# Adversarial Approaches to Bayesian Learning and Bayesian Approaches to Adversarial Robustness

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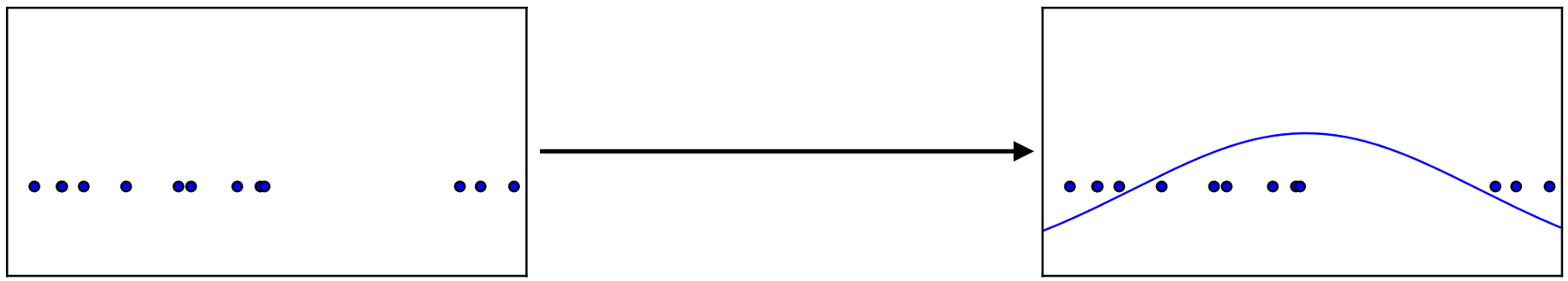
# Speculation on Three Topics

- Can we build a generative adversarial model of the posterior over parameters?
- Adversarial variants of variational Bayes
- Can Bayesian modeling solve adversarial examples?

# Generative Modeling

- Density estimation



- Sample generation

Training examples

Model samples

(Goodfellow 2016)

# Adversarial Nets Framework

$D(\mathrm{x})$ tries to be near 1

Differentiable function $D$

$x$ sampled from data

$D$ tries to make $D(G(z))$ near 0, $G$ tries to make $D(G(z))$ near 1

$D$

$x$ sampled from model

Differentiable function $G$

Input noise $z$

# Minimax Game

$$J^{(D)} = -\frac{1}{2}\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}} \log D(\boldsymbol{x}) - \frac{1}{2}\mathbb{E}_{\boldsymbol{z}} \log\left(1 - D\left(G(\boldsymbol{z})\right)\right)$$

