

# Constellation Rearrangement in Cooperative Relay-HARQ Network

**Abstract**—We study the constellation rearrangement (CoRe) problem in a relay-HARQ network to achieve symbol mapping diversity for reliable communication. Specifically, we formulate the bit error rate (BER) maximization into a quadratic three-dimensional assignment problem (Q3AP) and solve it with an efficient modified Iterated Local Search (ILS) method to find the optimal CoRe solution. Performance gains on various channel settings are demonstrated with simulations.

## I. INTRODUCTION

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ combined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ system can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a same series of bits are mapped to different constellation points across different links.

There are a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER maximization in [7]. CoRe is also studied in Nakagami- $m$  channel [8] and in combination with power allocation [9]. Nevertheless, all the abovementioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on a same time slot or band, resulting in low bandwidth efficiency. Moreover, since the CoRe problem is usually formulated into a NP-complete binary linear programming (BIP) problem, existing CoRe implementation are mostly based on fixed rearrangement [4], [9], heuristic approaches such as simulated annealing [6] and genetic algorithm [7], or by impractically dropping the binary constellation mapping constraints [10].

Historically, various CoRe problem for HARQ system fall within the realm of Quadratic Assignment Problem (QAP) or its extensions like Quadratic 3-dimensional Assignment Problem (Q3AP) [11]. Recent development in the numerical approaches to QAP/Q3AP [12] has enabled us to efficiently derive CoRe schemes with high quality.

In this work, we study the CoRe for cooperative relay-HARQ channel based on BER maximization. The main contributions of this paper are as follows:

- We propose to use CoRe for the relay-HARQ scheme

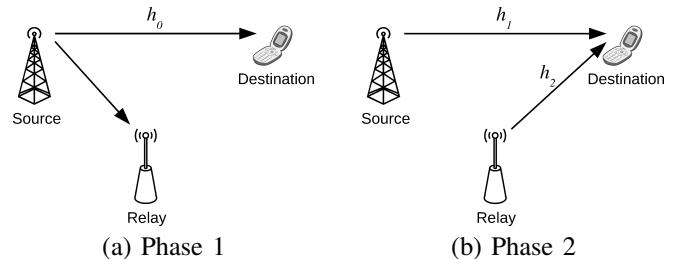


Fig. 1. Cooperative relay-HARQ networks.

similar to [13]. As depicted in Fig. 1, the source and the relay jointly perform the retransmission to the destination simultaneously, practically forming a 2-by-1 MIMO system.

- In our cooperative relay-HARQ settings, we formulate the CoRe design into a Q3AP problem. By taking advantage of the latest numerical solvers, we demonstrate significant performance gain of the optimized CoRe over non-CoRe and simple CoRe schemes for various channel settings.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into a Q3AP solution and provides a brief description of the numerical algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the cooperative relay-HARQ network depicted in Fig. 1. Denote  $\mathcal{C}$  as the constellation used by this relay network. In the first phase, the source convert a bit sequence of length  $\log_2 |\mathcal{C}|$  into symbols with Gray mapping  $\psi_0 : \{0, \dots, |\mathcal{C}| - 1\} \rightarrow \mathcal{C}$ . The bit sequence is indexed by its decimal equivalence  $p \in \{0, \dots, |\mathcal{C}| - 1\}$ . Then the source transmit  $\psi_0[p]$  to the destination via channel  $h_0$  which is also overheard by the decode-and-forward (DF) relay. We assume that relay is placed strategically so that it has negligible decoding error rate as in [1]. Upon receiving a request for retransmission, the second phase is started. This time the source and the relay adopt remap  $p$  into  $\psi_1[p]$  and  $\psi_2[p]$ , respectively, where potentially  $\psi_1 \neq \psi_0$  and  $\psi_2 \neq \psi_0$ . The remapped symbols are transmitted simultaneously on the same band to the destination via channel  $h_1$  and  $h_2$ . In summary, the received signal at the destination during the 2 phases are

$$y_1 = h_0 \psi_0[p] + v_1 \quad (1a)$$

$$y_2 = h_1 \psi_1[p] + h_2 \psi_2[p] + v_2 \quad (1b)$$

where  $v_1 \sim \mathcal{CN}(0, \sigma_v^2)$  and  $v_2 \sim \mathcal{CN}(0, \sigma_v^2)$  are the additive noise. Throughout this work, we assume the channels  $h_0$ ,  $h_1$  and  $h_2$  follows independent Rician distribution.

