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Mappings Among Number Words, Numerals, and Non-Symbolic Quantities in Preschoolers

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Abstract

In mathematically literate societies, numerical information is represented in three distinct codes: a verbal code (i.e., number words); a digital, symbolic code (e.g., Arabic numerals); and an analogical code (i.e., quantities, Dehaene, 1992). In order to communicate effectively using these numerical codes, our understanding of number must involve an understanding of each representation as well as how they map to other representations. In the current study, we looked at 3-and 4-year-old children's understanding of Arabic numerals in relation to both quantities and number words. Results suggest that the mapping between quantities and numerals is more difficult than the mapping between numerals and number words and between number words and quantities. Thus, we compare two competing models designed to investigate how children represent the meanings of Arabic numbers - whether numerals are mapped directly to the quantities they represent or instead if numerals are mapped to quantities indirectly via a direct mapping to number words. We find support for the latter suggesting that children may first map numerals to number words (another symbolic representation) and only through this mapping are numerals subsequently tied to the quantities they represent. In addition, unlike both mappings involving quantity, the mapping between the two symbolic representations of number (numerals and number words) was not set-size dependent providing further evidence that children may map symbols to other symbols in the absence of a quantity referent. Together results provide new insight into the important processes involved in how children acquire an understanding of symbolic representations of number.

Keywords: *Number word; Arabic numeral; non-symbolic number; symbolic number*

Mapping Among Number Words, Numerals, and Non-Symbolic Quantities in Preschoolers

Our society relies upon the ability to communicate numerical information on a regular basis. While paying for groceries, we see that the written number of items shown on the bill is equal to the number of physical items we placed in the bag, the total amount spoken by the cashier is equal to the total number printed on the bottom of the receipt, and so on. In this simple example, numerical information is interpreted in non-symbolic form (the quantity of items), a written symbolic form (the numerals printed on the receipt), and a verbal form (the number words spoken by the cashier; Dehaene, 1992). A complete understanding of number involves weaving together these representations in a meaningful way, a particularly difficult challenge for children who must come to understand (1) that the number words we hear and speak represent the physical quantities we see in the world (from here on, referred to as quantity-word mappings), (2) that symbolic written Arabic numerals map onto spoken number words (word-numeral mappings), and (3) that these Arabic numerals also represent physical quantities (quantity-numeral mappings).

Although infants, children, adults, and even non-human animals can track numerical information in their environment (as evidenced by their ability to compare and manipulate sets of items, e.g., an array of dots; Barth, La Mont, Lipton, & Spelke, 2005; Brannon, 2002; Cantlon & Brannon, 2006; Cordes & Brannon, 2008; McCrink, Dehaene, & Dehaene-Lambertz, 2007; McCrink & Wynn, 2004, 2007, 2009; Xu & Spelke, 2000; see Anderson & Cordes, 2013; Gallistel, 1990 for review), the use of a sophisticated symbolic system for representing number is a human-specific cultural invention that must be learned. Thus, one of the central questions in numerical cognition research is how children learn to map symbolic representations of number (i.e., verbal number words and written Arabic numerals) with non-symbolic representations of number (i.e., quantity)[1]. Although substantial research has investigated how children learn quantity-word mappings (e.g., Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Odic, Le Corre, & Halberda, 2015), substantially less work has investigated children's learning of Arabic numerals.

Symbolic Arabic Numerals

Given the ubiquitous nature of written communication, children must come to learn the meaning of printed communication in many different contexts, including Arabic numerals for numbers, letters for written language, and other specialized symbolic systems (e.g., maps). Researchers have investigated how young preschool-aged children spontaneously represent quantity in written form, even before learning the conventional number system (i.e., Arabic numerals). These studies suggest that children gradually move from using idiosyncratic non-representational written forms (e.g., scribbles) to using more iconic representations (e.g., five tally marks representing "5") and finally to using the conventional numeral system (e.g., Hughes, 1986; Sinclair, Siegrist, & Sinclair, 1983). Despite children's early abilities to represent written symbols for quantity, truly understanding the symbolic nature of print is not a straightforward task and young children show fundamental misunderstandings about the nature of written symbols (Bialystok, 2000; Bialystok & Martin, 2003). For example, in the Moving Word paradigm, an experimenter places a card with a printed word in front of an object then labels the object and reads the word on the card (e.g., the child sees a toy dog and a place card that says "dog", and the experimenter would say "this says dog"). However, when the card is moved to a different object, children often respond that the word on the card now indicates the name of the

[1] Although there is substantial debate about whether the symbolic representations are mapped to the approximate number system (ANS) or the object file system and how this mapping occurs (e.g., Gelman & Gallistel, 1978; Le Corre & Carey, 2007), we will remain agnostic about these claims and simply refer to "quantity" which may be represented via either/both of these systems.

new object, despite no change to the written symbols on the card (e.g., the child expects that same card to say “car” when placed in front of a car; Bialystok & Martin, 2003). This lack of understanding of the stability of symbolic representations has been shown for both the meaning of written words and the meaning of Arabic numerals (i.e., quantity-numeral mappings), suggesting that early on, children’s understandings of printed symbols may be particularly fragile. Eventually, however, children are able to map between Arabic numerals and spatial representations of quantity (e.g., number lines Siegler & Opfer, 2003; Barth & Paladino, 2011) and between discrete quantity (e.g., arrays of dots) and paired (i.e., simultaneously presented) Arabic numerals and number words (Mundy & Gilmore, 2009). Yet little is known about when and how children acquire an understanding of Arabic numerals, and how word-numeral mappings may be related to children’s quantity-numeral understandings.

Given the typical progression of verbal versus written language, children are introduced to spoken number words before they are introduced to written numerals. Furthermore, research looking directly at quantity-word and quantity-numeral mappings suggests that quantity-word mappings develop earlier than mappings involving numerals (Benoit et al., 2013), giving rise to the possibility of distinct pathways for the acquisition of these mappings. That is, since children typically acquire quantity-word mappings prior to learning Arabic numerals, it is possible that their newfound understanding of number words may play a role in the acquisition of Arabic numerals. In the current study, we compare two potential mechanisms for learning the quantity-numeral mapping. On the one hand, given that the purpose of both Arabic numerals and number words is to represent quantity, it may be that children learn to map these representations in a similar way: children may learn the mapping between numerals and the quantities they represent directly, just as they had previously learned how number words represent quantities. That is, quantity-numeral mappings may be acquired independently of word-numeral mappings (we will refer to this as the Quantity account). On the other hand, given that symbolic representations are more precise and children learn Arabic numerals after having learned verbal number words, children may rely upon word-numeral mappings to indirectly form quantity-numeral mappings. That is, children may use their understanding of number words to bridge the gap in understanding how Arabic numerals represent non-symbolic quantities (we will refer to this as the Symbolic account).

