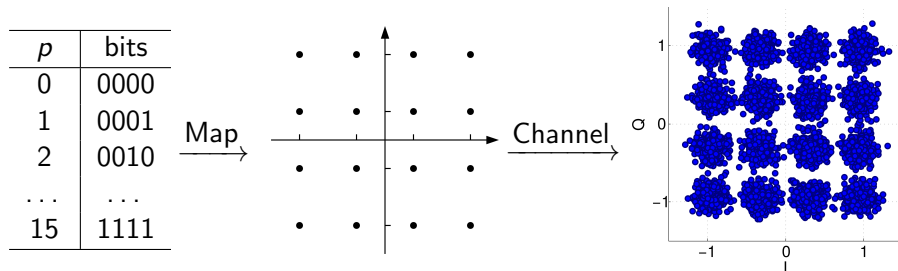


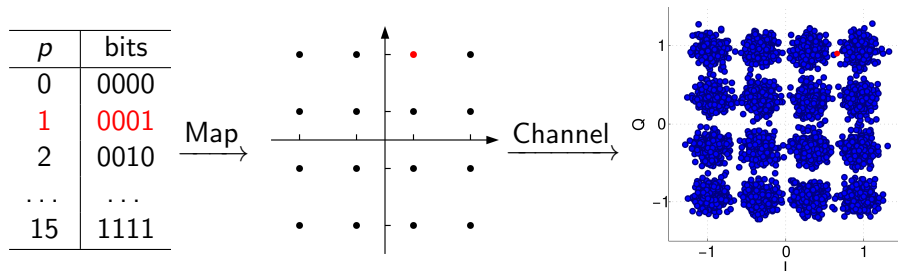
# Modulation Mapping



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# Single Transmission: Gray-mapping

## Strategy (Gray-mapping)

Neighboring constellation points (**horizontally** or **vertically**) differ only by 1 bit, so as to minimize the Bit Error Rate (BER).

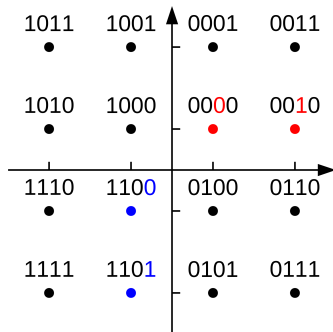


Figure : Gray-mapping for 16-QAM, 3GPP TS 25.213.

# HARQ with Constellation Rearrangement (CoRe)

## Hybrid Automatic Repeat reQuest (HARQ)

- ▶ Same piece of information is retransmitted again and again, and combined at the receiver until it is decoded successfully or expiration.
- ▶ An error control scheme widely used in modern wireless systems such as HSPA, WiMAX, LTE, etc.

## Constellation Rearrangement (CoRe)

- ▶ For each round of retransmission, different modulation mappings are used (explained next).
- ▶ Exploit the Modulation Diversity (MoDiv).

# An Example of CoRe

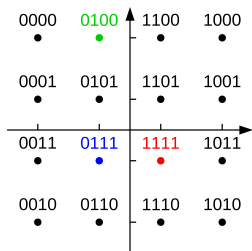


Figure : Original transmission.

- Original transmission: 0111 is easily confused with 1111, but well distinguished from 0100.

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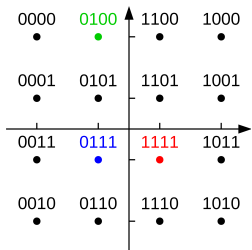


Figure : Original transmission.

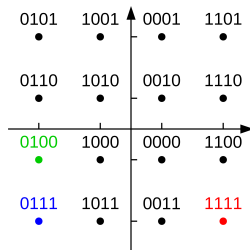


Figure : First retransmission.

- ▶ Original transmission: 0111 is easily confused with 1111, but well distinguished from 0100.
- ▶ First retransmission: 0111 should now be mapped far away from 1111, but can be close to 0100.