Assuming that the destination has perfect channel state information (CSI), it decides the index  $p$  with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_1 - \hat{h}_0 \psi_0[\hat{p}]|^2 + |y_2 - \hat{h}_1 \psi_1[\hat{p}] - \hat{h}_2 \psi_2[\hat{p}]|^2. \quad (2)$$

### III. OPTIMAL CONSTELLATION REARRANGEMENT

In this section we first formulate the min-BER CoRe design into a Q3AP problem, and explain the numerical approach to compute the input cost matrix to Q3AP problem. Then we provide an efficient algorithm to solve the Q3AP solution.

#### A. BER Maximization via Q3AP solution

Assume that the information-bearing index  $p$  follows a uniform distribution, the BER can be upper-bounded using pair-wise error probability (PEP) []

$$P_{BER} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p, q]}{Q} P_{PEP}(q|p) \quad (3)$$

where  $B[p, q]$  is the Hamming distance between the binary representation of  $p$  and  $q$  and  $P_{PEP}(q|p)$  is the probability for the ML decoder to prefer  $q$  over  $p$  when  $p$  is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, q, \psi_1, \psi_2) < 0\} \quad (4)$$

i.e. given indices  $p, q$  and the remapping scheme  $\psi_1, \psi_2$ , the probability of random variable  $\delta < 0$  evaluated over the random variables  $h_0, h_1, h_2, v_0, v_1$ , and  $\delta$  is defined as

$$\delta = |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + |h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_1|^2 - |v_1|^2. \quad (5)$$

In order to formulate the Q3AP problem, we denote binary variable  $x_{pij} = 1$  if  $\psi_1[p] = \psi_0[i]$  and  $\psi_2[p] = \psi_0[j]$  and  $x_{pij} = 0$  otherwise. Denote  $\mathbf{x} = \{x_{pij}|p, i, j = 0, \dots, Q-1\}$ , and the constraint sets:

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\} \quad (6a)$$

$$\mathcal{I} = \left\{ \mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\} \quad (6b)$$

$$\mathcal{J} = \left\{ \mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\}. \quad (6c)$$

Then from (3)(4)(9), the BER-minimization CoRe scheme  $\min_{\psi_1, \psi_2} P_{BER}$  can be reformulated as

$$\begin{aligned} \min_{\mathbf{x}} & \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} c_{pijql} x_{pij} x_{qkl} \\ \text{s.t. } & \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J} \end{aligned} \quad (7)$$

in which

$$c_{pijql} = \frac{B[p, q]}{Q} P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} \quad (8)$$

$$\begin{aligned} \delta &= |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + \\ &|h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_1|^2 - |v_1|^2. \end{aligned} \quad (9)$$

#### B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters  $\{c_{pijql}\}$  of Q3AP problem. According to 8, the key to compute  $c_{pijql}$  lies in the evaluation of  $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l)\}$ , i.e. the CDF of random variable  $\delta(p, i, j, q, k, l)$  as in (9). For the general Rician channel assumption  $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma_{h_m}^2)$ ,  $m = 0, 1, 2$ , we extend the method in [] to compute  $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$ :

$$\begin{aligned} &P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} \\ &\approx \frac{1}{2v} \sum_{t=1}^v \Re \{ \Phi_{\delta}(\xi + j\xi\tau_t) \} + \tau_t \Im \{ \Phi_{\delta}(\xi + j\xi\tau_t) \} \end{aligned} \quad (10)$$

where the moment generating function (MGF)  $\Phi_{\delta}(\omega) = \mathbb{E}_{\delta}[\exp(-\omega\delta)]$ ,  $\tau_k = \tan((k-1/2)\pi/v)$  and  $\Re\{\cdot\}$ ,  $\Im\{\cdot\}$  denotes the real and image part, respectively. Parameter  $\xi$  is selected to ensure convergence of the integration and [14] suggested to use  $\xi = 1/4$ . The size  $u$  of the expansion (10) needs to be larger when  $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$  is smaller in order to maintain an acceptable numerical accuracy.