The Quantity Account: Number words are the first symbolic representations of number that children acquire[2] (e.g., Le Corre & Carey, 2007), suggesting that number words must be mapped directly to quantity. As such, children may acquire Arabic numerals in the same way, by forming a direct mapping between numerals and the quantities they represent. For example, children may learn that the word "*four*" represents the set of four items ({0 0 0 0}), and independently, learn that the numeral "4" represents the set of four items. Only after acquiring these two symbolic representations, would children understand how they are mapped together, learning that the Arabic numeral "4" represents the same quantity as the word "*four*".

Evidence from comparison tasks with both adults and children provide some support for this view. When adults and children are given two Arabic numerals and asked to decide which is largest, their responses depend upon the distance between and the size of the quantities that the numerals represent (termed the distance effect or ratio effect; Moyer & Landauer, 1967, 1973), suggesting a (direct) mapping between numerals and quantity. Moreover, other work has found that preschoolers perform better on tasks assessing quantity-numeral mappings than on those assessing word-numeral mappings, suggesting that translating between the two symbolic

[2] This acquisition may occur via verbal counting, however, that is not the focus of this study.

representations may be more difficult than that between symbolic and non-symbolic representations (Benoit et al., 2013). Together, these studies support the Quantity Account of Arabic numeral acquisition, positing that children acquire a direct mapping between quantities and Arabic numerals, independent of their understanding of the number words used to represent the printed symbols.

If it is the case that children learn number words and Arabic numerals by independently mapping each symbol to quantity, then we would expect any relation between children's performance on tasks tapping word-numeral mappings and tasks tapping quantity-numeral mappings to be mediated by their performance on tests of quantity-word mappings. In other words, children's understanding of how number words represent numerals should only predict their understanding of how numerals represent quantities to the extent that they also understand how number words represent quantities. In this way, quantities form the basis for understanding both symbolic representations.

In addition, if children's understanding of quantity-numeral mappings and their understanding of quantity-word mappings both hinge upon independent mappings to quantity then children should show similar developmental progressions for acquiring both symbolic systems. Prior literature characterizing how children map number words to quantities can generate specific hypotheses for how children may later perform on quantity-numeral mappings. In particular, despite extensive experience with numerical information and even the ability to recite the count list up to five or ten by the age of 3 (Baroody & Price, 1983; Miller, Smith, Zhu, & Zhang, 1995; Sarnecka & Lee, 2009), many studies find that the process of learning that number words in their count list represent specific quantities is a laborious one (Palmer & Baroody, 2011; Condry & Spelke, 2008; Mix, 2009; Opfer, Thompson, & Furlong, 2010; Wynn, 1992). For example, children typically first show they understand that the word “one” refers to a specific quantity sometime between the ages of two and three years, yet it can take an additional 1-1.5 years for them to show the same understanding for the number words “two”, “three”, and “four” (Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Carey, 2008; Sarnecka & Lee, 2009; Wagner & Johnson, 2011; Wynn, 1990, 1992). Furthermore it is often up to two years after children begin talking about number that they demonstrate an understanding of the Cardinal Principle: that a set’s cardinality is indexed by the last number word that they speak in a counting list (Frye, Braisby, Lowe, Maroudas, & Jon, 1989; Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Schaeffer, Eggleston, & Scott, 1974; Wynn, 1990). Together, these results suggest that quantity-word mappings are acquired in a set-size dependent manner, such that children learn the number words for smaller quantities first and in order. If, in parallel, children learn quantity-numeral mappings directly, then we would expect that children’s performance on all three mappings (quantity-numeral, quantity-word, and word-numeral) would also be set-size dependent. That is, if Arabic numerals are mapped directly to quantity then children should learn to map Arabic numerals to small quantities before learning to map Arabic numerals to larger quantities, as has been shown for children’s quantity-word mappings. Furthermore, since both Arabic numerals and number words are mapped directly to quantity then the secondary word-numeral mapping should be set-size dependent, such that children learn the words for Arabic numerals representing smaller set-sizes before learning the words for Arabic numerals representing larger set sizes.

In sum, if children learn the quantity-numeral mappings independent of the acquisition of quantity-word mappings, in line with the Quantity Account, then (1) quantity-word mappings should mediate the relation between children’s performance on tasks tapping quantity-numeral understandings and those tapping word-numeral mappings; and (2) both quantity-numeral and word-numeral mappings should be acquired in a set-size dependent manner.

The Symbolic Account: Alternatively, children may instead acquire an understanding of Arabic numerals via their knowledge of the verbal number words. That is, children may first learn the names for the Arabic numerals and then deduce that these Arabic numerals must also represent the quantities that the number words represent. More concretely, children may learn that "*four*" represents a set of four items and that "*four*" also represents the symbol "4", then only through transitivity do they surmise that the symbol "4" also represents the set of four things.

Indirect evidence in favor of this account is drawn from work with the Moving Word paradigm (Bialystok, 2000). As noted above, preschoolers typically expect a written symbol (i.e., Arabic numeral) to represent the size of the set it is placed in front of, even when the symbol is moved from one set to another (that is, they do not understand that each Arabic numeral represents a unique set size). Importantly, however, preschoolers do not make this error when they know the name of the Arabic numeral (i.e., they know the number word that numeral represents; Bialystok, 2000) suggesting that having a word-numeral mapping may allow children to better understand the stability of quantity-numeral mappings. As such, these findings suggest that children's word-numeral mappings may provide a critical foundation for children's understandings of how Arabic numerals represent quantity, in line with the Symbolic Account.

Moreover, work with adults suggests that although symbolic representations are at least implicitly associated with the underlying quantity they represent (as evidenced by ratio effects; Moyer & Landauer, 1967, 1973), Arabic numerals may actually be more directly connected to number words than to non-symbolic quantities (e.g., an array of dots; Lyons, Ansari, & Beilock, 2012). Lyons et al. (2012) showed that when adults were asked to compare a symbol to a dot array (non-symbolic quantity) their performance was lower than when comparisons involved two symbols or two quantities. This was true even when the two symbolic forms being compared were different formats (i.e., a number word and an Arabic numeral). Thus, at least for adults, quantity-numeral mappings may be particularly difficult, more so than word-numeral mappings, which involve two symbolic representations. However, it is unclear if the cost of quantity-numeral mappings demonstrated in adults is related to the way children initially learn to integrate Arabic numerals into their existing number knowledge (in which case, the difficulty should be evident early in development) or alternatively, if this pattern of difficulty seen in adults is not indicative of the acquisition process, but instead arises after extensive experience with Arabic numerals (in which case, it should be absent early in development). In particular, the Symbolic Account of numeral understanding would suggest that adults' relative difficulty with quantity-numeral mappings (Lyons et al., 2012) does come from the way numerals are initially integrated into children's number knowledge and not a product of extensive experience.

