Percolation, sliding, localization and relaxation in glassy circuits

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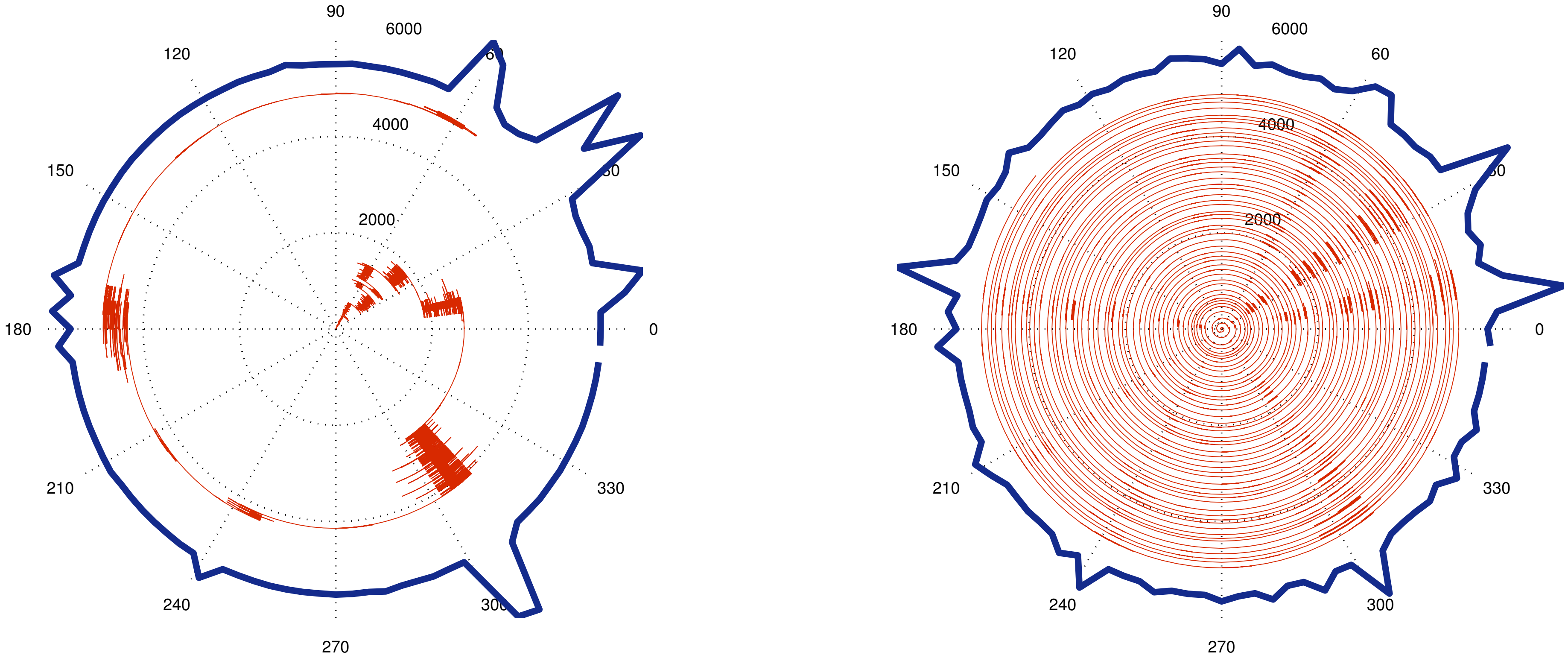
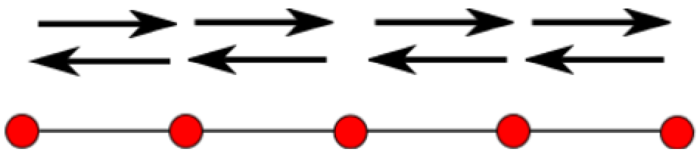


D. Hurowitz and D. Cohen, `arXiv` (2014)

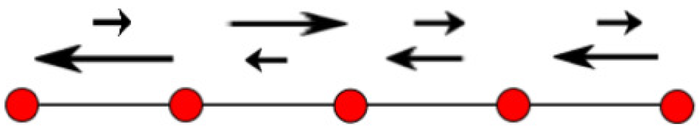
Brownian motion

Simple random walk [Einstein]
uniform lattice - all rates are equal



Random walk on disordered lattice [Alexander et. al]
random lattice - random, symmetric rates

Random walk in random environment [Sinai, Derrida]
rates allowed to be asymmetric
Dynamics described conservative rate equation