To compute  $\Phi_{\delta}(\omega)$ , denote Gaussian random vectors  $\mathbf{z}_1 = [h_0, v_0]^T$ ,  $\mathbf{z}_2 = [h_1, h_2, v_1]^T$ , such that  $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$ ,  $m = 1, 2$ , where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T \quad (11)$$

$$\boldsymbol{\Sigma}_1 = \text{diag}(\sigma_{h_0}^2, \sigma_v^2), \boldsymbol{\Sigma}_2 = \text{diag}(\sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_v^2) \quad (12)$$

Then (9) can be rewritten as  $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$ , where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix} \quad (13a)$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix} \quad (13b)$$

in which where  $e_{ab} = \psi_0[a] - \psi_0[b]$ . Then the MGF can be computed as [15]

$$\Phi_{\delta}(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \quad (14)$$

When the Rician-fading channel reduces to Rayleigh-fading channel, i.e. when  $\mu_{h_m} = 0$ ,  $m = 0, 1, 2$  there is a simple upperbound of  $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$  similar to [4]. Note that

$$\begin{aligned} &P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} \\ &= \mathbb{E}_{h_0, h_1, h_2} \left\{ Q \left( \sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma_v^2}} \right) \right\} \end{aligned} \quad (15)$$

Applying the same relaxation as in [4], we have

$$\begin{aligned} & P_{h_0, h_1, h_2, v_0, v_1} \{ \delta(p, i, j, q, k, l) < 0 \} \\ & \leq \frac{3\sigma_v^4}{\sigma_{h_0}^2 |e_{pq}|^2 (\sigma_{h_1}^2 |e_{ik}|^2 + \sigma_{h_2}^2 |e_{jl}|^2)} \end{aligned} \quad (16)$$

which is tight in high-SNR regime.

### C. Q3AP Solution

## IV. NUMERICAL RESULTS

In this section we present the numerical results of CoRe under various channel settings. In our simulation, all Rician channels are assume to have the same Rician parameter  $K$ . Also we assume that during the second phase, the phase of channel  $h_1$  and  $h_2$  can be aligned at the source and relay, respectively. Consequently, we define  $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$ ,  $\mu_{h_2} = a\sqrt{K/(K+1)}$ ,  $\sigma_{h_0}^2 = \sigma_{h_1}^2 = 1/(K+1)$  and  $\sigma_{h_2}^2 = |a|^2/(K+1)$ , where  $a$  represents the ratio between the amplitude of the line of sight (LOS) components of the relay-to-destination and the source-to-destination link. Throughout the simulation we consider 16-QAM constellation thus  $Q = 16$ . The noise power is parameterized with  $E_b/N_0$  of the source-to-destination link.

Firstly we provide an example of Q3AP optimized CoRe. When  $E_b/N_0 = 2\text{dB}$ ,  $K = 10$  and  $a = 1$ , the remapping scheme of  $\psi_0$  (Gray-mapping),  $\psi_1$  and  $\psi_2$  is depicted in Fig. 2. We note that for the chase combining HARQ scheme and the joint ML demodulator, the optimal  $\psi_1$  and  $\psi_2$  are no longer Gray mapping. A general pattern of  $\psi_1$  and  $\psi_2$  is that the indices whose constellation points are close to each other during the first phase are remapped to constellation points far away from each other during the second phase, and vice versa. Both  $\psi_1$  and  $\psi_2$  are essentially the same as Seddik's heuristic result in [4], denoted as  $\psi_S$ . However, it is noted  $\psi_1 \neq \psi_2$ . In this special example or Rician channel with strong LOS component, an explanation is that the cooperative transmission is equivalent to transmission from a single transmitter with mapping  $\psi_1 + \psi_2$ . In this aspect, CoRe in effect results in a new constellation different from 16-QAM, while the individual constellations at the source and relay are kept unchanged.