$$J^{(G)} = -J^{(D)}$$

-Equilibrium is a saddle point of the discriminator loss

-Resembles Jensen-Shannon divergence

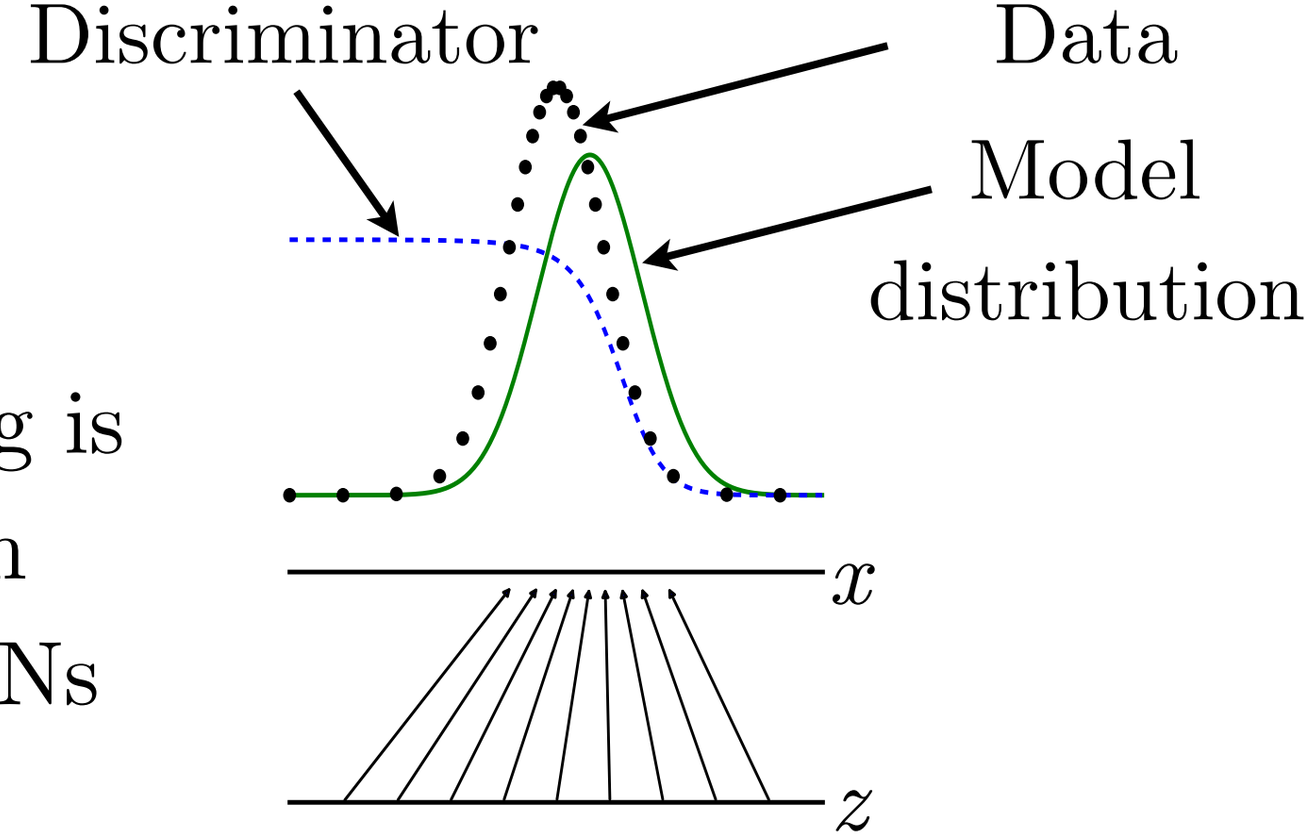
-Generator minimizes the log-probability of the discriminator being correct

# Discriminator Strategy

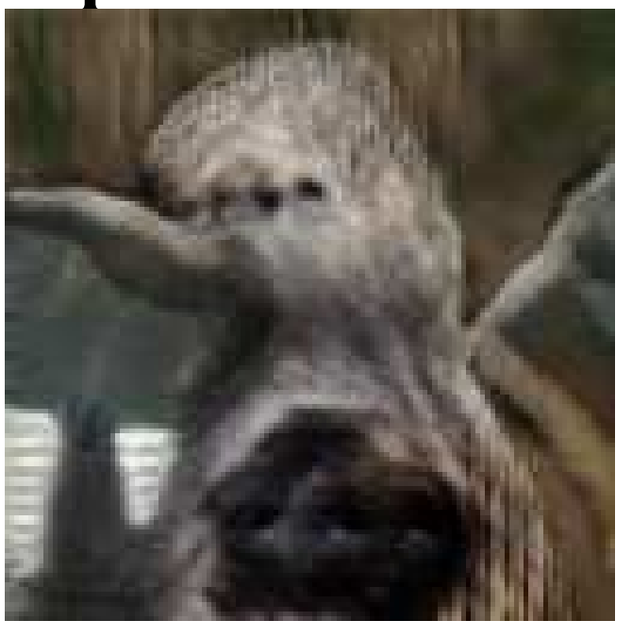
Optimal $D(\boldsymbol{x})$ for any $p_{\text{data}}(\boldsymbol{x})$ and $p_{\text{model}}(\boldsymbol{x})$ is always

