Modulation Diversity Design in Cooperative Relay-HARQ Network

Abstract—**Modulation diversity (MoDiv) is a practically useful diversity enhancement technique that utilizes different modulation mappings from bits to constellation symbols. This approach is particularly effective when retransmitting packets in hybrid-ARQ (HARQ) systems. In this paper, we study the optimization of MoDiv in a cooperative relay-HARQ network where the source and relay jointly share the task of retransmission through coordination. By allowing different nodes to adopt different modulation mapping, we can better exploit signal space diversity to reduce the bit error rate (BER) at the HARQ receiver. To minimize the receiver BER, the MoDiv design optimization problem becomes a quadratic three-dimensional assignment problem (Q3AP). In order to solve the NP-complete Q3AP, we adopt an efficient modified Iterated local search method. Numerical results under various channel settings demonstrate the Performance gains of the proposed MoDiv scheme for HARQ.**

I. Introduction

In modern wireless communication, high rate data transmission often leads to reception errors. To overcome packet loss due to transmission errors, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are important mechanism for better reliability. HARQ in conjunction with relay networks has attracted great deal of research interest in recent years [1]. Because symbols transmitted in practice often utilize linear modulations of finite-size constellation (e.g., Q-ary PSK, or Q-ary QAM), the performance of cooperative relay-HARQ systems can benefit from Modulation Diversity (MoDiv) [2], [3], in which each group of $\log_2 Q$ bits are mapped to different constellation points across different links and in different transmissions.

A number of research works have already focused on MoDiv for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, MoDiv was designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate-optimized MoDiv is studied in [6]. For relay-HARQ systems, MoDiv is designed for BER minimization in [7]. MoDiv is also studied under wireless Nakagami-m channels [8] and in combination with power allocation [9]. Note that the aforementioned works assume cooperative relay-HARQ schemes with channel orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links. In other words, the (re)transmissions on the S-D link and the R-D links occupy orthogonal bands or orthogonal time slots, resulting in low bandwidth efficiency. On the other hand, three-terminal relay networks where the S-D and R-D links are additive instead of orthogonal have long been proposed in [10] and have been extensively studied by[11], [12]. It would be interesting to extend MoDiv for HARQ to these co-channel transmission relay networks.

One major difficulty of MoDiv optimization originates from the fact that they are hard to solve exactly. The MoDiv design problems for HARQ belong to the realm of the Quadratic Assignment Problem (QAP) [13], which has various applications including VLSI design, facility planning, or analysis of chemical reactions,[14]. As a special case of binary programming, QAP is known to be NP-hard and so is finding an approximate solution [15]. Some existing MoDiv implementations circumvent the difficulty by adopting a fixed remapping scheme [4], [9], or by neglecting the binary constellation mapping constraints [16]. Other heuristic or metaheuristic approaches applied simulated annealing (SA) [6] and genetic algorithms [7]. It has been reported that some heuristic QAP solvers [17], [18], [19] provide very high-quality solutions over the cases in QAPLIB [20].

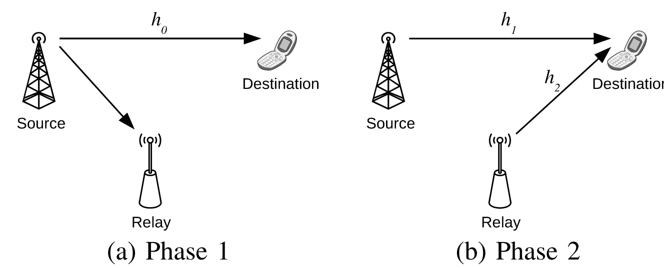


Fig. 1. Cooperative relay-HARQ networks.

We note that the joint optimization of two modulation diversity mappings on S-D link and R-D link, respectively, leads to an even harder Quadratic 3-dimensional Assignment Problem (Q3AP). It is more challenging to find an efficient Q3AP algorithm that generates high-quality solutions. In this work, we study the MoDiv for cooperative relay-HARQ networks based on BER minimization. Our main contributions in this paper are as follows:

- We propose a novel MoDiv scheme for a relay-HARQ network similarly considered in [21]. As depicted in Fig. 1, the source and the relay jointly execute retransmission to the destination in a coordinated fashion, practically forming a 2-by-1 multiple-input single-output (MISO) diversity system.
- In our cooperative relay-HARQ environment, we formulate the MoDiv design into a Q3AP problem and solve it with an efficient iterated local search (ILS) method. We demonstrate the performance gains by exploiting the signal-space diversity with the optimized MoDiv mapping for various channel settings.

