Primary School Mathematics: The Case for Enlightened Timed Testing to Achieve Fluency in Single-Digit Arithmetic

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Introduction

I teach mathematics at RMC primarily to engineering and science students. Generally, they are not proficient at arithmetic and my colleagues would not disagree. That is not to say they are not bright. They are. And those at the top are exceptional. It's just that some have an arithmetic deficiency and their corresponding number sense is poor. Some of my students can tell me what 4×8 is but they require a calculator to get 40% of 800.

The news from our primary school system is not encouraging.[1] The evidence on the performance of Canadian primary school students on recent international tests suggests that things are not getting better:

"67% of Grade 3 students met the standard in 2014 compared to 71% in 2010, indicating a four-percentage-point decrease."[2]

Not surprisingly, this performance has been given considerable coverage in the popular media.[3] These reports constituted my introduction to "the math wars," the on-going debate about how best to teach children mathematics.

My focus has been on how children learn the basic arithmetic facts. There is general agreement that children should be "fluent," that is, when confronted with, say, 8×7, a child should know that it's 56 within a short period of time.[4] The disagreement lies in how best to get children to fluency.

I grew up in the sixties, in an era of flashcards and timed tests. Nowadays, there are mathematics education researchers who see these tools as counter-

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[1] A very good summary of what is happening in Canadian mathematics education can be found in a Stokke, Anna (2015) "What to Do About Canada's Declining Math Scores," C D Howe Institute, Commentary No. 427, Toronto, ON.

[2] See *http://www.eqao.com/NR/ReleaseViewer.aspx?Lang=E&release=b14R003*

[3] See, for example, Erin Anderssen's *Globe and Mail* article "Why the War Over Math is Distracting and Futile," *(*March 1, 2014, available at *http://www.theglobeandmail.com/news/national/education/why-the-war-over-math-is-distracting-and-futile/article17178295/).*

[4] The pedagogy in vogue these days is called the "discovery" pedagogy, a variant of "inquiry based learning." Good references include Boaler ("What's Math Got to Do with It," 2008, Penguin, New York), Small ("Big Ideas from Dr Small," 2010, Nelson Education), and Van de Walle et al. ("Teaching Student-Centered Mathematics," 2014, Pearson Education). Among these researchers, there are different meanings for fluency and their standards are not mine. For instance, the US Common Core standard, as interpreted by most States, is that a child be "efficient and accurate" when it comes to single-digit arithmetic. John Van de Walle suggests that a child should be able to come to the answer of a single-digit arithmetic question within 3 seconds (see his book cited above). It's my feeling that a child should know single-digit arithmetic cold. There should be no delay in recognizing that 8×7 is 56.

productive. For instance, Boaler says this: "The damage starts early in this country [the United States], with school districts requiring young children to take timed math tests from the age of 5. This is despite research that has shown that timed tests are the direct cause of the early onset of math anxiety."[5]

So if not timed tests (and other forms of abuse), how should we get students to fluency? In a different article, Boaler argues for an emphasis on teaching "number sense":

"Mathematics facts are important but the memorization of math facts through times table repetition, practice and timed testing is unnecessary and damaging. The English minister's mistake when he was asked 7×8 prompted calls for more memorization. [Earlier in the piece, Boaler had explained that an English cabinet minister, in a public forum, had mistakenly said that 7×8 was 54.] This was ironic as his mistake revealed the limitations of memorization without 'number sense'. People with number sense are those who can use numbers flexibly. When asked to solve 7×8 someone with number sense may have memorized 56 but they would also be able to work out that 7×7 is 49 and then add 7 to make 56, or they may work out ten 7's and subtract two 7's $(70 - 14)$. They would not have to rely on a distant memory."[6]

Gosh, she is being hard on the minister. It's not clear how the minister got himself into the situation of having to recite an arithmetic fact in public. But I'd rather take a stab at 7×8 with a faulty memory and get it wrong than tackle it with some "number sense," a time delay to do the mental gymnastics, and then get it wrong. But more to the point, Boaler is arguing that we should have a way to get the answer to an arithmetic fact if it is not fluent.