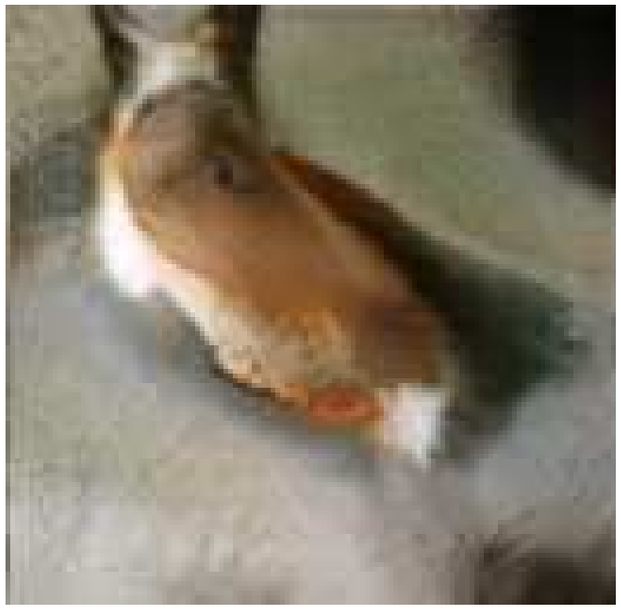
$$D(x) = \frac{p_{\text{data}}(x)}{p_{\text{data}}(x) + p_{\text{model}}(x)}$$

Estimating this ratio using supervised learning is the key approximation mechanism used by GANs



# High quality samples from complicated distributions

(Goodfellow 2016)

# Speculative idea: generator nets for sampling from the posterior

- Practical obstacle:
    - Parameters lie in a much higher dimensional space than observed inputs
- Possible solution:
    - Maybe the posterior does not need to be extremely complicated
    - HyperNetworks (Ha et al 2016) seem to be able to model a distribution on parameters

# Theoretical problems

- A naive application of GANs to generating parameters would require samples of the parameters from the true posterior

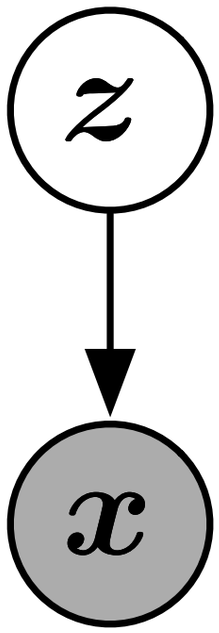
- We only have samples of the data that were generated using the true posterior

# HMC approach?

$$\frac{p(\boldsymbol{X} \mid \boldsymbol{\theta})}{p(\boldsymbol{X} \mid \boldsymbol{\theta}^*)} = \Pi_i \frac{p(\boldsymbol{x}^{(i)} \mid \boldsymbol{\theta})}{p(\boldsymbol{x}^{(i)} \mid \boldsymbol{\theta}^*)}$$

- Allows estimation of unnormalized likelihoods via discriminator
- Drawbacks:
  - Discriminator needs to be re-optimized after visiting each new parameter value
  - For the likelihood estimate to be a function of the parameters, we must include the discriminator learning process in the graph for the estimate, as in unrolled GANs (Metz et al 2016)