If it is the case that children learn the quantity-numeral mappings via their knowledge of word-numeral mappings, then the relation between how well children perform on a quantity-word mapping task and their performance on a quantity-numeral mapping task should be mediated by their ability to perform a word-numeral mapping task.

Moreover, the symbolic account also makes unique predictions about whether these mappings are magnitude-dependent. In contrast to the Quantity account, if number words and Arabic numerals are directly mapped to each other, then performance on symbolic-symbolic mapping tasks (word-numeral tasks) should not necessarily be quantity-dependent; that is, performance can be independent of the set sizes described by the numerals and number words involved. If this were the case, then although mappings involving quantity may be learned in a magnitude-dependent order (i.e., smaller sets are easier than larger sets; Condry & Spelke, 2008; Wynn, 1990), mappings involving only symbols may not necessarily show this same magnitude-dependent performance. Instead, the order in which children acquire these word-numeral mappings may be more dependent upon caregiver input and may be shaped by arbitrary experiences such as their age on their birthday, numbers in their address, phone numbers, or sports jerseys. Number talk in the home appears to play a role in children's early quantity-word mapping abilities (e.g., Gunderson & Levine, 2011), making it likely the case that environmental input, including naming numerals (though infrequent; Susperreguy & Davis-Kean, 2015), may also impact children's word-numeral mapping as well.

The current study aims to compare these two contrasting hypotheses (the Quantity account and the Symbolic account) about how preschoolers learn to map Arabic numerals to other symbols (verbal number words) and to quantities (non-symbolic dot arrays) in order to speak to how children may come to learn the meanings of Arabic numerals.

Directional Mappings

In addition to understanding how two distinct symbolic representations (number words and Arabic numerals) represent non-symbolic quantity, sophisticated numerical reasoning must also involve bidirectional mappings between each of these representations. For example, children must understand both that the written number "4" means the quantity of four items and that the quantity of four items can be represented using the written number "4". Some evidence suggests that children acquire the quantity-word mapping in a bidirectional manner, such that when children learn one direction (e.g., "*four*" refers to four items) the other direction immediately follows (e.g., four items can be referred to as "*four*"; Benoit et al., 2013; Fuson, 1988; Gelman &

Gallistel, 1978; Le Corre et al., 2006; Wynn, 1990, 1992). On the other hand, when the direct mappings between symbolic (paired Arabic numerals and number words) and non-symbolic (arrays of dots) number were investigated with approximate quantities (i.e., involving large values and/or in cases where children were unable to verbally count the items), children performed better on tasks requiring the conversion of a non-symbolic representation to a symbolic representation (quantity-to-word or quantity-to-numeral), compared to the reverse (e.g., Mundy & Gilmore, 2009; Odic et al., 2015). In light of these conflicting results across studies, it is unclear whether performance differences will also be seen across tasks assessing distinct directions of the mappings between number words, Arabic numerals, and quantities.

Furthermore, given that the current study used an exact, small number matching task in preschoolers, we were able to look at children's use of counting across different symbolic representations (Arabic numerals and number words) and across distinct directions (from a given quantity versus to a specific quantity). When children do engage in spontaneous counting during numerical tasks, their performance tends to improve relative to children who do not spontaneously use counting (e.g., Bar-David, Compton, Drennan, Finder, Grogan, & Leonard, 2009; Posid & Cordes, 2014). Yet, not all contexts are equally likely to elicit counting (Gelman & Tucker, 1975; Goldstein, Cole, & Cordes, 2016; Mix, Sandhofer, Moore, & Russell, 2012; Wylie, Jordan, & Mulhern, 2012). For example, when reading to their children, parents do not count all sets equally, but instead seem to be less likely to count smaller sets than larger sets (Goldstein et al., 2016; Mix et al., 2012). However, less is known about the contexts that give rise to spontaneous counting in young children and in particular whether different representations (words and numerals) are equally likely to evoke counting behavior.

The Current Study

In the current study, three- and four-year-old children were presented with six tasks assessing the mappings between verbal number words, written Arabic numerals, and non-symbolic dot displays for the quantities 1-5. To assess bidirectionality, tasks assessed both directions of each mapping (e.g., the mapping from quantities-to-numerals and the mapping from numerals-to-quantities). By investigating mappings involving both types of symbolic representations and each mapping direction, we address two specific questions: (1) Are Arabic numerals mapped directly to non-symbolic representations of quantity or are Arabic numerals more directly mapped to symbolic verbal number words? (2) Are the mappings between Arabic numerals, number words, and quantities symmetric and bidirectional in preschoolers?

Method

Participants

Participants included 24 three-year-olds (M = 3;0, Range = 2;4 to 3;6, 12 males) and 24 four-year-olds (M = 4;1, Range = 3;7 to 4;6, 15 males) with complete datasets. An additional ten 3-year-olds and two 4-year-olds participated in the study but were excluded because they did not complete the majority of the tasks (10) or because of experimenter error (2). Participants were recruited from the Boston, MA area, including from local preschools and museums. Signed consent was obtained from the parents of participating children and children received a small gift for participating.

Design

In order to investigate how children map between different representations of number, researchers have used many different production, estimation, and choice tasks (e.g.,Fuson, 1988; Gelman & Gallistel, 1978; LeCorre & Carey, 2007; Mix, 2008; Schaeffer et al., 1974; Wynn, 1990, 1992). In the current study, we adapted these classic tasks in order to make the non-numerical aspects of each task as consistent as possible across distinct mappings. One major adaption was to provide children with the same five options (magnitudes of 1-5) in all tasks, which allowed us to equate the tasks in several important ways. First, we did not want the questions to be open ended (i.e., no options) such that children could respond in a different form or modality than we anticipated (e.g., writing down two tick marks for “two” rather than the symbol 2; Bialystok & Codd, 1996; Hughes, 1983). Furthermore, we wanted to equate all tasks as best as possible by providing five of the same options each time. By offering five choices, we were able to make all tasks identical in that chance level was 20%, allowing for a direct comparison of performance across tasks. Although other work suggests that some of these larger sets may be very difficult for these young children who have likely not yet acquired the Cardinal Principle (e.g., LeCorre & Carey, 2007), we were interested in seeing differences in performance within a small range that was still likely to generate variability. Thus, based on prior work suggesting that children are actively learning the magnitudes of 1-5 around this age (approximately 2.5-4.5; Le Corre et al., 2006; Sarnecka & Carey, 2008; Wynn, 1990, 1992) and an attempt to match all tasks as best as possible, we decided to provide all five options on each trial. In the Stimuli and Materials section, we will outline the specific stimuli used for each type of task. In the Procedure section, we will outline the specific procedure for each task as a whole (see Figure 1 for an example of each task).