We organize the manuscript as follows. Section II describes the underlying cooperative relay-HARQ system model. Section III formulates the MoDiv design into a Q3AP solution and provides an outline of the modified ILS algorithm. Section IV present numerical results as performance demonstration. Finally, Section V concludes the paper.

II. System Model

We consider the cooperative relay-HARQ network shown in Fig. 1. There are two phases in this transmission system. In phase 1, source node broadcasts its packet that can be received by both the destination and the relay. In phase 2 of this HARQ setup, upon packet loss, both the relay and the source node cooperatively retransmit the lost packet information to the destination. Our goal is to design the H-ARQ constellation mapping for modulation diversity such that the packet BER at the receiver is minimized.

Denote $\mathcal{C}$ as the constellation used by this relay network whose size is $Q = |\mathcal{C}|$. In the first phase, the source converts a bit sequence of length $\log_2 Q$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, Q-1\} \rightarrow \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \ldots, Q-1\}$. The source transmits $\psi_0[p]$ to the destination via channel h_0 which is also overheard by the decode-and-forward (DF) relay. We assume that this relay is placed strategically such that it has negligible decoding error rate as in [8], [7]. Upon receiving a request for retransmission, the second phase begins with the source and the relay remapping p into $\psi_1[p]$ and $\psi_2[p]$, respectively. In general, $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are transmitted simultaneously on the same frequency band to the destination via channels h_1 and h_2. In summary, the received signals at the destination during the two phases are, respectively,

$$y_1 = h_0\psi_0[p] + v_1, \tag{1a}$$
$$y_2 = h_1\psi_1[p] + h_2\psi_2[p] + v_2, \tag{1b}$$

where $v_1 \sim \mathcal{CN}(0, \sigma_v^2)$ and $v_2 \sim \mathcal{CN}(0, \sigma_v^2)$ are additive channel noises. Throughout this work, we assume the channels h_0, h_1 and h_2 follows independent Rician distribution.

Assuming that the destination has perfect channel state information (CSI). Based on the received symbols y_1 and y_2 from the two phases, the destination decodes the data by identifying the index p via maximum likelihood (ML) detection:

$$\min_{\hat{p}} |y_1 - h_0\psi_0[\hat{p}]|^2 + |y_2 - h_1\psi_1[\hat{p}] - h_2\psi_2[\hat{p}]|^2. \tag{2}$$

III. Optimal Constellation Mapping for Modulation Diversity

In this section we first formulate the minimum BER design of MoDiv into a Q3AP problem. We then elaborate on the numerical approach for computing the input cost matrix of the Q3AP problem. We then provide an efficient algorithm to obtain the Q3AP solution.

A. BER Minimization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded and approximated using the pair-wise error probability (PEP) [13]

$$P_{\mathrm{BER}} \approx \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{\mathrm{PEP}}(q|p), \tag{3}$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q and $P_{\mathrm{PEP}}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{\mathrm{PEP}}(q|p) = P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,q,\psi_1,\psi_2) < 0\}. \tag{4}$$

In other words, given indices p, q and the remapping scheme ψ_1, ψ_2, the probability of the random variable $\delta < 0$ being evaluated over the random variables h_0, h_1, h_2, v_1, v_2, and δ is defined as

$$\begin{aligned}\delta = &|h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 + \\ &|h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_2|^2 - |v_2|^2.\end{aligned} \tag{5}$$

In order to formulate the Q3AP problem, we introduce binary variables

$$x_{pij} = \begin{cases} 1, & \text{if } \psi_1[p] = \psi_0[i] \text{ and } \psi_2[p] = \psi_0[j] \\ 0, & \text{otherwise.} \end{cases}$$

Denote sets

$$\mathbf{x} = \{x_{pij} | \quad p, i, j = 0, \ldots, Q-1\} \tag{6a}$$
$$\mathcal{P} = \left\{\mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}, \tag{6b}$$
$$\mathcal{I} = \left\{\mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}, \tag{6c}$$
$$\mathcal{J} = \left\{\mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}. \tag{6d}$$

