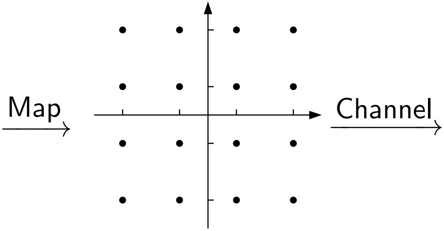
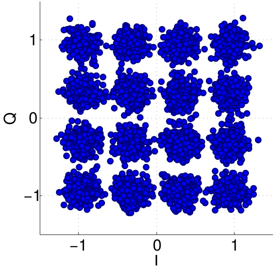
Modulation Mapping

p	bits
0	0000
1	0001
2	0010
...	...
15	1111





- Unideal wireless channel tends to cause demodulation errors.
- Constellation points closer to each other are more likely to be confused.

Modulation mapping needs to be carefully designed!

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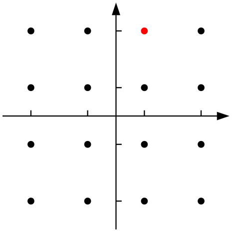
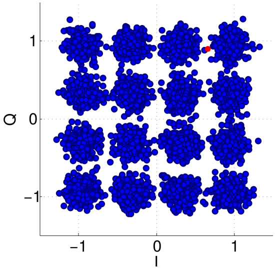
Map $\longrightarrow$



Channel $\longrightarrow$



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Single Transmission: Gray-mapping

Strategy (Gray-mapping)

Neighboring constellation points (horizontally or vertically) differ only by 1 bit, so as to minimize the Bit Error Rate (BER).

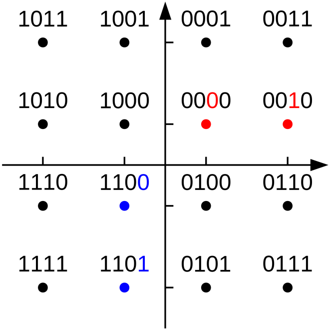


Figure : Gray-mapping for 16-QAM, 3GPP TS 25.213.

HARQ with Constellation Rearrangement (CoRe)

Hybrid Automatic Repeat reQuest (HARQ)

- Same piece of information is retransmitted again and again, and combined at the receiver until it is decoded successfully or expiration.
- An error control scheme widely used in modern wireless systems such as HSPA, WiMAX, LTE, etc.

Constellation Rearrangement (CoRe)

- For each round of retransmission, different modulation mappings are used (explained next).
- Exploit the Modulation Diversity (MoDiv).

An Example of CoRe

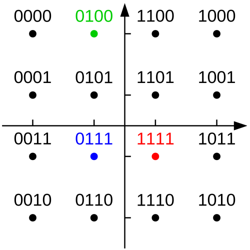


Figure : Original transmission.

- Original transmission: 0111 is easily confused with 1111, but well distinguished from 0100.

An Example of CoRe

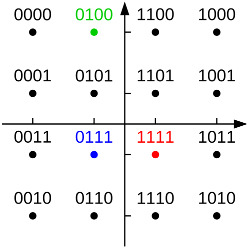


Figure : Original transmission.

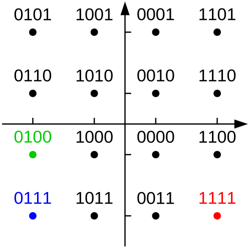


Figure : First retransmission.

- Original transmission: 0111 is easily confused with 1111, but well distinguished from 0100.
- First retransmission: 0111 should now be mapped far away from 1111, but can be close to 0100.

General Design of MoDiv Through CoRe

Challenges

1. More than 1 retransmissions?
2. More general wireless channel models?
3. Larger constellations (e.g. 64-QAM)?

We formulated 2 different MoDiv design problems into Quadratic Assignment Problems (QAPs) and demonstrate the performance gain over existing CoRe schemes.

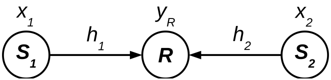
Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- System components: 2 sources (S_1, S_2) communicate with each other with the help of 1 relay (R).