[Insert Figure 1 Approximately Here]

Stimuli and Materials

In each mapping task, the child was presented with a single target item presented either visually or verbally (depending on the trial type) by the experimenter and a separate set of five items presented as options to choose from (either visually or visually). Thus, in each case, the child was asked to map *from* a single item (numeral or dot array presented on a small, white, laminated target card (9.0 cm x 12.3 cm) or a single number word) *to* one of the five options (five numerals or five dot arrays on a large, white, laminated choice card (20.32 cm x 27.94 cm) or five number words spoken aloud). The large choice card contained five options, presented horizontally with a thick black rectangular border and 0.5 cm of white space between each image.

Matching Practice: The matching practice involved images from basic categories, including common animals and objects. Three different choice cards were created, each containing five images - one image from each of five basic categories of common animals and objects (e.g., shoes). The small target card contained an image from one of the five categories (e.g., a bulldog was to be matched to the beagle on the larger card). There were three different cards, each with a different exemplar from a different category. Each image covered approximately the same area (M = 8.28 cm^2, SD = 2.29).

Numeral Target Stimuli: The Numeral-to-Quantity and Numeral-to-Word tasks used single target cards containing a single numeral. Five different single target cards were created, one for each numeral (from 1 to 5). Numerals were approximately the same height (M = 4.40 cm, SD = 0.19) and width (M = 2.74 cm, SD = 0.33). Numerals used the same black font on all cards.

Numeral Choice Stimuli: In the Word-to-Numeral and Quantity-to-Numeral tasks, the experimenter presented the child with a large choice card containing five numerals (the numbers 1 through 5) presented in a random order. Five different choice cards (each containing the five numerals) were created, each displaying a unique random order of the numerals one through five. Each choice card used a single color and font for all numerals, but the five cards each used different fonts and colors (red, orange, yellow, green, or blue). The height (M = 2.56 cm, SD = 0.19) and width (M = 1.94 cm, SD = 0.35) of numerals was similar between cards.

Quantity Target Stimuli: The Quantity-to-Word and the Quantity-to-Numeral tasks presented children a target card containing a single array of dots. Five different target cards were created, one for each dot magnitude (from 1 to 5 dots). The location of dots on a card was randomly determined for each card. Dots were colored in black and were heterogeneously sized on a card. Individual dots on a card varied between five diameter sizes (0.6cm, 1.2cm, 1.9cm, 2.6cm, and 3.2cm). The cumulative area of dots on the cards was 4.3 cm^2, 6.0 cm^2, 5.8 cm^2, 10.7 cm^2, and 9.7 cm^2 for the 1 dot-, 2 dot-, 3 dot-, 4 dot-, and 5 dot-cards respectively.

Quantity Choice Stimuli: During the Word-to-Quantity and Numeral-to-Quantity tasks the experimenter presented the child with a large choice card containing five dot arrays, each representing a distinct set size of one to five dots, presented in random order. There were five choice cards, each presenting the dot arrays in a unique random order and unique configurations. Each choice card used a single color for all dot arrays, but the five choice cards each used different colors (red, orange, yellow, green, or blue). Dots were randomly placed within each array. Dots were heterogeneously sized within each array and varied between five diameter sizes (0.6cm, 1.2cm, 1.9cm, 2.6cm, and 3.2cm). For each card, cumulative area was randomly varied such that it was not a reliable cue to determine the numerical sizes of the arrays.

Procedure

Children were first oriented to the matching game via two practice trials involving non-numerical stimuli. Children were asked to find the picture in the choice set that matched the target picture.

Following practice, children participated in six tasks (each containing five unique trials) in a set order such that the major task of interest (quantity-numeral mappings) was presented first, followed by the remaining mappings: Numeral-to-Quantity, Quantity-to-Numeral, Quantity-to-Word, Numeral-to-Word, Word-to-Quantity, and Word-to-Numeral.[3] Counting was neither prevented nor encouraged on any task and general positive feedback (e.g., "Nice job") was given after a child's response, regardless of response accuracy.

For all visual matching tasks (i.e., tasks *not* involving number words), the same general procedure was used. The experimenter randomly selected one of the large choice cards containing five distinct images and used her finger to circle each picture while saying, "See, there's a [number / group of dots] here, a [number / group of dots] here, …" until all five pictures were highlighted. Then, the experimenter randomly selected one of the target cards (containing a single numeral or a single image), showed it to participants, and asked, "Can you point at the [number/group of dots] that matches [or goes with] this [number/group of dots]?" Each task (other than the practice) consisted of five trials involving the target magnitudes from 1-5 presented in a random order, with a different choice (large) card and a different target (small) card on each trial.

The verbal matching tasks (i.e., task involving number words) were very similar in structure except the target number word or the choices of number words were provided verbally without a visual picture. It is also worth noting that unlike the visual tasks, the verbal tasks presented the five options sequentially and in a set order (as a count list) as opposed to simultaneously in a random order (like the visual cards). This was done to approximately equate (as well as possible) the demands of each task, restricting children's responses across all tasks to one of five possible responses.

[3] Nine children completed tasks in a slightly different order. However, performance on each task was not significantly different between task orders (p's>0.1).

Numeral-to-Quantity: This task (modeled after Siegel, 1974) was used to assess how well children mapped numerals onto quantities. Children were asked to identify the "group of dots" (from the choice card) that matched the "number" shown on the target card.

Quantity-to-Numeral: This task (modeled after Huntley-Fenner, 2001) was used to assess how well children mapped quantities onto numerals. Children were asked to identify the "number" (from the choice card) that matched the "group of dots" presented on the target card.

Quantity-to-Word: This task (modeled after the "What's on this Card" task; e.g., Gelman, 1993; Gelman & Gallistel, 1978; Le Corre et al., 2006) was used to assess the mapping of quantities onto number words. On every trial, children were shown a target card with an array of dots and asked: "How many dots do you see on the card: one, two, three, four, or five dots?"

Word-to-Quantity: This task (modeled after Condry & Spelke, 2008) assessed the mapping of number words onto quantities. On every trial, children were shown a choice card depicting five different dot arrays and asked: "Can you point at the group of dots with [Number Word] dots?"