On the matter of 7×8, last week I pulled one of my first-year engineering calculus students aside and asked him what 7×8 was. He looked off into space and after about 4-5 seconds responded 57. I immediately asked him how he got it and he responded: "Well, sir, I knew that 7×7 was 49. I then added 8 to get 57." Obviously he should have added 7 but it's easy to see

[5] See "Timed Tests and the Development of Math Anxiety," *Education Week*, July 3, 2012, available online at *www.edweek.org*.

[6] See Boaler's piece "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts," with Cathy Williams and Amanda Confer, updated Jan 2015, available at: *https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf*

how one might make the slip he did.[7]

Boaler's position is at odds with my experience. I did flashcards and timed tests as a child and don't recall them being particularly onerous. More recently, I've used them with my boys and other children and found them to be very effective. I certainly don't feel I'm doing any lasting damage to the children I'm teaching.

It seems to me that Boaler has in mind a *public* timed test, one where not only the child sees a bad result but also the child's peers. I agree that such tests are counterproductive for those who don't do well on them.

But is the solution to get rid of timed tests? I say no. I'll argue that there is a role for what I term *enlightened timed tests* (eTTs). A timed test is one where a child does as many basic arithmetic questions as he or she can in a fixed period of time, usually a minute. An eTT has the following additional characteristics:

1. *eTTs are executed on a computer.* A child logs onto a website, signs in, hits the start button, and undergoes the test. Answers are clicked with a mouse.

2. *An eTT Program is individualized.* An eTT Program is a sequence of timed tests grouped into modules and submodules. To "pass" a particular submodule and move on to the next, a child will have to satisfy whatever pass criterion a teacher has specified. And since children learn at different rates, they will proceed through the eTT Program at different speeds. In this way the eTT Program is *individualized.*

3. *eTT results are private.* The only other person to see the child's result is the teacher.

4. *eTTs don't require the teacher to mark 25-30 tests each time an eTT is written.* The computer marks the tests and goes over errors with students. A teacher monitors results but does not have to mark. This time saving allows the teacher to spend more face to face time with students.

5. *eTTs are done frequently and in such quantity as is required for a student to become fluent.* The science of learning suggests that frequent

[7] It's also true that this student does not have good number sense. Any time I multiply two numbers and one of them is even, the answer has to be even.

short tests are preferred to less frequent longer tests. I have in mind that, when a child is ready, he or she would plug in once a day for at most five minutes. Furthermore, since children learn at different speeds, students simply keep doing these tests until a standard the teacher defines is reached. Thereafter, students would do a maintenance program to keep the skill at the standard.

6. *eTTs are only given once a student understands how to manufacture unknown arithmetic facts from known facts.* The primary school curricula that I have observed are very good at teaching children strategies to get facts they don't know from facts they do. For instance, take $8 + 7$. If a child is not fluent with this fact but knows that $8 + 8 = 16$ then he or she could conclude that $8 + 7$ is 1 less than $8 + 8$ and therefore 15. I will argue below that, once a child is able to use a strategy to get all of the single-digit facts for an operation, he or she is ready for eTTs. The only exception to this is the multiplication table which is inherently more difficult to learn.

7. *eTTs have an absolute requirement that mistakes be reviewed.* The general rule is that we learn from our mistakes regardless of what we are doing. This is certainly true when it comes to single-digit arithmetic. A mistake gives the teacher a chance to review the fundamental strategies a student could use to calculate the fact in error.

In sum, eTTs have two essential characteristics. First they are *private*. Nobody has to know a student's results except that student and the teacher. And second, they are *individualized*: each student has his or her program and continues to work at it in short bursts until fluency is achieved. There is no need for all students to proceed lock-step, each doing the same timed test at each sitting. Nor is an eTT program a race. Ideally, a student should keep working at whatever eTT Program is put in place until fluency is achieved.

In the case where some students are anxious about the clock even with the private nature of eTTs, a teacher can think about recording the time for a student to do a fixed number of questions.[8] In such tests, the child does not see or sense a clock.

[8] This idea was suggested to me by my colleague David Wehlau who found that his daughter in Grade 2 had an absolute disdain for a clock that showed up on the computer screen as she did timed tests.

It is worth remarking that eTTs as I've described them are consistent with the role for drill as described by Van de Walle et al. in their book "Teaching Student-Centered Mathematics."[9]

But there is more to my argument. I think there is justification for timed tests based in cognitive science and, in particular, the science of *deep learning* and I'll make this point more fully below.