$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{W}\boldsymbol{p}, \qquad \boldsymbol{W} = \begin{bmatrix} -\gamma_1 & w_{1,2} & 0 & \dots \\ w_{2,1} & -\gamma_2 & w_{2,3} & \dots \\ 0 & w_{3,2} & -\gamma_3 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}, \qquad \sum_n w_{nm} = 0$$

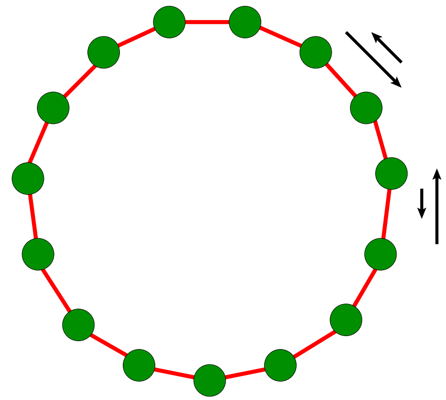
transition rates across n^{th} bond $w_n \mathrm{e}^{\pm \mathcal{E}_n/2}$

Stochastic field $\mathcal{E}_n \sim [s-\sigma, s+\sigma]$
Affinity / Bias $s = \frac{1}{N}\sum \mathcal{E}_n \equiv \frac{\mathcal{S}_{\circlearrowleft}}{N}$
Resistor network $P(w) \propto w^{\alpha-1}$

How do spectral properties of $\boldsymbol{W}$ depend on (α, σ, s)?

Sinai spreading

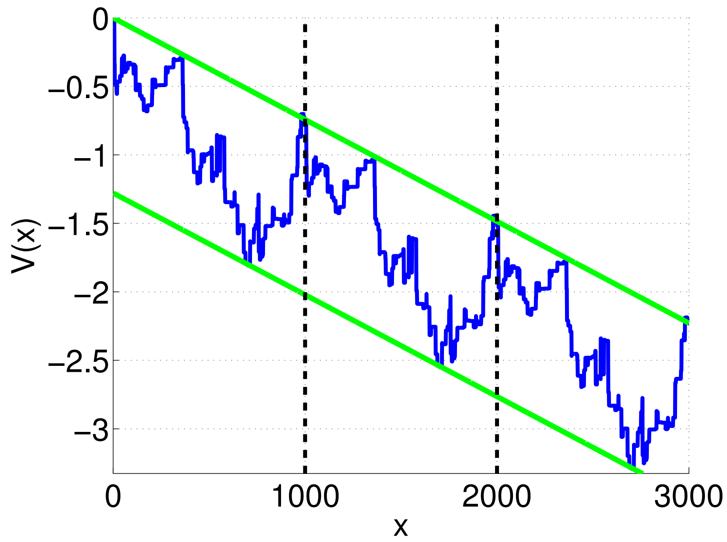
Stochastic field: $\mathcal{E}_n \equiv \ln\left[\frac{w_{n+1,n}}{w_{n,n+1}}\right]$, $\sigma = \sqrt{\mathrm{Var}(\mathcal{S}_n)}$



Stochastic Motive Force: $\mathcal{S}_{\circlearrowleft} = \sum_{n\in\mathrm{ring}} \ln\left[\frac{\overrightarrow{w}_n}{\overleftarrow{w}_n}\right]$

If $\frac{\overrightarrow{w}_n}{\overleftarrow{w}_n} = \exp\left[-\frac{E_n - E_{n-1}}{T}\right] \rightsquigarrow \mathcal{S}_{\circlearrowleft} = 0$