Then from (3)(4)(5), the BER minimization MoDiv scheme $\min_{\psi_1,\psi_2} P_{\mathrm{BER}}$ can be reformulated as

$$\begin{aligned}&\min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl}x_{pij}x_{qkl}, \\ &\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}.\end{aligned} \tag{7}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}, \tag{8}$$

$$\begin{aligned}\delta = &|h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_2|^2 \\ &+ |h_0(\psi_0[p] - \psi_0[q]) + v_1|^2 - |v_1|^2 - |v_2|^2.\end{aligned} \tag{9}$$

Here a Q3AP instance is fully defined by the 6-dimensional matrix $\{c_{pijqkl}\}$ which must be determined for our MoDiv design.

B. Computation of the Pair-wise Symbol Error Rate

In this section we focus on the computation of the parameters $\{c_{pijqkl}\}$ of the Q3AP problem. According to (8), the key to compute c_{pijqkl} lies in the evaluation of $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l)$, i.e. the cumulative distribution function (CDF) of the random variable $\delta(p,i,j,q,k,l)$ as in (5).

Define the moment generating function (MGF)

$$\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)].$$

Under the well known Rician channel model, $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma^2_{h_m}), m = 0, 1, 2$, we can extend the method in [13], [22] to compute $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$:

$$P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\} \approx \frac{1}{2v}\sum_{t=1}^{v} \mathrm{Re}\left\{\Phi_\delta(\xi + \mathrm{j}\xi\tau_t)\right\} + \tau_t \mathrm{Im}\left\{\Phi_\delta(\xi + \mathrm{j}\xi\tau_t)\right\}, \quad (10)$$

where $\tau_t = \tan((t-1/2)\pi/v)$ and $\mathrm{Re}\{\cdot\}$, $\mathrm{Im}\{\cdot\}$ denote the real and image parts, respectively. The parameter ξ is selected to ensure convergence of the integration and $\xi = 1/4$ was suggested in [22]. The size v of the expansion (10) needs to be large when $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$ is small in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote the Gaussian random vectors $\mathbf{z}_1 = [h_0, v_1]^T$, $\mathbf{z}_2 = [h_1, h_2, v_2]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1, 2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T, \quad (11)$$

$$\boldsymbol{\Sigma}_1 = \mathrm{diag}\left(\sigma^2_{h_0}, \sigma^2_v\right), \, \boldsymbol{\Sigma}_2 = \mathrm{diag}\left(\sigma^2_{h_1}, \sigma^2_{h_2}, \sigma^2_v\right). \quad (12)$$

Then (5) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e^*_{pq} \\ e_{pq} & 0 \end{bmatrix}, \quad (13a)$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e^*_{ik}e_{jl} & e^*_{ik} \\ e_{ik}e^*_{jl} & |e_{jl}|^2 & e^*_{jl} \\ e_{ik} & e_{jl} & 0 \end{bmatrix}, \quad (13b)$$

here $e_{ab} = \psi_0[a] - \psi_0[b]$, for which the MGF can be computed as [23]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \quad (14)$$

Note that for any instance of Q3AP a total number of Q^6 coefficients must be computed. Fortunately, the MoDiv design is based on statistical CSI which does not require online computation of the Q^6 coefficients. In fact, the optimized MoDiv can be precomputed and stored a priori in our network nodes.

In our simulation, we implement the above procedure for 16-QAM and 32-QAM with Armadillo library [24] on a workstation with 48 cores and finished the computation in several days for a $Q = 32$ case and a few hours for a $16-QAM$ case. For larger constellation such as 64-QAM, however, the time and spacial complexity may still be too high. We will address in future works to reduce this complexity by adding a few rules to restrict the remapping schemes.

