MAE386 Ideation, Thought Media, and Mathematics

COURSE PLAN

January 2026

The following table defines the first day of the week for each week this term:

Week	Week of	Mon(2)	Wed(1)
1	05-Jan	Lecture	
2	12-Jan	Lecture	
3	19-Jan	Lecture	
4	26-Jan	Lecture	
5	02-Feb	Lecture	
6	09-Feb	Lecture	
	16-Feb		**Reading Week**
7	23-Feb	Lecture	
8	02-Mar	Lecture	
9	09-Mar	Lecture	
10	16-Mar	Lecture	
11	23-Mar	Lecture	
12	30-Mar	Lecture	
13	06-Apr	Stat Holiday	Lecture

Most of your reading will be from the book *Catching Unicorns: The Exographic Revolution and the Rise of Techno-Literate Culture* (Bill Hurley and David Hurley, Wild Road Books, 2026). I'll refer to it as "CU."

The general weekly plan is as follows. At the beginning of the lecture, we will do what I call "Mr. Fixit Exercises." I will give you a couple of sentences that are not well written and we will try to fix them. There is a supporting file for this work, *MAE386 MR FIXIT EXERCISES Jan 2026.pdf*, available at the MAE386 Google Drive site. After these exercises, we will discuss your answers to the Notebook questions. Each week you will be assigned Notebook questions to answer based on your reading of CU. Drafts of your answers should be in your Notebook by the beginning of the next week's lecture. It's during this lecture that we will discuss your answers to the questions. Once we finish that discussion, I will give a brief overview of what we will cover the following week.

Week 1

Preparation for Class

none

Reading

Read CU, Introduction (pp. 3-12)

Notebook Questions

When you write a book, you usually mention all of the important concepts developed over the course of the book in the introductory chapter. That's what we've tried to do in

CU. Hence, I think it behooves us to spend some time on this chapter to understand these concepts. The following questions are to be completed by the beginning of the Week 2 lecture.

- In footnote 2 on p. 4, we describe Merlin Donald's characterization of *engrams* and *exograms*.
    - Define both terms and then explain his view of the advantages of exograms.
    - Can you think of any other advantages of exograms?
    - As we know, both engrams and exograms require effort to form and store. Discuss the relative efforts required for each mode of memory.
    - Are there engrams that cannot be stored as exograms?
    - An important characteristic of memory is speed of recall. What are the relative speeds of recall for engrams and exograms?
    - As a part of your education, you are forced to internalize a large list of engrams. For example, a mathematics student must understand how to differentiate an exponential function. With the technologies we now have to search and access our great quantity of exograms (i.e., Google, ChatGPT, etc.), is it still necessary to require students to form engrams?
- We've argued that a word written on a page is symbol. Is a spoken word also a symbol? Argue why or why not.
- On pp. 5-6, the characteristics of a techno-literate culture are defined. Briefly describe these 4 characteristics. Is the CAF a techno-literate culture?
- A short time ago in evolutionary time (10,000 years ago), we lived in small hunter-gatherer groups. Today our techno-literate groups are very large and, for the most part, urbanized. It's been argued that we're able to live in large anonymous groups because we've self-domesticated (sheep, cows, and other animals have been domesticated with selective breeding) and this fosters cooperation. Why is cooperation so important to Homo sapiens?
- Use a counter-example to show that the following statement is false: All ideas discoverable by the human mind can be discovered without exographics.
- In the book, we refer to concepts without a real-world referent as abstract objects. So, the number 23 is an abstract object whereas a baseball is a concrete object. Explain why the mathematical concept of multiplication is an abstract object.

Week 2

Preparation for Class

Complete the Week 1 Notebook Questions and be prepared to discuss them in class.

Reading

Read CU, Introduction (pp. 12-21)

Notebook Questions

The following questions are to be completed by the beginning of the Week 3 lecture. You may have to do some internet research to answer some of them.

