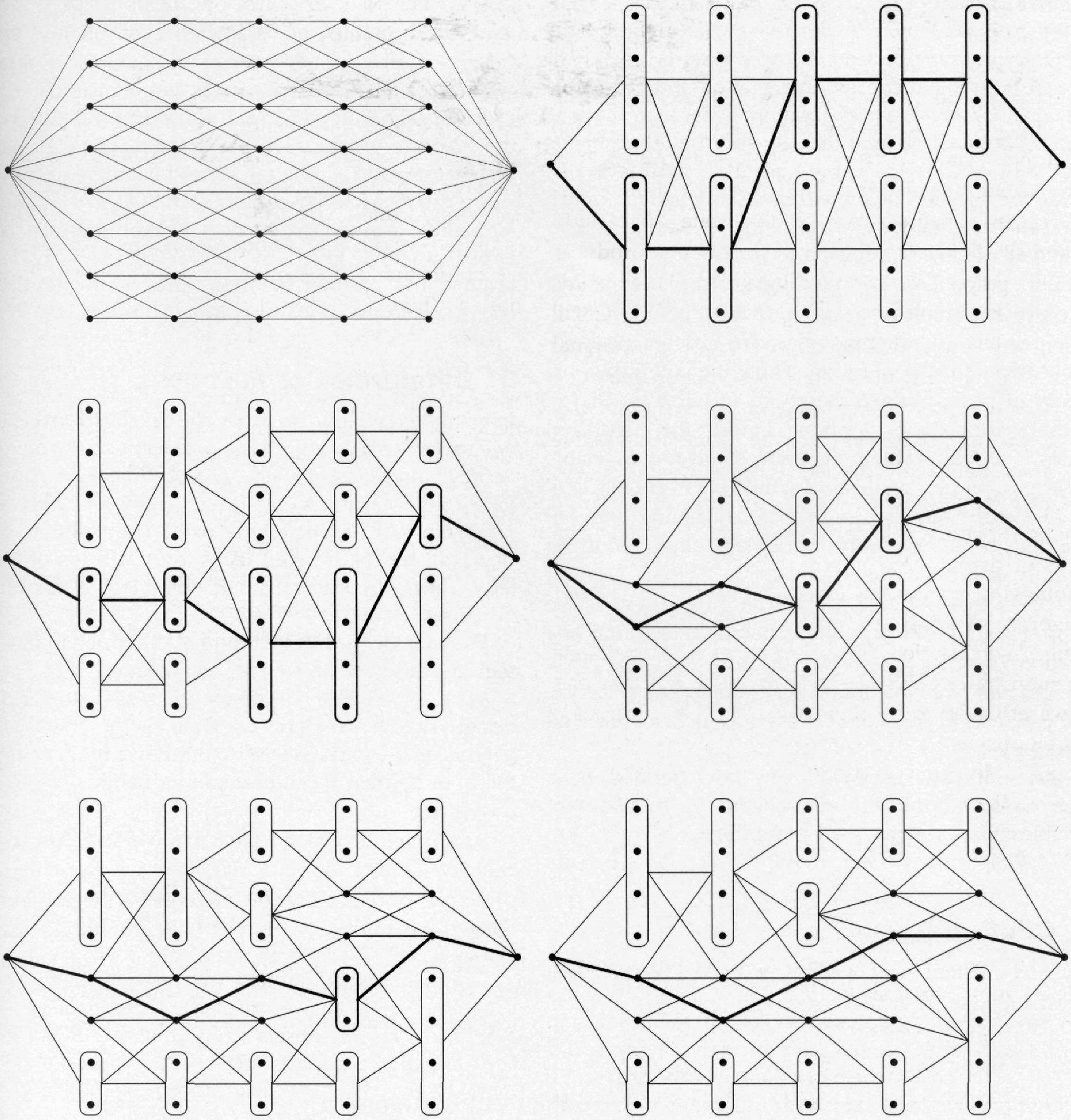

Fig. 1. The upper-left figure is the original dynamic programming setup. The five remaining figures show a possible progression of our coarse-to-fine dynamic programming algorithm. The optimal path (shown in bold lines) in the final figure in the lower-right must also be the optimal path in the original trellis.

$$C^k(S,T) = \min_{\{(s,t):s\in S, t\in T,(s,t)\in\mathcal{E}\}} C(s,t)$$

$$(S_0^k,\dots,S_N^k) = \arg\min_{\{(S_0,\dots,S_N)\in\pi(\mathcal{G}^k)\}} C^k(S_0,\dots,S_N)$$

where $P(S_n^k)$ is any partition of S_n^k. Note that $(S_0^k,\dots,S_N^k)$ defined above can be computed through DP.

Proposition. *If* $|\mathcal{S}| < \infty$ *let*

$$k^* = \min\{k : |S_n^k| = 1, n = 0,\dots,N\}$$

and let $S_n^{k^*} = \{s_n^{k^*}\}$, $n = 0,\dots,N$. *Then,*

$$C(s_0^{k^*},\dots,s_N^{k^*}) = \min_{\{(s_0,\dots,s_N)\in\pi(\mathcal{G})\}} C(s_0,\dots,s_N)$$

Proof. Since $|\mathcal{S}| < \infty$ only a finite number of iterations are possible before $\mathcal{S}^k$ is composed entirely of singleton sets so k^* is well-defined.

Let $(s_0,\dots,s_N) \in \pi(\mathcal{G})$. Then, there exist $S_n \in \mathcal{S}^{k^*}$, $n = 0,\dots,N$ such that $s_n \in S_n$ since $\mathcal{S}^{k^*}$ partitions $\mathcal{S}$. Also, note that $(s_n, s_{n+1}) \in \mathcal{E}$ for $n = 0,\dots,N-1$ implies $(S_n, S_{n+1}) \in \mathcal{E}^{k^*}$ and $(S_0,\dots,S_N) \in \pi(\mathcal{G}^{k^*})$. Then,

$$C(s_0,\dots,s_N) \geq C^{k^*}(S_0,\dots,S_N) \geq$$

$$C^{k^*}(S_0^{k^*},\dots,S_N^{k^*}) = C(s_0^{k^*},\dots,s_N^{k^*}).$$

□

It is worth noting that one can substitute a *lower bound*, $\tilde{C}^k(S,T)$, for the true minimum arc cost, $C^k(S,T)$, between two superstates without affecting the correctness of the