Modulation Diversity Design in Cooperative Relay and HARQ Transmission

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Abstract

Modulation diversity (MoDiv) is a simple and practical transmission enhancement technique that utilizes different modulation mappings to reduce packet loss rate and achieve higher link throughput. MoDiv is particularly meaningful and effective in hybrid-ARQ (HARQ) systems. In this paper, we study the deployment and optimization of MoDiv in a coordinated relay-HARQ network to mitigate packet loss. We formulate the design optimization of MoDiv into a quadratic three-dimensional assignment problem (Q3AP), which we solve using a modified iterated local search (ILS) method. Numerical results demonstrate clear performance gain over simple relay/retransmissions and over a heuristic design under fading channels.

Index Terms

Modulation diversity, relay, HARQ, Q3AP.

I. Introduction

In wireless data communication systems, high rate transmission under poor channel conditions often lead to reception errors. To recover lost packet due to transmission errors and packet loss. To recover lost packets, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are important mechanisms for improving reliability at both network layer [1] and PHY layer [2]. HARQ in conjunction with relay networks has attracted substantial research interest in recent years [3]. Because data symbols in practical transmission often utilize linear modulations of finite-size constellation (e.g., Q-ary PSK, or Q-ary QAM), the performance of cooperative relay-HARQ systems can benefit from Modulation Diversity (MoDiv) [4], in which each group of $\log_2 Q$ bits

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are mapped to different constellation points across different links and in different transmissions to better exploit channel diversity.

There are several known research works on MoDiv for HARQ [5], relay networks [6], [7] and relay-HARQ systems [8], [9]. Despite the promising performance gain, these works are developed for a relay network setting that tend to consume substantial bandwidth for which (re)transmissions on the Source-Destination (S-D) link and the Relay-Destination (R-D) links occupy orthogonal channels (frequency bands or time slots). In order to achieve better spectral efficiency, we study MoDiv design for the relay-HARQ configuration of Fig. 1 in which the S-D and R-D links are co-channel and additive as in the classic scenario [10], [11]. Such configuration specifically forms a 2-by-1 multiple-input single-output (MISO) diversity transmission. For this system configuration, we formulate the bit error rate (BER) minimization MoDiv design into a quadratic three-dimensional assignment problem (Q3AP). As Q3AP is NP-hard, our MoDiv design cannot be solved exactly even for a moderate size constellation such as 16-QAM. Fortunately, there exist a number of heuristic algorithms [12] for its reduced form, known as the Quadratic Assignment Problem (QAP), that results in high-quality solutions over the QAPLIB dataset. In this work, we adopt a modified Iterated Local Search (ILS) method that is efficient for solving the Q3AP formulated from the MoDiv design problem for minimizing reception BER. Our numerical results demonstrate significant BER reduction over non-MoDiv (simple) HARQ re-transmissions. Moreover, we also show that when network channels are in heavy fading conditions, comparable performance gain can also be achieved by a low complexity heuristic MoDiv scheme.

In this manuscript, Section II first describes the relay-HARQ cooperation model. Section III presents the optimal MoDiv design problem and propose different solutions. Section IV provides and compares the numerical results to illustrate the benefit of MoDiv. Finally, Section V concludes this work.

II. System Model

We consider the cooperative relay-HARQ network shown in Fig. 1, in which there are two transmission phases. In phase 1, source node broadcasts its packet to both destination and relay nodes. In phase 2 of this HARQ setup, upon packet loss notification, both relay and source nodes cooperatively retransmit the lost packet information to the destination. Our goal is to optimize H-ARQ constellation mappings to improve modulation diversity so as to minimize the packet BER at the receiver.

(a) Phase 1 (b) Phase 2

Fig. 1. Cooperative relay-HARQ networks.