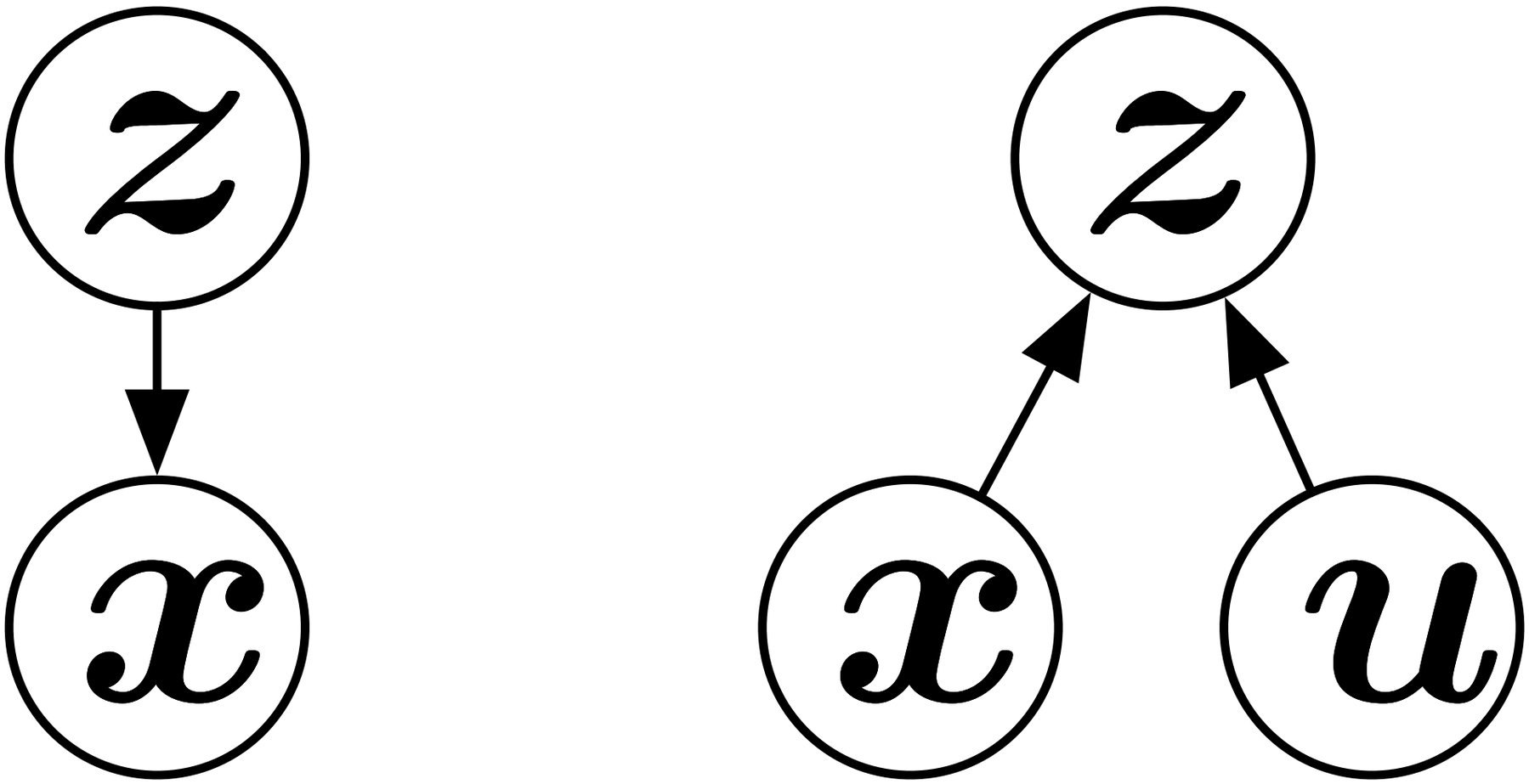
# Variational Bayes




$$\log p(\boldsymbol{x}) \geq \log p(\boldsymbol{x}) - D_{\mathrm{KL}}\left(q(\boldsymbol{z}) \| p(\boldsymbol{z} \mid \boldsymbol{x})\right)$$
$$= \mathbb{E}_{\boldsymbol{z} \sim q} \log p(\boldsymbol{x}, \boldsymbol{z}) + H(q)$$

- Same graphical model structure as GANs
- Often limited by expressivity of $q$

# Arbitrary capacity posterior via backwards GAN




Generation process

Posterior sampling process

# Related variants

- Adversarial autoencoder (Makhzani et al 2015)
  - Variational lower bound for training decoder
  - Adversarial training of encoder
    - Restricted encoder
    - Makes aggregate approximate posterior indistinguishable from prior, rather than approximate posterior indistinguishable from true posterior
    - Uses variational lower bound for training decoder

