Constellation Rearrangement in Cooperative Relay-HARQ Network

***Abstract*—We study the constellation rearrangement (CoRe) problem in a relay-HARQ network to achieve symbol mapping diversity for reliable communication. Specifically, we formulate the bit error rate (BER) maximization into a quadratic three-dimensional assignment problem (Q3AP) and make use of the recent development of numerical method to find the optimal CoRe solution. Performance gains on various channel settings are demonstrated with simulations.**

I. Introduction

In modern wireless communication systems, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are recognized as key technologies for reliable transmission. HARQ comined with relay networks has attracted great research interest in recent years [1]. Since in practice the transmitted symbols are modulated from a finite-size constellation (e.g., PSK, QAM), the performance of cooperative relay-HARQ system can be further enhanced with Constellation Rearrangement (CoRe) [2], [3], in which a same series of bits are mapped to different constellation points across different links.

There are a wide variety of works on CoRe for cooperative relay systems with different channel settings and design criteria. For the simple three-node single hop relay network, CoRe is designed to minimize symbol error rate (SER) in [4] and the bit error rate (BER) in [5]. The rate optimized CoRe is studied in [6]. For relay-HARQ systems, CoRe is designed based on BER maximization in [7]. CoRe is also studied in Nakagami-m channel [8] and in combination with power allocation [9]. Nevertheless, all the abovementioned works assume cooperative relay-HARQ schemes with orthogonality between the source-to-destination (S-D) link and the relay-to-destination (R-D) links, i.e. the (re)transmissions on the S-D link and the R-D links can not be on a same time slot or band, resulting in low bandwidth efficiency. Moreover, since the CoRe problem is usually formulated into a NP-complete binary linear programming (BIP) problem, existing CoRe implementation are mostly based on fixed rearrangement [4], [9], heurisitc approaches such as simulated annealing [6] and genetic algorithm [7], or by impractically dropping the binary constellation mapping constraints [10].

Historically, various CoRe problem for HARQ system fall within the realm of Quadratic Assignment Problem (QAP) or its extensions like Quadratic 3-dimensional Assignment Problem (Q3AP) [11]. Recent development in the numerical approaches to QAP/Q3AP [12] has enbaled us to efficiently derive CoRe schemes with high quality.

In this work, we study the CoRe for cooperative relay-HARQ channel based on BER maximization. The main contributions of this paper are as follows:

- We propose to use CoRe for the relay-HARQ scheme similar to [13]. As depicted in Fig. 1, the source and the relay jointly perform the retransmission to the destination simultaneously, practically forming a 2-by-1 MIMO system.
- In our cooperative relay-HARQ settings, we formulate the CoRe design into a Q3AP problem. By taking advantage of the latest numerical solvers, we demonstrate significant performance gain of the optimized CoRe over non-CoRe and simple CoRe schemes for various channel settings.

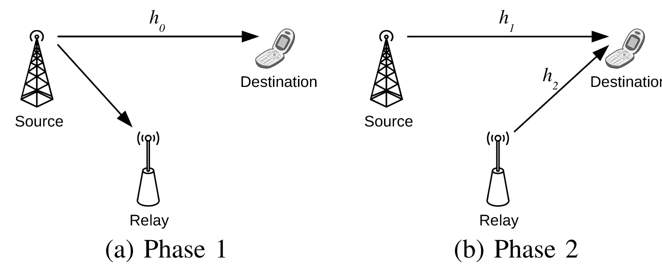

Fig. 1. Cooperative relay-HARQ networks.

The rest of this paper is organized as follows. Section II describes the cooperative relay-HARQ system model. Section III formulates the CoRe design into a Q3AP solution and provides a brief description of the numerical algorithm. The numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. System Model and Problem Formulation

We consider the cooperative relay-HARQ network depicted in Fig. 1. Denote $\mathcal{C}$ as the constellation used by this relay network. In the first phase, the source convert a bit sequence of length $\log_2 |\mathcal{C}|$ into symbols with Gray mapping $\psi_0 : \{0, \ldots, |\mathcal{C}| - 1\} \rightarrow \mathcal{C}$. The bit sequence is indexed by its decimal equivalence $p \in \{0, \ldots, |\mathcal{C}| - 1\}$. Then the source transmit $\psi_0[p]$ to the destination via channel h_0 which is also overheard by the decode-and-forward (DF) relay. We assume that relay is placed strategically so that it has negligible decoding error rate as in []. Upon receiving a request for retransmission, the second phase is started. This time the source and the relay adop remap p into $\psi_1[p]$ and $\psi_2[p]$, respectively, where potentially $\psi_1 \neq \psi_0$ and $\psi_2 \neq \psi_0$. The remapped symbols are transmitted simultaneously on the same band to the destination via channel h_1 and h_2. In summary, the received signal at the destination during the 2 phases are

$$y_1 = h_0\psi_0[p] + v_1 \tag{1a}$$
$$y_2 = h_1\psi_1[p] + h_2\psi_2[p] + v_2 \tag{1b}$$

where $v_1 \sim \mathcal{CN}(0, \sigma_v^2)$ and $v_2 \sim \mathcal{CN}(0, \sigma_v^2)$ are the additive noise. Thoughout this work, we assume the channels h_0, h_1 and h_2 follows independent Rician distribution.