- Define the terms cultural object, made world, and lifeways. What is the relationship among these terms?
- In the book, we argue that a cultural object is equivalent to a set of ideas. Is Vermeer's *Girl with a Pearl Earring* a cultural object? Is it equivalent to a set of ideas?
- Define the Ideasphere. Is the *Bible* in the Ideasphere? What about hamburgs and hockey sticks?
- Define the e-Class. Determine, as best you can, whether the following cultural objects are in the e-Class:
    - beer
    - Robert Rauschenberg's *White Paintings*
    - the Burj Khalifa in Dubai
    - a hammer
    - the pulley
    - yarn
    - ChatGPT
    - the M777 howitzer
    - the McLaren MCL39, the motorcar that powered Lando Norris to the F1 Drivers' Championship in 2025
    - George Gershwin's *Rhapsody in Blue*.
- We define serious ideation to be the thinking that an individual or group does, usually over an extended period of time, to arrive at a cultural object. For example, the Wright brothers worked for about 4 years to figure out how to control the flight of a fixed-wing aircraft. We conceive serious ideation as a process which repeats two basic steps: a thinking step and an action step. We term this process *the diaconatic*. For this question, I would like you to take a phenomenological approach to assess how you determine the solutions, if any, to the equation $2x^2 - 12x - 7 = 0$. If you are like me, there will be periods where you *think*, and then periods where you engage in *action* (i.e., writing on a page). Record what you do and then argue in detail that you used *the diaconatic*. If you did use the diaconatic, your answer should follow this pattern: [Think1, Action 1], [Think 2, Action2], and so on. For each item in that sequence, describe what happened. Do not include the thinking and actions required of the phenomenological approach. Simply focus on what you do to solve the equation.

- Until relatively recently, we attributed our long-term physiological changes to biology. Mutations within our genetic material at reproduction would lead to physiological changes and if those changes were advantageous given the environment we happened to be in, they generally spread to the population. There is strong archeological evidence that some 3 million years ago, one of our ancestor species was a bipedal, small-headed ape. Now our heads are 3-4 times as big. This is biology at work. But there is another force that acts on us and that is our cultural evolution. Sometime our interaction with the world results in a cultural advance that eventually changes our biology. Give an example of this.
- Before setting out on the research that led to CU, I was a confirmed neurocentrist. I felt that I only needed my head to arrive at ideas. Give an example that demonstrates that some ideation has to be embodied (i.e., it requires muscle movement .. you can use an arithmetic example if you like).

Week 3

Preparation for Class

Complete the Notebook Questions for Week 2 and be prepared to discuss them in class.

Reading

Read CU, Chapter 1 (pp. 25-38)

Notebook Questions

- Classify the following examples as glottographic writing and/or semasiographic writing (your notebook answer should argue why you chose the categorization you did and there is no need to echo the pictures in your notebook):
  - The cat in the hat.
  - These notations, scratched in mud:
  - $E = mc^2$
  - This sentence: An elliptic curve over **Q** is said to be modular if it has a finite covering by a modular curve of the form $X_0(R)$. (This is the first sentence of Andrew Wiles's 108-page paper wherein he gives the proof of Fermat's Last Theorem, one of the great accomplishments of 20^{th} century mathematics.)
  - Christopher Wool's painting *Apocalypse Now*:

    It reads "Sell the house, sell the car, sell the kids."
  - This centuries-old snippet of sheet music:
  - A page from Darwin's notebook showing his so-called "Tree of Life."