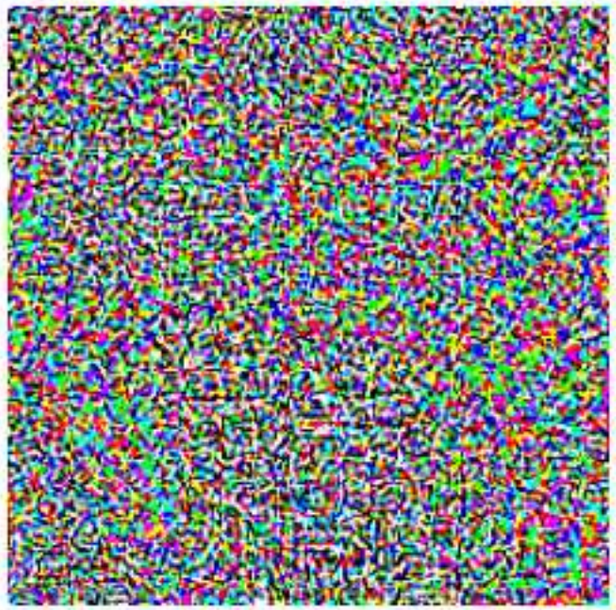
# ALI / BiGAN

- Adversarially Learned Inference (Dumoulin et al 2016)
  - Gaussian encoder
- BiGAN (Donahue et al 2016)
  - Deterministic encoder