Affinity : $s = \frac{\mathcal{S}_{\circlearrowleft}}{N}$



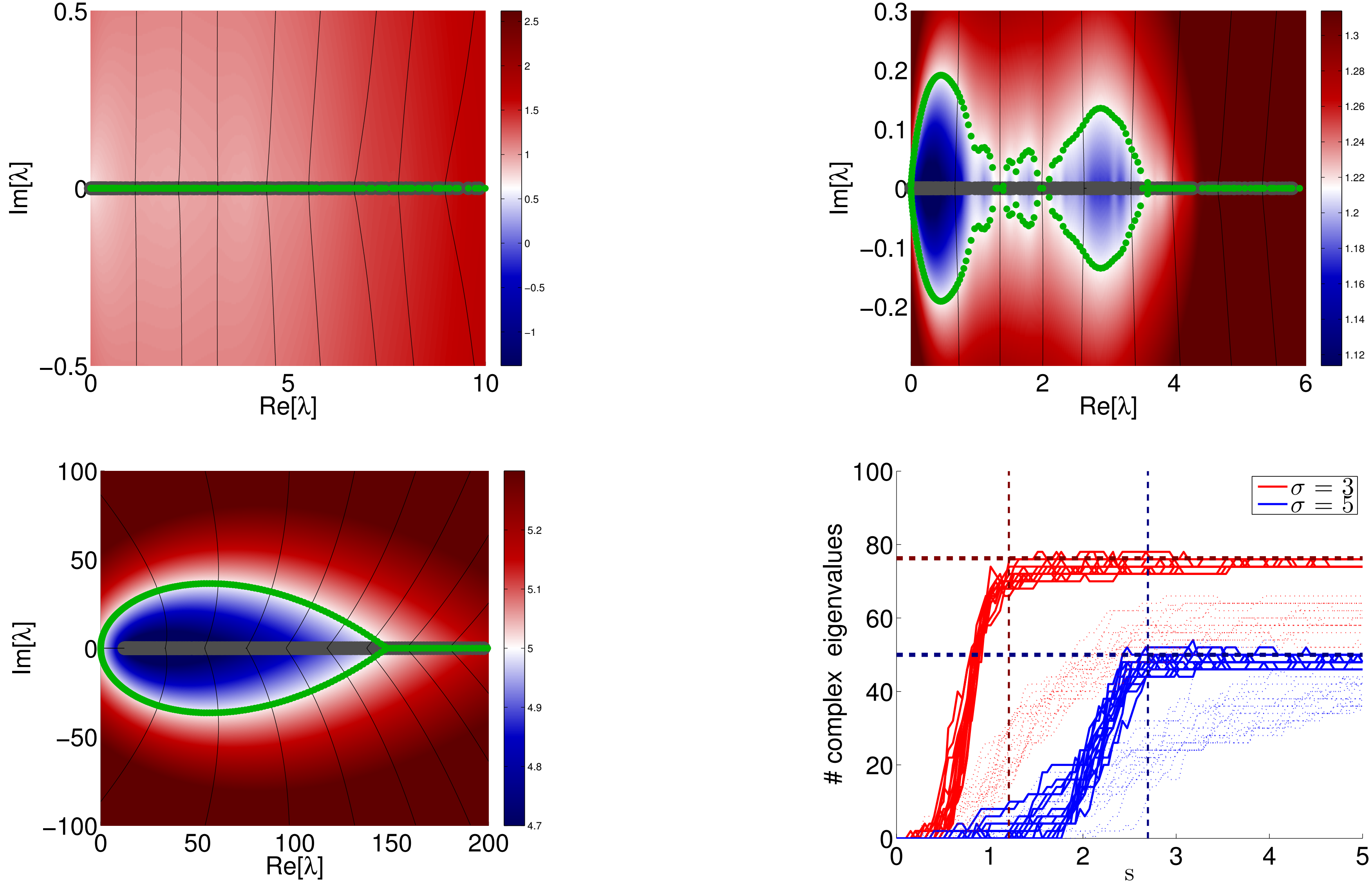
For small s [1]:
Sub-diffusive spreading $x \sim [\log(t)]^2$,
Exponentially small drift $v \sim e^{-\sqrt{N}}$.

For arbitrary s [2,3]:
Complicated expressions for v and D.
sliding transition

[1] **Sinai** (1982)
[2] **Derrida** (1983)
[3] **Aslangul, Pottier, Saint-James** (1989)

ESR is violated for large s

The spectrum



What is the threshold bias s_c for complex eigenvalues (delocalization)?

How is s_c related to the percolation transition? to the sliding transition?

The spectral equation

Gauge away disorder $\tilde{\boldsymbol{W}} = \mathrm{e}^{\boldsymbol{U}/2} \boldsymbol{W} \mathrm{e}^{-\boldsymbol{U}/2}$

Associated hermitian matrix $\boldsymbol{H}$ with real eigenvalues $\epsilon_k(s)$ by setting $\mathcal{S}_{\circlearrowleft} = 0$

Spectral determinant for complex eigenvalues z

$$\prod_{k=1}^{N} \left(\frac{z + \epsilon_k(s)}{\overline{w}} \right) \quad = \quad 2 \left[\cosh\left(\frac{S_{\circlearrowleft}}{2} \right) - 1 \right]$$

Electrostatic pitcture

$$\Psi(z) \quad = \quad \sum_k \ln\left(z - \epsilon_k\right) \quad \equiv \quad V(x,y) + iA(x,y)$$