Why Fluency Is Important

It's important to understand why fluency and number sense are crucial. First, they are critical to functioning in a technologically advanced society. There are many occupations that require a knowledge of arithmetic. These range from trades (carpenters, plumbers, millwrights, etc.) to the professions (accountants, lawyers, doctors, pharmacists, teachers, etc.). But most importantly for my argument, fluency is required to be able to do higher levels of mathematics more easily. I'll illustrate with two examples: fractions and the solution of simple equations.

First fractions. Consider a primary school student who has yet to achieve fluency in single-digit arithmetic and has now embarked upon fractions, surely one of the most difficult topics in primary school mathematics. Eventually the student will come to problems like this:

$$\frac{5}{9} - \frac{2}{7} = ?$$

Let's look at what's required to solve this problem.

First, a student needs to have a way of dealing with it and generally this would either be by getting a common denominator or by cross-multiplication. If we were to cross-multiply, we would get

$$\frac{5}{9} - \frac{3}{7} = \frac{(5 \times 7) - (9 \times 3)}{(9 \times 7)},$$

and absent fluency, the student would have to use a calculator to do the three products on the rhs. This would yield

$$\frac{(5 \times 7) - (9 \times 3)}{(9 \times 7)} = \frac{35 - 27}{63}$$

[9] See pages 168-172 in Van de Walle, John, LouAnn H. Lovin, Karen S. Karp, and Jennifer M. Bay-Williams, *Teaching Student Centered Mathematics*, Volume 1, Pearson, New York, 2014.

and the student would likely have to use a calculator to get the difference $35 - 27$.

All in, this student would need to do 4 computations with a calculator.[10] These take time but, most importantly, they are time away from the focus required to understand the original problem. This time away is a distraction and makes learning the higher level problem more difficult. There are many studies in early mathematics education which demonstrate this.[11] For students not fluent in arithmetic, the problem of fractions becomes two problems.

Now algebra. Suppose a student is asked to solve this equation:

$$4(8x - 4) - 5(3x + 12) = 9x - 4.$$

First it would be expanded

$$32x - 16 - 15x - 60 = 9x - 4$$

giving

$$17x - 76 = 9x - 4$$

and this simplifies to

$$17x - 9x = 76 - 4.$$

Finally we have

$$8x = 72$$

and therefore

$$x = 9.$$

Here is a list of the arithmetic calculations required:

$$\begin{array}{cc} 4 \times 8 & 32 - 15 \\ 4 \times 4 & 60 + 16 \\ 5 \times 3 & 17 - 9 \\ 5 \times 12 & 76 - 4 \\ & 72 \div 8 \end{array}$$

[10] The same computations are requiired if a common deminator is sought.

[11] See J. Sweller (1988). "Cognitive Load During Problem Solving: Effects on Learning," *Cognitive Science*, 12, 257- 285 and Tronsky, L. N., M. McManus, and E. C. Anderson, (2008). "Strategy Use in Mental Subtraction Determines Central Executive Load," *American Journal of Psychology*, 121(2), 189-207.

A student not fluent in arithmetic would have to spend a considerable amount of time with a calculator. Moreover, a lot has to go right to get a correct answer for this relatively simple problem. If we assume the student has a 95% chance of getting each of these facts correctly, he or she has only a 2 in 3 chance of getting the correct answer. It really helps to know arithmetic.

So two summary points. Modern life requires that a citizen have a working knowledge of arithmetic to make his or her way in the world. And second, a child fluent in arithmetic is in a much better position to do higher levels of mathematics.[12]

Cognitive Aspects of Arithmetic Fluency

The Role of Memory

Learning mathematics is not easy. We now have an extremely sophisticated number system that took literally millennia to develop. I continue to marvel at the genius of the placeholder concept. And if you don't share my enthusiasm, try doing some arithmetic with Roman numerals.[13] It shouldn't be surprising that it takes a considerable effort for children to learn our number system and the four arithmetic operations with these numbers.

The primary difficulty is the nature of our memories. It's no small feat to store $7 \times 8 = 56$ in long-term memory. For whatever reason, evolution has not bestowed us with a long-term memory that is easy to write.

