The Non Equilibrium Steady State of Sparse Systems: Energy Absorption and Current

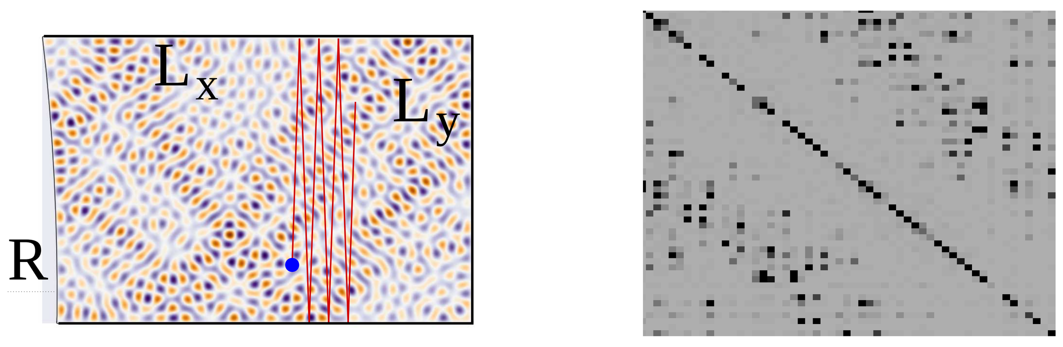
Daniel Hurowitz [1,2]

Ben Gurion University in the Negev, Beer Sheva, Israel

What is a "sparse" matrix?

- The majority of matrix elements are small, yet some are very large.
- The elements have a log-wide distribution with a median that is much smaller compared to the average.

Example: Weakly chaotic billiard [3]



Why sparsity is interesing

- Energy absorption rate (EAR) calculation goes beyond linear response theory (requires resistor network calculation) [1].
- Leads to a novel NESS that has "glassy" nature [1].
- Novel quantum saturation effect [1].
- Current vs. driving goes beyond LRT (emergence of a Sinai regime) [2].

Driven system + bath : A paradigm for NESS

The Hamiltonian:

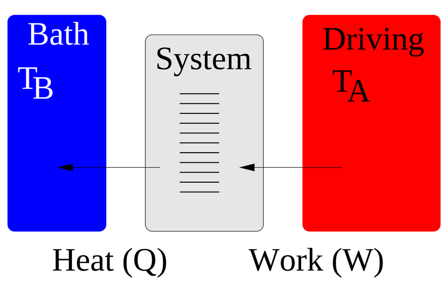
$$\mathcal{H}_{\text{total}} = \mathcal{H}_0 - f(t)\{V_{nm}\} + F(t)\{W_{nm}\} + \mathcal{H}_{Bath}$$

Definitions:

$$\langle f(t)f(t')\rangle = \varepsilon^2\delta(t-t')$$

$T_A = \infty \equiv$ Temperature of A

$T_B \equiv$ Temperature of B

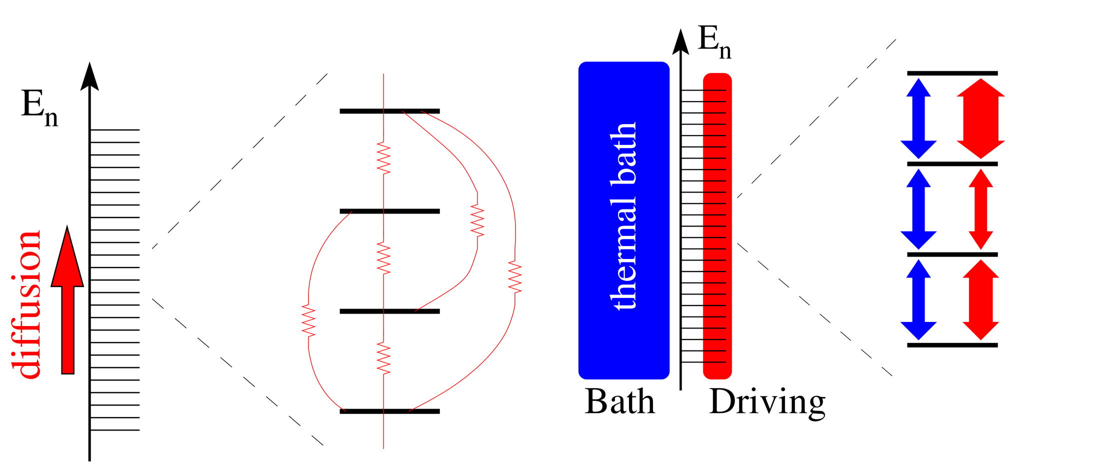


Steady state:

$$\dot{E} = \dot{W} - \dot{Q} = 0$$

$T_A \neq T_B$: NESS is non-canonical

Crossover from Linear response to semi-linear response



$D_{SLRT} = \left[1/w_n\right]^{-1}$ $\qquad D_{LRT} = \overline{w_n}$

Master equation description of dynamics

Quantum master equation for the reduced probability matrix:

$$\frac{d\rho}{dt} = -i[\mathcal{H}_0, \rho] - \frac{\varepsilon^2}{2}[V,[V,\rho]] + \mathcal{W}^{\beta}\rho$$