Numeral-to-Word: This task (modeled after Bialystok & Codd, 1996) assessed the mapping of numerals onto number words. On every trial, children were shown a single numeral target card and asked: "Is this a one, two, three, four, or five?"

Word-to-Numeral: This task assessed the mapping of number words onto specific numerals. On every trial, children were shown a choice card depicting the numerals 1-5 and asked: "Can you point at the number [Number Word]?"

Data Coding and Analyses

Correct responses received a score of 1 and incorrect responses or refusals to respond received a score of 0. Children failed to provide a response on only 0.8% of trials. Scores were summed (for a total of 5) on each of the mapping tasks. Participant's responses were live scored and/or scored after the fact using a video recording of the child's responses. When non-parametric tests were warranted, Wilcoxon Signed Ranks tests were used. However, all patterns of analyses remain the same when using parametric and non-parametric statistics.

In order to look at counting behavior as a possible post-hoc explanation for some of the findings, 50% of the videos (12 three-year-olds and 12 four-year-olds) were coded offline for whether or not the child engaged in counting behavior on the quantity tasks. Two independent coders coded the videos and agreed on 97% of the trials (across the four tasks). A third independent coder settled the disagreements (on 3% of trials).

Results

Task Performance: We investigated performance across each mapping direction (see Table 1) and across small and large set sizes (see Table 2) by comparing performance to chance. Overall, four-year-olds performed above chance (on average) in all tasks and for all set sizes. Three-year-olds performed above chance on all tasks except for large values in the quantity-numeral mappings.

[Insert Table 1 Approximately Here]

[Insert Table 2 Approximately Here]

Analysis of Bidirectional Mappings: First, we compared performance on the two tasks for each mapping dyad (e.g., numeral-to-word and word-to-numeral) to determine whether children were better at mapping in one direction than the other. Since the scores could range from 0 to 5 on each task and children participated in all tasks, Wilcoxon Signed Ranks tests were used. We found no significant difference between the two directions in either quantity-numeral task (quantity-to-numeral M=2.8 (SD=1.8); numeral-to-quantity M=2.56 (SD=1.8): Z=-1.27, p=0.2) or word-numeral tasks (words-to-numerals M=4.0 (SD=1.3); numerals-to-words M=3.9 (SD=1.6): Z=-0.60, p=0.5). However, there was a marginal difference for the quantity-word tasks (quantity-to-word M=4.12 (SD=1.3); word-to-quantity M=3.8 (SD=1.3): Z=-1.94, p=0.052), with children showing slightly better performance on the task requiring the mapping of a given quantity to a specific number word relative to the task mapping a given number word to a specific quantity.

Given that substantial prior work suggests that children perform better on numerical tasks in which they spontaneously engage counting strategies (e.g., Bar-David et al., 2009; Posid & Cordes, 2014) and spontaneous counting is not equally elicited across distinct contexts (Fuson, Secada, & Hall, 1983; Michie, 1984), we hypothesized post-hoc that children may have engaged in more spontaneous counting during quantity-to-word tasks relative to word-to-quantity tasks. In particular, in the word-to-quantity task children were presented with five distinct sets of dots, which may have been overwhelming. In turn, this may have resulted in a lower likelihood of engaging a complex counting strategy in the word-to-quantity task relative to the quantity-to-word task, in which they saw only a single set. Thus, we decided post-hoc to investigate spontaneous counting behavior as a potential factor explaining the improved performance in the quantity-to-word task. Children counted on significantly more trials when mapping from a single quantity to a number word (average of 56% of trials) compared to mapping from a number word to a quantity (average of 9% of trials; p<0.001). However, this same difference was not found on the numeral-to-quantity mappings, which had relatively low rates of counting behavior overall (from a single quantity to a numeral, M=0.09%; from a single numeral to a quantity, M=14%; p=0.2).

Lastly, additional analyses explored individual differences in directional mapping scores that may have been missed by looking only at group averages. That is, if half of the children acquired the word-to-quantity mapping before the quantity-to-word mapping and the other half of children acquired the quantity-to-word mapping first, then on average, across all participants, no directional differences would be observed. To explore this possibility, difference scores were computed as the difference in performance across both tasks in a given dyad for each participant (e.g., performance on numeral-to-quantity task – performance on the quantity-to-numeral task), resulting in three scores which could vary anywhere from -5 to +5 (with a score of 0 indicating identical performance across the two tasks). If children acquired one mapping prior to the other, then histograms would reveal bimodal distributions, with the majority of children falling at either end of the spectrum. This was not the case. Rather, individual children generally performed comparably on tasks assessing both directions of each dyad; the majority of children performed identically or by a difference of one point in each of the three dyads (see Figure 2 for histograms), with no evidence of significant skew (using skew/SE < ±1.96; Cramer & Howitt, 2004). On the quantities-words tasks, 75% of children performed within one point (skewness of -0.62 (SE = 0.34)); on the words-numerals tasks, 91.7% of children performed within one point (skewness of 0.44 (SE = 0.34)); and on the quantities-numerals tasks, 77% of children performed within one point (skewness of -0.27 (SE = 0.34). Thus, the evidence at both the group level and at the individual level supports claims that children show symmetrical, bidirectional performance on each of these mappings.

[Insert Figure 2 Approximately Here]

Relative Difficulty of the Mappings: To explore differences between the three mapping dyads, we summed scores in each direction to form three distinct dyad scores: (1) Quantity-

Numeral (quantity-to-numeral + numeral-to-quantity), (2) Quantity-Word (quantity-to-word + word-to-quantity), and (3) Word-Numeral (word-to-numeral + numeral-to-word), each ranging from possible values of 0 to 10 (10 trials assessing each mapping). We then performed a mixed-measures ANOVA exploring the impacts of Age Group (2; between-subjects) and Dyad (3; within-subjects) on these summed scores. Results revealed a main effect of dyad ($F(2,92)=27.85$, $p<0.001$, $\eta_p^2=0.4$) and a main effect of age group ($F(1,46)=19.35$, $p<0.001$, $\eta_p^2=0.3$), but no interaction ($F(2,92)=0.88$, $p=0.4$, $\eta_p^2=0.02$; see Figure 3). Follow up t-tests revealed that performance on the Quantity-Numeral tasks ($M=5.4$) was significantly lower than on the Quantity-Word tasks ($M=7.88$; $t(47)=5.873$, $p<0.001$; Cohen's $d=0.65$) and on the Word-Numeral tasks ($M=7.9$; $t(47)=6.44$, $p<0.001$; Cohen's $d=0.68$). However, performance did not differ between the Word-Numeral tasks and the Quantity-Word tasks ($p=0.9$; Cohen's $d<0.05$). Although four-year olds ($M=8.3$) outperformed three-year olds ($M=5.8$) overall, both age groups performed better on mapping tasks involving number words (with both quantities and numerals) compared to mapping tasks not involving number words (mapping quantity and numerals directly; see Figure 3).

