

Constellation Rearrangement in Cooperative Relay-HARQ Network

Abstract—We study the constellation rearrangement (CoRe) problem in a multi-relay-HARQ network to achieve symbol mapping diversity for reliable communication. Specifically, we propose a framework of three relay-HARQ protocol to be chosen from according to the channel settings and the capability of cooperation of the relay network. The corresponding bit error rate (BER) based CoRe can be formulated into a quadratic three-dimensional assignment problem (Q3AP) or traditional quadratic assignment problem (QAP) and can be numerically solved with an efficient modified Iterated Local Search (ILS) method. Performance gains on various channel settings are demonstrated with comprehensive simulations.

I. INTRODUCTION

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ comined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ system can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a same series of bits are mapped to different constellation points across different links.

There are a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER maximization in [7]. CoRe is also studied in Nakagami- m channel [8] and in combination with power allocation [9]. Nevertheless, all the abovementioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on a same time slot or band, resulting in low bandwidth efficiency.

Historically, various CoRe problem for HARQ system fall within the realm of Quadratic Assignment Problem (QAP) or its extensions like Quadratic 3-dimensional Assignment Problem (Q3AP) [10]. Since the CoRe problem is usually formulated into a NP-complete binary linear programming (BIP) problem, existing CoRe implementation are mostly based on fixed rearrangement [4], [9], or by impractically dropping the binary constellation mapping constraints [11]. Other works apply heuristic/metaheuristic approaches such as simulated annealing [6] and genetic algorithm [7]. It has been reported that some heuristic QAP solvers [] provides very high-quality solutions over the cases in QAPLIB [12].

In this work, we study the CoRe for cooperative relay-HARQ channel based on BER maximization. The main contributions of this paper are as follows:

- We propose a framework of three relay-HARQ protocol to be chosen from according to the channel settings and the capability of cooperation of the relay network. Similar to the relay-HARQ scheme in [13], multiple-input single-output (MISO) retransmission to the destination is allowed. As depicted in Fig. 1, in a multi-relay networks, when the source and the relay have joint transmission capability, different relays and the source rearrange the constellation mapping differently for the Chase Combining retransmission in order to provide additional signal-space gain. The CoRe design based on BER maximization can be formulated into a Q3AP problem. A simpler scheme where the relays and the source adopt a same constellation rearrangement is also allowed when the performance loss compared with the previous scheme is not significant. When the source and the relay does not have joint transmission capability, a single relay or the source with the best channel can be selected for the retransmission. The latter 2 schemes have a conventional QAP formulation [10].
- Since Q3AP/QAP formulation is NP-complete, we adopt an efficient iterated local search (ILS) method to solve it numerically. Simulation results demonstrate that the optimized mapping rearrangement provides significant performance gain by exploring the signal-space diversity.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into Q3AP/QAP problem and provides a brief description of the numerical algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the cooperative relay-HARQ network depicted in Fig. 1. Denote \mathcal{C} as the constellation used by this relay network whose size is $Q = |\mathcal{C}|$. In the first phase, the source convert a bit sequence of length $\log_2 |\mathcal{C}|$ into symbols with Gray mapping $\psi_0 : \{0, \dots, Q-1\} \rightarrow \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \dots, Q-1\}$. Then the source transmit $\psi_0[p]$ to the destination via channel g which is also received by N decode-and-forward (DF) relays, as shown in Fig. 1(a). We assume that the relays are placed strategically so that it has negligible decoding error rate as in [].

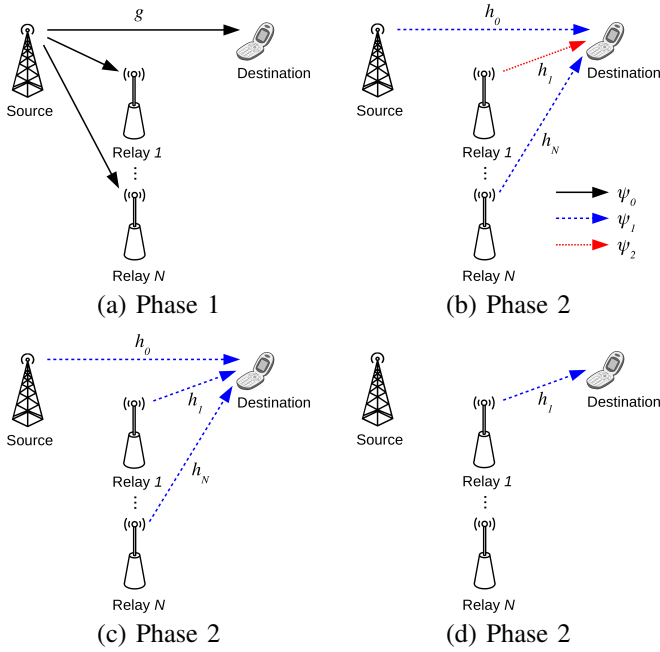


Fig. 1. Cooperative relay-HARQ networks.