Assuming that the destination has perfect channel state information (CSI), it decides the index p with the maximum likelihood (ML) rule

$$\min_{\hat{p}} |y_1 - \hat{h}_0\psi_0[\hat{p}]|^2 + |y_2 - \hat{h}_1\psi_1[\hat{p}] - \hat{h}_2\psi_2[\hat{p}]|^2. \quad (2)$$

III. Optimal Constellation Rearrangement

In this section we first formulate the min-BER CoRe design into a Q3AP problem, and explain the numerical approach to compute the input cost matrix to Q3AP problem. Then we provide an efficient algorithm to solve the Q3AP solution.

A. BER Maximization via Q3AP solution

Assume that the information-bearing index p follows a uniform distribution, the BER can be upper-bounded using pair-wise error probability (PEP) []

$$P_{BER} = \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}(q|p) \quad (3)$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q and $P_{PEP}(q|p)$ is the probability for the ML decoder to prefer q over p when p is actually transmitted. According to (2), we have

$$P_{PEP}(q|p) = P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,q,\psi_1,\psi_2) < 0\} \quad (4)$$

i.e. given indices p, q and the remapping scheme ψ_1, ψ_2, the probability of random variable $\delta < 0$ evaluated over the random variables h_0, h_1, h_2, v_0, v_1, and δ is defined as

$$\delta = |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + \\ |h_1(\psi_1[p] - \psi_1[q]) + h_2(\psi_2[p] - \psi_2[q]) + v_1|^2 - |v_1|^2. \quad (5)$$

In order to formulate the Q3AP problem, we denote binary variable $x_{pij} = 1$ if $\psi_1[p] = \psi_0[i]$ and $\psi_2[p] = \psi_0[j]$ and $x_{pij} = 0$ otherwise. Denote $\mathbf{x} = \{x_{pij}|p, i, j = 0, \dots, Q-1\}$, and the constraint sets:

$$\mathcal{P} = \left\{\mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\} \quad (6a)$$

$$\mathcal{I} = \left\{\mathbf{x} : \sum_{i=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\} \quad (6b)$$

$$\mathcal{J} = \left\{\mathbf{x} : \sum_{j=0}^{Q-1} x_{pij} = 1, x_{pij} \in \{0,1\}\right\}. \quad (6c)$$

Then from (3)(4)(9), the BER-minimization CoRe scheme $\min_{\psi_1,\psi_2} P_{BER}$ can be reformulated as

$$\min_{\mathbf{x}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} c_{pijqkl} x_{pij} x_{qkl} \quad (7)$$
$$\text{s.t. } \mathbf{x} \in \mathcal{P} \cap \mathcal{I} \cap \mathcal{J}$$

in which

$$c_{pijqkl} = \frac{B[p,q]}{Q} P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \quad (8)$$

$$\delta = |h_0(\psi_0[p] - \psi_0[q]) + v_0|^2 - |v_0|^2 + \\ |h_1(\psi_0[i] - \psi_0[k]) + h_2(\psi_0[j] - \psi_0[l]) + v_1|^2 - |v_1|^2. \quad (9)$$

B. Computation of the Pair-wise Symbol Rate

In this section we focus on the computation of the parameters $\{c_{pijqkl}\}$ of Q3AP problem. According to 8, the key to compute c_{pijqkl} lies in the evaluation of $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l)$, i.e. the CDF of random variable $\delta(p,i,j,q,k,l)$ as in (9). For the general Rician channel assumption $h_m \sim \mathcal{CN}(\mu_{h_m}, \sigma_{h_m}^2), m = 0,1,2$, we extend the method in [] to compute $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$:

$$P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\ \approx \frac{1}{2v}\sum_{t=1}^{v} \Re\{\Phi_\delta(\xi + j\xi\tau_t)\} + \tau_t \Im\{\Phi_\delta(\xi + j\xi\tau_t)\} \quad (10)$$

where the moment generating function (MGF) $\Phi_\delta(\omega) = \mathbb{E}_\delta[\exp(-\omega\delta)]$, $\tau_k = \tan((k-1/2)\pi/v)$ and $\Re\{\cdot\}$, $\Im\{\cdot\}$ denotes the real and image part, respectively. Parameter ξ is selected to ensure convergence of the integration and [14] suggested to use $\xi = 1/4$. The size u of the expansion (10) needs to be larger when $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$ is smaller in order to maintain an acceptable numerical accuracy.