- Define exographics.
- In our definition of exographics, we used the term *persistent* by which we meant that the exographic symbols are fixed or stationary. Consider a person signing (i.e., using sign language). Can signing be considered exographics? Explain why or why not.
- Explain the *memory extension purpose* of exographics (see pp. 29-30). In your explanation be sure to explain clearly the cognitive weakness exographics helps us to overcome. You might consider using an arithmetic example in your explanation.
- Had we not invented exographics, would it be possible to discover 847 x 86? (See the discussion of *exophonics* (pp. 32-35)). Clearly, we would never use exophonics to discover an idea because exographics is a much better technology for that purpose. Nonetheless, the discussion of exophonics makes an important point. What is it?
- In CU, we distinguish *finished writing* and *rough writing*. How would you respond to this statement: "Between finished writing and rough writing, finished writing is more important because, generally, you are communicating your idea to others with finished writing."
- Generally, our species has a hard-wired desire to consume story and this likely goes back to oral story-telling soon after we began to speak. As things progressed, we told increasingly complicated stories. In this regard, the Homeric epics (*The Odyssey* and *The Iliad*) are striking. We know that Homer composed both orally and they were performed (or, more accurately, improvised) by poets at religious festivals and other kinds of gatherings. Some argue that the primary purpose for the invention of the Greek alphabet and written language was to record the epics. But once the alphabet appeared, the quantity of Greek literature exploded. Why?
- What is the reification purpose of exographics (pp. 35-36)?

Summary. So, we've identified two purposes for exographics. One, exographics makes it much easier to form longer arguments in our working memories (memory extension purpose). And two, it's a way to get this symbolic information into our visual fields to begin with so that we can then do the manipulations (reification purpose). Both are required to discover Pirsig's ghosts but reification is crucial because you have to have these symbols in your visual field before you can begin to manipulate them.

Preparation for Class

Complete the Notebook Questions for Week 3 and be prepared to discuss them in class.

Reading

Read CU, Chapter 2 (pp. 39-44)

Read CU, Chapter 3, (pp. 54-60)

Notebook Questions

- Study Harvey Leyman's growth curves in Figure 2.2 at the top of page 42. These curves plot the increase in ideas over time (time is measured in centuries). Moreover, each curve is for a particular discipline and these disciplines generally require exographics to discover ideas. For example, it would be impossible to develop mathematics without exographics. Hence these curves track grow in the e-Class. What general functional form do these curves follow?
- Suppose that the rate of increase in the number of new ideas discovered over a short period of time is proportional to the stock of ideas at that point in time. Show that this assumption is consistent with an exponential increase in the number of new ideas over time. (See footnote 3, p. 41)
- Study Max Roser's graphic on page 40 closely. You could argue that this curve tracks the growth in the Ideasphere. Identify the approximate year where the slope of this curve really starts to increase significantly. Do you see any issue with the slope of the curve after the year 2000?
- Have university curricula got more complex over the last, say, 500 years? What does this say about the growth in the e-Class?
- In the solution of a *complex* Raven's Progressive Matrix problem, we usually have to conjecture a sequence of rules until we eventually get one that works. We have been arguing generally that exographics is necessary to discover certain kinds of ideas. With Raven's Progessive Matrices, is exographics required to come up with candidate rules to consider? (Keep in mind that coming up with a rule is different than determining whether a particular rule works.) What does this tell us about the role of exographics in human imagination and ideation?
- Write a short note on the experimental work done with chimpanzees to solve Match-to-Sample problems where the subject is asked to discover the underlying abstract relation. It appears that minds that function in a symbolic world are able to solve these problems whereas those that don't, cannot. In the book, we have not explained why. Can you think of an explanation?
- When I coach students how to write a math exam, I generally urge them to spend a brief amount of time looking at all problems right away. Based on your reading in Chapter 3, is there a reason why this might be a good idea?

Week 5

Preparation for Class

Complete the Notebook Questions for Week 4 and be prepared to discuss them in class. I have purposely given only a short reading and just a few questions. This is to give you a chance to bring your notebooks up to date.

Reading

Read CU, Chapter 5 (pp. 69-77)

Notebook Questions

- The ancient Greeks, particularly Plato, were not fans of the new technology of writing. In fact, they believed in an oral approach called *the dialectic*. What is the dialectic?
- The chapter presents clear evidence that intellectual skill requires a significant investment in the wiring of your mind. Basically, it's an effort to load your long-term memory so your imagination can stir and manipulate that memory content. On the other hand, we've developed the e-Library to the point where we can now search and access a large quantity of external information, and hence one could argue that the e-Library reduces what needs to be stored in long-term memory. Assess whether we will get to the stage where nothing needs to be stored in long-term memory.
- Define *the diagraphic* and discuss how it is similar to the dialectic.