C. Rayleigh Fading Channel Model

When the Rician-fading channel reduces to the Rayleigh-fading channel, i.e. when $\mu_{h_m} = 0, m = 0, 1, 2$ there is a simple upper bound of $P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\}$ similar to [4]. Note that

$$P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\} = \mathbb{E}_{h_0,h_1,h_2}\left\{Q\left(\sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma^2_v}}\right)\right\}. \quad (15)$$

Applying the same relaxation as in [4], we have

$$P_{h_0,h_1,h_2,v_1,v_2}\{\delta(p,i,j,q,k,l) < 0\} \le \frac{3\sigma^4_v}{\sigma^2_{h_0}|e_{pq}|^2(\sigma^2_{h_1}|e_{ik}|^2 + \sigma^2_{h_2}|e_{jl}|^2)}. \quad (16)$$

which is tight in the high-SNR regime and simplifies the evaluation of the Q^6 coefficients.

D. Q3AP Solution

For practical-sized constellation such as 16-QAM and 32-QAM, it is impractical to apply the exact branch-and-bound algorithm [25]. Also, our tests show that even the smaller 16-QAM does not have enough symmetry to exploit for faster solution as does the 16-PSK constellation [26]. Consequently, the MoDiv problem is solved with the ILS method [25] extended from its QAP version [27].

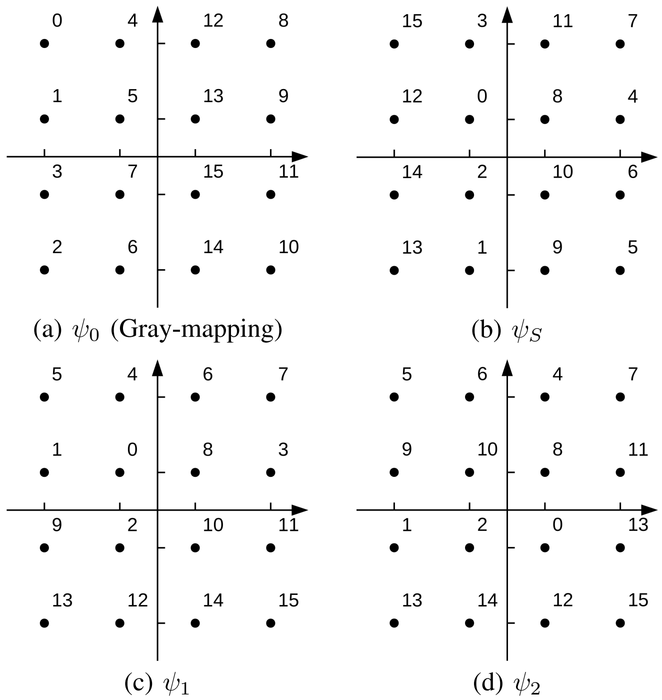
The basic procedure of ILS is outlined as follows:

- S1 Initialization: starting from a random mapping $\psi_1^{(0)}, \psi_2^{(0)}$.
- S2 Local search: find a locally optimum solution $\psi_1^{(0)*}, \psi_2^{(0)*}$. Set $n = 0$
- S3 Perturbation: from the last locally optimum solution $\psi_1^{(n)*}, \psi_2^{(n)*}$, generate a new mapping $\psi_1^{(n+1)}, \psi_2^{(n+1)}$.
- S4 Local search: from $\psi_1^{(n+1)}, \psi_2^{(n+1)}$, find a new locally optimum solution $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$.
- S5 Examination of the acceptance criterion: compare $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ with $\psi_1^{(n)*}, \psi_2^{(n)*}$. If the new mapping is not accepted, reset $\psi_1^{(n+1)*} = \psi_1^{(n)*}$ and $\psi_2^{(n+1)*} = \psi_2^{(n)*}$. Set $n \leftarrow n + 1$.
- S6 Return to S3 until the stopping criterion is satisfied.

The local search in steps S2 and S4 attempts to exchange the mapping of exactly 2 indices in ψ_1 or ψ_2 in order to lower the objective function, i.e. a 2-opt neighborhood search. A first-improvement rule is used, which means that whenever a reduction in the objective function is made the mapping is updated. The perturbation in S3 is executed by exchanging the mapping of k_p indices, where $k_p \in [k_{p,min}, k_{p,max}]$ is adaptively increased if the perturbation does not produce a better solution, or it is reset to $k_{p,min}$ otherwise. For the acceptance criterion examination in S5, $\psi_1^{(n+1)*}, \psi_2^{(n+1)*}$ is accepted if the objective function $f^{(n+1)} < f^{(n)}$, or accepted with probability $\exp(-[f^{(n+1)} - f^{(n)}]/T)$, where T is the temperature parameter similar to that of simulated annealing.

