



# 线性代数

$Ax = \lambda x$   
 $A$ :  $n \times n$  Matrix  
 $x$ : "Eigen Vector"  
 $\lambda$ : "Eigen Value"  
 same thing  $\rightarrow (A - \lambda I)x = 0$   
 $\det(A - \lambda I) = 0$  ← "Characteristic Equation of A"  
 $(A - \lambda I)$  is singular  
 $(A - \lambda I)$  is not invertible  
 $P(\lambda) = \det(A - \lambda I)$   
 "Characteristic Polynomial"

$A = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix}$      $A - I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 25 \end{bmatrix}$   
 $\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$   
 $A - \lambda I = \begin{bmatrix} 4 & 8 \\ 6 & 26 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{bmatrix}$   
 $\det(A - \lambda I) = \begin{vmatrix} 4-\lambda & 8 \\ 6 & 26-\lambda \end{vmatrix} = (4-\lambda)(26-\lambda) - (8)(6) = 104 - 30\lambda + \lambda^2 - 48 = \lambda^2 - 30\lambda + 56 = (\lambda - 28)(\lambda - 2) = 0$   
 $\lambda_1 = 28$   
 $\lambda_2 = 2$   
 $N(A - \lambda_2 I) = \begin{bmatrix} 4-\lambda_2 & 8 & | & 0 \\ 6 & 26-\lambda_2 & | & 0 \end{bmatrix} = \begin{bmatrix} 4-2 & 8 & | & 0 \\ 6 & 26-2 & | & 0 \end{bmatrix} = \begin{bmatrix} 2 & 8 & | & 0 \\ 6 & 24 & | & 0 \end{bmatrix} \xrightarrow{R1/2} \begin{bmatrix} 1 & 4 & | & 0 \\ 6 & 24 & | & 0 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 1 & 4 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$   
 $x_1 + 4x_2 = 0 \rightarrow x_1 = -4x_2 \rightarrow \begin{bmatrix} -4x_2 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} -4 \\ 1 \end{bmatrix} x_2 \rightarrow x \begin{bmatrix} -4 \\ 1 \end{bmatrix}$   
 Eigenspace  $\rightarrow$  Eigen Vector

# 标量



- 简单操作

$$c = a + b$$

$$c = a \cdot b$$

$$c = \sin a$$

- 长度

$$|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{otherwise} \end{cases}$$

$$|a + b| \leq |a| + |b|$$

$$|a \cdot b| = |a| \cdot |b|$$

# 向量



- 简单操作

$$c = a + b \quad \text{where } c_i = a_i + b_i$$

$$c = \alpha \cdot b \quad \text{where } c_i = \alpha b_i$$

$$c = \sin a \quad \text{where } c_i = \sin a_i$$

- 长度

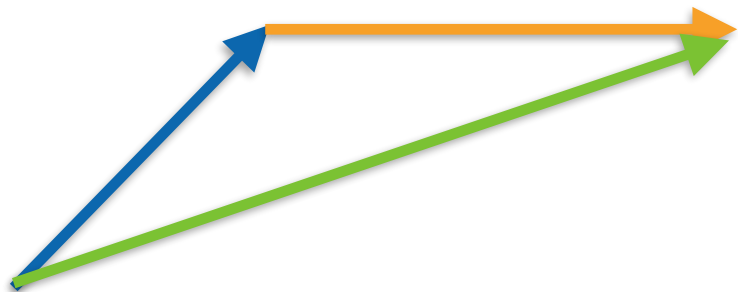
$$\|a\|_2 = \left[ \sum_{i=1}^m a_i^2 \right]^{\frac{1}{2}}$$