Upon receiving a request for retransmission, the second phase is started. We assume that source and the N relays are capable of joint transmission and are considered a set $\mathcal{T} = \{t_0, t_1, \dots, t_N\}$ as a whole, where t_0 represents the source and t_n represents the n -th relay, $1 \leq n \leq N$. To better explore the signal-space diversity during the retransmission, \mathcal{T} is further partitioned into two subsets \mathcal{T}_1 , \mathcal{T}_2 , and nodes in \mathcal{T}_1 , \mathcal{T}_2 remap p into $\psi_1[p]$ and $\psi_2[p]$, respectively. Potentially $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are then transmitted jointly on the same band to the destination via channel h_0, h_1, \dots, h_N , as shown in Fig. 1(b). In summary, the received signal at the destination during the 2 phases are

$$y_t = g\psi_0[p] + v_t \quad (1a)$$

$$y_r = \left(\sum_{n: t_n \in \mathcal{T}_1} h_n \right) \psi_1[p] + \left(\sum_{n: t_n \in \mathcal{T}_2} h_n \right) \psi_2[p] + v_r \\ = g_1\psi_1[p] + g_2\psi_2[p] + v_r, \quad (1b)$$

respectively, where $v_t, v_r \sim \mathcal{CN}(0, \sigma_v^2)$ are the additive noise. Throughout this work, we assume the channels g and h_n , $n = 0, \dots, N$ follow independent Rician distribution, thus g_1 and g_2 are also Rician distributed. Assuming that the destination has perfect channel state information (CSI), it decides the index p with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_t - g\psi_0[\hat{p}]|^2 + |y_r - g_1\psi_1[\hat{p}] - g_2\psi_2[\hat{p}]|^2. \quad (2)$$

Note that theoretically \mathcal{T} can be partitioned into K subsets where $K > 2$ and the nodes in each individual subset uses a specific mapping. However, the resulting formulation would be a $Q(K+1)$ AP problem which is intractable for medium size constellation like 16-QAM. On the other hand, as we will show in Section III, by setting $K = 1$, i.e. $\psi_1 = \psi_2$ as in Fig. 1(c), the CoRe design problem can be simplified from Q3AP to QAP, and our numerical results in Section IV

will demonstrate that under certain channel conditions the performance loss is negligible compared to the case where $K = 2$. When the multi-relay network is not capable of joint transmission, we can select a single node with the best channel for retransmission, the formulation of which is essentially the same as [] (Fig. 1(d)).

III. OPTIMAL CONSTELLATION REARRANGEMENT

In this section we first formulate the min-BER CoRe design into a Q3AP problem, and explain the numerical approach to compute the input cost matrix to Q3AP problem. Then we provide an efficient algorithm to solve the Q3AP solution.

A. BER Maximization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded using pair-wise error probability (PEP) []

$$P_{BER} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p, q]}{Q} P_{PEP}(q|p) \quad (3)$$

where $B[p, q]$ is the Hamming distance between the binary representation of p and q and $P_{PEP}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{h_0, h_1, h_2, v_0, v_1} \{ \delta(p, q, \psi_1, \psi_2) < 0 \} \quad (4)$$

i.e. given indices p, q and the remapping scheme ψ_1, ψ_2 , the probability of random variable $\delta < 0$ evaluated over the random variables h_0, h_1, h_2, v_0, v_1 , and δ is defined as

$$\delta = |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + |h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_1|^2 - |v_1|^2. \quad (5)$$

In order to formulate the Q3AP problem, we denote binary variable $x_{pij} = 1$ if $\psi_1[p] = \psi_0[i]$ and $\psi_2[p] = \psi_0[j]$ and $x_{pij} = 0$ otherwise. Denote $\mathbf{x} = \{x_{pij}|p, i, j = 0, \dots, Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\} \quad (6a)$$

$$\mathcal{I} = \left\{ \mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\} \quad (6b)$$

$$\mathcal{J} = \left\{ \mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0, 1\} \right\}. \quad (6c)$$

Then from (3)(4)(9), the BER-minimization CoRe scheme $\min_{\psi_1, \psi_2} P_{BER}$ can be reformulated as

$$\min_{\mathbf{x}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} c_{pijql} x_{pij} x_{qkl} \quad (7)$$

$$\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}$$

in which

$$c_{pijql} = \frac{B[p, q]}{Q} P_{h_0, h_1, h_2, v_0, v_1} \{ \delta(p, i, j, q, k, l) < 0 \} \quad (8)$$

$$\delta = |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + |h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_1|^2 - |v_1|^2. \quad (9)$$