Denote $\mathcal{C}$ as the primary constellation used by this relay network whose cardinality equals $Q = |\mathcal{C}|$. In the first phase, the source converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \rightarrow \mathcal{C}$. The bit sequence can be indexed by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. The source transmits $\psi_0[p]$ to the destination via channel h_0, though $\psi_0[p]$ is simultaneously received by the decode-and-forward (DF) relay. We assume the relay to be placed strategically such that it has negligible decoding error rate as in [9], [8]. Upon receiving a request for retransmission, the second phase begins with the source and the relay re-mapping p into $\psi_1[p]$ and $\psi_2[p]$, respectively. In general, we have 3 distinct mappings $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are transmitted simultaneously on the same channel band to the destination via channels h_1 and h_2. In summary, the received signals at the destination during the two phases are, respectively,

$$y_1 = h_0 \psi_0[p] + v_1, \tag{1a}$$

$$y_2 = h_1 \psi_1[p] + h_2 \psi_2[p] + v_2, \tag{1b}$$

where $v_1, v_2 \sim \mathcal{CN}(0, \sigma_v^2)$ are additive channel noises. Throughout this work, we assume fading wireless channels h_0, h_1, and h_2 to follow independent Rician distribution.

Assuming that the destination acquires perfect channel state information (CSI). Based on the received symbols y_1 and y_2, the destination decodes the data by identifying index p via maximum likelihood (ML) detection:

$$\min_{\hat{p}} |y_1 - h_0 \psi_0[\hat{p}]|^2 + |y_2 - h_1 \psi_1[\hat{p}] - h_2 \psi_2[\hat{p}]|^2. \tag{2}$$

III. Optimal Constellation Mapping for Modulation Diversity

In this section we first formulate the minimum BER design of MoDiv into a Q3AP problem. We then elaborate on the numerical approach for computing the input cost matrix of the Q3AP problem. We then provide an efficient algorithm to obtain numerical Q3AP solution.

A. BER Minimization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded and approximated using the pair-wise error probability (PEP) [5]

$$P_{\mathrm{BER}} \approx \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{\mathrm{PEP}}(q|p), \tag{3}$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q normalized by $\log_2 Q$ and $P_{\mathrm{PEP}}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{\mathrm{PEP}}(q|p) = P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,q,\psi_1,\psi_2) < 0\}. \tag{4}$$

in which, given random channels and random noise variables h_0, h_1, h_2, v_1, v_2, δ is defined as

$$\begin{aligned} \delta = & |h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 + \\ & |h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_2|^2 - |v_2|^2. \end{aligned} \tag{5}$$

In other words, given indices p, q and the remapping scheme ψ_1, ψ_2, the pairwise error event is equivalent to $\delta < 0$. In order to formulate the Q3AP problem, we introduce binary variables

$$x_{pij} = \begin{cases} 1, & \text{if } \psi_1[p] = \psi_0[i] \text{ and } \psi_2[p] = \psi_0[j] \\ 0, & \text{otherwise.} \end{cases}$$

Denote $\mathbf{x} = \{x_{pij} |\, p, i, j = 0, \ldots, Q-1\}$ and constraint set

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\} \right\}. \tag{6}$$

We further denote $\mathcal{I}$ and $\mathcal{J}$ as in (6) by replacing the summation index p with i and j, respectively. Then from (3)(4)(5), the BER minimization MoDiv scheme $\min_{\psi_1,\psi_2} P_{\mathrm{BER}}$ becomes

$$\begin{aligned} & \min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl} x_{pij} x_{qkl}, \\ & \text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}. \end{aligned} \tag{7}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}, \tag{8}$$

$$\begin{aligned} \delta = & |h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_2|^2 \\ & + |h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 - |v_2|^2. \end{aligned} \tag{9}$$

B. Computation of the Pair-wise Symbol Error Rate

In this section we focus on computing parameters $\{c_{pijqkl}\}$ of the Q3AP problem. According to (8), the key to evaluating c_{pijqkl} lies in the evaluation of $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l)$, i.e. the cumulative distribution function (CDF) of the random variable $\delta(p,i,j,q,k,l)$ of (5). Define the moment generating function (MGF)

$$\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)].$$

Under the well known Rician channel model, $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma^2_{h_m}), m = 0,1,2$, we can extend the method proposed in [5], [13] to compute $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$:

$$P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\} \approx \frac{1}{2v}\sum_{t=1}^{v} \mathrm{Re}\left\{\Phi_\delta(\xi + \mathrm{j}\xi\tau_{\mathrm{t}})\right\} + \tau_{\mathrm{t}}\mathrm{Im}\left\{\Phi_\delta(\xi + \mathrm{j}\xi\tau_{\mathrm{t}})\right\}, \tag{10}$$

where $\tau_t = \tan((t-1/2)\pi/v)$ and $\mathrm{Re}\{\cdot\}$, $\mathrm{Im}\{\cdot\}$ denote the real and imaginary parts, respectively. The parameter ξ is selected to ensure convergence of the integration and $\xi = 1/4$ was suggested in [13]. The size v of the expansion (10) needs to be large when $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$ is small in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, let us denote Gaussian random vectors $\mathbf{z}_1 = [h_0, v_1]^T$, $\mathbf{z}_2 = [h_1, h_2, v_2]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1,2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T, \tag{11}$$

$$\boldsymbol{\Sigma}_1 = \mathrm{diag}\left(\sigma^2_{h_0}, \sigma^2_v\right), \boldsymbol{\Sigma}_2 = \mathrm{diag}\left(\sigma^2_{h_1}, \sigma^2_{h_2}, \sigma^2_v\right). \tag{12}$$

Then (5) can be rewritten as $\delta = \mathbf{z}_1^H\mathbf{A}_1\mathbf{z}_1 + \mathbf{z}_2^H\mathbf{A}_2\mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e^*_{pq} \\ e_{pq} & 0 \end{bmatrix}, \ \mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e^*_{ik}e_{jl} & e^*_{ik} \\ e_{ik}e^*_{jl} & |e_{jl}|^2 & e^*_{jl} \\ e_{ik} & e_{jl} & 0 \end{bmatrix}, \tag{13}$$

in which the notation $e_{ab} = \psi_0[a] - \psi_0[b]$. Thus, the MGF is

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \tag{14}$$

Note that for any instance of Q3AP a total number of Q^6 coefficients must be computed. Fortunately, the MoDiv design is based on statistical CSI which does not require online computation of the Q^6 coefficients in real time. In fact, the optimized MoDiv can be precomputed and stored a priori in our network nodes. In our simulation, we implement the above procedure for 16-QAM and 32-QAM with in C++ on a workstation with 48 cores and finished the computation in several days for a $Q = 32$ case and a few hours for a 16-QAM case. For larger constellation such as 64-QAM, however, the time and spacial complexity may still be impractical. We will address in future works new means to reduce this complexity by imposing rules to restrict the remapping schemes.

C. Q3AP Solution

For modest constellation sizes such as 16-QAM and 32-QAM, it is impractical to apply the exact branch-and-bound algorithm [14]. Also, our tests show that they do not have enough symmetry to exploit for faster solution as does the 16-PSK constellation [15]. Consequently, the MoDiv problem is solved with the ILS method [14] extended from its QAP version [16]. Starting from two random initial mappings $\psi_1^{(0)}, \psi_2^{(0)}$, the algorithm executes a local search by exchanging the mappings of exactly 2 indices whenever a reduction in the objective function is made. This approach can lower the objective function and update the mapping locally. When the process hits a local minimum, it executes a perturbation step by exchanging the mappings of k_p indices, where integer k_p is adaptively adjusted within a range $[k_{p,min}, k_{p,max}]$. The perturbation is accepted with a probability defined as in simulated annealing, after which the local search is restarted from the new mappings until the stopping criterion is satisfied.