[Insert Figure 3 Approximately Here]

Next, given that each of the contrasting hypotheses provide distinct predictions for which mappings may be set-size dependent, mapping scores in each dyad were separated into proportion correct for small numbers (1, 2, and 3) and for large numbers (4 and 5). Children performed better on small numbers, relative to large numbers, on the Quantity-Word tasks (M_{Small}=87%, M_{large}=66%; Z=4.42, $p<0.001$) and on the Quantity-Numeral tasks (M_{Small}=60%, M_{large}=44%; Z=3.30, $p<0.001$), but performed equally on the small and large numbers in the Word-Numeral tasks (M_{Small}=80%, M_{large}=79%; $Z=0.31$, $p=0.8$). Thus, in both dyads involving quantity (Quantity-Word tasks and Quantity-Numeral tasks), performance was dependent upon numerical size, with better performance on trials involving small numbers relative to large numbers. In contrast, performance on the Word-Numeral tasks, which required translation between two symbols, appeared to be independent of the numerical values presented, with performance on small numbers matching that of large numbers.

Mediation Analyses: Both age groups were least accurate on the Quantity-Numeral dyad, relative to the other dyads, suggesting that this mapping may be particularly difficult. However, how children go about acquiring this mapping could be accounted for by two contrasting accounts: by acquiring the mapping directly between numerals and quantities (Quantity Account) or instead, by acquiring the mapping between numerals and number words and then, through this mapping, inferring the relation between numerals and quantities (Symbolic Account; see Figure 4 for the models tested for each account). In order to test predictions of these distinct hypotheses, we tested two mediation analyses using regression with the PROCESS version 2.15 package in SPSS with 1000 bootstrap samples for corrected confidence intervals (Hayes, 2013). In all analyses we controlled for categorical age[4]. See Table 3 for the bivariate correlations between variables.

[Insert Figure 4 Approximately Here]

First, we wanted to establish that there was a significant relation between performance on the Word-Numeral dyad and the Quantity-Numeral dyad (controlling for age) before including the mediator. The direct regression was significant, R^2=0.44, F(2,45)=17.59, p<0.001, with Word-Numeral performance providing significant unique variance (B=0.58, p<0.001) in predicting Quantity-Numeral performance, over and above age. Next, to test the Quantity account, we tested whether performance on the Quantity-Word dyad mediated this relation

[4] Age was treated as a categorical variable for consistency within all the analyses. However, the pattern and significance of results are identical when age is treated continuously.

between performance on the Word-Numeral dyad and the Quantity-Numeral dyad (see Figure 4, Panel A) by including Quantity-Word performance in the model. There were significant paths from Word-Numeral performance to Quantity-Word performance (controlling for age; B=0.38, $p<0.001$) and to Quantity-Numeral performance (controlling for age and Quantity-Word performance; B=0.52, $p<0.01$). The path from Quantity-Word performance to Quantity-Numeral performance was not significant however (controlling for age and Word-Numeral performance; B=0.17, $p=0.4$). Critically, the indirect path from Numeral-Word to Quantity-Numeral performance, mediated by Quantity-Word performance, was not significant, B=0.06, 95% CI: (-0.09,0.24). Thus, we did not find evidence in favor of the Quantity account; that is, analyses revealed that the relation between children's understanding of how number words are associated with Arabic numerals and their understanding of how quantities are associated Arabic numerals was not mediated by their ability to map between quantities and number words.

On the other hand, tests of the Symbolic account yielded a different pattern. Again, we first established that there was a significant relation between performance on the Quantity-Word dyad and the Quantity-Numeral dyad ($R^2=0.32$, $F(2,45)=10.5$, $p<0.001$) with Quantity-Word performance providing significant independent variance, B=0.496, $p<0.02$, over and above age. Thus, we next tested whether performance on the Word-Numeral dyad mediated the relation between performance on the Quantity-Word dyad and the Quantity-Numeral dyad (see Figure 4, Panel B) by including Word-Numeral performance in the model. There was a significant direct path from Quantity-Word performance to Word-Numeral performance (controlling for age; B=0.63, $p<0.001$) and from Word-Numeral performance to Quantity-Numeral performance (controlling for age and Quantity-Word performance; B=0.52, $p<0.01$). There was not a significant direct path from Quantity-Word performance to Quantity-Numeral performance (controlling for age and Word-Numeral performance; B=0.17, *p*=0.4), after including Word-Numeral performance in the model. Critically, however, there was a significant indirect effect from Quantity-Word performance to Quantity-Numeral performance through Word-Numeral performance, controlling for age: B=0.33, 95% CI (0.1,0.77). Thus, we do find evidence for an indirect relation between children's number quantity-word mapping ability and their quantity-numeral mapping ability, through their ability to map between the two symbols (number words and numerals), consistent with the Symbolic account.

[Insert Table 3 Approximately Here]

Discussion

Overall, our results replicate critical findings involving quantity-word mappings, but also extend this prior literature by incorporating children's understanding of Arabic numerals. In particular, we investigated (1) how children think about the numerical values represented by Arabic numerals and (2) whether the mappings between Arabic numerals, number words, and quantities are bidirectional in preschoolers.

Directional Mappings

First, our data suggest that children show bidirectional performance on both quantity-numeral and word-numeral mappings, consistent with other recent work (Benoit et al., 2013). However, there was a slight tendency for children to score higher on their quantity-to-word mapping relative to their word-to-quantity mapping, consistent with findings from other mapping tasks using larger magnitudes and/or more approximate mapping tasks (Mundy & Gilmore, 2009; Odic et al., 2015). In this previous work, researchers have suggested that the pattern may reflect the imprecise nature of non-symbolic representations of quantity since mapping from a quantity requires estimating only a single set (the target), while having quantities as options requires estimating more than one set (all the choices). Given that our task used much smaller sets (magnitudes of 1-5, as opposed to quantities in the 13-80 range used in Mundy & Gilmore, 2009) that allowed children to engage in counting if they wanted to (as opposed to the fast, approximate mapping in Odic et al., 2015), the pattern of findings in our data may stem from differences in spontaneous use of counting strategies rather than (or in addition to) differences caused by the approximate nature of estimation. That is, we found that children were less likely to engage in counting behaviors when there were five non-symbolic quantities in the task (i.e., when mapping from a number word to a quantity) compared to when only a single non-symbolic quantity was involved (i.e., when mapping from a single quantity to a number word). It may be that having five non-symbolic sets was daunting or overwhelming for children to try to count and so they opted not to use this strategy (or did not readily think to use this strategy), where as contexts that contained only a single more readily elicited a counting strategy. Furthermore, the finding that children performed better on tasks that elicited higher rates of spontaneous counting is consistent with substantial literature showing the benefits of spontaneous counting (Bar-David et al., 2009; Le Corre et al., 2006; Mix, 2008; Posid & Cordes, 2014).