IV. Numerical Results

In this section we present the numerical results of MoDiv for the relay-HARQ network under various channel settings. In our simulation, all Rician channels are assumed to have the same Rician parameter K. Also we assume that during the second phase, the phases of the line of sight (LOS) components of channels h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma^2_{h_0} = \sigma^2_{h_1} = 1/(K+1)$ and $\sigma^2_{h_2} = |a|^2/(K+1)$, where a represents the ratio



(a) ψ_0 (Gray-mapping) (b) ψ_S

(c) ψ_1 (d) ψ_2

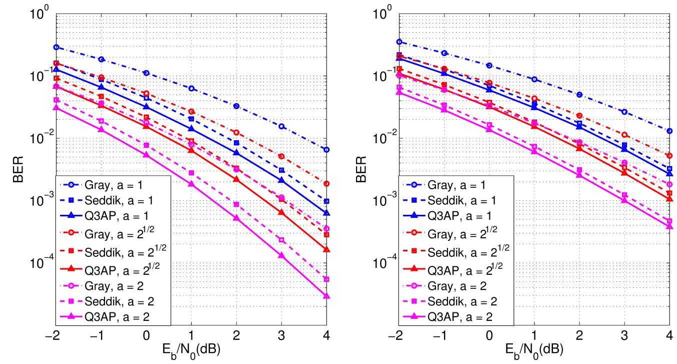
Fig. 2. Q3AP optimized MoDiv schemes.

between the amplitude of the LOS component of the relay-to-destination and the S-D link. Throughout the simulation we consider 16-QAM constellation, i.e., $Q = 16$. The noise power is parameterized with E_b/N_0 of the S-D link.

First, we provide an example of Q3AP optimized MoDiv. When $E_b/N_0 = 2$dB, $K = 10$ and $a = 1$, the remapping scheme of ψ_0 (using Gray-mapping), ψ_1 and ψ_2 is depicted in Fig. 2. We note that for the cooperative HARQ combining scheme and the joint ML demodulator, the optimal ψ_1 and ψ_2 are no longer a Gray mapping. A general pattern of ψ_1 and ψ_2 is that the indices whose constellation points were close to on other during the first phase are then remapped to constellation points far apart from each other during the second phase, and vice versa. Both ψ_1 and ψ_2 are essentially very close to the heuristic result proposed by Seddik in [4], which we denote as ψ_S. However, we note that $\psi_1 \neq \psi_2$.

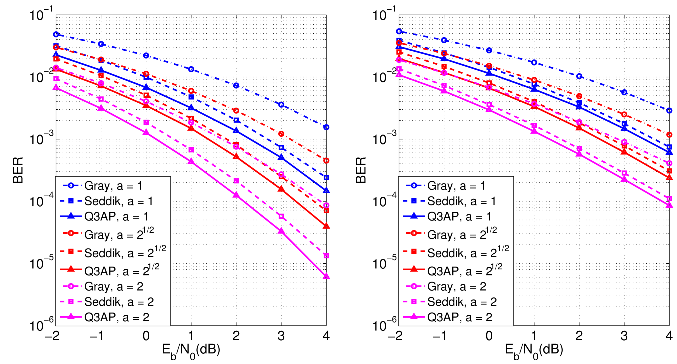
In this special example of Rician channel model with strong LOS component, an interpretation is that the cooperative transmission is equivalent to transmission from a single transmitter with mapping $\psi_1 + \psi_2$. In this aspect, MoDiv in effect results in a new constellation different from 16-QAM, while the individual constellations at the source and relay are kept unchanged.

Next we compare the BER upper bound P_{BER}, which is also the Q3AP objective function, for three mapping schemes: the Q3AP optimized MoDiv, the extension of Seddik's remapping $\psi_1 = \psi_2 = \psi_S$, and no MoDiv at all, i.e. $\psi_1 = \psi_2 = \psi_0$. In Fig. 3, there is a considerable performance gain for MoDiv versus no MoDiv at all. Moreover, the gain of the Q3AP optimized MoDiv over $\psi_1 = \psi_2 = \psi_S$ indicates that in cooperative relay-HARQ networks, the BER performance can be further improved by using different rearrangements at the source and relay during the cooperative transmission. This gain increases as K increases, i.e. when the channel becomes less random.