Week 6

Preparation for Class

Complete the Notebook Questions for Week 5 and be prepared to discuss them in class.

Reading

Read CU, Chapter 6 (pp. 79-88)

Read CU, Chapter 7 (p. 93)

Notebook Questions

This is an important chapter. To this point, we have been concerned with individual minds and how exographics can support the imaginations of these minds on the way to idea discovery. But the main strength of our species is our ability to collaborate and cooperate. An individual mind can come up with Special Relativity but it takes a *network of minds* (over time and space) to refine technologies like the MRI machine, electricity, and jet aircraft. Most of what we do these days requires an army of us. Think of our joint efforts to get to the moon, to build Large Language Models, and to manufacture a car.

We evolved some wonderful skills to collaborate. Unlike other species, we communicate in a very sophisticated way (speaking and writing), we've self-domesticated (which enables strangers to cooperate willingly), we have an unparalleled ability to learn from one another, and we have extraordinary abilities to manipulate imaginary things that we represent with symbols.

- Define Theory of Mind.

- Explain Michael Tomasello's concept of *shared intentionality* using the example of the little boy who helps the experimenter with the magazines.
- Why is our mimetic skill important?
- Esther Herrmann and her colleagues did experiments which enabled them to compare the general intelligence and social intelligence of young children and chimpanzees. What did they find?
- One of the fortunate outcomes of human evolution is that we developed the ability to make about 40 different sounds which we string together to form to speak words and communicate. In fact, we communicate in an extraordinarily refined way relative to all other species. Had we not evolved the ability to make different sounds, would we be communicating as effectively as we do today?
- Some writers have compared our minds to computers. In the context of language acquisition, explain why the computer analogy is not very good. (See p. 93)

Week 7

Preparation for Class

Complete the Notebook Questions for Week 6 and be prepared to discuss them in class.

Reading

Read CU, Chapter 8 (pp. 99-117)

Notebook Questions

- In this chapter, we document the transition in Europe from the Roman Number System to the Hindu-Arabic system we use today. One of the reasons for the switch was that the Hindu-Arabic system was easier to use cognitively. What is the generality we can draw about exographic representation systems? (You might begin by noting that both number representation systems are equivalent in the sense that they represent the same objects but use different representations to do so. For example, 23 and XXIII represent the same number.
- In the section on music, we document the adoption of staff notation. What was the major benefit of this exographic system?
- We discussed the advantages of phonetic writing systems over ideographic/pictographic systems (where the symbol is basically a picture of the concept or word). What is the major advantage of phonetic writing systems?
- Is it possible to extend our vision to see distant objects with exographics? Give an example.
- The Oksapmin people of Papua New Guinea have no exographic system. But they use a body-counting system for their numbers. Here is a picture of the association of their numbers with body parts:

They are able to do simple addition and subtraction with this system. Clearly the use of their bodies constitutes a mnemonic for their numbers. Can you think of any mnemonic system you were taught to remember important aspects of our representation systems?

Week 8

Preparation for Class

Complete the Notebook Questions for Week 7 and be prepared to discuss them in class.

Reading

Read CU, Chapter 9 (pp. 121-128)

Notebook Questions

- Pick any day you want. Document *every* instance where you deal with literacy (exographics) and/or measurement from the start of the day to the end.
- In the chapter, we make the point that good ideas seem to have a tendency to go global. Look at 3 cultural objects (ideas) that we have not mentioned in the text and document their reach. Would you say they are currently global, and if not, do they have the potential to go global.

Week 9

Preparation for Class

Complete the Notebook Questions for Week 8 and be prepared to discuss them in class.