D. A Heuristic MoDiv Scheme

For Rician fading channels, there is a strong Line Of Sight (LOS) component in h_1 and h_2. Our Q3AP solution benefits from 2 different gains. Firstly, by allowing $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$, we achieves the signal space diversity gain just as existing MoDiv schemes. Secondly, by jointly designing ψ_1 and ψ_2 and allowing $\psi_1 \neq \psi_2$, we achieve the cooperative gain between the source and the relay. When the channels are independently Rayleigh, however, there is little cooperative gain to exploit and we can greatly simplify the MoDiv design problem by forcing equal HARQ mapping, i.e., $\psi_1 = \psi_2$. Though we can solve the resulting QAP problem rigorously, from (5), we can take a simple heuristic approach noticing that the two indices mapped to two symbols close to each other in Phase 1 should be mapped to two symbols far apart in Phase 2. Based on such heuristic, $\psi_1 = \psi_2$ can be designed by adapting the trans-modulation proposal of Seddik [6] for 16-QAM and 64-QAM. For 32-QAM constellation, a similar heuristic MoDiv design may be extended by remapping the 3 Most Significant Bits (MSBs) and the 2 Least Significant Bits (LSBs) separately. We will show in the next section that, when channels experience deep fading, such heuristic remapping method offers comparable performance gain as our Q3AP-based MoDiv.

IV. Numerical Results

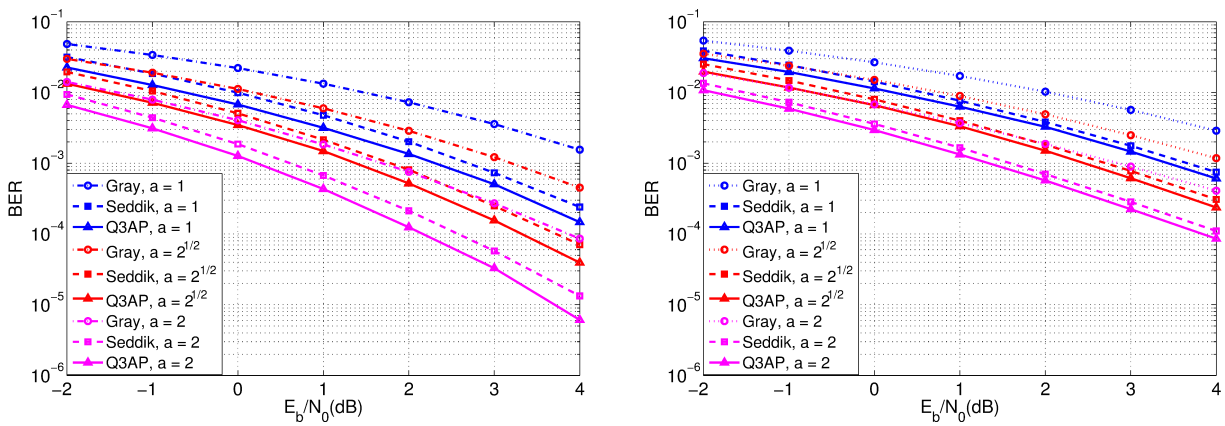
In our simulation, all Rician fading channels are assumed to have the same Rician parameter K. Also we assume that during the second phase, the phases of the line of sight (LOS) components of channels h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma^2_{h_0} = \sigma^2_{h_1} = 1/(K+1)$ and $\sigma^2_{h_2} = |a|^2/(K+1)$, where a denotes the ratio between the amplitude of the LOS component of the relay-to-destination and the S-D link. The noise power is parameterized with E_b/N_0 of the S-D link.

First, we provide an example of Q3AP optimized MoDiv for 16-QAM. For $E_b/N_0 = 2$dB, $K = 10$ and $a = 1$, the remapping scheme ψ_1 and ψ_2 are depicted in Fig. 2. The Gray mapping ψ_0 and the heuristic remapping $\psi_1 = \psi_2 = \psi_S$ are defined according to [6]. The results support the use of heuristics discussed in Section III-D as both ψ_1 and ψ_2 are essentially very close to ψ_S. However, we note that in general $\psi_1 \neq \psi_2$.

Next we compare the empirical BER for three mapping schemes: Q3AP-optimized MoDiv, the heuristic MoDiv $\psi_1 = \psi_2 = \psi_S$ and no MoDiv at all, i.e. $\psi_1 = \psi_2 = \psi_0$. The empirical BER is evaluated by applying the ML demodulator in (2) and over $M = 10^7$ randomly generated channels, noise instances and data symbols. In Fig. 3, there is a considerable performance gain for MoDiv versus no MoDiv. Moreover, the gain of the Q3AP optimized MoDiv over $\psi_1 = \psi_2 = \psi_S$ indicates that different rearrangements at the source and relay during the cooperative transmission can further reduce BER and boost performance. However, as K decreases, heuristic MoDiv

(a) ψ_1 (b) ψ_2

Fig. 2. Q3AP optimized MoDiv schemes.