$$\|a\| \geq 0 \text{ for all } a$$

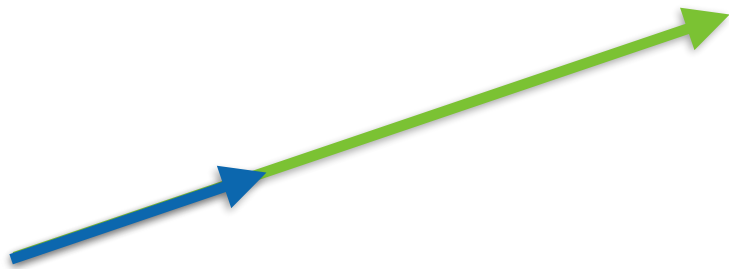
$$\|a + b\| \leq \|a\| + \|b\|$$

$$\|a \cdot b\| = |a| \cdot \|b\|$$

# 向量



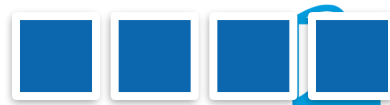
$$c = a + b$$



$$c = \alpha \cdot b$$

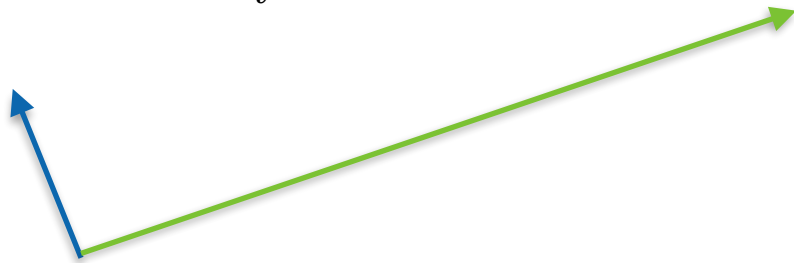
数学家的 '**parallel for all do**'