Reading

Read CU, Chapter 9 (pp. 130-146)

Notebook Questions

- There were 5 ancient "river" civilizations and each appears to have invented their own form of writing. For this question, focus on the Sumerians. It seems reasonable that, once a civilization invents a writing system, that they might also begin to add to the e-Class. Assess that hypothesis for the ancient Sumerians.
- We suggest in CU that the Shang used a positional number system. However, since that writing, I have found the following information on the internet. The Shang had representations for individual numbers as shown in this table:

The number, 3568, is represented:

Note that it is the concatenation of the symbol for 3000, 500, 60, and 8. If this internet source is accurate, this is an example of an additive-subtractive system. The Roman number system is also an additive-subtractive system. Our number system,

the Hindu-Arabic system is a positional number system. What are the advantages of a positional number system?

- If you are using an additive-subtractive number system and try to do multiplication the way we do in our Hindu-Arabic positional system, you soon run into a nightmare. How did the ancient Romans multiply, say, 325 by 64?
- The ancient Sumerians invented positional number systems but they did not have a symbol for 0. Of course this presents a difficulty for positional number systems. For example, how would you represent the number 3008? How did the Sumerians deal with not having a place-holder value? In other words, how would they have represented 3008?
- Ancient measures of length tended to follow somatic standards. For example, the cubit was defined to be the length between the tip of an elbow and the tip of the middle finger. [In class, we will do this exercise .. I will bring in a measuring tape and I want each of you to measure your cubit length in inches (you might want to pair up to execute these measurements).] For this question, let's consider the ancient Egyptians building the pyramids. This was a massive undertaking that took thousands of labourers and engineers working for years. What would be the problem with using someone's actual cubit measure each time a measure of length was required? (For example, the blocks at the base of each pyramid were roughly 2 cubits by 2 cubits by 1 cubit). What is the solution of this problem?
- Use internet research tools to write a brief history of how we have standardized the measure of a meter.
- Is it possible to do any kind of standardized measurement without exographics? Can I argue that modern measurement would be impossible without exographics?
- Based on pp. 143-145, it appears that a large civilization developed in what is now Ukraine as evidenced by the Trypillia megasites. Strangely, there is no evidence of any writing or measurement. What is the likely reason?

Week 10

Preparation for Class

Complete the Notebook Questions for Week 9 and be prepared to discuss them in class.

Reading

Read CU, Chapter 11 (pp. 147-153)

Notebook Questions

I have entitled this chapter "The Greek Miracle." It's a phrase that many historians of ideas use because the ancient Greeks made such stunning contributions to the e-Class and Ideasphere.

- Is it fair to say that the ancient Greeks invented the phonetic system for

glottographic writing?

- Discuss the role of exographics, measurement, and knowledge of the heavens in the construction of the Antikythera mechanism.
- Consider a triangle and its opposite side bisectors (from each vertex a line is drawn that divides the opposite side into lines of equal length):

Ancient Greek mathematicians noticed that these lines intersected at a single point. More generally, they invented a system of analysis with *definitions* (i.e., points, lines, triangles, and so on), *axioms* (i.e., parallel lines do not intersect), and then *theorems* (the opposite side bisectors of any triangle intersect at one point). This is the same system we use today in mathematics. Would it have been possible for ancient Greek mathematicians to prove that a triangle's opposite-side bisectors intersected at a single point without using exographics? Explain why.

- **Bonus Problem**: Consider the previous problem and suppose the vertices of a triangle are at (6,6), (12,6), and (6,18). An opposite-side bisector is called a *median*. Let the *centroid* be given by the average of the three vertices: ([6+12+6]/3, [6+6+18]/3) = (8,10). Can you show that each of the three medians goes through this centroid. Once you have shown this, assess whether you could have done it without pencil and paper. Keep in mind that this is just an instance and does not constitute a general proof. Such a proof would begin with the vertices **a**, **b**, and **c** where **a** is a vector representing the vertex (a_1, a_2) and **b** and **c** are defined in a similar way. Try to do this general proof. You might want to study this proof:

- There can be no doubt that the Greeks made intellectual contributions that dwarfed those of the other ancient cultures. These were in the subject areas of mathematics, philosophy, history, science, politics, art, and architecture. What do I suggest is the reason for this advance?

