well as heavy flavours. Here we also consider light- and heavy- quark masses, but focus on lattice results and discuss them in more detail. We do not discuss the top quark, however, because it decays weakly before it can hadronize, and the nonperturbative QCD dynamics described by present day lattice simulations is not relevant. The lattice determination of light- (up, down, strange), charm- and bottom-quark masses is considered in Sects. 3.1, 3.2, and 3.3, respectively.

Quark masses cannot be measured directly in experiment because quarks cannot be isolated, as they are confined inside hadrons. On the other hand, quark masses are free parameters of the theory and, as such, cannot be obtained on the basis of purely theoretical considerations. Their values can only be determined by comparing the theoretical prediction for an observable, which depends on the quark mass of interest, with the corresponding experimental value.

In the last edition of this review [2], quark-mass determinations came from two- and three-flavour QCD calculations. Moreover, these calculations were most often performed in the isospin limit, where the up- and down-quark masses (especially those in the sea) are set equal. In addition, some of the results retained in our light-quark mass averages were based on simulations performed at values of m_{ud} which were still substantially larger than its physical value imposing a significant extrapolation to reach the physical up- and down-quark mass point. Among the calculations performed near physical m_{ud} by PACS-CS [93–95], BMW [7,8] and RBC/UKQCD [31], only the ones in Refs. [7,8] did so while controlling all other sources of systematic error.

Today, however, the effects of the charm quark in the sea are more and more systematically considered and most of the new quark-mass results discussed below have been obtained in $N_f = 2+1+1$ simulations by ETM [4], HPQCD [14] and FNAL/MILC [5]. In addition, RBC/UKQCD [10], HPQCD [14] and FNAL/MILC [5] are extending their calculations down to up-down-quark masses at or very close to their physical values while still controlling other sources of systematic error. Another aspect that is being increasingly addressed are electromagnetic and $(m_d - m_u)$, strong isospin-breaking effects. As we will see below these are particularly important for determining the individual up- and down-quark masses. But with the level of precision being reached in calculations, these effects are also becoming important for other quark masses.

Three-flavour QCD has four free parameters: the strong coupling, α_s (alternatively $\Lambda_{\rm QCD}$) and the up-, down- and strange-quark masses, m_u, m_d and m_s. Four-flavour calculations have an additional parameter, the charm-quark mass m_c. When the calculations are performed in the isospin limit, up- and down-quark masses are replaced by a single parameter: the isospin-averaged up- and down-quark mass, $m_{ud} = \frac{1}{2}(m_u + m_d)$. A lattice determination of these parameters, and in particular of the quark masses, proceeds in two steps:

1. One computes as many experimentally measurable quantities as there are quark masses. These observables should obviously be sensitive to the masses of interest, preferably straightforward to compute and obtainable with high precision. They are usually computed for a variety of input values of the quark masses which are then adjusted to reproduce experiment. Another observable, such as the pion decay constant or the mass of a member of the baryon octet, must be used to fix the overall scale. Note that the mass of a quark, such as the b, which is not accounted for in the generation of gauge configurations, can still be determined. For that an additional valence-quark observable containing this quark must be computed and the mass of that quark must be tuned to reproduce experiment.
2. The input quark masses are bare parameters which depend on the lattice spacing and particulars of the lattice regularization used in the calculation. To compare their values at different lattice spacings and to allow a continuum extrapolation they must be renormalized. This renormalization is a short-distance calculation, which may be performed perturbatively. Experience shows that one-loop calculations are unreliable for the renormalization of quark masses: usually at least two loops are required to have trustworthy results. Therefore, it is best to perform the renormalizations nonperturbatively to avoid potentially large perturbative uncertainties due to neglected higher-order terms. Nevertheless we will include in our averages one-loop results if they carry a solid estimate of the systematic uncertainty due to the truncation of the series.

In the absence of electromagnetic corrections, the renormalization factors for all quark masses are the same at a given lattice spacing. Thus, uncertainties due to renormalization are absent in ratios of quark masses if the tuning of the masses to their physical values can be done lattice spacing by lattice spacing and significantly reduced otherwise.

We mention that lattice QCD calculations of the b-quark mass have an additional complication which is not present in the case of the charm- and light-quarks. At the lattice spacings currently used in numerical simulations the direct treatment of the b quark with the fermionic actions commonly used for light quarks will result in large cutoff effects, because the b-quark mass is of order one in lattice units. There are a few widely used approaches to treat the b quark on the lattice, which have been already discussed in the FLAG 13 review (see Section 8 of Ref. [2]). Those relevant for the