To compute $\Phi_\delta(\omega)$, denote Gaussian random vectors $\mathbf{z}_1 = [h_0, v_0]^T$, $\mathbf{z}_2 = [h_1, h_2, v_1]^T$, such that $\mathbf{z}_m \sim \mathcal{CN}(\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m)$, $m = 1, 2$, where

$$\boldsymbol{\mu}_1 = [\mu_{h_0}, 0]^T, \boldsymbol{\mu}_2 = [\mu_{h_1}, \mu_{h_2}, 0]^T \quad (11)$$

$$\boldsymbol{\Sigma}_1 = \text{diag}\left(\sigma_{h_0}^2, \sigma_v^2\right), \boldsymbol{\Sigma}_2 = \text{diag}\left(\sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_v^2\right) \quad (12)$$

Then (9) can be rewritten as $\delta = \mathbf{z}_1^H \mathbf{A}_1 \mathbf{z}_1 + \mathbf{z}_2^H \mathbf{A}_2 \mathbf{z}_2$, where

$$\mathbf{A}_1 = \begin{bmatrix} |e_{pq}|^2 & e_{pq}^* \\ e_{pq} & 0 \end{bmatrix} \quad (13a)$$

$$\mathbf{A}_2 = \begin{bmatrix} |e_{ik}|^2 & e_{ik}^* e_{jl} & e_{ik}^* \\ e_{ik} e_{jl}^* & |e_{jl}|^2 & e_{jl}^* \\ e_{ik} & e_{jl} & 0 \end{bmatrix} \quad (13b)$$

in which where $e_{ab} = \psi_0[a] - \psi_0[b]$. Then the MGF can be computed as [15]

$$\Phi_\delta(\omega) = \sum_{m=1,2} \frac{\exp(-\omega \boldsymbol{\mu}_m^H \mathbf{A}_m (\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)^{-1} \boldsymbol{\mu}_m)}{\det(\mathbf{I} + \omega \boldsymbol{\Sigma}_m \mathbf{A}_m)}. \quad (14)$$

When the Rician-fading channel reduces to Rayleigh-fading channel, i.e. when $\mu_{h_m} = 0, m = 0,1,2$ there is a simple upperbound of $P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\}$ similar to [4]. Note that

$$P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\ = \mathbb{E}_{h_0,h_1,h_2}\left\{Q\left(\sqrt{\frac{|h_0 e_{pq}|^2 + |h_1 e_{ik} + h_2 e_{jl}|^2}{2\sigma_v^2}}\right)\right\} \quad (15)$$

(a) ψ_0 (Gray-mapping)

(b) ψ_1 (c) ψ_2

Fig. 2. Q3AP optimized CoRe schemes.

Applying the same relaxation as in [4], we have

$$\begin{aligned} & P_{h_0,h_1,h_2,v_0,v_1}\{\delta(p,i,j,q,k,l) < 0\} \\ & \leq \frac{3\sigma_v^4}{\sigma_{h_0}^2 |e_{pq}|^2 (\sigma_{h_1}^2 |e_{ik}|^2 + \sigma_{h_2}^2 |e_{jl}|^2)} \end{aligned} \tag{16}$$

which is tight in high-SNR regime.

C. Q3AP Solution

IV. Numerical Results

In this section we present the numerical results of CoRe under various channel settings. In our simulation, all Rician channels are assume to have the same Rician parameter K. Also we assume that during the second phase, the phase of channel h_1 and h_2 can be aligned at the source and relay, respectively. Consequently, we define $\mu_{h_0} = \mu_{h_1} = \sqrt{K/(K+1)}$, $\mu_{h_2} = a\sqrt{K/(K+1)}$, $\sigma_{h_0}^2 = \sigma_{h_1}^2 = 1/(K+1)$ and $\sigma_{h_2}^2 = a^2/(K+1)$, where $a \in \mathbb{R}$ represents the ratio between the amplitude of the line of sight (LOS) components of the relay-to-destination and the source-to-destination link. Throughout the simulation we consider 16-QAM constellation thus $Q = 16$. The noise power is parameterized with $E_b/N0$ of the source-to-destination link.

Firstly we provide an example of Q3AP optimized CoRe. When $E_b/N0 = dB$, $K = 10$ and $a = 1$, the remapping scheme of ψ_0 (Gray-mapping), ψ_1 and ψ_2 is depicted in Fig. 2.

V. Conclusion

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