Hursh Desai

Production Planning

The ABC Co. manufactures heavy duty air compressors for the home and light industrial markets. ABC is presently trying to plan its production and inventory levels for the next six months. Because of seasonal fluctuations in utility and raw material costs, the per unit cost of producing air compressors varies from month to month due to differences in the number of working days, vacations, and schedule maintenance and training. The following table summarizes the monthly production costs, demands, and production capacity that ABC's management expects to face over the next six months.

	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
Units Production Cost	$240	$250	$265	$285	$280	$260
Units Demanded	1000	4500	6000	5500	3500	4000
Maximum Production	4000	3500	4000	4500	4000	3500

Given the size of ABC's warehouse, a maximum of 6,000 units can be held in inventory at the end of any month. The owner of the company likes to keep at least 1,500 units in inventory as safety stock to meet unexpected demand contingencies. To maintain a stable workforce, the company wants to produce no less than one-half of its maximum production capacity each month. ABC's controller estimates that the cost of carrying a unit in any given month is approximately equal to 1.5% of the unit production cost in the same month. ABC estimates the number of units carried in inventory each month by averaging the beginning and ending inventory for each month. There are 2,750 units currently in inventory. ABC wants to identify the production and inventory plan for the next six months that will meet the expected demand each month (on time) while minimizing production and inventory cost.

Conclusion and Recommendation

ABC Co. can minimize the total cost by producing these many heavy-duty air compressors per month:

Month	Production
1	4000
2	3500
3	4000
4	4250
5	4000
6	3500

Managerial Problem Definition

Decisions to be made – How much to produce each month.

Objective – Minimize Total Cost

Restrictions – Production per month has to be greater than one-half of maximum production and less than maximum production. Ending Inventory for the month has to be greater than 1,500 and less than 6,000.

Model Formulation

Model 1 (Ending Inventory as Intermediate Variable)

Decision Variables:

X_1 : amount of production in Month 1

X_2: amount of production in Month 2

X_3: amount of production in Month 3

X_4: amount of production in Month 4

X_5: amount of production in Month 5

X_6: amount of production in Month 6

Objective Function:

Minimize $(240X_1 + 250X_2 + 265X_3 + 285X_4 + 280X_5 + 260X_6)$

Constraints:

$X_1 >= 2000$

$X_2 >= 1750$

$X_3 >= 2000$

$X_4 >= 2250$

$X_5 >= 2000$

$X_6 >= 1750$

$1500 >= I_1 >= 6000$

$1500 >= I_2 >= 6000$

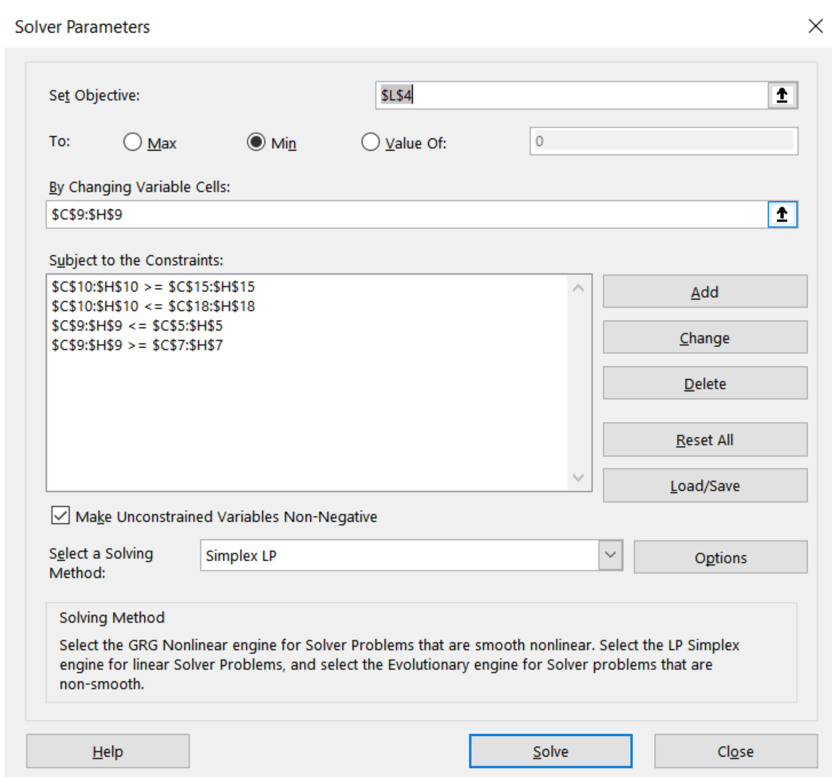
$1500 >= I_3 >= 6000$

$1500 >= I_4 >= 6000$

$1500 >= I_5 >= 6000$

$1500 >= I_6 >= 6000$

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	**Production schedule**												=SUMPRODUCT(C9:H9,C3:H3)
2	Month	0	1	2	3	4	5	6			Production Cost	$6,136,250.00	=SUMPRODUCT(C12:H12,C3:H3*0.015)
3	Unit Production Cost		240	250	265	285	280	260			Holding cost	$73,153.13	=SUM(L1:L3)
4	Demand		1000	4500	6000	5500	3500	4000			Total Cost	$6,209,403.13	
5	Maximum Production		4000	3500	4000	4500	4000	3500					
6									=0.5*H5				
7	Minimum Production		2000	1750	2000	2250	2000	1750					
8			>=	>=	>=	>=	>=	>=					
9	Production		4000	3500	4000	4250	4000	3500	=G10+H9-H4				
10	Ending inventory	2750	5750	4750	2750	1500	2000	1500					
11									=AVERAGE(G10,H10)				
12	Avg. Ending Inventory		4250	5250	3750	2125	1750	1750					
13													
14			>=	>=	>=	>=	>=	>=					
15			1500	1500	1500	1500	1500	1500					
16													
17			<=	<=	<=	<=	<=	<=					
18			6000	6000	6000	6000	6000	6000					



Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C9	Production >=	4000	-33.3375	261.9	33.3375	1E+30
D9	Production >=	3500	-27.0125	268.225	27.0125	1E+30
E9	Production >=	4000	-15.875	279.3625	15.875	1E+30
F9	Production >=	4250	0	295.2375	1E+30	9.2375
G9	Production >=	4000	-9.2375	286	9.2375	1E+30
H9	Production >=	3500	-33.2875	261.95	33.2875	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C10	Ending inventory >=	5750	0	1500	4250	1E+30
D10	Ending inventory >=	4750	0	1500	3250	1E+30
E10	Ending inventory >=	2750	0	1500	1250	1E+30
F10	Ending inventory >=	1500	0	1500	0	1E+30
G10	Ending inventory >=	2000	0	1500	500	1E+30
H10	Ending inventory >=	1500	295.2375	1500	250	0
C10	Ending inventory >=	5750	0	6000	1E+30	250
D10	Ending inventory >=	4750	0	6000	1E+30	1250
E10	Ending inventory >=	2750	0	6000	1E+30	3250
F10	Ending inventory >=	1500	0	6000	1E+30	4500
G10	Ending inventory >=	2000	0	6000	1E+30	4000
H10	Ending inventory >=	1500	0	6000	1E+30	4500

Model 2 (Ending Inventory as Decision Variable)

Decision Variables:

X_1 : amount of production in Month 1

X_2: amount of production in Month 2

X_3: amount of production in Month 3

X_4: amount of production in Month 4

X_5: amount of production in Month 5

X_6: amount of production in Month 6

I_1: amount of ending inventory after month 1

I_2: amount of ending inventory after month 2

I_3: amount of ending inventory after month 3

I_4: amount of ending inventory after month 4

I_5: amount of ending inventory after month 5

I_6: amount of ending inventory after month 6

<u>Objective Function:</u>

Minimize: $(X_1*240 + X_2*250 + X_3*265 + X_4*285 + X_5*280 + X_6*260) + [3.6*((2750+I_1)/2) + 3.75*((I_1 + I_2)/2) + 3.98*((I_2 + I_3)/2) + 4.28*((I_3 + I_4)/2) + 4.20*((I_4 + I_5)/2) + 3.9*((I_5 + I_6)/2)]$