(a) $K = 10$ (b) $K = 5$

Fig. 3. BER upper bounds of (1) Q3AP optimized MoDiv (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.



(a) $K = 10$ (b) $K = 5$

Fig. 4. Monte-Carlo simulated BER of (1) Q3AP optimized MoDiv (2) $\psi_1 = \psi_2 = \psi_S$ (3) $\psi_1 = \psi_2 = \psi_0$ for $K = 5, 10$, $a = 1, \sqrt{2}, 2$.

The performance gain of Q3AP optimized MoDiv for the cooperative relay-HARQ system is further verified in Fig. 4. In this figure, we evaluate the actual BER by running a Monte-Carlo simulation. We implement the ML demodulator in (2) and evaluate the average BER of $M = 10^7$ randomly generated indices p. The actual BER curves demonstrate a similar trend as the BER upper bound curves in Fig. 3.

Finally, we test the robustness of the Q3AP MoDiv solution against mismatch in channel information. The intuition is that in practical systems, the CSI may not be perfectly accurate and the phase of the LOS component of the S-D and R-D links may not be perfectly aligned. Also, the Q3AP MoDiv scheme can only be computed off-line for a limited number of channel settings a priori, instead of for all possible combinations. We deliberately test the BER performance of the Q3AP MoDiv solution for channel parameters $a = 1$, $K = 5$ while the actual channel setting is $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$. The BER upper bound and Monte-Carlo simulated BER are plotted in Fig. 5. Apparently, the Q3AP MoDiv solution is not sensitive to the relative amplitude and the phase alignment error between S-D and R-D links.

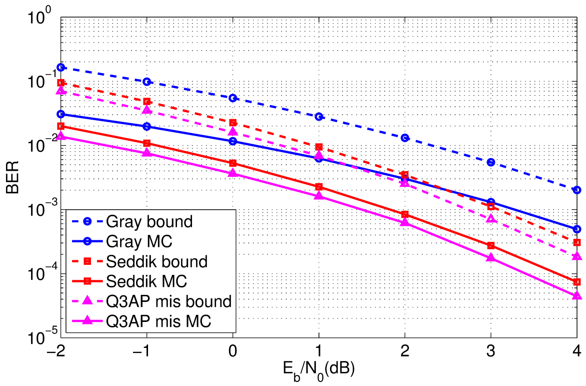


Fig. 5. BER performance under channel mismatch. Q3AP MoDiv solution for CSI $a = 1$, $K = 5$ while the actual CSIX is $a = \sqrt{2}\exp(j\pi/12)$, $K = 10$.

V. Conclusion

In this work, we investigated the modulation diversity (MoDiv) design problem in a three-node cooperative relay-HARQ systems featured by the coordinated retransmission from both the source and the relay. Aiming to minimized the bit error rate (BER) upper bound, we formulated the MoDiv design into a quadratic three-dimensional assignment problem (Q3AP), and presented an efficient modified iterative local search (ILS) solution. Our numerical tests have demonstrated the performance advantage of the Q3AP-based MoDiv design. By allowing the source and relay to use two different remapping schemes during the retransmission, instead of repeating the use of Gray mapping, we demonstrated the effective reduction of BER and the method's robustness against channel state information (CSI) inaccuracy.

References

[1] H. A. Ngo and L. Hanzo, "Hybrid Automatic-Repeat-reQuest systems for cooperative wireless communications," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 25–45, First Quarter 2014.

[2] G. Benelli, "A new method for the integration of modulation and channel coding in an ARQ protocol," *IEEE Trans. Commun.*, vol. 40, no. 10, pp. 1594–1606, Oct 1992.

[3] C. Wengerter, A. von Elbwart, E. Seidel, G. Velev, and M. Schmitt, "Advanced hybrid ARQ technique employing a signal constellation rearrangement," in *Proc. Vehic. Technol. Conf. (VTC)*, vol. 4, Sep 2002, pp. 2002–2006.

[4] K. Seddik, A. Ibrahim, and K. Liu, "Trans-modulation in wireless relay networks," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 170–172, Mar 2008.

[5] M. Khormuji and E. Larsson, "Improving collaborative transmit diversity by using constellation rearrangement," in *Proc. IEEE Wireless Commun. Networking Conf. (WCNC)*, Mar 2007, pp. 803–807.