# Adversarial Examples

panda
58% confidence

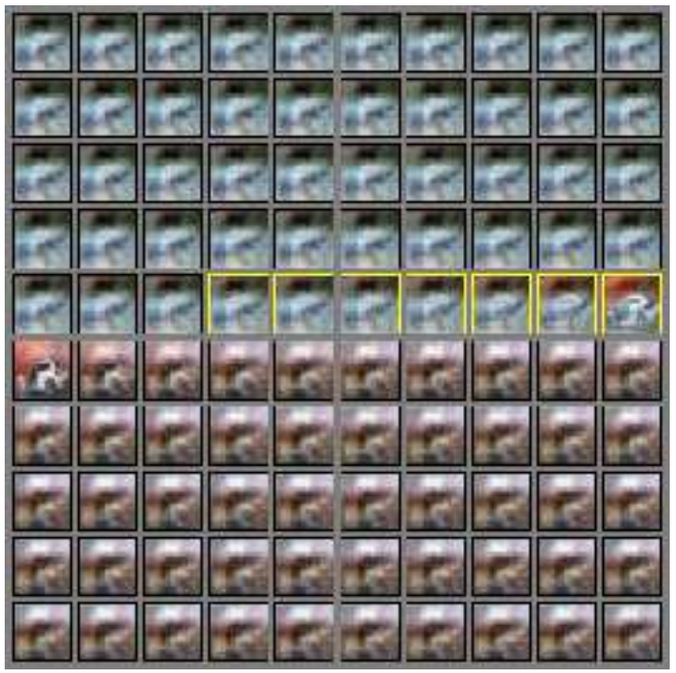
$+ .007 \times$

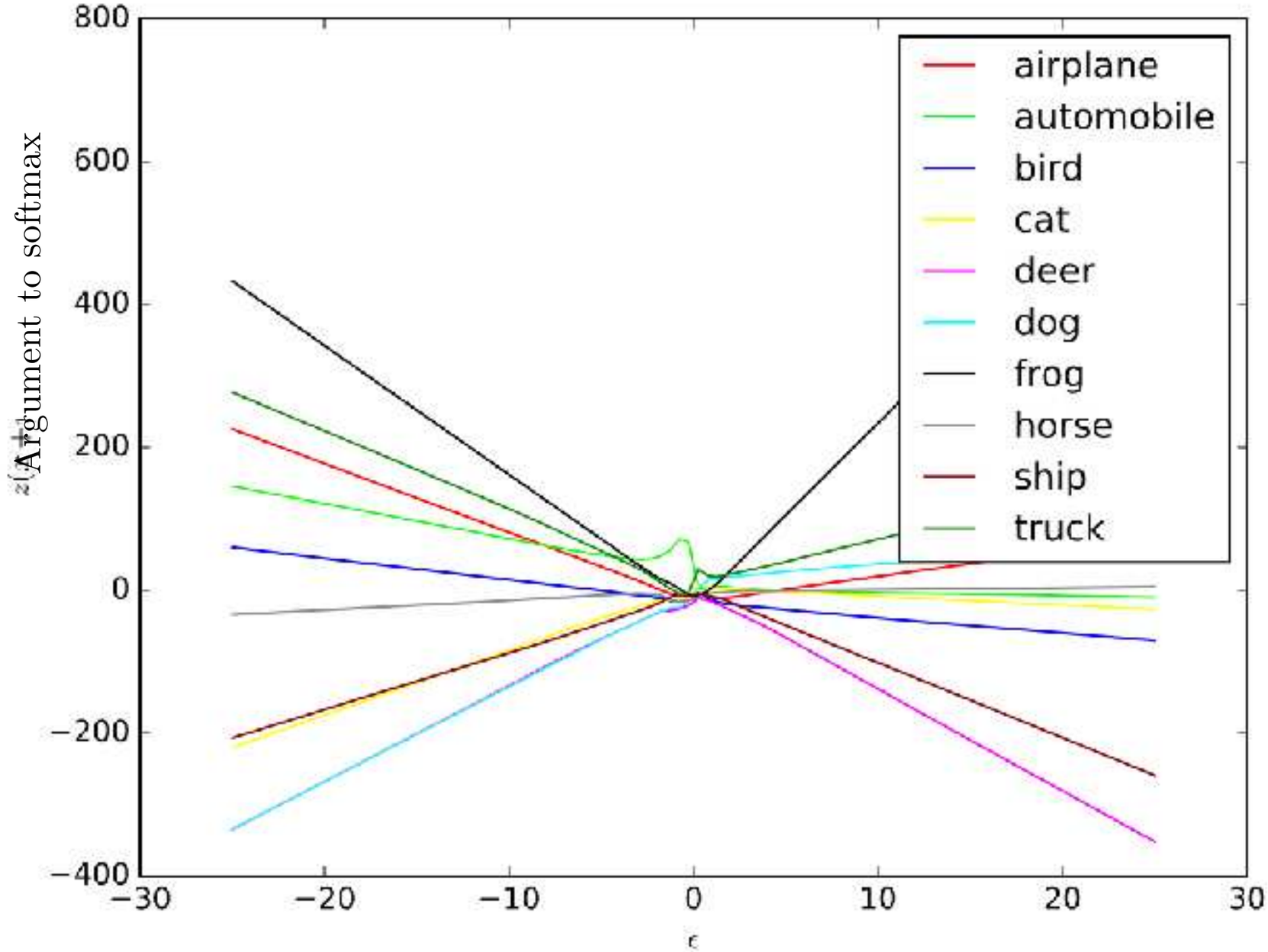