Learning Arabic Numerals

However, the most critical findings in the current manuscript are those involving Arabic numerals. First, children performed worse on quantity-numeral mapping tasks compared to both of the other mapping tasks. This finding is consistent with research looking at parent-child interactions, which suggest that parents tend to engage in quantity-word mappings much more frequently than labeling or writing numerals with their preschool aged children (e.g., Gunderson & Levine, 2011; Susperreguy & Davis-Kean, 2015). In addition, when we consider intuitions about the contexts in which numerals are used, children likely encounter more word-numeral associations without the relevant quantity (e.g., hear an adult label “2” as “*two*” without a corresponding set of two items nearby – for example, an address number) than quantity-numeral associations in the absence of a number word (e.g., see “2” associated with two ducks without an adult also saying “*two*”). This observed pattern, however, is in contrast to a recent study providing evidence that French children learn to map numerals with quantities before they learn to map numerals with words (Benoit et al., 2013). While it is possible that cultural differences contributed to our distinct patterns of findings, it seems more likely that this discrepancy may be attributed to the presentation format of the quantities used. That is, Benoit et al. presented quantity via canonical forms (i.e., the pattern found on dice), which may have been a familiar format to the children. As such, children may have been able to recognize these representations, providing them with an advantage in the quantity mapping tasks (e.g., Mandler & Shebo, 1982), even without a more general understanding of the relation between numerals and arbitrary visual representations of quantity. Our task, in contrast, presented quantities using random configurations which varied from trial-to-trial, making it unlikely that previous familiarity with specific configurations of the items contributed to the pattern of results found, making responses necessarily dependent upon an ability to identify the numerosity of the quantities presented. Future research is needed to clarify whether cultural differences, stimuli differences, or other unidentified factors may have contributed to the disparate pattern of findings observed across these two studies to clarify the developmental trajectory of these mappings.

Furthermore, although children performed better on trials involving small numbers (1,2,3) than on those involving large numbers (4,5) on dyads involving quantity (quantity-word, quantity-numeral; Le Corre & Carey, 2007; Sarnecka & Carey, 2008), performance on the word-numeral tasks (involving symbols exclusively) was not dependent upon number size. This finding suggests that the mapping between two symbolic forms, written and verbal (at least for relatively small numbers between 1 and 5), may not depend on the magnitude of the quantities the symbols represent. Instead, this finding supports the Symbolic account outlined in the Introduction and suggests that children can map directly between the two symbolic representations of number without reference to the quantity those symbols represent. Our mediation analyses provide further support for this account, suggesting that children do not map directly between Arabic numerals and quantities, even early in the learning of numerical knowledge. Rather, preschoolers may rely upon a direct word-numeral mapping in order to form a quantity-numeral mapping. That is, it may be that once children already have one mapping between a symbolic representation (i.e., number words) and quantity, they integrate the second symbolic representation (i.e., Arabic numerals) by mapping it to the existing symbolic structure and use the existing quantity-word mapping to understand the quantity-numeral mapping, rather than creating a distinct mapping to quantity. This finding is consistent with recent work showing that adults similarly have a privileged mapping between Arabic numerals and number words (two symbols) compared to Arabic numerals and quantities (Lyons et al., 2012). Although this finding may appear to be at odds with work showing ratio dependent responding even for Arabic numerals (Moyer & Landauer, 1973), we do not believe these findings are inconsistent. Instead, it may be that the mapping between Arabic numerals and quantities (leading to ratio dependent responding) is indirect and occurs primarily through mapping Arabic numerals and number words.

Lastly, children were more likely to engage in counting behavior when the task involved a number word compared to when the task involved an Arabic numeral. Thus, although (as mediation analyses suggest) children may have translated the Arabic numeral into a number word to perform the task – the Arabic numeral may still have impacted how they performed the task. That is, children did not engage in the same strategies when encountering Arabic numerals and number words. It may be that the verbal counting routine was less salient when the experimenter did not use any number words in the task, but in contrast, when the experimenter explicitly used number words (as in the quantity-word tasks), an important component of the verbal counting procedure, this may have signaled to children that the counting routine should be used. Relatedly, it may be that children are more likely that the mere presentation of Arabic numerals may have taxed the children's attentional capacities more, making it less likely that they had the cognitive resources available to engage in a long and complicated counting procedure. Regardless, this pattern of results suggests that more research is needed to investigate how children connect the counting routine to their understanding of Arabic numerals. Although children may be able to map between Arabic numerals and number words, this does not imply that children are also equally likely to see the counting routine as being relevant to both Arabic numerals and number words.

Implications

How symbolic representations come to represent quantity is a central aspect of numerical cognition, as well as many other fields. Termed the "symbol-grounding problem" (e.g., Harnad, 1990), this discussion has lead to the formation of distinct, competing theories and much discussion and debate (e.g., Leibovich & Ansari, 2016; Nunez & Lakoff, 2000; Odic, Le Corre, & Halberda, 2015). Much of this debate surrounds the involvement of the approximate number system (ANS) in symbolic representations. Given that the current study involved exact representations of small quantities (meaning, children could count and were not asked to order or manipulate the quantities), we cannot directly speak to the on-going debate about the role of the

ANS in symbolic numerical representations. However, our data do suggest two key points that may influence the theoretical foundations of the debate. First, there may be critical differences between how children process Arabic numerals and number words (akin to differences in vocabulary, word-fluency, and reading comprehension in early readers; Pikulski & Chard, 2005; Tannenbaum, Torgesen, & Wagner, 2006). Although both are symbolic representations of numerical information, the current data suggest that children's understanding of Arabic numerals may be grounded in their understanding of number words, rather than their understanding of quantity. In other words, since Arabic numerals are learned after number words, number words are likely learned via a mapping to quantity and Arabic numerals are then mapped into this already existing system. Second, although children and adults may be able to associate symbolic representations with the underlying quantities they represent, there may also be cases where the symbolic information is processed without the underlying quantity. In particular, children did not show differences in performance across set size when mapping between two symbolic representations (word-numeral mappings). This might suggest that the learning of Arabic numerals does not occur in a magnitude specific order (i.e., not dependent upon number size), but rather that children learn the associations between numerals and number words in a fashion that is not necessarily ordered. For example, children may first learn the numbers for their age, their jersey number, or their phone number, which would not predict that children would necessarily learn 1 before 2, followed by 3, etc. Alternatively, it may be that children do learn the mappings in numerical order (i.e., learning 1 before 2, followed by 3, etc.) but that the mappings themselves are not more difficult for higher numbers than for lower numbers. That is, the precision of these symbolic representations in an exact matching task results in non-systematic errors, such that when children make errors, these errors are equally likely across the magnitudes 1-5.