<u>Constraints:</u>

$X_1 >= 2000$

$X_2 >= 1750$

$X_3 >= 2000$

$X_4 >= 2250$

$X_5 >= 2000$

$X_6 >= 1750$

$1500 >= I_1 >= 6000$

$1500 >= I_2 >= 6000$

$1500 >= I_3 >= 6000$

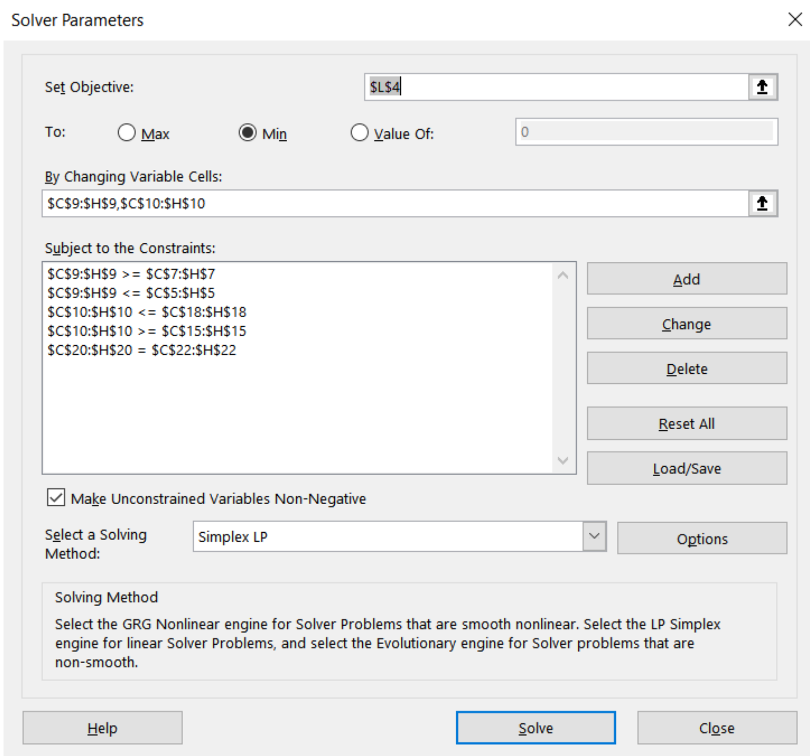
$1500 >= I_4 >= 6000$

$1500 >= I_5 >= 6000$

$1500 >= I_6 >= 6000$

$I_{i-1} + P_i - I_i = D_i$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	**Production schedule**												=SUMPRODUCT(C9:H9,C3:H3)			
2	Month	0	1	2	3	4	5	6			Production Cost	$6,136,250.00	=SUMPRODUCT(C12:H12,C3:H3*0.015)			
3	Unit Production Cost		240	250	265	285	280	260			Holding cost	$73,153.13	=SUM(L1:L3)			
4	Demand		1000	4500	6000	5500	3500	4000			Total Cost	$6,209,403.13				
5	Maximum Production		4000	3500	4000	4500	4000	3500								
6									=0.5*H5							
7	Minimum Production		2000	1750	2000	2250	2000	1750								
8			>=	>=	>=	>=	>=	>=								
9	Production		4000	3500	4000	4250	4000	3500								
10	Ending inventory	2750	5750	4750	2750	1500	2000	1500								
11									=AVERAGE(H10,G10)							
12	Avg Ending Inventory		4250	5250	3750	2125	1750	1750								
13																
14			>=	>=	>=	>=	>=	>=								
15			1500	1500	1500	1500	1500	1500								
16																
17			<=	<=	<=	<=	<=	<=								
18			6000	6000	6000	6000	6000	6000								
19									=G10+H9-H10							
20	Demand		1000	4500	6000	5500	3500	4000								
21			=	=	=	=	=	=								
22			1000	4500	6000	5500	3500	4000								



Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C9	Production >=	4000	-33.3375	240	33.3375	1E+30
D9	Production >=	3500	-27.0125	250	27.0125	1E+30
E9	Production >=	4000	-15.875	265	15.875	1E+30
F9	Production >=	4250	0	285	1E+30	9.2375
G9	Production >=	4000	-9.2375	280	9.2375	1E+30
H9	Production >=	3500	-33.2875	260	33.2875	1E+30
C10	Ending inventory >=	5750	0	3.675	33.3375	1E+30
D10	Ending inventory >=	4750	0	3.8625	27.0125	1E+30
E10	Ending inventory >=	2750	0	4.125	15.875	1E+30
F10	Ending inventory >=	1500	0	4.2375	1E+30	9.2375
G10	Ending inventory >=	2000	0	4.05	1E+30	33.2875
H10	Ending inventory >=	1500	295.2375	1.95	1E+30	295.2375

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C20	Production to meet Demand <=	1000	273.3375	1000	250	250
D20	Production to meet Demand <=	4500	277.0125	4500	250	1250
E20	Production to meet Demand <=	6000	280.875	6000	250	2000
F20	Production to meet Demand <=	5500	285	5500	250	2000
G20	Production to meet Demand <=	3500	289.2375	3500	250	0
H20	Production to meet Demand <=	4000	293.2875	4000	250	0

Question 7

Part A/B:

The optimal solution of both models are degenerate because at least one of the constraints' RHS has an allowable increase or decrease of 0. And, because they are both degenerate that means that it is also an unique solution.

Part C:

The constraint on the production quantity of month 1 is 4,000 and because we produce 4,000 in the month 1 as part of the optimal solution, that means that constraint is binding. Since these bounds are binding at optimality that means the reduced cost of -33.3375 tells us that if the bounds on this decision variable is relaxed by 1 that it will decrease the total cost by 33.3375.

Part D:

The reduced cost of the inventory variable is 0 because the bounds are not binding in Month 1. This means that if you change the bounds for this variable the total cost will not change.

Part E:

The reduced cost of the inventory variable in Month 6 is 295.23749. The bounds on the variable are 1,500 and because the ending inventory in that month is also 1,500 that means these bounds are binding. This means that if the bounds for this variable are relaxed by 1 then the total cost will increase by 295.23749.

Part F:

Because the shadow price of the demand in the second model is 273.3375 that means if the demand increases by 1 the optimal cost will increase by 273.3375. So when demand increases by 100 the optimal cost increases by $27,333.75. However, the allowable increase is only 250 so based solely off of the sensitivity report we do not know what happens to the optimal cost.

Part G:

In model 1 the shadow price of the safety stock constraint in month 1 is zero. And in model 2 the reduced cost of the ending inventory variable for month 1 is also zero.