# 向量



- 点乘  $a^\top b = \sum_i a_i b_i$

- 正交  $a^\top b = \sum_i a_i b_i = 0$



# 矩阵



- 简单操作

$$C = A + B$$

where  $C_{ij} = A_{ij} + B_{ij}$

$$C = \alpha \cdot B$$

where  $C_{ij} = \alpha B_{ij}$

$$C = \sin A$$

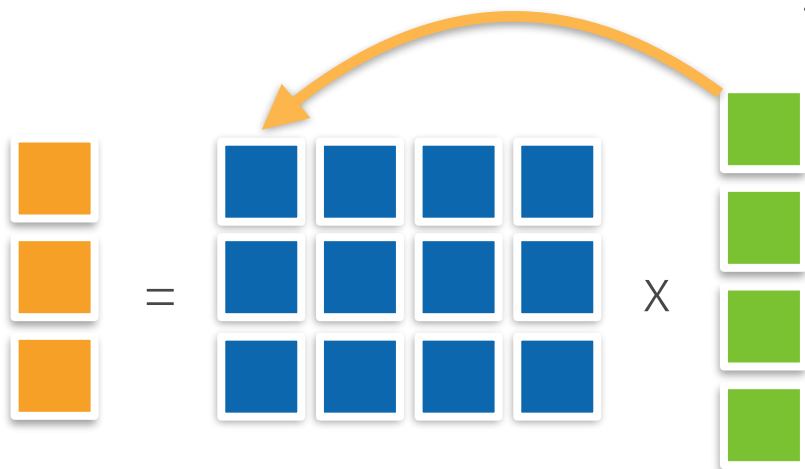
where  $C_{ij} = \sin A_{ij}$

# 矩阵



- 乘法（矩阵乘以向量）

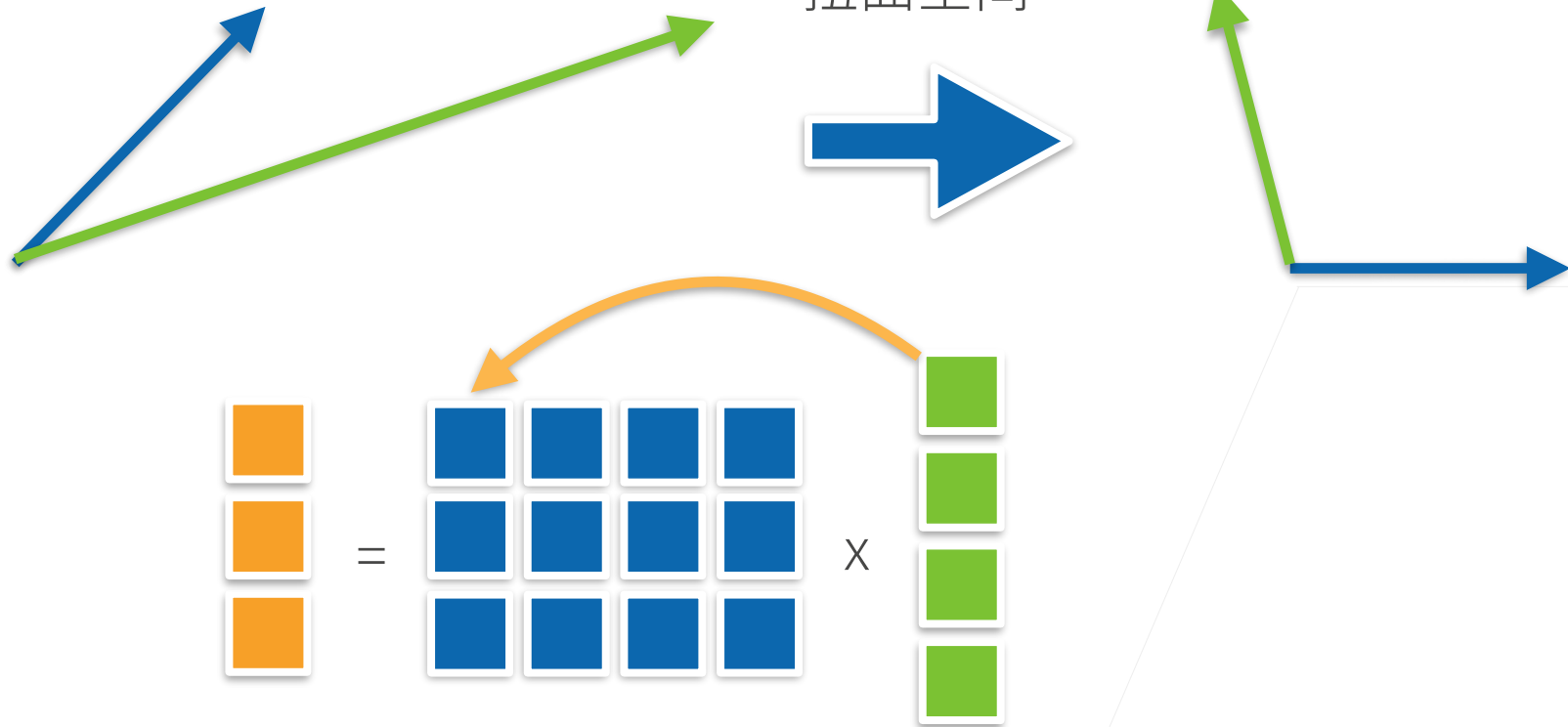
$$c = Ab \text{ where } c_i = \sum_j A_{ij} b_j$$