Week 11

Preparation for Class

Complete the Notebook Questions for Week 10 and be prepared to discuss them in class.

Reading

Read CU, Chapter 14 (pp. 171-188)

Notebook Questions

Alfred Crosby's question in this chapter's epigraph is a good one: "How, between the ninth and the sixteenth centuries, had these bumpkins managed all that?" The centers of the intellectual world in the 9th century were Baghdad in the east and Constantinople (now Istanbul, Turkey) in the west. Central and northern Europe were

backwaters. Yet by the 16th century, Europe had caught up and moved substantially ahead. In the chapter, I address that issue.

- After the fall of Rome, the Roman Catholic Church expanded across Europe and, for most of the period we are considering, was the largest political organization in Europe. We know that all large bureaucratic organizations require exographics to manage and the same was true of the Church. Is there an argument that the Church was a techno-literate culture? In your answer, make sure to assess the four characteristics that define a techno-literate culture.
- Let's think about the invention of the printing press in 1450. Reconsider Table 8.2 in CU on p. 105 that shows the volume of books published over 50-year periods. Post-Gutenberg, there is a substantial increase in the number of books sold. Over the first 50 years, these tended to be religious books (about 50%); other genres included science, law, medicine, and reference books (dictionaries, grammars, calendars, etc.). The printing press brought down the cost of books by quite a bit and this would surely have spurred demand. The population of Europe at the time was about 50 million and it's been estimated that 5-10% of the European population was literate. Those buying books would have included the clergy, nobility, merchants, and urban elites. Is the staggering increase in the number of books sold inconsistent with these facts?
- Let's consider the effects of the Protestant Revolution. Luthor proclaimed the doctrine of *sola scriptura*, the idea that the interpretation of scripture was an individual responsibility whereas, for the Church, it was dictated and regulated by Rome. What effect would *sola scriptura* have had on the quantity of e-Class ideas produced?
- In Europe, we also observe the proliferation of universities. What effect would this have on additions to the e-Class?
- Discuss the role of standardization of measurement in this great European e-Class advance.

Week 12

Preparation for Class

Complete the Notebook Questions for Week 11 and be prepared to discuss them in class.

Reading

Read CU, Chapter 16 (pp. 195-207)

Notebook Questions

This chapter presents our reasoning for why exographics explains "The Great Divide," the phrase given to the great cultural differences that exist among today's societies. These range from the primitive cultures you find in some African or South American

cultures to those of the advanced techno-literate societies.

- The idea that literacy explained the Great Divide was put forward in the 1960s and subsequently was criticized severely. Patricia Greenfield said this:

"[Their work] should rid us once and for all of the ethnocentric and arrogant view that a single technology [exographics] suffices to create in its users a distinct, let alone superior, set of cognitive processes." (Patricia Greenfield. "Book Review of 'The Psychology of Literacy'". *Harvard Educational Review* 53, no. 2 (1983): 216–220.)

You can also look at the Halverson critique on p. 204. How are these critiques incorrect?

- What does Alexander Luria's work tell us about education and literacy?
- Explain what Scribner and Cole's work tells us about the effects of education and literacy.
- What does Boyd, Richerson, and Henrich's work tell us about the intelligence of non-literate peoples?
- What does Leibenberg's work tell us about the abilities of non-literate peoples to store vast amounts of knowledge.
- Is David Olson correct when he argues that an oral culture would be capable of discovering Euclid's geometry?
- In the face of this evidence, what is our argument that exographics explains The Great Divide.

Week 13

Preparation for Class

Complete the Notebook Questions for Week 12 and be prepared to discuss them in class.

Reading

Read the short note in *Week13 Note LLMs and AGI.pdf* available on the course Google Drive. I will speak to this note in class.

Notebook Questions

None.