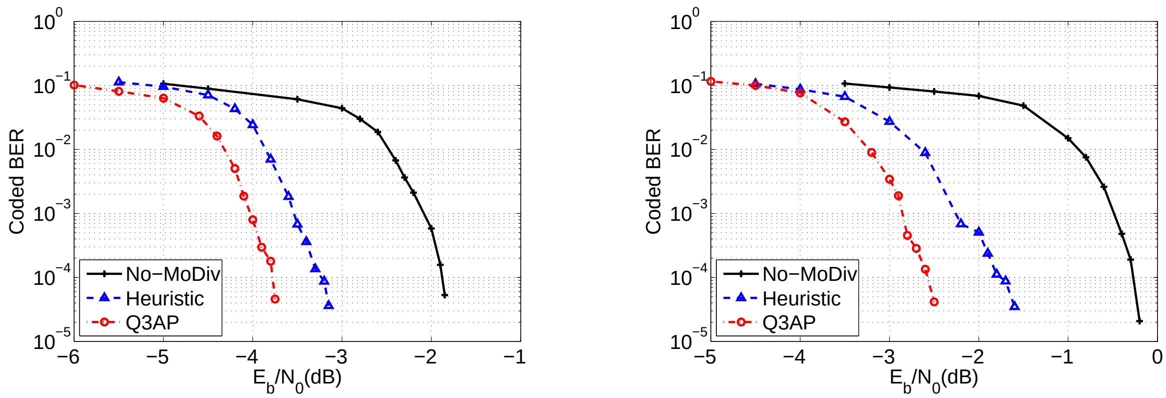


(a) $K = 10$ (b) $K = 5$

Fig. 3. Monte-Carlo simulated BER of (1) Q3AP optimized MoDiv (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

becomes a good approximate solution with diminishing performance gap from Q3AP-optimized MoDiv.

Finally, we test the coded BER performance and the robustness of the Q3AP MoDiv design with a typical LDPC coded system based on [17]. We use a LDPC code with length $L = 2400$, coding rate of 0.75 and a Monte-Carlo run of up to 2000 LDPC frames. Although our Q3AP MoDiv scheme depends on noise power and statistical CSI, in this simulation we only use Q3AP MoDiv schemes designed for the specific channel condition of $E_b/N_0 = -2dB$ under AWGN channel. We test the MoDiv design on Rician channels of $K = 10$, $a = 1$, and various E_b/N_0 to demonstrate its robustness. The coded BER results for 16-QAM and 32-QAM are plotted in Fig. **??** and Fig. **??**, respectively. We can see that the Q3AP MoDiv solution provides additional



(a) 16-QAM

(b) 32-QAM

Fig. 4. Monte-Carlo simulated coded BER.

gain over the heuristic MoDiv and no MoDiv even for mismatched design conditions. Also we notice that the Q3AP MoDiv provides larger performance gain for 32-QAM than for 16-QAM constellation.

The robustness is important to scalable extensions of the present simple network model. For multiple cooperative relays, our natural extension is to allow different relay nodes to randomly select between the two Q3AP-optimized mappings ψ_1 and ψ_2 to ensure sufficient MoDiv and channel diversity.

V. Conclusion

In this work, we investigated the modulation diversity (MoDiv) design problem in a three-node cooperative relay-HARQ system driven by coordinated retransmission from both the source and the relay. Aiming to minimize the bit error rate (BER) upper bound, we formulated the MoDiv design into a quadratic three-dimensional assignment problem (Q3AP), and presented an efficient modified iterative local search (ILS) solution. Our numerical tests demonstrate the performance advantage and robustness of the Q3AP-based MoDiv design over simply repeated use of Gray mapping. When the channel experiences deep fades, a heuristic MoDiv design can also achieve comparable performance gains at low complexity.

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