[6] ——, "Rate-optimized constellation rearrangement for the relay channel," *IEEE Commun. Lett.*, vol. 12, no. 9, pp. 618–620, Sep 2008.

[7] J. W. Kim, H. Lee, J. Ahn, and C. Kang, "Design of signal constellation rearrangement (CoRe) for multiple relay links," in *Proc. IEEE GLOBECOM*, Nov 2009, pp. 1–6.

[8] H.-S. Ryu, J.-S. Lee, and C. Kang, "BER analysis of constellation rearrangement for cooperative relaying networks over Nakagami-m fading channel," in *Proc. IEEE Int. Commun. Conf. (ICC)*, Jun 2011, pp. 1–5.

[9] T.-W. Yu, W.-H. Sheen, and C.-H. Wang, "Power allocation for cooperative bit-interleaved coded modulation systems with decode-remap-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1712–1721, May 2012.

[10] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, Sep 1979.

[11] R. Nabar, H. Bolcskei, and F. Kneubuhler, "Fading relay channels: performance limits and space-time signal design," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1099–1109, Aug 2004.

[12] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4152–4172, Dec 2005.

[13] H. Samra, Z. Ding, and P. Hahn, "Symbol mapping diversity design for multiple packet transmissions," *IEEE Trans. Commun.*, vol. 53, no. 5, pp. 810–817, May 2005.

[14] G. Finke, R. E. Burkard, and F. Rendl, "Quadratic assignment problems," *North-Holland Mathematics Studies*, vol. 132, pp. 61–82, 1987.

[15] S. Sahni and T. Gonzalez, "P-complete approximation problems," *J. ACM*, vol. 23, no. 3, pp. 555–565, Jul. 1976.

[16] A. Bin Sediq, P. Djukic, H. Yanikomeroglu, and J. Zhang, "Optimized nonuniform constellation rearrangement for cooperative relaying," *IEEE Trans. Veh. Technol.*, vol. 60, no. 5, pp. 2340–2347, Jun 2011.

[17] Z. Drezner, "Extensive experiments with hybrid genetic algorithms for the solution of the quadratic assignment problem," *Computers & Operations Research*, vol. 35, no. 3, pp. 717–736, 2008.

[18] T. James, C. Rego, and F. Glover, "Multistart tabu search and diversification strategies for the quadratic assignment problem," *IEEE Trans. Syst., Man, Cybern. A*, vol. 39, no. 3, pp. 579–596, May 2009.

[19] U. Benlic and J.-K. Hao, "Memetic search for the quadratic assignment problem," *Expert Systems with Applications*, vol. 42, no. 1, pp. 584–595, 2015.

[20] R. E. Burkard, S. E. Karisch, and F. Rendl, "QAPLIB–a quadratic assignment problem library," *Journal of Global optimization*, vol. 10, no. 4, pp. 391–403, 1997.

[21] Y. Qi, R. Hoshyar, M. Imran, and R. Tafazolli, "H2-ARQ-relaying: Spectrum and energy efficiency perspectives," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1547–1558, Sep 2011.

[22] G. Taricco and E. Biglieri, "Exact pairwise error probability of space-time codes," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 510–513, Feb 2002.

[23] M. Schwartz, W. R. Bennett, and S. Stein, *Communication systems and techniques*. John Wiley & Sons, 1995.

[24] C. Sanderson, "Armadillo: An open source C++ linear algebra library for fast prototyping and computationally intensive experiments," 2010.

[25] P. M. Hahn, B.-J. Kim, T. Sttzle, S. Kanthak, W. L. Hightower, H. Samra, Z. Ding, and M. Guignard, "The quadratic three-dimensional assignment problem: Exact and approximate solution methods," *European Journal of Operational Research*, vol. 184, no. 2, pp. 416 – 428, 2008.

[26] H. D. Mittelmann and D. Salvagnin, "On solving a hard quadratic 3-dimensional assignment problem," *Mathematical Programming Computation*, pp. 1–16, 2015.

[27] T. Stützle, "Iterated local search for the quadratic assignment problem," *European Journal of Operational Research*, vol. 174, no. 3, pp. 1519 – 1539, 2006.