# General Design of MoDiv Through CoRe

## Challenges

1. More than 1 retransmissions?
2. More general wireless channel models?
3. Larger constellations (e.g. 64-QAM)?

We formulated 2 different MoDiv design problems into **Quadratic Assignment Problems (QAPs)** and demonstrate the performance gain over existing CoRe schemes.

# Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- ▶ System components: 2 sources ( $S_1$ ,  $S_2$ ) communicate with each other with the help of 1 relay ( $R$ ).

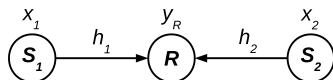


Figure : TWRC-ANC channel.



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- ▶ System components: 2 sources ( $S_1$ ,  $S_2$ ) communicate with each other with the help of 1 relay ( $R$ ).
- ▶ Alternating between 2 phases:
  - ▶ Multiple-Access Channel (MAC) phase: the 2 sources transmit to the relay simultaneously.

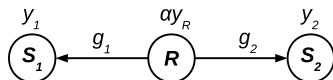


$$y_R = h_1 x_1 + h_2 x_2 + n_R$$

Figure : TWRC-ANC channel.

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- ▶ Alternating between 2 phases:
  - ▶ Multiple-Access Channel (MAC) phase: the 2 sources transmit to the relay simultaneously.
  - ▶ Broadcast Channel (BC) phase: the relay amplify and broadcast the signal received during the MAC phase back to the 2 sources



$$y_1 = \alpha g_1 y_R + n_1,$$
$$y_2 = \alpha g_2 y_R + n_2$$

Figure : TWRC-ANC channel.

# HARQ-Chase Combining (CC) Protocol

- ▶  $Q$ : size of the constellation.
- ▶  $M$ : maximum number of retransmissions.
- ▶  $\psi_m[p]$ ,  $m = 0, \dots, M$ ,  $p = 0, \dots, Q - 1$ : constellation mapping function between “label”  $p$  to a constellation point for the  $m$ -th retransmission.

Due to symmetry of the channel, consider the transmission from  $S_1$  to  $S_2$  only. The received signal during the  $m$ -th retransmission of label  $p$  is:

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} (h_1^{(m)} \psi_m[p] + h_2^{(\tilde{m})} \psi_{\tilde{m}}[\tilde{p}] + n_R^{(m)}) + n_2^{(m)},$$

# HARQ-Chase Combining (CC) Protocol

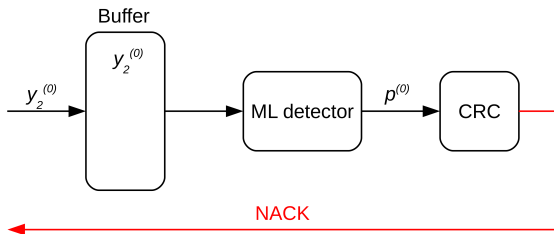
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# HARQ-Chase Combining (CC) Protocol (Continued)

The receiver combines all the received symbols across all retransmissions so far until decoding is determined successful.

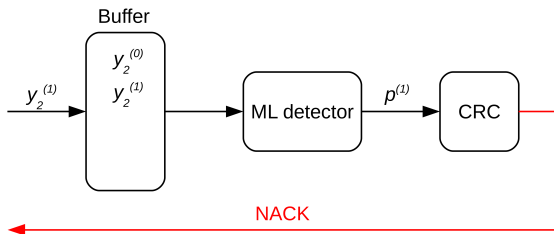


## Maximum Likelihood (ML) detector

$$p^* = \arg \min_p \sum_{k=0}^m \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}.$$

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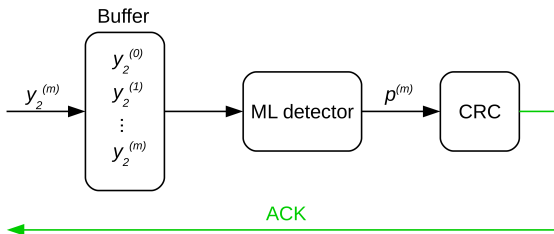