B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters $\{c_{pijqkl}\}$ of Q3AP problem. According to 8, the key to compute c_{pijqkl} lies in the evaluation of $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l)\}$, i.e. the CDF of random variable $\delta(p, i, j, q, k, l)$ as in (9). For the general Rician channel assumption $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma_{h_m}^2)$, $m = 0, 1, 2$, we extend the method in [1] to compute $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$:

$$P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} \approx \frac{1}{2v} \sum_{t=1}^v \Re \{ \Phi_\delta(\xi + j\xi\tau_t) \} + \tau_t \Im \{ \Phi_\delta(\xi + j\xi\tau_t) \} \quad (10)$$

where the moment generating function (MGF) $\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)]$, $\tau_k = \tan((k-1/2)\pi/v)$ and $\Re\{\cdot\}$, $\Im\{\cdot\}$ denotes the real and image part, respectively. Parameter ξ is selected to ensure convergence of the integration and [14] suggested to use $\xi = 1/4$. The size u of the expansion (10) needs to be larger when $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$ is smaller in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote Gaussian random vectors $\mathbf{z}_1 = [h_0, v_0]^T$, $\mathbf{z}_2 = [h_1, h_2, v_1]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1, 2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T \quad (11)$$

$$\boldsymbol{\Sigma}_1 = \text{diag}(\sigma_{h_0}^2, \sigma_v^2), \boldsymbol{\Sigma}_2 = \text{diag}(\sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_v^2) \quad (12)$$

Then (9) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix} \quad (13a)$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix} \quad (13b)$$

in which where $e_{ab} = \psi_0[a] - \psi_0[b]$. Then the MGF can be computed as [15]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)} \quad (14)$$

When the Rician-fading channel reduces to Rayleigh-fading channel, i.e. when $\mu_{h_m} = 0, m = 0, 1, 2$ there is a simple upperbound of $P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\}$ similar to [4]. Note that

$$P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} = \mathbb{E}_{h_0, h_1, h_2} \left\{ Q \left(\sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma_v^2}} \right) \right\} \quad (15)$$

Applying the same relaxation as in [4], we have

$$P_{h_0, h_1, h_2, v_0, v_1} \{\delta(p, i, j, q, k, l) < 0\} \leq \frac{3\sigma_v^4}{\sigma_{h_0}^2 |e_{pq}|^2 (\sigma_{h_1}^2 |e_{ik}|^2 + \sigma_{h_2}^2 |e_{jl}|^2)} \quad (16)$$

which is tight in high-SNR regime.

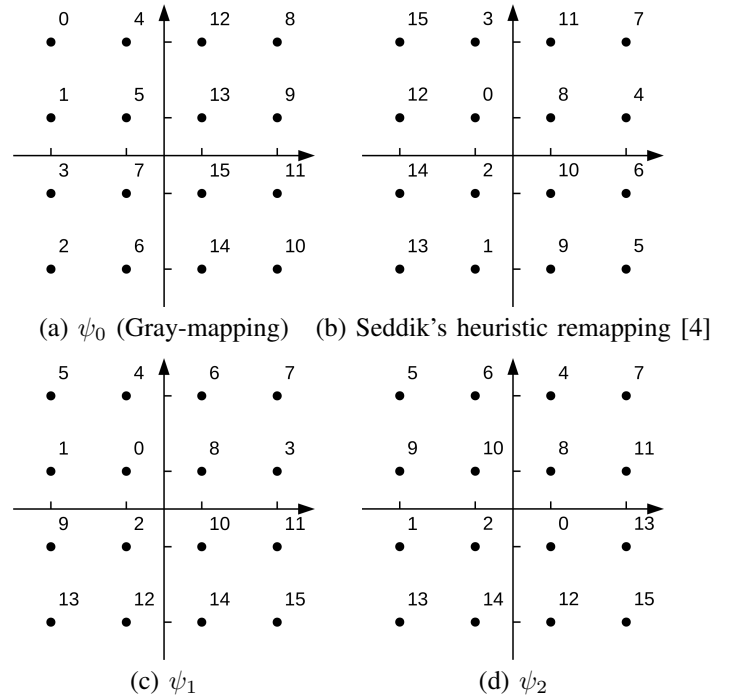


Fig. 2. Q3AP optimized CoRe schemes.

C. Q3AP Solution

IV. NUMERICAL RESULTS

In this section we present the numerical results of CoRe under various channel settings. In our simulation, all Rician channels are assume to have the same Rician parameter K . Also we assume that during the second phase, the phase of channel h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma_{h_0}^2 = \sigma_{h_1}^2 = 1/(K+1)$ and $\sigma_{h_2}^2 = |a|^2/(K+1)$, where a represents the ratio between the amplitude of the line of sight (LOS) components of the relay-to-destination and the source-to-destination link. Throughout the simulation we consider 16-QAM constellation thus $Q = 16$. The noise power is parameterized with E_b/N_0 of the source-to-destination link.