Fluency and Deep Learning

Cognitive science and neuroscience are in their infancies. No one really knows exactly how our minds work. Certainly the physical structure and it's neuronal/electrical nature suggest a function similar to a computer and, almost certainly, one that is massively parallel. But we have, at best, only a rough idea of its inner workings.

There have been some interesting developments in the machine learning and pattern recognition literature which have some implications for arith-

[12] An excellent summary paper on this second point is Wu, H. (1999). "Basic Skills Versus Conceptual Understanding," *American Educator*, 1-7.

[13] Our placeholder system came to the West via India and the Middle East. The predominant number system of European trade and commerce in the 11th and 12th centuries was the Roman numeral system. In order to do calculations, merchants needed a special board and a bag of counting discs. This changed with the arrival of the Hindu-Arabic system. Merchants soon figured out that calculations could be made with pen and paper only. And, over a couple of centuries, the Hindu-Arabic placeholder system replaced the Roman system.

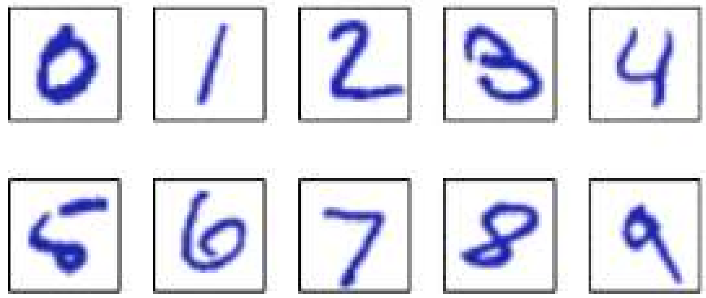

Figure 1: Hand-written Digits 0-9.

metic fluency. Recently "deep learning" algorithms have been applied successfully to some very difficult recognition/classification problems.

Here is an example. Suppose we wanted to have a machine look at a single handwritten digit to determine which of the 10 digits it is. Examples of hand written digits are shown in Figure 1. Deep learning algorithms would first "train" an algorithm on a dataset consisting of a very large number of pictures of hand-written digits, each with the identification of which digit it is. By "training" we mean that the picture data is used to estimate the parameters of a sophisticated neural network algorithm (NN), one that, at least physically, appears to have the same functional characteristics as our brains. The hallmark feature is a layered network of neurons, with each neuron having a threshold function that depends on what happens in other neurons. Once trained, the NN would be tried on a number of pictures of digits to see how well it performed. As it turns out, these algorithms have been astonishingly successful on some very difficult recognition problems including image recognition and speech recognition problems.

This is not to say that, when we think, the neuronal structures in our brains follow deep learning algorithms. But I'd be surprised if there was not a kernel of truth in the analogy. Assuming there is some similarity, the implication is that our brain circuits for single-digit arithmetic need to be trained. We need to see $8 + 7$ as many times as is required to burn a "deep learning" neural circuit for it in permanent memory. This would suggest that eTTs have their place in early mathematics education. But there is more to it as I will explain in the next section.

Understanding and Fluency

Learning arithmetic has an interesting parallel with learning how to read. Suppose a student is taught how to read phonetically. He or she first learns the sounds associated with each of the 26 letters (there are more than 26 sounds). Then the student learns how to decode words by pronouncing the sound associated with each letter of a word and then making a guess as to what the word is. When the student finally gets to reading, the context will give additional clues. For instance, if the sentence is "The dog runs after the cat", once a child gets to the last word "cat", and makes the hard "c" sound, he or she could then make a reasonable guess without decoding the last two letters. So, the mnemonics (or memory aids) are the *sounds* of the letters and *context*. Beginning readers will get increasingly adept at decoding the more they read. Ultimately a whole word is recognized without decoding. In fact, there are many examples of passages with scrambled words that mature readers are able to read despite severe misspellings. Our reading circuitry is special indeed.[14]

I think a similar thing happens when developing fluency in single-digit arithmetic. In class, a student will be taught strategies to derive new facts from ones he or she already knows using well know strategies. For instance, suppose that a child is learning addition facts. The simplest strategy is the "Count On" strategy. For example, if a child had to get $2+6$, he or she could start at the higher number, 6, and "count on" 2 from there. So the child would say "six, seven, eight" possibly using two fingers to count the seven and eight. Another is the "Break to 10" strategy. For example, suppose a child had to get $8 + 6$. The child could first break 6 into two parts, $2 + 4$, and the required sum would be $8 + 2 + 4$. So you can see what is happening. The 2 gets the student to 10 and, from there, it's easy to add 4 to 10 to get 14.[15]