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# MoDiv Design: Criterion

Bit Error Rate (BER) upperbound after  $m$ -th retransmission

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{D[p, q]}{Q \log_2 Q} P_{PEP}^{(m)}(q|p),$$

- ▶  $D[p, q]$ : hamming distance between the bit representation of label  $p$  and  $q$ .
- ▶  $P_{PEP}^{(m)}(q|p)$ : pairwise error probability (PEP), the probability that when label  $p$  is transmitted, but the receiver decides  $q$  is more likely than  $p$  after  $m$ -th retransmission.



# MoDiv Design: Criterion (Continued)

Is minimizing  $P_{BER}^{(m)}$  over the mappings  $\psi_1[\cdot], \dots, \psi_m[\cdot]$  directly a good idea?

1. No one knows how many retransmissions is needed in advance (value of  $m$ ).
2. Jointly designing all  $m$  mappings is prohibitively complex.
3.  $P_{PEP}^{(m)}(q|p)$  can only be evaluated numerically, very slow and could be inaccurate.

# MoDiv Design: Modified Criterion

1. Successive optimization instead of joint optimization.

$$\text{Joint: } \min_{\psi^{(k)}, k=0, \dots, m} P_{BER}^{(m)}, m = 1, \dots, M$$

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2. A closed-form approximation to  $P_{PEP}^{(m)}(q|p)$  that can be iteratively updated for growing  $m$ .

$$\begin{aligned}\tilde{P}_{PEP}^{(m)}(q|p) &= \tilde{P}_{PEP}^{(m-1)}(q|p) \tilde{E}_k[p, q] \\ \tilde{P}_{PEP}^{(-1)}(q|p) &= 1/2\end{aligned}$$

# Representation of CoRe

Representing  $\psi_m[\cdot]$  with  $Q^2$  binary variables:

$$x_{pi}^{(m)} = \begin{cases} 1 & \text{if } \psi_m[p] = \psi_0[i] \\ 0 & \text{otherwise.} \end{cases} \quad p, i = 0, \dots, Q-1$$

$\psi_0$  represents Gray-mapping for the original transmission (fixed).

Constraints:  $\psi_m[\cdot]$  as a permutation of  $0, \dots, Q-1$

$$\sum_{p=0}^{Q-1} x_{pi} = 1$$

$$\sum_{i=0}^{Q-1} x_{pi} = 1$$

	$i = 0$	$i = 1$	$i = 2$	$i = 3$
$p = 0$	0	1	0	0
$p = 1$	0	0	1	0
$p = 2$	1	0	0	0
$p = 3$	0	0	0	1

# A Successive KB-QAP Formulation

## MoDiv design via successive KB-QAP

1. Set  $m = 1$ . Initialize the “distance” matrix and the approximated PEP, for  $i, j, p, q = 0, \dots, Q - 1$ :

$$d_{ij} = \tilde{E}_k[i, j], \quad \tilde{P}_{PEP}^{(0)}(q|p) = d_{pq}/2$$

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3. Solve the  $m$ -th KB-QAP problem:

$$\min_{\{x_{pi}^{(m)}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f_{pq}^{(m)} d_{ij} x_{pi}^{(m)} x_{qj}^{(m)}$$



# A Successive KB-QAP Formulation (Continued)

## MoDiv design via successive KB-QAP

### 4. Update PEP:

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)}$$

where  $\hat{x}_{pi}^{(m)}$  is the solution from Step 3.

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5. Increase  $m$  by 1, return to Step 2 if  $m \leq M$ .

# Numerical Results: Simulation Settings

- ▶ 64-QAM constellation ( $Q = 64$ ).

---

<sup>1</sup>E. Taillard, "Robust taboo search for the quadratic assignment problem," Parallel Computing, vol.17, no.4, pp.443-455, 1991.