Corresponding stochastic rate equation:

$$\frac{dp_n}{dt} = \sum_m w_{nm}p_m - w_{mn}p_n$$

The transition rates

$$w_{nm} = w^{\varepsilon}_{nm} + w^{\beta}_{nm}$$
$$w^{\varepsilon}_{nm} = w^{\varepsilon}_{mn} \propto \varepsilon^2$$
$$\frac{w^{\beta}_{nm}}{w^{\beta}_{mn}} = \exp\left[-\frac{E_n - E_m}{T_B}\right]$$

Steady state equation:

$$\dot{\rho} = \mathcal{W}\rho = 0$$

The generalized fluctuation - dissipation phenomenology

$$\dot{W} = \text{rate of heating} = \frac{D}{T_{\text{system}}}$$
$$\dot{Q} = \text{rate of cooling} = \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$

At the NESS:

$$T_{\text{system}} = \left(1 + \frac{D(\varepsilon)}{D_B}\right)T_B$$
$$\dot{Q} = \dot{W} = \frac{1/T_B}{D_B^{-1} + D(\varepsilon)^{-1}}$$

Experimental determination of response:

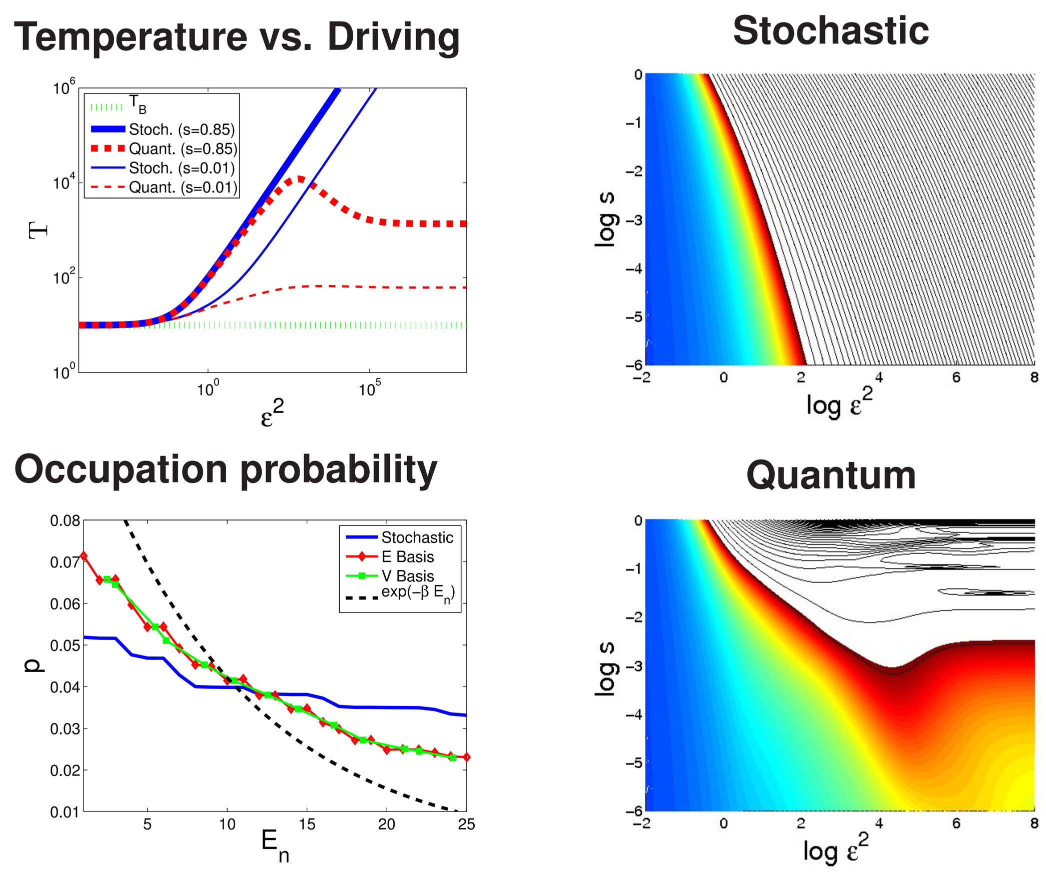
$$D(\varepsilon) = \frac{\dot{Q}(\varepsilon)}{\dot{Q}(\infty) - \dot{Q}(\varepsilon)}D_B$$