Figure : TWRC-ANC channel.

Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- System components: 2 sources (S_1, S_2) communicate with each other with the help of 1 relay (R).
- Alternating between 2 phases:
  - Multiple-Access Channel (MAC) phase: the 2 sources transmit to the relay simultaneously.



$$y_R = h_1 x_1 + h_2 x_2 + n_R$$

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  - Broadcast Channel (BC) phase: the relay amplify and broadcast the signal received during the MAC phase back to the 2 sources

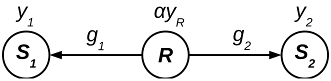

Figure : TWRC-ANC channel.

$$y_1 = \alpha g_1 y_R + n_1,$$
$$y_2 = \alpha g_2 y_R + n_2$$

HARQ-Chase Combining (CC) Protocol

- Q: size of the constellation.
- M: maximum number of retransmissions.
- $\psi_m[p]$, $m = 0, \ldots, M$, $p = 0, \ldots, Q-1$: constellation mapping function between "label" p to a constellation point for the m-th retransission.

Due to symmetry of the channel, consider the transmission from S_1 to S_2 only. The received signal during the m-th retransmission of label p is:

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} (h_1^{(m)} \psi_m[p] + h_2^{(\tilde{m})} \psi_{\tilde{m}}[\tilde{p}] + n_R^{(m)}) + n_2^{(m)},$$

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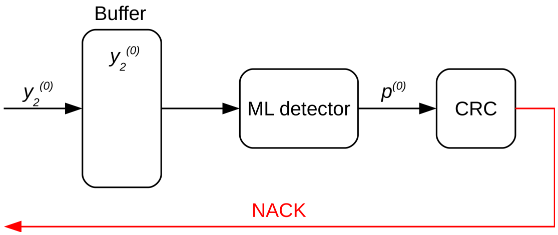
HARQ-Chase Combining (CC) Protocol (Continued)

The receiver combines all the received symbols across all retransmissions so far until decoding is determined successful.



Maximum Likelihood (ML) detector

$$p^* = \arg\min_{p} \sum_{k=0}^{m} \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}.$$

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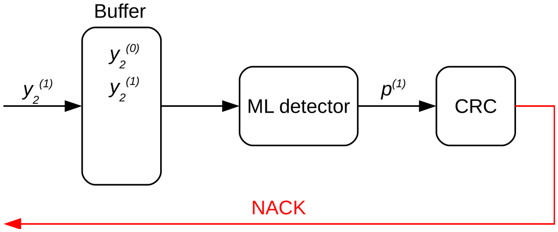

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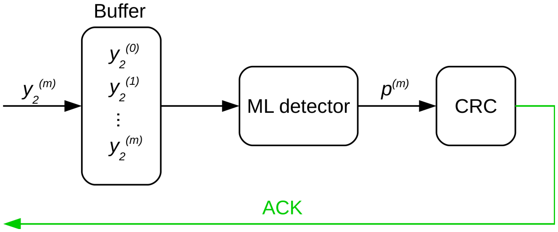

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MoDiv Design: Criterion

Bit Error Rate (BER) upperbound after m-th retransmission

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{D[p,q]}{Q \log_2 Q} P_{PEP}^{(m)}(q|p),$$

- $D[p,q]$: hamming distance between the bit representation of label p and q.
- $P_{PEP}^{(m)}(q|p)$: pairwise error probability (PEP), the probability that when label p is transmitted, but the receiver decides q is more likely than p after m-th retransmission.

MoDiv Design: Criterion (Continued)

Is minimizing $P_{BER}^{(m)}$ over the mappings $\psi_1[\cdot], \ldots, \psi_m[\cdot]$ directly a good idea?

1. No one knows how many retransmissions is needed in advance (value of m).
2. Jointly designing all m mappings is prohibitively complex.
3. $P_{PEP}^{(m)}(q|p)$ can only be evaluated numerically, very slow and could be inaccurate.