Hence at any stage before complete fluency, a child will be fluent in some basic facts and have a number of strategies to get non-fluent facts from fluent facts. When a child can figure out a non-fluent fact using a strategy and fluent

[14] As a point of comparison, the Chinese and Japanese languages, in essence, are not phonetic languages in the sense that a word symbol cannot be sounded as it can in English. This, then, is part of the explanation of why Chinese and Japanese children must work more intensively to learn their languages. Their languages do not have the phonetic mnemonic that English has.

[15] Alternatively the student could break 8 into $4+4$ to arrive at $4+4+6 = 4+10 = 14$. Both of these strategies take advantage of 10 as an anchor.

facts, we say the fact is *calculable*. When all of the facts for a given operation are calculable, the operation is said to be *calculable*.

As I see it, this is the time to begin eTTs. Suppose we think of the facts for this child as partitioned into those that are fluent and those that are calculable. Then eTTs will do two things. First they will reinforce the facts the child already knows. And second, the child will get more adept at the mental calculus he or she uses to get a non-fluent fact. Eventually, the child will become fluent in a calculable fact and so on subsequent tests the calculus will no longer be required. In this way, for a given fact, we can think of two neural circuits: one is the circuit to do the calculus; and the other is the fluent one that gives the fact immediately. With eTTs, it's my belief that both these circuits are strengthened. Of course, our minds may not work like this at all. There may not be dedicated circuits. But, based on my feeling of how we learn to read phonetically, I think the analogy is reasonable.

Boaler and Number Sense

Let me come back to Boaler's point about number sense. In her view, at a given point in time, a student will have the single-digit facts partitioned. There will be facts that are fluent and facts that are calculable. For example, for a period of time, a student might get 7×8 by doing a quick mental calculation, say with $7 \times 7 + 7$ or $8 \times 8 - 8$. Let's take the former (her example). The student needs to be fluent with 7×7 and then he or she has to be able to add 7 to 49 to get the answer. This is clearly a better mnemonic than, say, the "22" mnemonic where I simply subtract 2 from each digit in 78 to get 56. Eventually, after running into 7×8 a sufficient number of times, the fact would transition from one that is calculable to one that is fluent. That is certainly one way to go.

With this approach, teachers need to be careful about the use of calculators. If a student is at the stage where 7×8 is calculable and subsequently uses a calculator in all instances where 7×8 arises, then the $7 \times 7 + 7$ mnemonic will not be used and, consequently, there will some depreciation when the student tries to remember it. But I think that such calculator exercise has to be useful in burning the 7×8 circuit.

But what about situations when the student does not have access to a calculator. In this case, if a student is not fluent with 7×8, he or she will use up valuable working memory to execute the $7 \times 7 + 7$ calculation, the answer may be wrong (as with my student)[16], and more generally, having to

[16] In the absence of a calculator, I guess the key question is whether a student would

do use the $7 \times 7 + 7$ circuit distracts the student from the original problem.

Or what about the case where a student has to do a number of exerices with single-digit fractions. Take for example the difference introduced earlier:

$$\frac{5}{9} - \frac{2}{7}.$$

We've already been over the single-digit products required to get an answer. Certainly a student could use a calculator. But it would be a lot quicker if these products were fluent.

All in, it's not clear to me that it's best to leave the student with some single-digit facts that are calculable. The risk is that some of them never become fluent. On the other hand, with eTTs employed only when a student is in a position where the facts are calculable, the student will be strengthening his or her calculable circuits as well as his or her fluency circuits.

The Efficiency of eTTs

The discovery pedagogy appears to take the following approach to fluency. Initially students get to a position where all of the facts are calculable with some fluent. At this point, a sort of longer-period osmosis begins. The student will move on to more complicated math (like fractions and algebraic manipulation) where, as a result of doing lots of problems some of which require single-digit arithmetic, the calculable circuits and the fluent circuits for all of the single-digit facts are strengthened to the point where a student becomes fluent. But over this same period, a student who was fluent when the osmosis period began for other students will do either the required work in less time or more work in the same time, and his or her working memory will not be strangled by the 4-5 seconds he or she requires to execute each of the required single-digit arthmetic calculations a particular problem requires. So there are advantages to early fluency.