Firstly we provide an example of Q3AP optimized CoRe. When $E_b/N_0 = 2\text{dB}$, $K = 10$ and $a = 1$, the remapping scheme of ψ_0 (Gray-mapping), ψ_1 and ψ_2 is depicted in Fig. 2. We note that for the chase combining HARQ scheme and the joint ML demodulator, the optimal ψ_1 and ψ_2 are no longer Gray mapping. A general pattern of ψ_1 and ψ_2 is that the indices whose constellation points are close to each other during the first phase are remapped to constellation points far away from each other during the second phase, and vice versa. Both ψ_1 and ψ_2 are essentially the same as Seddik's heuristic result in [4], denoted as ψ_S . However, it is noted $\psi_1 \neq \psi_2$. In this special example of Rician channel with strong LOS component, an explanation is that the cooperative transmission is equivalent to transmission from a single transmitter with mapping $\psi_1 + \psi_2$. In this aspect, CoRe in effect results in a new constellation different from 16-QAM, while the individual constellations at the source and relay are kept unchanged.

Fig. 3. BER upper bounds of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

Fig. 4. Monte-Carlo simulated BER of (1) Q3AP optimized CoRe (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

Next we compare the BER upperbound P_{BER} , which is also the Q3AP objective function, for 3 mapping schemes: the Q3AP optimized CoRe, the extension of Seddik's remapping $\psi_1 = \psi_2 = \psi_S$, and no CoRe at all i.e. $\psi_1 = \psi_2 = \psi_0$. In Fig. 3, there is a considerable performance gain for CoRe versus no CoRe at all. Moreover, the gain of Q3AP optimized CoRe over $\psi_1 = \psi_2 = \psi_S$ indicates that in cooperative relay-HARQ network, the BER performance can be further improved by using different rearrangement at the source and relay during the cooperative transmission. This gain increases as K increase, i.e. when the channel becomes less random.

The performance gain of Q3AP optimized CoRe for the cooperative relay-HARQ system is further justified in Fig. 4, where we evaluate the actual BER by running a Monte-Carlo simulation. We implement the ML demodulator in (2) and evaluate the average BER of $M = 10^7$ randomly generated indices p . The actual BER curves demonstrate a similar pattern as the BER upperbound curves in Fig. 3.

Finally, we test the robustness of the Q3AP CoRe solution against mismatch in channel settings. The intuition is that

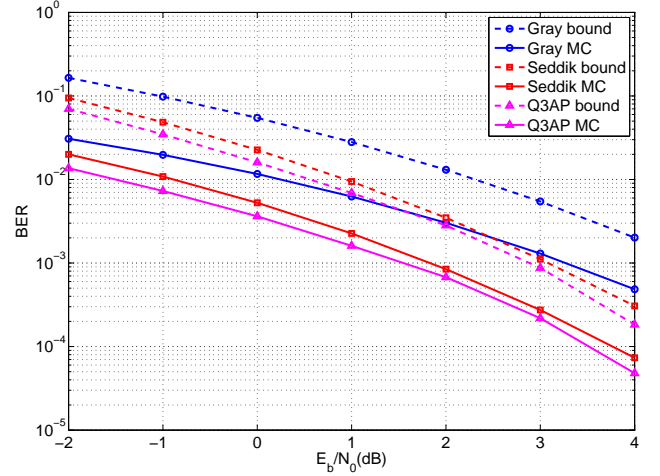


Fig. 5. BER performance under channel mismatch. Q3AP CoRe solution for $a = 1$, $K = 10$ in the actual channel settings $a = \sqrt{2} \exp(j\pi/12)$, $K = 10$.

in practical, the CSI may not be perfectly accurate. Also, Q3AP CoRe scheme can only be computed off-line for a finite number of channel settings. We deliberately test the BER performance of the Q3AP CoRe solution for $a = 1$, $K = 10$ in the actual channel settings $a = \sqrt{2} \exp(j\pi/12)$, $K = 10$. The BER upperbound and Monte-Carlo simulated BER are plotted in Fig. 5. Apparently, the Q3AP CoRe solution is not sensitive in the relative amplitude and the phase alignment error between the relay-to-destination and source-to-destination link.

V. CONCLUSION

In this work we investigated the constellation rearrangement (CoRe) problem in cooperative relay-HARQ system. Based on the maximization of bit error rate (BER) upper bound, the CoRe design was formulated into a quadratic three-dimensional assignment problem (Q3AP), then solved with an efficient modified iterative local search (ILS) method. Our numerical tests demonstrated the performance gain of the Q3AP-based CoRe which allows the source and relay to use different remapping schemes during the retransmission other than Gray mapping and verified its robustness against channel state information (CSI) imperfection.

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