MoDiv Design: Modified Criterion

1. Successive optimization instead of joint optimization.

$$\text{Joint:} \quad \min_{\psi^{(k)}, k=0,\dots,m} P_{BER}^{(m)}, \ m = 1, \dots, M$$

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$$\text{Successive:} \quad \min_{\psi^{(m)} | \psi^{(k)}, k=0,\ldots,m-1} \tilde{P}_{BER}^{(m)}, \; m = 1, \ldots, M$$

2. A closed-form approximation to $P_{PEP}^{(m)}(q|p)$ that can be iteratively updated for growing m.

$$\tilde{P}_{PEP}^{(m)}(q|p) = \tilde{P}_{PEP}^{(m-1)}(q|p) \tilde{E}_k[p, q]$$
$$\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2$$

Representation of CoRe

Representing $\psi_m[\cdot]$ with Q^2 binary variables:

$$x_{pi}^{(m)} = \begin{cases} 1 & \text{if } \psi_m[p] = \psi_0[i] \\ 0 & \text{otherwise.} \end{cases} \quad p, i = 0, \ldots, Q-1$$

ψ_0 represents Gray-mapping for the original transmission (fixed).

Constraints: $\psi_m[\cdot]$ as a permutation of $0, \ldots, Q-1$

$$\sum_{p=0}^{Q-1} x_{pi} = 1$$

$$\sum_{i=0}^{Q-1} x_{pi} = 1$$

	$i=0$	$i=1$	$i=2$	$i=3$
$p=0$	0	1	0	0
$p=1$	0	0	1	0
$p=2$	1	0	0	0
$p=3$	0	0	0	1

A Successive KB-QAP Formulation

MoDiv design via successive KB-QAP

1. Set $m = 1$. Initialize the "distance" matrix and the approximated PEP, for $i, j, p, q = 0, \ldots, Q-1$:

$$d_{ij} = \tilde{E}_k[i,j],\ \tilde{P}^{(0)}_{PEP}(q|p) = d_{pq}/2$$

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3. Solve the m-th KB-QAP problem:

$$\min_{\{x^{(m)}_{pi}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f^{(m)}_{pq} d_{ij} x^{(m)}_{pi} x^{(m)}_{qj}$$

A Successive KB-QAP Formulation (Continued)

MoDiv design via successive KB-QAP

4. Update PEP:

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)}$$

where $\hat{x}_{pi}^{(m)}$ is the solution from Step 3.

A Successive KB-QAP Formulation (Continued)

MoDiv design via successive KB-QAP

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   where $\hat{x}^{(m)}_{pi}$ is the solution from Step 3.

5. Increase m by 1, return to Step 2 if $m \leq M$.

Numerical Results: Simulation Settings

- 64-QAM constellation ($Q = 64$).

[1] E. Taillard, “Robust taboo search for the quadratic assignment problem,” Parallel Computing, vol.17, no.4, pp.443-455, 1991.

[2] “Enhanced HARQ Method with Signal Constellation Rearrangement,” in 3rd Generation Partnership Project (3GPP), Technical Specification TSGR1#19(01)0237, Mar. 2001.

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  3. QAP-based solution (QAP)

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Numerical Results: Uncoded BER

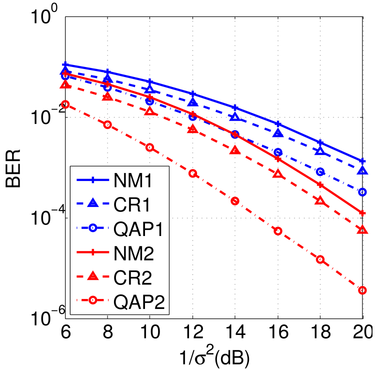


Figure : $m = 1, 2.$

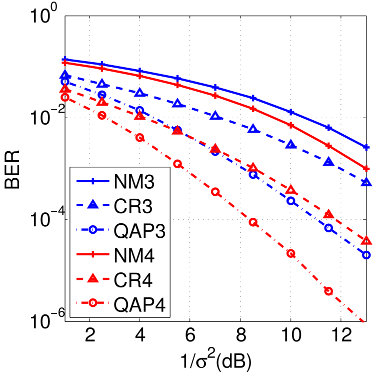
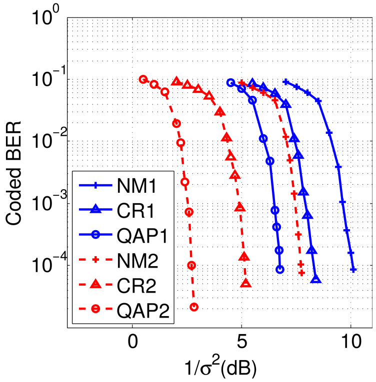