But what is the cost of developing the fluency? I happen to think that, with eTTs, it's minimal and not damaging. My experience is that a child does not find five minutes a day for three to four months particularly onerous. In fact, most of the children I've taught looked forward to the tests when they started to pick up speed.[17] They also have a real sense of satisfaction

make more mistakes when a fact is calculable versus when it is fluent. I'm not sure what the answer is but I'd guess the former.

[17] I don't have data, but my experience has been that there is a mildly exponential growth in the number of questions answered on eTTs over time. The relationship is certainly not linear.

when all of the facts have been mastered.

All in, its my view that the mininal investment required for fluency pays a big dividend when more complicated math is undertaken.

Young Students are Resourceful

Carraher et al. (1985) document an interesting case of child street vendors in Recife, a city of 1.5 million on the north-eastern coast of Brazil.[18] Typically, the parents of these children are the street vendors but the children help out and sometimes have to handle transactions themselves. The subjects were 5 children ranging in age from 9 to 15 with varying levels of education and coming from very poor economic backgrounds.

The researchers approached these children in their street vendor settings and engaged them in some hypothetical transactions. Here is an example:

"Customer: How much is one coconut?
M (a subject): 35
Customer: I'd like ten. How much is that?
M: (Pause) Three will be 105; with three more that will be 210. (Pause) I need four more. That is ... (pause) 315 ... I think it is 350."

Subsequent to these street interactions, the researchers engaged these children in a formal test usually at the child's house. The problems given the children were based on the initial interaction with the child. Here are examples one and two from the paper (page 26):

"**First example (M, 12 years)**

Informal Test
Customer: I'm going to take four coconuts. How much is that?
Child: Three will be 105, plus 30, that's 135 ... one coconut is 35 ... that is ... 140!

Formal Test
Child solves the item explaining out loud:
4 times 5 is 20, carry the 2; 2 plus 3 is five, times 4 is 20.
Answer written: 200

Second example (MD, 9 years)

Informal Test

[18] See Terezinha N. Carraher, David William Carraher and Analucia D. Schliemann, "Mathematics in the Streets and the Schools," *British Journal of Developmental Psychology*, 3, (1985): 21-29.

Customer: OK, I'll take three coconuts (at the price of Cr$ 40.00 each). How much is that?
Child: (Without gestures, calculates out loud) 40, 80, 120.

Formal Test
Child solves the item and obtains 70. She then explains the procedure 'Lower the zero; 4 and 3 is 7.' "

By way of summary statistics, the children answered 98.2% of the 63 informal test problems correctly but only 73.7% of the formal test problems. In other words, the children were able to find a way to do the required arithmetic accurately because it was essential they did. Moreover, it's clear that they found ways that were different than the ways they were learning at school.

The clear takeaway is that children are quite resourceful and they will find a way. Of course this is exactly what we would expect to find as a child goes from facts that are calculable to those that are fluent. They find a preferred strategy for a given fact and over a series of timed tests where this fact appears, they would do the calculus more quickly until eventually, with fluency, it is not required anymore.

The 5-Minute Arithmetic System (5MA System)

We are currently building an eTT web app which could be used by any monitor/student pair.[19] A monitor could be a teacher, parent, or anybody capable of supervising the student. Once a student is ready for eTTs (i.e. when all the facts are either fluent or calculable), he or she would plug into the app once a day for at most 5 minutes. Ideally the student would take a tablet, sign in, and undertake an eTT.

To emphasize, eTTs should not begin until a child is ready for them. It goes without saying that the complete set of single-digit facts should be partitioned into manageable chunks and this will vary student to student. For instance, a monitor might decide to begin addition by working on only the addition facts up to $5 + 5$ (i.e. no addend higher than 5). At this point, the monitor will proceed with what I term *Development Sessions*, sessions designed to teach strategies for getting known facts from unknown facts.