$=$

gibbon
99% confidence

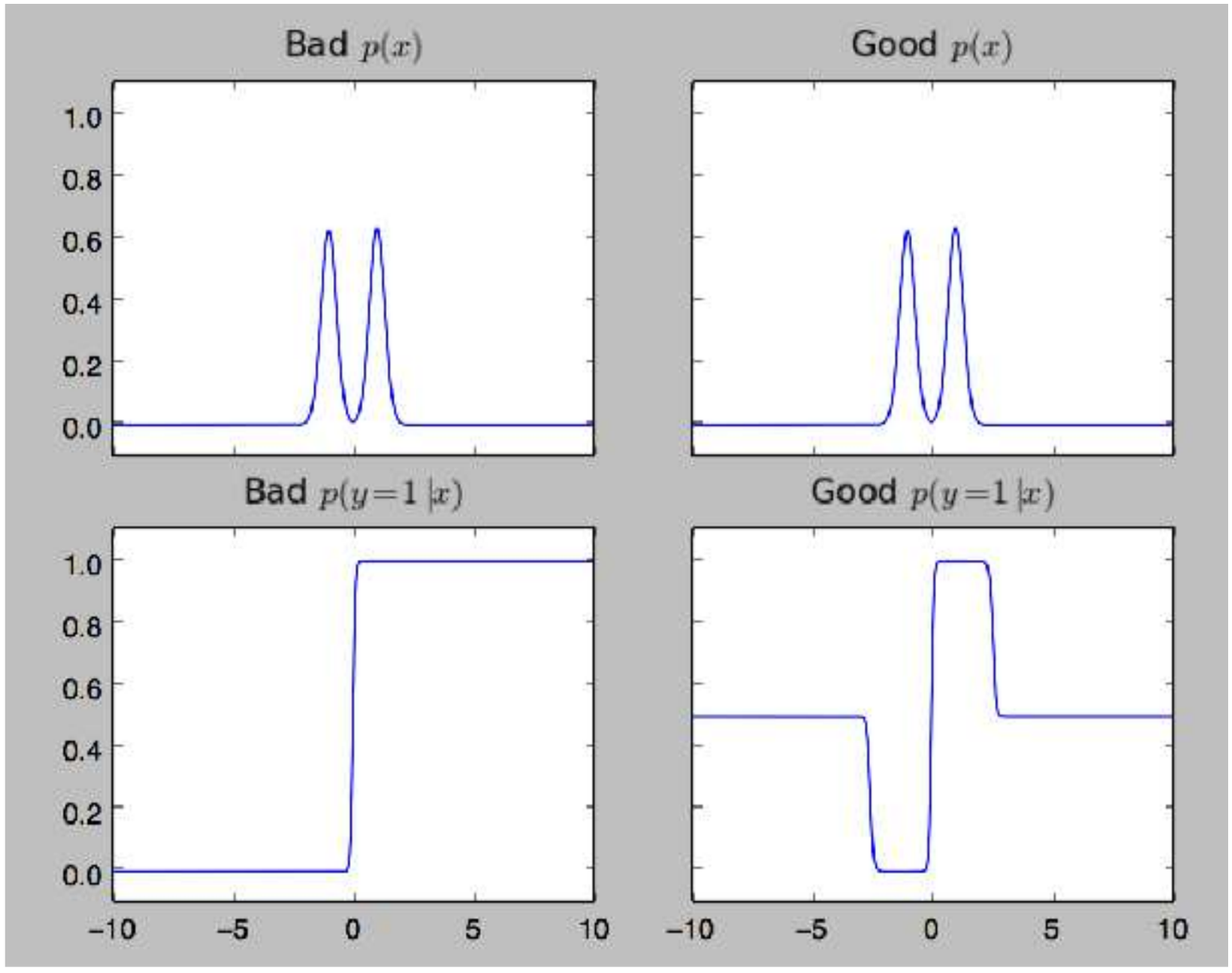
# Overly linear, increasingly confident extrapolation