The steady state temperature

Microscopic and macroscopic Temperature

$$\frac{p_n}{p_m} = \exp\left(-\frac{E_n - E_m}{T_{nm}}\right), \quad T_{\text{system}} = \text{average}[T_{nm}]$$

This resembles a glassy state



The quantum saturation effect

For very strong driving, the NESS is a mixture of V eigenstates:

$$p_r \sim \exp(-\langle E\rangle_r / T_{mix})$$

For non-sparse V, the eigenstates are extended in energy space - $T_\infty \to \infty$.

If V is sparse, the states are localized. As $s \to 0$, $T_\infty \sim T_{mix} \sim T_B$.

$$T_B < T_\infty < \infty \qquad \textbf{[depends on the sparsity]}$$

Derivation of the cooling rate formula

cooling rate:

$$\dot{Q} = -\sum_{n,m}(E_n - E_m)\, w^{\beta}_{nm}\, p_m$$

occupation imbalance:

$$p_n - p_m = \left[2\tanh\left(-\frac{E_n - E_m}{2T_{nm}}\right)\right]\bar{p}_{nm}$$

up/down transitions imbalance

$$w^{\beta}_{nm} - w^{\beta}_{mn} = \left[2\tanh\left(-\frac{E_n - E_m}{2T_B}\right)\right]\bar{w}^{\beta}_{nm}$$

$$\dot{Q} = \frac{1}{2}\sum_{n,m}(E_n - E_m)^2\frac{\bar{w}^{\beta}_{nm}}{T_B}\bar{p}_{nm} - \frac{1}{2}\sum_{n,m}(E_n - E_m)^2\frac{\bar{w}^{\beta}_{nm}}{T_{nm}}\bar{p}_{nm}$$
$$= \frac{D_B}{T_B} - \frac{D_B}{T_{\text{system}}}$$

definition of the diffusion coefficient:

$$D_B \equiv \left[\frac{1}{2}\sum_n (E_n - E_m)^2\, \bar{w}^{\beta}_{nm}\right]$$

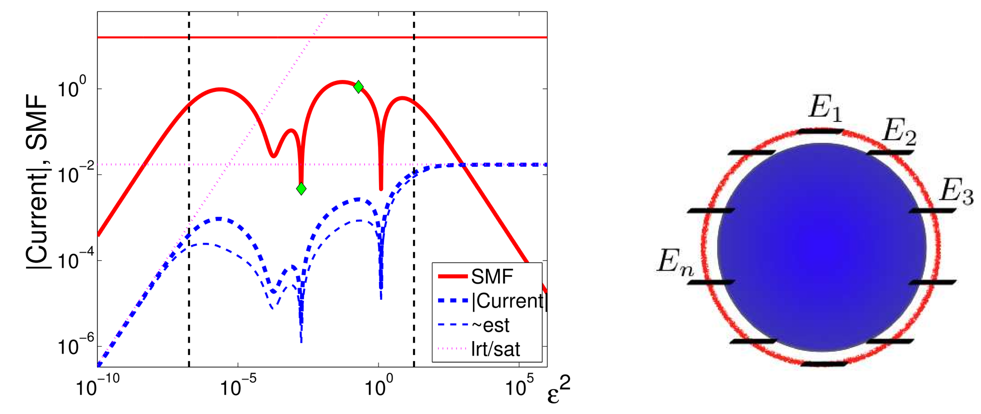
definition of effective system temperature:

$$\frac{1}{T_{\text{system}}} \equiv \left[\frac{1}{T_{nm}}\right]$$

Current vs. driving

Driving $\rightsquigarrow$ Stochastic Motive Force $\rightsquigarrow$ Current

Regimes: LRT regime, Sinai regime, Saturation regime



$$I = w_{\overrightarrow{n}}p_{n-1} - w_{\overleftarrow{n}}p_n, \qquad \sum_n p_n = 1$$

$$I \sim \frac{1}{N}\,\overline{w}\,\exp\left[-\frac{\mathcal{E}_{\cap}}{2}\right] 2\sinh\left(\frac{\mathcal{E}_{\circlearrowleft}}{2}\right)$$

