$$P(\bar{x}|\bar{y}) = \prod_{i=1}^{n} P(x_i|y_i) = \prod_{i=1}^{n} P(x_i|y_i, z_i) = P(\bar{x}|\bar{y}, \hat{z}). \quad (33)$$

Therefore,

$$\frac{P(\bar{x}, \bar{y})}{P(\bar{y})} = \frac{P(\bar{x}, \bar{y}, \hat{z})}{P(\bar{y}, \hat{z})}.$$

Each term of (32) may be written as:

$$\sum_{\bar{y} \in C(\hat{y})} \frac{P(\bar{x}, \bar{y}, \hat{z})}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}', \hat{z})} = \sum_{\bar{y} \in C(\hat{y})} \frac{P(\bar{y}, \hat{z}) \frac{P(\bar{x}, \bar{y})}{P(\bar{y})}}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}', \hat{z})} \quad (34)$$

because, from (33), $P(\bar{x}|\bar{y}) = P(\bar{x}|\bar{y}, \hat{z})$. Similarly, each term of (31) may be written as:

$$\sum_{\bar{y} \in C(\hat{y})} \frac{P(\bar{x}, \bar{y})}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}')} = \sum_{\bar{y} \in C(\hat{y})} \frac{P(\bar{x}, \bar{y})}{P(\bar{y})} \frac{P(\bar{y})}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}')}. \quad (35)$$

Now, noting that, for $\bar{y} \in C(\hat{y}), P(\bar{y}, \hat{y}) = P(\bar{y})$, and because of Lemma 1,

$$\frac{P(\bar{y})}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}')} = \frac{P(\bar{y}, \widehat{y})}{P(\widehat{y})} = P(\bar{y}|\widehat{y}) =$$

$$= P(\bar{y}|\widehat{y}, \hat{z}) = \frac{P(\bar{y}, \hat{z})}{\sum_{\bar{y}' \in C(\hat{y})} P(\bar{y}', \hat{z}))},$$

which implies that each term in the right hand side of (34) is equal to the corresponding term of (35), from which the theorem follows. □

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