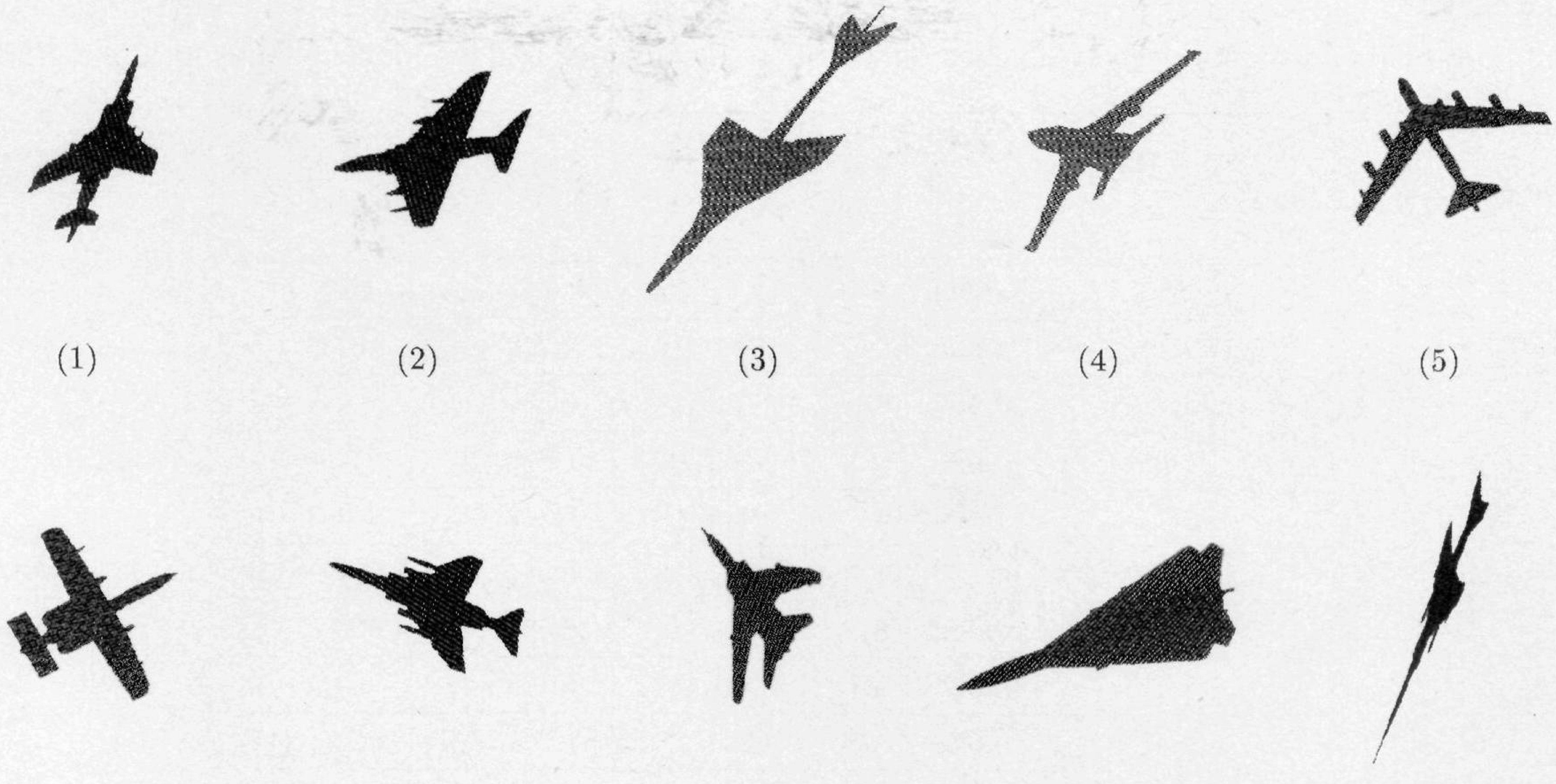

Fig. 5. The test images.

function with some traditional methods on synthetic data. The final experiment compared the proposed invariant function with a wavelet-based method. The wavelet function defined in [32] is used to calculate the dyadic wavelet transform in these experiments.

5.1 A Real Image Recognition Test

This experiment investigates the recognition power of the invariant function defined in (22).

$$I(t) = \frac{\eta_{3,4,5,6,7,8}(t)}{\eta_{7,8,9,10,11,12}(t)}. \tag{22}$$

Fig. 4 shows 20 models of airplanes. Some of these models represent different objects; others represent similar objects (e.g., models 7 and 20 or models 18 and 19). The former case shows the discrimination power of the invariant function, in general, and the latter shows its ability to describe small variations. Fig. 5 shows 10 test images. Table 1 gives the model images that have been transformed to produce the test images. These test images are obtained from different random affine transformations. A small uniformly distributed random noise has been added to the boundaries of the airplanes. The signal to noise ratio (SNR) is defined as the ratio of the energy of the boundary function ($x(t)$ and $y(t)$) and the variance of the uniformly distributed white noise [24]. The signal to noise ratio in this case is about 50 dB. The correlation function defined in (23) is used to measure the similarity between two invariant functions, say $I_1(t)$ and $I_2(t)$.

$$R(I_1(t), I_2(t)) = \frac{\int I_1(t) I_2(t) dt}{\|I_1\| \|I_2\|}, \tag{23}$$

where the integral is calculated over the interval of the functions $I_1(t)$ and $I_2(t)$. The invariant function calculated from the boundary of each test image is compared with the invariant functions calculated from the models using the correlation function defined in (23). Table 2 gives the best four matches for each test image. The best four matches have been listed in four columns, where the best match is listed in the first column in bold. For each test, the value of the correlation function between this test image and a model (the model number is the number between parentheses) is listed. The results show that the test images are identified correctly. Besides, there is a relatively large difference between the best match and the next one (the first and second columns). This means that the proposed invariant function has a good discrimination power.

The effect of adding large uniformly distributed random noise has been studied. The signal to noise ratio in this case is about 20 dB. Table 3 gives the best four matches for each test image. This table shows that the invariant function can successfully discriminate between objects in spite of the large level of noise.

TABLE 1
This Table Shows the Model Images Used to Produce the Test Images

Test image	1	2	3	4	5	6	7	8	9	10
Model image	6	2	19	15	5	1	8	16	20	10