

# Assignment 2

Sta242 Winter 2015  
Duncan Temple Lang

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**Due:** Wednesday, 29th April, 4pm

**Send electronic version (PDF and R package in separate files) and the address of the git repository to `dtemplelang@ucdavis.edu` with the exact subject STA242 Assignment 2**  
**Put a printed copy of the PDF and the code in Nick's mailbox.**

This project involves a simulation of a simple traffic flow model that exhibits a phase transition.

This homework focuses on several programming topics. One is writing *good*, flexible, reusable R functions and testing them to verify they are correct. The second topic is to use the S3 class mechanism to define classes and methods for printing and plotting objects representing the simulation. The third element is to make the code efficient, e.g., by profiling and using different computational approaches. The final aspect is to create an R package containing your code, help files and anything else. This will involve

1. Writing Functions,
2. Testing Functions,
3. Debugging,
4. Vectorized computations (rather than loops, if possible),
5. Profiling,
6. S3 classes and methods,
7. R packages.

You are to use git for the assignment to manage the code.

## 1 The Simulation Process

The simulation is about cars moving on a grid. We have two types of cars, “blue” and “red”. These move on a two-dimensional grid of size  $r$  by  $c$ . We populate the grid by placing  $\rho \times r \times c$  (with  $0 < \rho < 1$ ) cars at random positions in the  $r \times c$  cells, but with no two cars occupying the same cell. The type of each car is selected randomly from “red” and “blue” with equal probability. ( You can explore different probabilities, or forcing the numbers to be the same.)

Now the cars can move. In our configuration, the blue cars move at time periods  $t = 1, 3, 5, \dots$  and the red cars move at time periods  $t = 2, 4, 6, \dots$ , i.e., they alternate in time. The “blue” cars move vertically upward. The “red” cars move horizontally rightwards. When a car gets to the edge of the grid, it “wraps” around, i.e., when a blue car gets to the top row, the next time it moves it goes to the bottom row of the same column. Similarly a red car that gets to the right edge of the grid will move next to the first column of the grid, i.e., the extreme left.

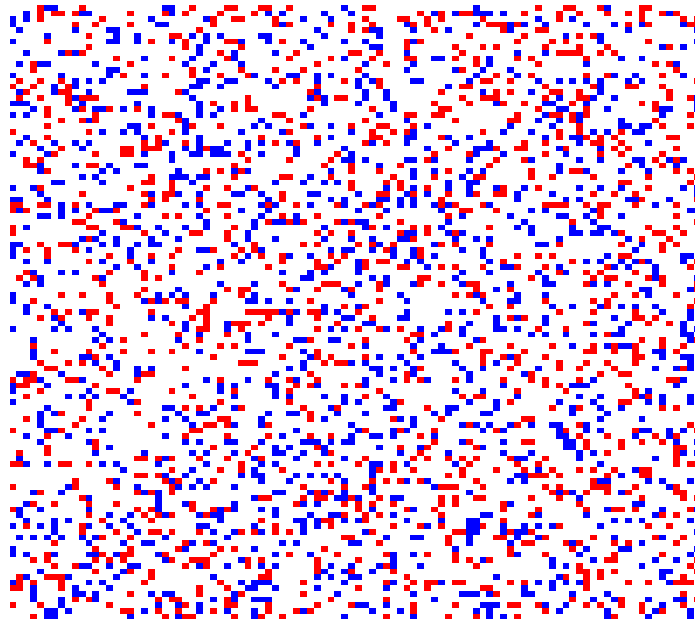
A car cannot move to cell if that cells is already occupied by another car (of any color).

This process is called the Biham-Middleton-Levine Traffic Model and is of interest because it is one of the very simplest processes that exhibits a phase-transition, and also self-organizing behavior. As  $\rho$  changes from .2 to .5, there

is a point at which the behavior dramatically shifts. And recent research has identified sub-structures. Additionally, while each car has a simple rule to move that only depends on whether another car is in its target location, the cars form a collective group and establish a clear pattern of self-organization motion.

## 2 Tasks

1. You are to write functions to simulate this process. You want to allow the caller specify  $\rho$  (or the number of cars or the number of red and number of blue cars). Also, she should be able to specify the dimension of the grid.
2. Your function should return an object with a class to identify it as a BML grid.
3. You can consider how to represent the state of grid so as to facilitate the computations to move the cars in each time step. Different representations will lead to different computations which may be more or less efficient.
4. Write a **method** for the function **plot()** to display the grid. Similarly define a method for **summary()** to provide a summary of the current state of the grid. One approach for the plot method is to produce something like



The functions **image()** and **rect()** may be useful.

5. Write one or more functions to move the cars on the grid for a time period  $t$ . Verify that it is correct, i.e. do some computations, perhaps write a function that compares the new positions with the old. Make certain to test the “degenerate” cases.
6. Use these functions to simulate the process for different grid sizes and values of  $\rho$ , ranging from .2 to .7. Interpret the results.
7. Time these runs and profile them (using **Rprof()** and **summaryRprof()**) to find where the bottlenecks are occurring. Use this information to refine your code or introduce an entirely new algorithm/approach (think vectorization!)

8. Write functions to be able to compute the number of cars that moved, that were blocked, and the average velocity at each time step. You might want to do this by passing the function(s) two grids from time  $t$  and  $t + 1$  respectively and computing the quantities from the difference.
9. Write up your findings both about the stochastic process and also the profiling. Hand in a PDF file with your writeup and an R package. Document the primary functions in the package. Use `R CMD check` to verify the package is valid.

Your package should make (at least) 2 functions available: **createBMLGrid()** and **runBMLGrid()**. The **createBMLGrid()** function should be callable as

```
g = createBMLGrid(r = 100, c = 99, ncars = c(red = 100, blue = 100))
```

where `r` and `c` are the dimensions of the grid and `ncars` specifies the number of red and blue cars.

The **runBMLGrid()** should be callable as

```
g.out = runBMLGrid(g, numSteps = 10000)
```

`g` is the grid (e.g., created with **createBMLGrid()**) and `numSteps` is the number of time steps.

You can make the signatures for these functions richer to accept the inputs in richer and more flexible ways, but they must allow us to call them in the manner shown above.

Take the time to review and refine your functions. Make certain to format and comment your code. Use class inheritance to customize methods, where appropriate.

Get started on this as soon as possible and ask lots of questions on Piazza and on the class mailing list.

*I strongly suggest that you implement a simple version of the code first and ensure it is working correctly. Then you can optimize it for efficiency. You will be able to use the correct version as a means of verifying any improved versions. Include all of the versions in the package.*