# 矩阵



扭曲空间



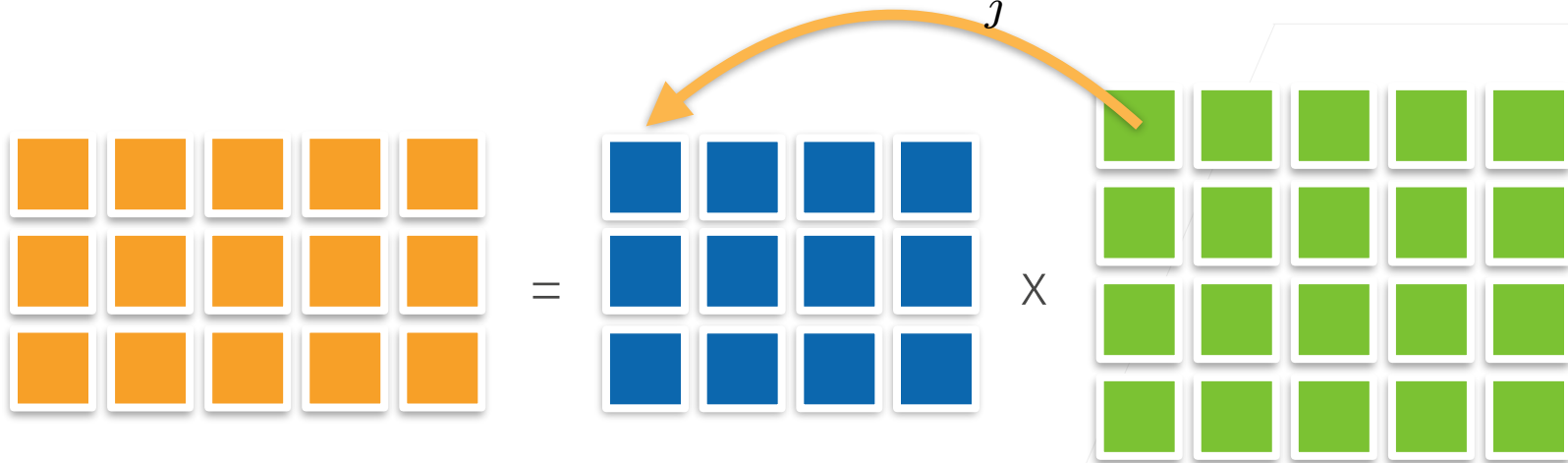


# 矩阵



- 乘法（矩阵乘以矩阵）

$$C = AB \text{ where } C_{ik} = \sum_j A_{ij} B_{jk}$$



# 矩阵



- 范数

$$c = A \cdot b \text{ hence } \|c\| \leq \|A\| \cdot \|b\|$$

- 取决于如何衡量  $b$  和  $c$  的长度
- 常见范数
  - 矩阵范数：最小的满足的上面公式的值
  - Frobenius 范数

$$\|A\|_{\text{Frob}} = \left[ \sum_{ij} A_{ij}^2 \right]^{\frac{1}{2}}$$

# 特殊矩阵



- 对称和反对称



$$A_{ij} = A_{ji} \text{ and } A_{ij} = -A_{ji}$$



- 正定

$$\|x\|^2 = x^\top x \geq 0 \text{ generalizes to } x^\top A x \geq 0$$



# 特殊矩阵

- 正交矩阵

- 所以行都相互正交

- 所有行都有单位长度

$U$  with  $\sum_j U_{ij}U_{kj} = \delta_{ik}$

- 可以写成  $UU^T = \mathbf{1}$

- 置换矩阵

$P$  where  $P_{ij} = 1$  if and only if  $j = \pi(i)$

- 置换矩阵是正交矩阵

# 矩阵



- 特征向量和特征值
  - 不被矩阵改变方向的向量

$$Ax = \lambda x$$

特征向量

- 对称矩阵总是可以找到特征向量

$x^3 + x^2 + y^3 + z^3 + xyz - 6 = 0$   
 $\text{grad} f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$   
 $\text{tg} x \cdot \text{cotg} x = 1$   
 $2x^2yy + y^2z = 2$   
 $\sin x$   
 $\cos x$   
 $Y_{i+1} = Y_i + b \cdot k_2$   
 $B = \begin{pmatrix} 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$   
 $a^2 = b^2 + c^2 - 2bc \cos \alpha$   
 $\sum_{i=0}^n (p_2(x_i) - y_i)^2$   
 $\text{tg} x = \frac{\sin x}{\cos x}$   
 $x - y + z = 1$   
 $x + \lambda y + z = \lambda$   
 $x + y + \lambda z = \lambda^2$   
 $x_2 = \begin{pmatrix} -\alpha \\ \beta \\ -\gamma \\ -\delta \end{pmatrix}$   
 $\iiint_M z dx dy dz = \int_0^{2\pi} \left( \int_0^2 \left( \int_{\frac{1}{2}}^1 n r dr \right) d\theta \right) dp$   
 $\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n^3 + 1} + n}{\sqrt[n]{3n^2 + 2n - 1}}$   
 $2 \arctg x - x = 0, I = (1, 10)$   
 $\int_{-\pi/2}^{\pi/2} \sin^4 x \cdot \cos^3 x dx$   
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$   
 $\frac{\partial z}{\partial x} = 2; \frac{\partial z}{\partial y} = 0$   
 $\vec{n} = (F'_x, F'_y, F'_z)$   
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$   
 $\sin 2x = 2 \sin x \cdot \cos x$   
 $|Z| = \sqrt{a^2 + b^2}$   
 $\frac{\partial f}{\partial x} = 16 - x^2 + 16y^2 - 4z > 0$   
 $\oint 3x^2 + 16x^{-0.12} dx \lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right)^n$   
 $\delta(p_2) = \sqrt{0.16}$   
 $c = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$   
 $\alpha, \beta, \gamma \in \mathbb{C}$   
 $f(x) = 2^{-x} + 1, \epsilon = 0.005$   
 $e^z - xyz = e; A[0; e; 1]$   
 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}$   
 $|x| + |y| \neq 0; y \neq 0$   
 $\frac{2x}{x^2 + 2y^2} = 2 \quad z = \frac{1}{x} \text{ and}$   
 $\sin(x+y) = \sin x \cos y + \cos x \sin y$   
 $y' - \frac{\sqrt{y}}{x+2} = 0; y|_0$   
 $\cos p = \frac{(1, 0)}{\sqrt{1}}$   
 $\lambda_2 = i\sqrt{14}$   
 $y = \text{tg} x$   
 $y = \text{cotg} x$   
 $\sin^2 x$   
 $A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}, x=0, y=1, z=2$   
 $A = [1; 0; 3]$

# 矩阵计算

动手学深度学习 v2  
李沐 · AWS