[19]Primarily, we have in mind either a teacher monoitoring a student, or a parent monitoring a child. But it needn't be just these two classes. I have used a manual version of the web app to teach older students quite successfully. One was a young man in his twenties who wanted to become a police officer. It took him less than a month to become fluent by working two five-minute sessions a day.

Once a monitor is confident that a student is at the stage where the facts are calculable, then eTTs can begin in order to burn the student's neural circuits for fluency.

The complete 5MA System comprises a number of *modules* and within each module there is a collection of *submodules*. Basically students will be working on submodules throughout. The idea is to have a monitor select the submodules that he or she feels are appropriate for his or her students. The main modules are:

1. *The Count Module.* Number recognition, order, and counting are fundamental and logically precede learning the operations.

2. *The Addition Module.* Generally addition is the first operation to tackle. We will have 4 submodules depending on the highest addend a monitor wants his or her students to tackle. *Add(3)* works on the first 3 integers; *Add(5)* works on the first 5 integers. We will also have an *Add(7)* and *Add(10)* submodule.

3. *The Subtraction Module.* Once a student is fluent with addition, subtract is tackled next. We plan a *Subtract(5)* and a *Subtract(10)* submodule.

4. *The Multiplication Module.* This is without question the toughest nut to crack. We see this module as comprising a submodule for each multiplier. What is important is the order in which these are done. It's also important that students understand the commutative property of multiplication and this will be emphasized in all Multiplication Development Sessions.

5. *The Division Module.* Once multiplication is mastered, division is straightforward. We plan *Div(5)* and *Div(10)* submodules.

In addition to selecting what submodules students will do, a monitor also selects pass criteria for moving to the next submodule. Typically this will be answering a threshold minimum number of questions and then a certain number of these correct over a number of successive timed tests.

For teachers and students without internet access, we plan a manual version of the 5MA System.

Motivation

In my experience with the primary school children I've taught, motivation is important. Children at this age do not appreciate the importance of becoming fluent. Hence a monitor should consider a series of incentives to encourage effort. Parents are in a good position to come up with effective incentives, teachers less so. But incentives are a must to get the proper effort and focus. One of the reasons for our insistence on short frequent work sessions is that children have difficulty focusing for periods any longer. What we require is the right set of incentives to encourage focus and effort for five minutes a day.

I think that fluency in single-digit arithmetic ought to be a goal of any reasonable curriculum in primary school mathematics. Fluency is easily measured with eTTs so it's straightforward to determine when a child reaches the standard. And once a child gets to fluency, there should be a formal celebration either at home or school or both. Becoming fluent is no easy task and we need to applaud children when they get there.

Finally, I want to emphasize that public timed tests are destructive and should not be used. The only person who should see a student's work is the monitor. It also shouldn't matter how quickly a student gets to fluency. The important thing is to get there.

Summary

I am a fan of the discovery pedagogy now dominating mathematics education in our primary schools. Anything that gets children excited about mathematics is good. But I also think that skill development is important. Students need to be able to do arithmetic and algebraic manipulation.

The current evidence would suggest that the discovery approach has yet to be successful. This shows up in provincial and international testing scores. It's most stark at the university level where there has been a marked depreciation in the mathematics skills of entering students. It could be that students have yet to see a full twelve years of the discovery pedagogy. But I have my doubts. And so do the administrations of just about all Canadian universities, administrations which have recognized a significant problem and are now taking steps to deal with it.[20] The recent announcement by the Government of Ontario would suggest that our legislators also think there is a

[20] See Borwn, Lois "Internet Teens Failing Math," *The Toronto Star*, available at: *http://www.thestar.com/life/parent/2009/09/06/internet_teens_failing_math.html*

problem.[21]

But coming back to primary school mathematics, I believe that knowing the basics is crucially important to higher level skill development. You can learn fractions and higher level arithmetic without single-digit fluency but it's more difficult. You can learn algebraic manipulation without single-digit fluency, but it's more difficult. A well built house needs a good foundation and so does the study of just about any discipline and particulary mathematics. We need to think really hard about how to develop these fundamental skills without turning children off. Along this line, I think eTTs are a good start to developing fluency without the attendant problems of tedium and turn-off.

[21] See the Press Release at *https://news.ontario.ca/edu/en/2016/04/ontario-dedicating-60-million-for-renewed-math-strategy.html*