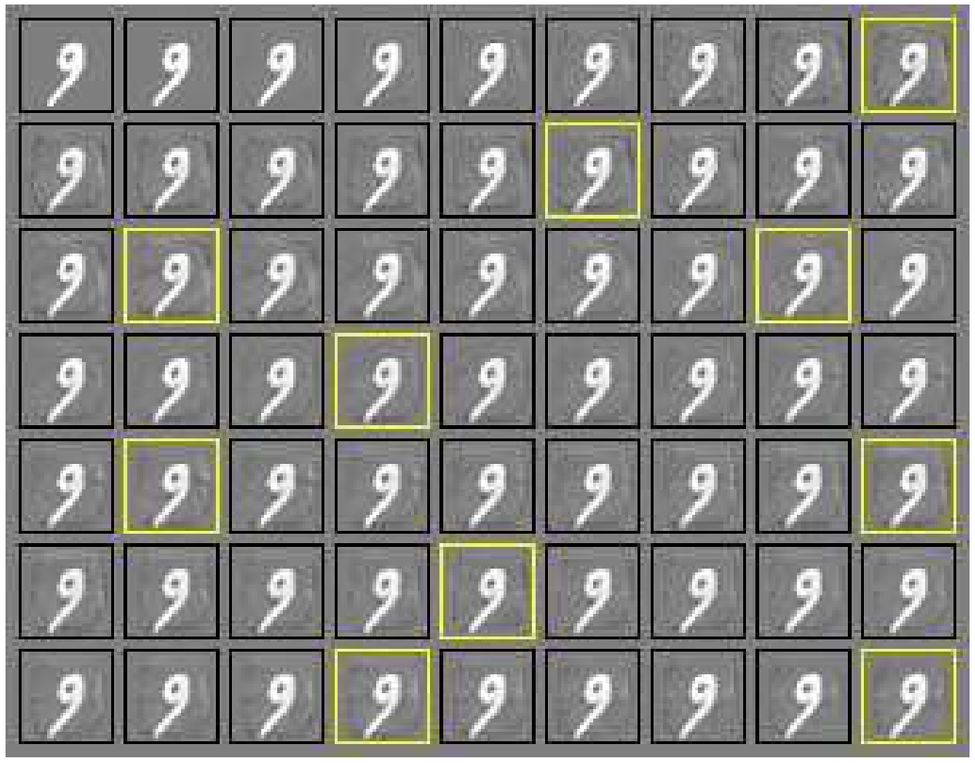


# Designing priors on latent factors

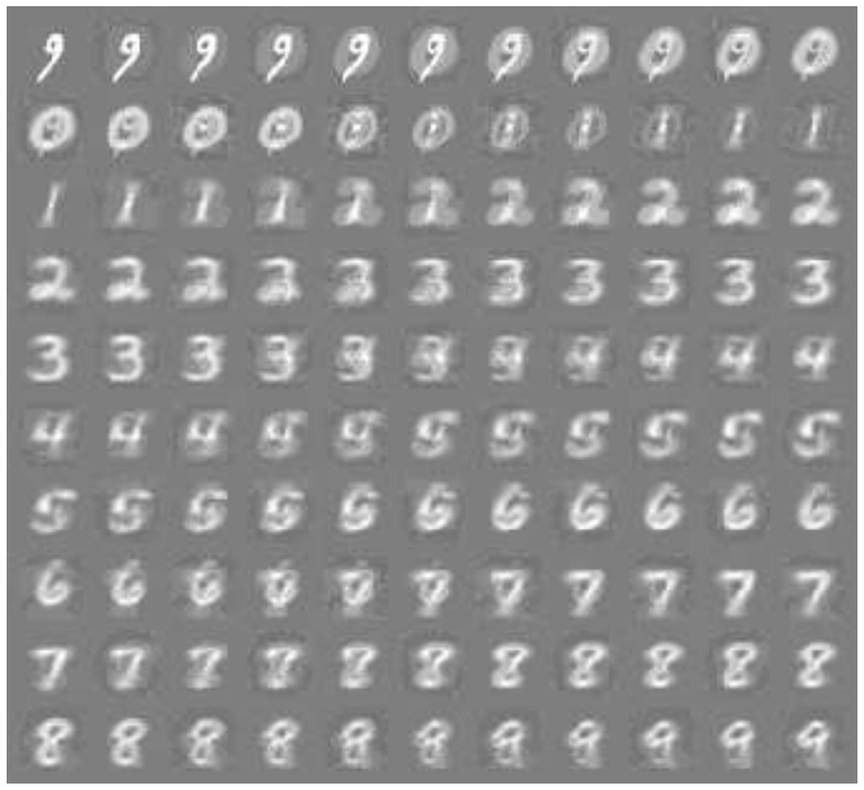
Both these two class mixture models implement roughly the same marginal over $x$, with very different posteriors over the classes. The likelihood criterion cannot strongly prefer one to the other, and in many cases will prefer the bad one.



# RBFs are better than linear models



Attacking a linear model



Attacking an RBF model

# Possible Bayesian solutions

- Bayesian neural network
    - Better confidence estimates might solve the problem
    - So far, has not worked, but may just need more effort
        - Variational approach
        - MC dropout
- Regularize neural network to emulate Bayesian model with RBF kernel (amortized inference of Bayesian model)

# Universal engineering machine (model-based optimization)

Make new inventions by finding input that maximizes model's predicted performance

Training data

Extrapolation

# Conclusion

- Generative adversarial nets may be able to
  - Sample from the Bayesian posterior over parameters
  - Implement an arbitrary capacity $q$ for variational Bayes
- Bayesian learning may be able to solve the adversarial example problem and unlock the potential of model-based optimization