fin which is in the form of a jet [9]. Because of the influence of the periodic excitation from the fin, a downstream moving staggered array of vortices, closely resembling a Karman vortex street but with reverse rotational direction, is formed in the wake [9]. The physics of thrust generation with oscillating motions was first presented by Durand [10] and after that observed in a number of experimental studies, for example, [11–13]. Rosen [14], Videler et al. [15], Lauder [5, 16], Drucker and Lauder [17], and Tytell and Lauder [18] are also some of the researchers who observed experimentally similar patterns in the wake of different fish species. However, numerical studies have provided more details on wake structures and effective parameters of undulatory motion of flexible body or fins [19–22] as well as vortical pattern in oscillating heaving foils [7, 9, 23–25].

Freymuth [11], Koochesfahani [12], Jones et al. [25], and Zhang et al. [26] showed through experiments and simulations that the wakes of oscillating airfoils can be characterized as drag-producing, neutral, or thrust producing mechanisms depending on the heaving/undulating frequency and amplitude. The general conclusion from the existing investigations is that the thrust force and propulsive efficiency strongly depend on the kinematic parameters of oscillations. Furthermore, the wake structure of the oscillatory mechanisms is closely related to the nondimensional parameter Strouhal number ($\mathrm{St} = 2fA/U$), where U is the free-stream velocity, A is the amplitude of oscillations, and f is the oscillating frequency.

In this paper a *NACA0012* foil, which oscillates with heaving and undulating mechanisms, is studied in three low, medium, and high Reynolds numbers, respectively, $\mathrm{Re} = 4000$, $\mathrm{Re} = 40000$, and $\mathrm{Re} = 400000$ ($\mathrm{Re} = UL/\nu$, where L is the characteristic length equal to the chord length of the foil and ν is the kinematic viscosity of the fluid). The oscillating frequency is varied from 0.001 to 0.05 for $\mathrm{Re} = 4000$, from 0.01 to 0.5 for $\mathrm{Re} = 40000$, and from 0.1 to 5 for $\mathrm{Re} = 400000$, which are equivalent to the St in the range of [0.05–2.5] with maximum amplitude excursion of $0.1L$. In present study, simulations are carried out for a typical foil geometry (*NACA0012*) which is commonly used in many applications as control surfaces or propulsors. In addition, a number of tests are accomplished to validate and evaluate the numerical solver in comparison with experimental data.

Several experimental and numerical works have separately studied the wake structure, energetics, and flow characteristics of undulating and heaving foils including the lift, drag, and propulsion efficiency [12, 13, 26–35]. However, this study attempts to present a comparison for the energetics, performance, and wake structures of the heaving and undulating mechanisms in an extended range of Strouhal numbers. Such comparisons could provide a wealth of data and consequences about the advantages and disadvantages of these mechanisms and also are helpful to select and design the appropriate mechanism for the specific applications. Furthermore, remarkable findings are brought out from this comparison which are presenting in Section 4. The range of Strouhal numbers reported here, that is, $0.05 < \mathrm{St} < 2.5$, for the undulating motion is wider than the well-known range in literature, that is, $0.2 < \mathrm{St} < 0.4$ [36]. This is in order for more observations and answering the following questions: why do the animals use the undulatory propulsion mechanisms in the reported range? And why do not they use lower or higher ranges of Strouhal numbers? In addition, the presented qualitative and quantitative data are useful in the designing and developing of bioinspired oscillatory mechanisms. Current research also brings out a comparison of the especial deformability of the undulating fins over the rigidity of the heaving fins which mostly exist in aquatic animals. Owing to the difficulty in testing the swimming parameters of live fish, experimental studies on the oscillating caudal fins are rare and hence such systematic numerical studies which can simulate the real conditions are essential.

The rest of this paper is organized as follows. The governing equations as well as methodology and implementation of numerical approach, including the enforcement of boundary conditions and the mesh motion and correction procedure, are described in the next section. In addition, the validation tests and calculation of parameters are presented. Then, the results including thrust, consumed power, efficiency, and wake structure of the heaving and undulating mechanisms are presented and discussed in detail. Finally, the major conclusions are presented.

2. Materials and Methods

2.1. Fluid-Solid Interaction Problem. The oscillations of bodies such as heaving and undulation of foils involve moving and deforming boundaries, respectively. The arbitrary Lagrangian-Eulerian (ALE) approach [37] is employed to encounter the fluid and moving body interactions. From the ALE viewpoint, the nodes of the computational domain mesh may be moved with the continuum, same as normal Lagrangian manner, be held fixed in Eulerian fashion, or be moved in some arbitrarily specified way to give a continuous rezoning capability [38]. In addition, the simulation of conservation equations on the moving meshes requires the geometric conservation law (GCL) to be satisfied [39].

2.2. Governing Equations of Fluid. The ALE formulations of the unsteady incompressible viscous flow, governed by the integral forms of Navier–Stokes equations, are as follows.

Conservation of mass:

$$\frac{\partial}{\partial t}\int_{V(t)} \rho V(t) + \int_{S(t)} \rho\left(\vec{v} - \vec{v}_g\right)\cdot \hat{n}\, dS = 0. \tag{1}$$

Conservation of momentum:

$$\frac{\partial}{\partial t}\int_{V(t)} \rho\vec{v}\, dV + \int_{S(t)} \rho\vec{v}\left(\vec{v} - \vec{v}_g\right)\cdot \hat{n}\, dS = \int_{V(t)} \left(\nabla\cdot\sigma + \rho\vec{f}_b\right) dV, \tag{2}$$

where, in the above equations, ρ is the fluid density, $\vec{v}$ is the Cartesian velocity vector of the fluid, $V(t)$ is an arbitrary