Figure : $m = 3, 4.$

Numerical Results: Coded BER

Add a Forward Error Correction (FEC) code so that the coded BER drop rapidly as the noise power is below a certain level. The result is termed "waterfall curve" which is commonly used to highlight the performance gain in dB.



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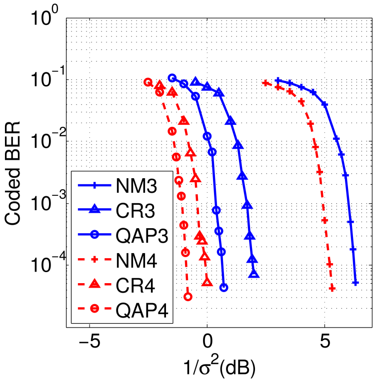


Figure : $m = 3, 4$.

Numerical Results: Average Throughput

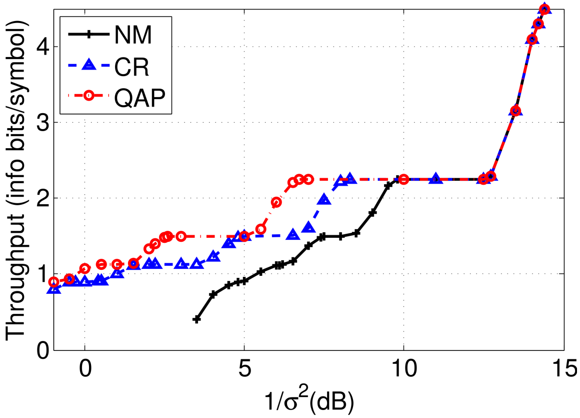


Figure : Throughput comparison.

Multiple-Input and Multiple-Output (MIMO) Channel

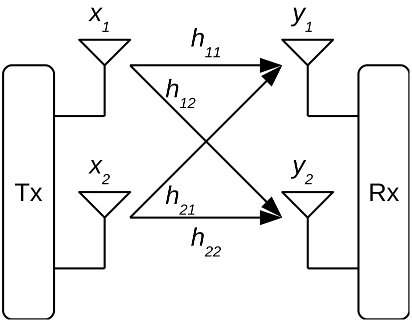
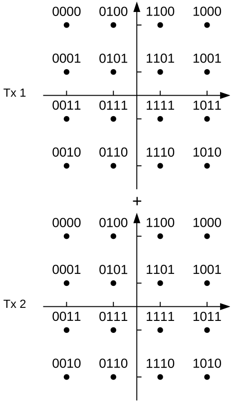


Figure : A 2×2 MIMO channel, $y_1 = h_{11}x_1 + h_{21}x_2 + n_1$, $y_2 = h_{12}x_1 + h_{22}x_2 + n_2$, or simply $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$.

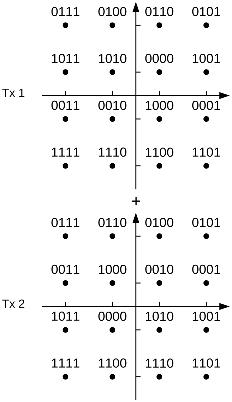
- An essential element in most modern wireless communication standards: Wi-Fi, HSPA+, LTE, WiMAX, etc.
- How do we generalize the idea of MoDiv design for MIMO channel?

An Example of CoRe for MIMO

- A 1×2 MIMO channel: $\mathbf{H} = [1, 1]$ (simple addition).
- Different mapping across the 2 transmitting antennas:
- Effective constellation seen by the receiver: $\psi_e = (\psi)_1 + (\psi)_2$.



Original transmission (Gray).



1st retransmission.