p \in [k_{p,min}, k_{p,max}]$ is adaptively increased if the perturbation does not produce a better solution or is reset to $k_{p,min}$ otherwise. For the acceptance criterion examination in S5, $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ is accepted if the objective function $f^{(n+1)} < f^{(n)}$, or accepted with probability $\exp(-[f(n+1) - f(n)]/T)$, where T is the temperature parameter similar to that of Simulated Annealing (SA).

IV. Numerical Results

In this section we present the numerical results of CoRe under various channel settings. In our simulation, all Rician channels are assumed to have the same Rician parameter K.



(a) ψ_0 (Gray-mapping) (b) Seddik's heuristic remapping [4]

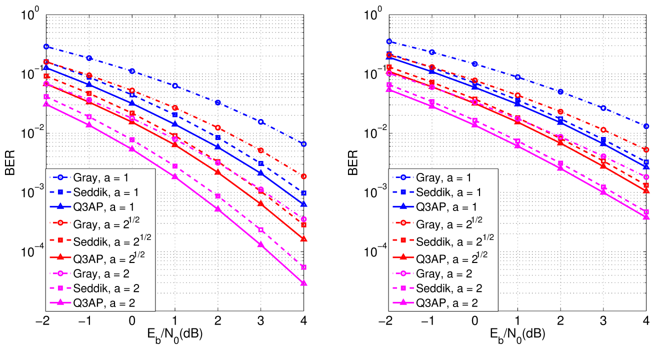
(c) ψ_1 (d) ψ_2

Fig. 2. Q3AP optimized CoRe schemes.

Also we assume that during the second phase, the phases of channels h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma_{h_0}^2 = \sigma_{h_1}^2 = 1/(K+1)$ and $\sigma_{h_2}^2 = |a|^2/(K+1)$, where a represents the ratio between the amplitude of the line of sight (LOS) components of the relay-to-destination and the source-to-destination link. Throughout the simulation we consider 16-QAM constellation thus $Q = 16$. The noise power is parameterized with E_b/N_0 of the source-to-destination link.

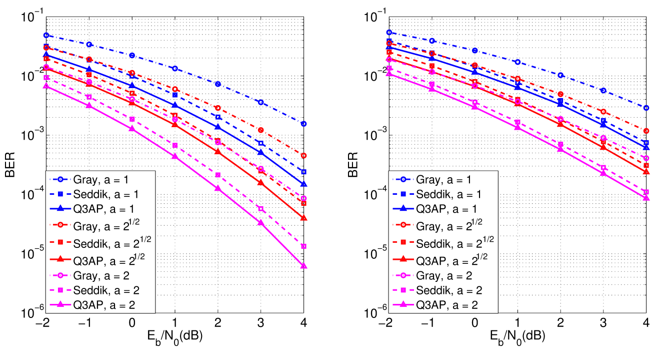
First, we provide an example of Q3AP optimized CoRe. When $E_b/N_0 = 2$dB, $K = 10$ and $a = 1$, the remapping scheme of ψ_0 (Gray-mapping), ψ_1 and ψ_2 is depicted in Fig. 2. We note that for the chase combining HARQ scheme and the joint ML demodulator, the optimal ψ_1 and ψ_2 are no longer a Gray mapping. A general pattern of ψ_1 and ψ_2 is that the indices whose constellation points are close to each other during the first phase are remapped to constellation points far away from each other during the second phase, and vice versa. Both ψ_1 and ψ_2 are essentially the same as Seddik's heuristic result in [4], denoted as ψ_S. However, it is noted $\psi_1 \neq \psi_2$. In this special example or Rician channel with strong LOS component, an explanation is that the cooperative transmission is equivalent to transmission from a single transmitter with mapping $\psi_1 + \psi_2$. In this aspect, CoRe in effect results in a new constellation different from 16-QAM, while the individual constellations at the source and relay are kept unchanged.

Next we compare the BER upper bound P_{BER}, which is also the Q3AP objective function, for three mapping schemes: the Q3AP optimized CoRe, the extension of Seddik's remapping $\psi_1 = \psi_2 = \psi_S$, and no CoRe at all i.e. $\psi_1 = \psi_2 = \psi_0$. In Fig. 3, there is a considerable performance gain for CoRe versus no CoRe at all. Moreover, the gain of the Q3AP



(a) $K = 10$ (b) $K = 5$

Fig. 3. BER upper bounds of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.



(a) $K = 10$ (b) $K = 5$

Fig. 4. Monte-Carlo simulated BER of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

optimized CoRe over $\psi_1 = \psi_2 = \psi_S$ indicates that in cooperative relay-HARQ networks, the BER performance can be further improved by using different rearrangements at the source and relay during the cooperative transmission. This gain increases as K increases, i.e. when the channel becomes less random.

The performance gain of Q3AP optimized CoRe for the cooperative relay-HARQ system is further justified in Fig. 4, where we evaluate the actual BER by running a Monte-Carlo simulation. We implement the ML demodulator in (2) and evaluate the average BER of $M = 10^7$ randomly generated indices p. The actual BER curves demonstrate a similar pattern as the BER upper bound curves in Fig. 3.

Finally, we test the robustness of the Q3AP CoRe solution against mismatch in channel settings. The intuition is that in practical, the CSI may not be perfectly accurate. Also, the Q3AP CoRe scheme can only be computed off-line for a finite number of channel settings. We deliberately test the BER performance of the Q3AP CoRe solution for $a = 1$, $K = 5$ in the actual channel settings $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$. The BER upper bound and Monte-Carlo simulated BER are plotted in Fig. 5. Apparently, the Q3AP CoRe solution is not sensitive to the relative amplitude and the phase alignment error between the relay-to-destination and source-to-destination links.

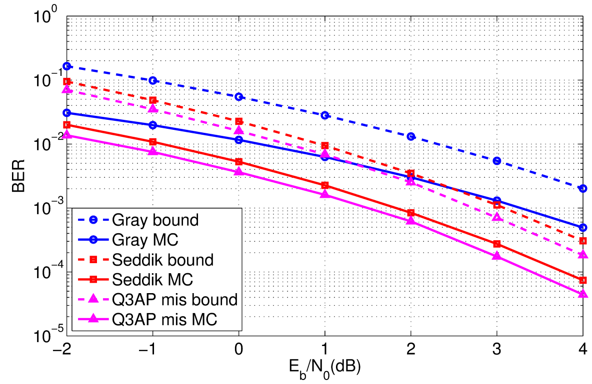


Fig. 5. BER performance under channel mismatch. Q3AP CoRe solution for $a = 1$, $K = 5$ in the actual channel settings $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$.

V. Conclusion

In this work we have investigated the constellation rearrangement (CoRe) problem in cooperative relay-HARQ systems. Based on the maximization of the bit error rate (BER) upper bound, the CoRe design was formulated into a quadratic three-dimensional assignment problem (Q3AP), then solved with an efficient modified iterated local search (ILS) method. Our numerical tests have demonstrated the performance gain of the Q3AP-based CoRe which allows the source and relay to use different remapping schemes during the retransmission other than Gray mapping and verified its robustness against channel state information (CSI) imperfection.

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