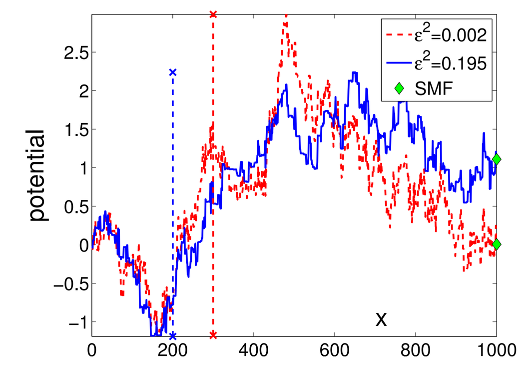
The Stochastic Motive Force (SMF)

If we had only a bath

$$\frac{w_{nm}}{w_{mn}} = \exp\left[-\frac{E_n - E_m}{T_B}\right]$$

We define a "field"

$$\mathcal{E}(x) \equiv \ln\left[\frac{w_{nm}}{w_{mn}}\right]$$



The "potential" variation along a segment

$$\mathcal{E}(x_1 \rightsquigarrow x_2) = \sum_{x=x_1}^{x_2}\mathcal{E}(x) = \int_{x_1}^{x_2}\mathcal{E}(x)dx$$
$$\mathcal{E}_{\cap} \equiv \text{maximum}\left\{|\mathcal{E}(x_1 \rightsquigarrow x_2)|\right\}$$
$$\mathcal{E}_{\circlearrowleft} \equiv \oint \mathcal{E}(x)dx \qquad \text{if no driving} = 0$$

Emergence of the "Sinai regime"

Sinai [1982]: Transport in a chain with random transition rates.
Assume transition rates are uncorrelated.
$\rightsquigarrow$ exponential build up of a potential barrier $\mathcal{E}_{\cap} \propto \sqrt{N}$
$\rightsquigarrow$ exponentially small current.

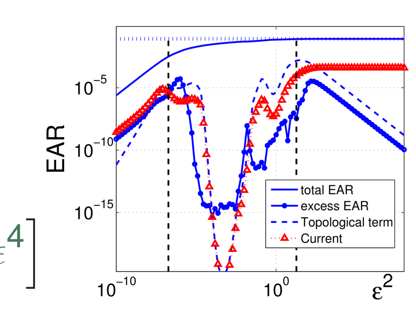
But... we have telescopic correlations:
$\mathcal{E}_{n,n+1} \sim \Delta_n \equiv (E_n - E_{n+1})$

$$\mathcal{E}_{\circlearrowleft} \approx -\sum_n\left[\frac{1}{1 + g_n\varepsilon^2}\right]\frac{\Delta_n}{T_B} \sim \frac{1}{T_B}\begin{cases}\varepsilon^2, & \varepsilon^2 < 1/g_{\max}\\ 1/\varepsilon^2, & \varepsilon^2 > 1/g_{\min}\\ [\pm]\sqrt{N}\Delta, & \text{otherwise}\end{cases}$$

Build up may occur if g_n are from a **log-wide** distribution.

Beyond fluctutation dissipation phenomenology: Topological term in EAR formula

$$\dot{Q} = \sum_n\left[w^{\beta}_{\overleftarrow{n}}p_n - w^{\beta}_{\overrightarrow{n}}p_{n-1}\right]\Delta_n$$
$$\approx \left[\frac{D_B}{T_B} - \frac{D_B}{T^{(0)}}\right] + T_B\mathcal{E}_{\circlearrowleft}\, I$$
$$\approx \frac{D_B}{T_B}\left[(g_n\varepsilon^2) - (g_n\varepsilon^2)^2 + \text{Var}(g)\varepsilon^4\right]$$



The EAR is correlated with the current.

Conclusions

1. The stochastic NESS resembles a glassy phase (wide distribution of microscopic temperatures).
2. Definition of effective NESS temperature and extension of FDR phenomenology.
3. Prediction of LRT→SLRT crossover.
4. For very strong driving, quantum saturation of effective NESS temperature.
5. Topological aspects: A wide distribution of rates is crucial for a Sinai regime.
6. Topological term in EAR is proportional to the entropy production, but sub-linear in driving intensity. The EAR is correlated with the current.

References

[1]D. Hurowitz, D. Cohen (EPL 2011)
[2]D. Hurowitz, S. Rahav, D. Cohen (EPL 2012)
[3]D. Cohen, T. Kottos, H. Schanz (JPA 2006)
[4]M. Wilkinson, B. Mehlig, D. Cohen (EPL 2006)
[5]A. Stotland, T. Kottos, D. Cohen (PRB 2010)
[6]A. Stotland, D. Cohen, N. Davidson (EPL 2009)

Mail: hurowits@gmail.com WWW: http://physics.bgu.ac.il/ hurowits