<sup>2</sup>"Enhanced HARQ Method with Signal Constellation Rearrangement," in 3rd Generation Partnership Project (3GPP), Technical Specification TSGR1#19(01)0237, Mar. 2001.

# Numerical Results: Simulation Settings

- ▶ 64-QAM constellation ( $Q = 64$ ).
- ▶ Maximum number of 4 retransmissions ( $M = 4$ ).

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- ▶ Compare 3 MoDiv schemes:
  1. No modulation diversity (NM)
  2. A heuristic CoRe scheme for HSPA<sup>2</sup>(CR)
  3. QAP-based solution (QAP)

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# Numerical Results: Uncoded BER

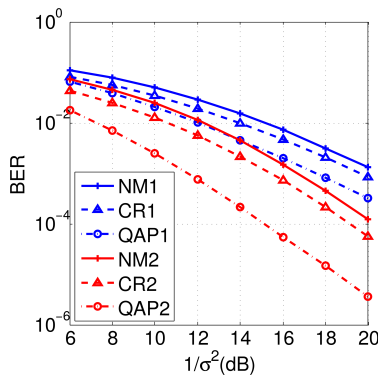


Figure :  $m = 1, 2$ .

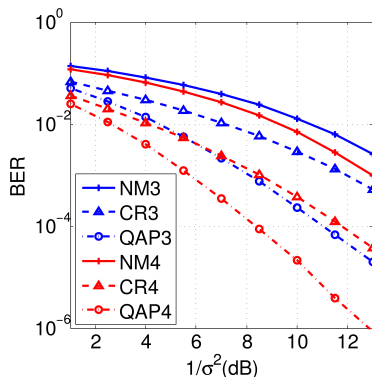


Figure :  $m = 3, 4$ .

## Numerical Results: Coded BER

Add a Forward Error Correction (FEC) code so that the coded BER drop rapidly as the noise power is below a certain level. The result is termed “waterfall curve” which is commonly used to highlight the performance gain in dB.

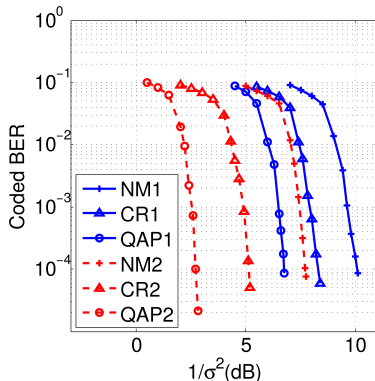


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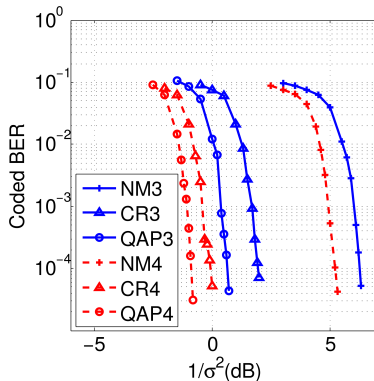


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# Numerical Results: Average Throughput

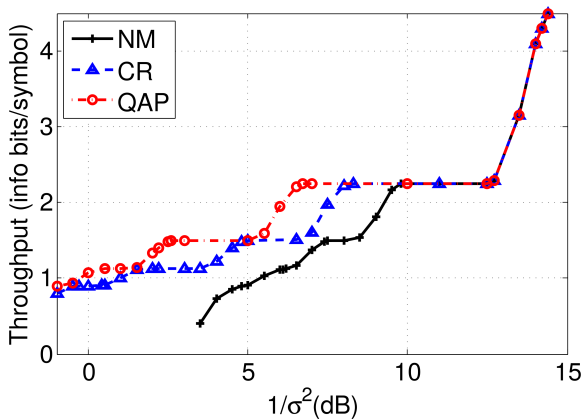


Figure : Throughput comparison.