Next we compare the BER upperbound  $P_{BER}$ , which is also the Q3AP objective function, for 3 mapping schemes: the Q3AP optimized CoRe, the extension of Seddik's remapping  $\psi_1 = \psi_2 = \psi_S$ , and no CoRe at all i.e.  $\psi_1 = \psi_2 = \psi_0$ . In Fig. 3, there is a considerable performance gain for CoRe versus no CoRe at all. Moreover, the gain of Q3AP optimized CoRe over  $\psi_1 = \psi_2 = \psi_S$  indicates that in cooperative relay-HARQ network, the BER performance can be further improved by using different rearrangement at the source and relay during the cooperative transmission. This gain increases as  $K$  increase, i.e. when the channel becomes less random.

The performance gain of Q3AP optimized CoRe for the cooperative relay-HARQ system is further justified in Fig. 4, where we evaluate the actual BER by running a Monte-Carlo simulation. We implement the ML demodulator in (2) and evaluate the average BER of  $M = 10^7$  randomly generated indices  $p$ . The actual BER curves demonstrate a similar pattern as the BER upperbound curves in Fig. 3.

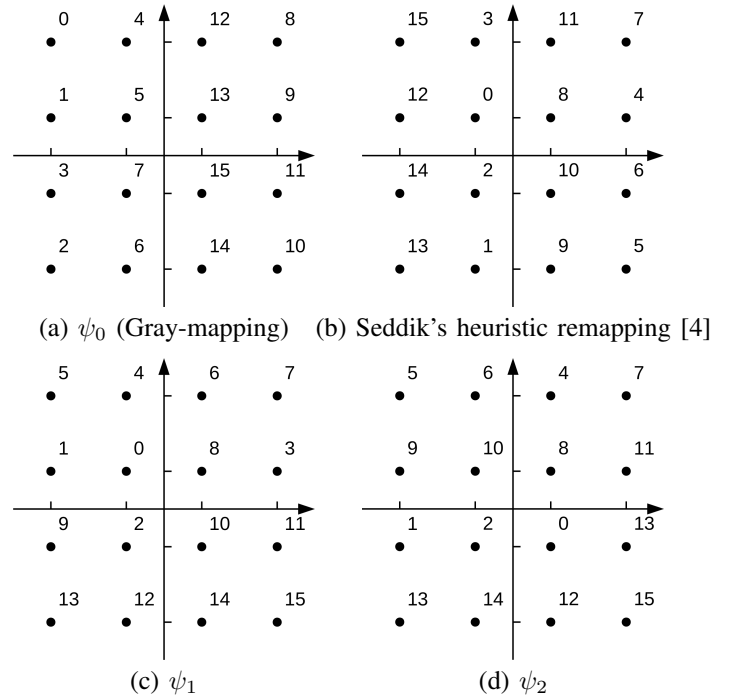


Fig. 2. Q3AP optimized CoRe schemes.

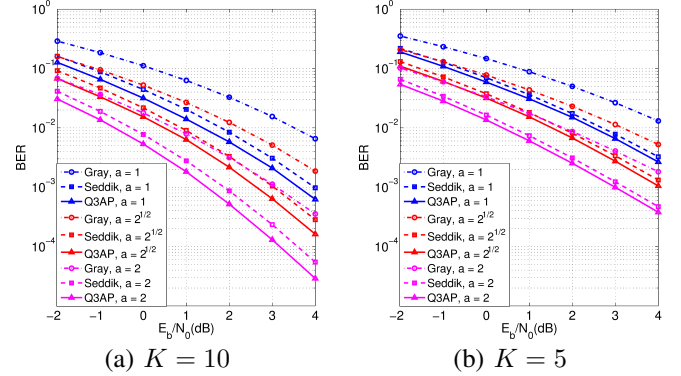


Fig. 3. BER upper bounds of (1) Q3AP optimized CoRe (2)  $\psi_1 = \psi_2 = \psi_S$  (3)  $\psi_1 = \psi_2 = \psi_0$  for  $K = 5, 10$ ,  $a = 1, \sqrt{2}, 2$ .

