Globally Optimal Regions and Boundaries as Minimum Ratio Weight Cycles

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Abstract—We describe a new form of energy functional for the modeling and identification of regions in images. The energy is defined on the space of boundaries in the image domain and can incorporate very general combinations of modeling information both from the boundary (intensity gradients, etc.) *and* from the interior of the region (texture, homogeneity, etc.). We describe two polynomial-time digraph algorithms for finding the *global* minima of this energy. One of the algorithms is completely general, minimizing the functional for any choice of modeling information. It runs in a few seconds on a 256x256 image. The other algorithm applies to a subclass of functionals, but has the advantage of being extremely parallelizable. Neither algorithm requires initialization.

Index Terms—Region identification, ratio, energy minimization, global optimum, active contour, snake, segmentation.

1 Introduction

One of the fundamental problems in image understanding is to identify regions in images with particular semantic content. It is safe to say that, if there was a mechanism to answer reliably the question, "What are the regions in the image that correspond to instances of *O*?" where *O* is some named class of objects, then many other tasks in image understanding would be greatly simplified.

Approaches to this problem, which of course is very far from being solved, tend to break into two categories. One approach is to segment the image globally, partitioning the image domain into labeled subsets based on some, usually generic criteria, with the hope that subsequent processing can group and split these regions using more sophisticated models and, hence, decompose the image domain into recognizable objects. These methods search a space of maps *from* the image domain to some other space.

Another approach to the same problem involves trying to identify objects in an image directly by searching a space of structures mapped *into* the image. Here, the emphasis is on modeling the properties of the regions occupied by objects from the outset, to various different levels of genericity. Examples of this approach are template-matching methods and the many variations on active contours.

Although these approaches are not entirely mathematically distinct (a region can always be modeled by its characteristic function, which is a field on the image), they do correspond conceptually to distinctions in the human visual system. It seems likely, for example [26], [29], that human perception of motion is based both on generic low-level computations, as well as on the identification and tracking of specific objects. They also lead to different visions of how to proceed: In one case, the development of mechanisms to perform further organization of the generic segmentation; in the other, the development of more sophisticated and specific models for individual regions. The two approaches are, in any case, complementary since generic segmentations can be used to inform the object models, as well as vice versa.

Many of these methods are explicitly or implicitly framed as optimization problems. The difficulty is that these optimization problems cannot, in general, be solved globally, meaning that the solutions that are found have an unknown dependence on initial conditions. In the case of segmentation methods, this usually takes the form of a choice of a number of region seeds, whereas for active contours an initial contour is necessary. For contours, the only problem that has up to this time been solved globally for general energies is that of finding an optimal curve joining two given points. A priori such curves do not allow the identification of regions, requiring further processing to group them into boundaries.

Unfortunately, the problem of finding a globally optimal boundary in an image without loss of descriptive power in the model cannot trivially be solved by the application of the same kind of techniques that work in the open curve case. The topological constraint of closure is not so easily incorporated into these local algorithms. In addition, there are obstructions to solving the problem of globally minimizing the linear form of energy typically used for active contours, to wit, the elimination of self-intersections and repeated segments, and the existence of trivial solutions. (We discuss these issues more fully in Section 4.1.) Therefore, it is of interest to have a model of region identification for which the global solution can be known in order to form some judgement of the relevance of the solution for the image as a whole, independent of the initial conditions.

In addition, many of the above models utilize only one of two possible sources of information about the region: the properties of the interior or of the boundary. Many segmentation methods partition the image domain based

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