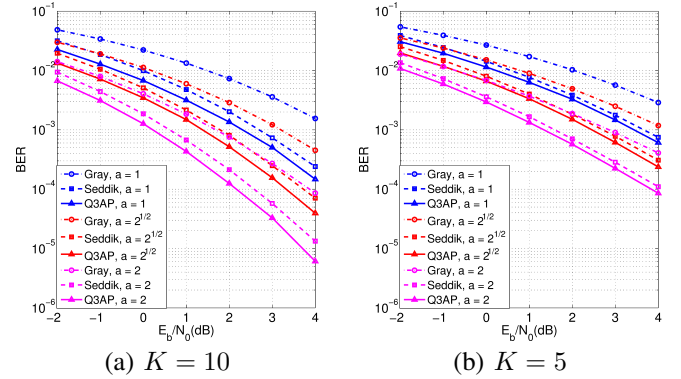


Fig. 4. Monte-Carlo simulated BER of (1) Q3AP optimized CoRe (2)  $\psi_1 = \psi_2 = \psi_S$  (3)  $\psi_1 = \psi_2 = \psi_0$  for  $K = 5, 10$ ,  $a = 1, \sqrt{2}, 2$ .

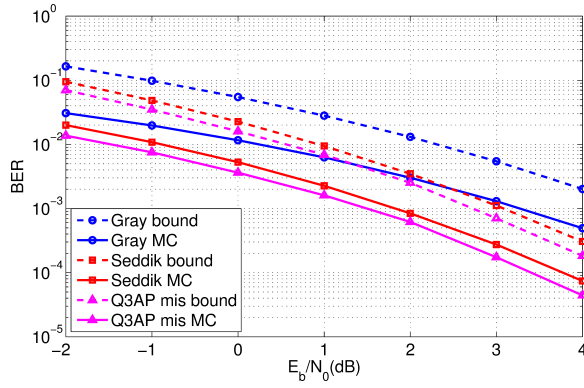


Fig. 5. BER performance under channel mismatch. Q3AP CoRe solution for  $a = 1$ ,  $K = 5$  in the actual channel settings  $a = \sqrt{2} \exp(j\pi/12)$ ,  $K = 10$ .

Finally, we test the robustness of the Q3AP CoRe solution against mismatch in channel settings. The intuition is that in practical, the CSI may not be perfectly accurate. Also, Q3AP CoRe scheme can only be computed off-line for a finite number of channel settings. We deliberately test the BER performance of the Q3AP CoRe solution for  $a = 1$ ,  $K = 5$  in the actual channel settings  $a = \sqrt{2} \exp(j\pi/12)$ ,  $K = 10$ . The BER upperbound and Monte-Carlo simulated BER are plotted in Fig. 5. Apparently, the Q3AP CoRe solution is not sensitive in the relative amplitude and the phase alignment error between the relay-to-destination and source-to-destination link.

## V. CONCLUSION

In this work we investigated the constellation rearrangement (CoRe) problem in cooperative relay-HARQ system. Based on the maximization of bit error rate (BER) upper bound, the CoRe design was formulated into a quadratic three-dimensional assignment problem (Q3AP), then solved with an efficient modified iterative local search (ILS) method. Our numerical tests demonstrated the performance gain of the Q3AP-based CoRe which allows the source and relay to use different remapping schemes during the retransmission other than Gray mapping and verified its robustness against channel state information (CSI) imperfection.

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