In conclusion, the acquisition of symbolic representations of number is critical for the acquisition of mathematical knowledge, as well as simply for communicating numerical information to others. Substantial research has investigated how children acquire and process numerical information in symbolic and non-symbolic forms. However, the current study extends these findings by directly comparing children's exact mapping using number words and Arabic numerals separately. By doing so, the current data provides insight into current theoretical discussions surrounding the integration of symbolic representations of number with non-symbolic representations of numerical information, while leaving open questions that could further illuminate this developmental process.

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Table 1

Mean Performance in Each Task by Age

	Quantity-Word		Quantity-Numeral		Word-Numeral	
	Quantity-to-Word	Word-to-Quantity	Quantity-to-Numeral	Numeral-to-Quantity	Word-to-Numeral	Numeral-to-Word
3-year-olds	0.7*	0.63*	0.39	0.38	0.7*	0.67*
4-year-olds	0.95*	0.87*	0.74*	0.65*	0.9*	0.9*

* *p*<0.05, compared to chance (0.2) using One-Sample Wilcoxon Signed Rank Tests.

Table 2

Mean Performance in Each Mapping Dyad by Age and Magnitude Size

	Quantity-Word		Quantity-Numeral		Word-Numeral	
	Small Values	Large Values	Small Values	Large Values	Small Values	Large Values
3-year-olds	0.78*	0.49*	0.46*	0.27	0.67*	0.71*
4-year-olds	0.96*	0.83*	0.75*	0.61*	0.92*	0.88*

* *p*<0.05, compared to chance (0.2) using One-Sample Wilcoxon Signed Rank Tests.

Table 3

Bivariate Correlations

	Age Group	Numerals and Words	Quantities and Words	Quantities and Numerals
Age Group	-	0.385 (*p*=0.007)	0.522 (*p*<0.001)	0.479 (*p*=0.001)
Numerals and Words		-	0.583 (*p*<0.001)	0.607 (*p*<0.001)
Quantities and Words			-	0.506 (*p*<0.001)
Quantities and Numerals				-

Simple correlations (p-values) for all variables included in the regression analyses.

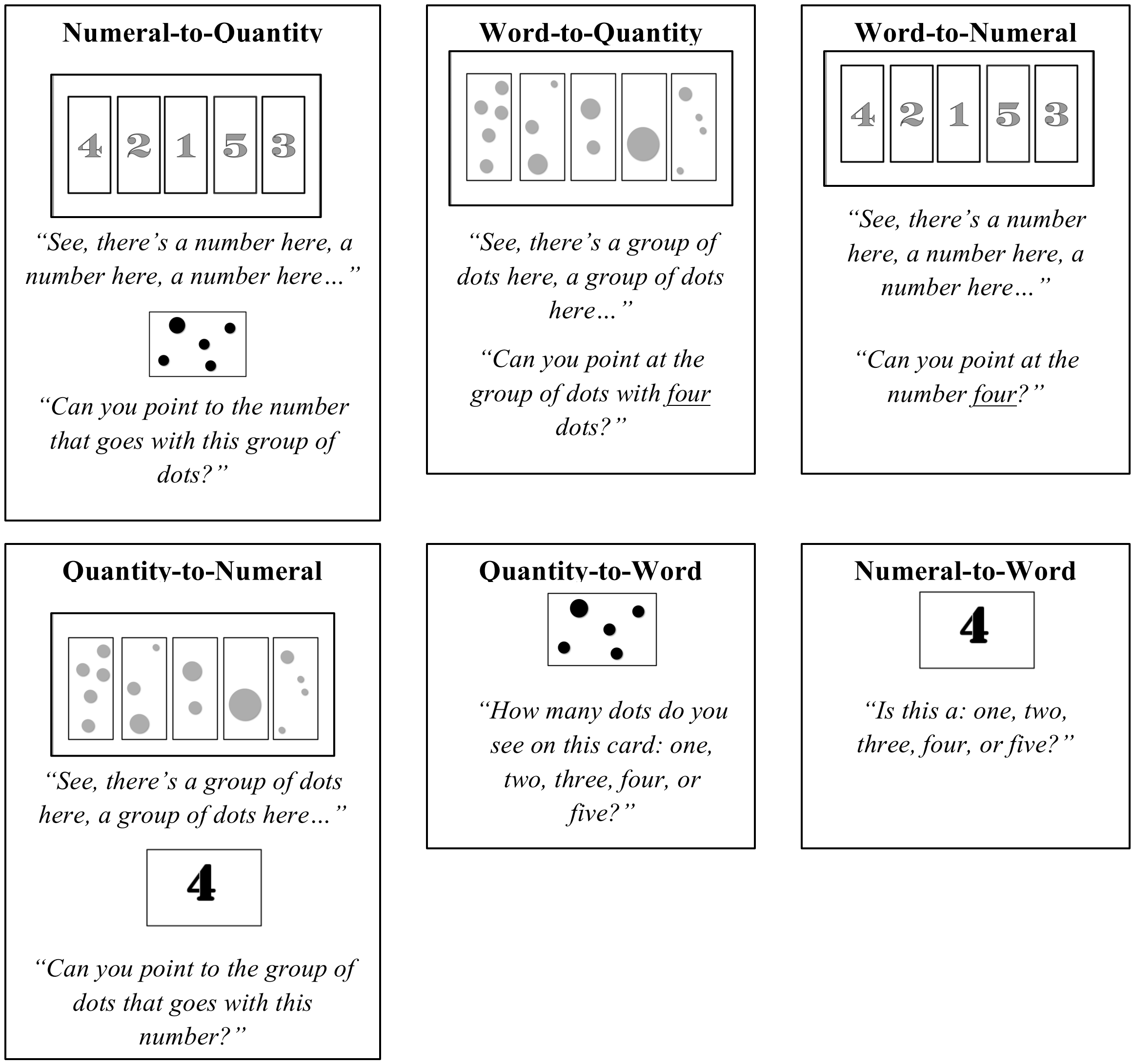

Figure 1. Examples of each of the six mapping tasks.

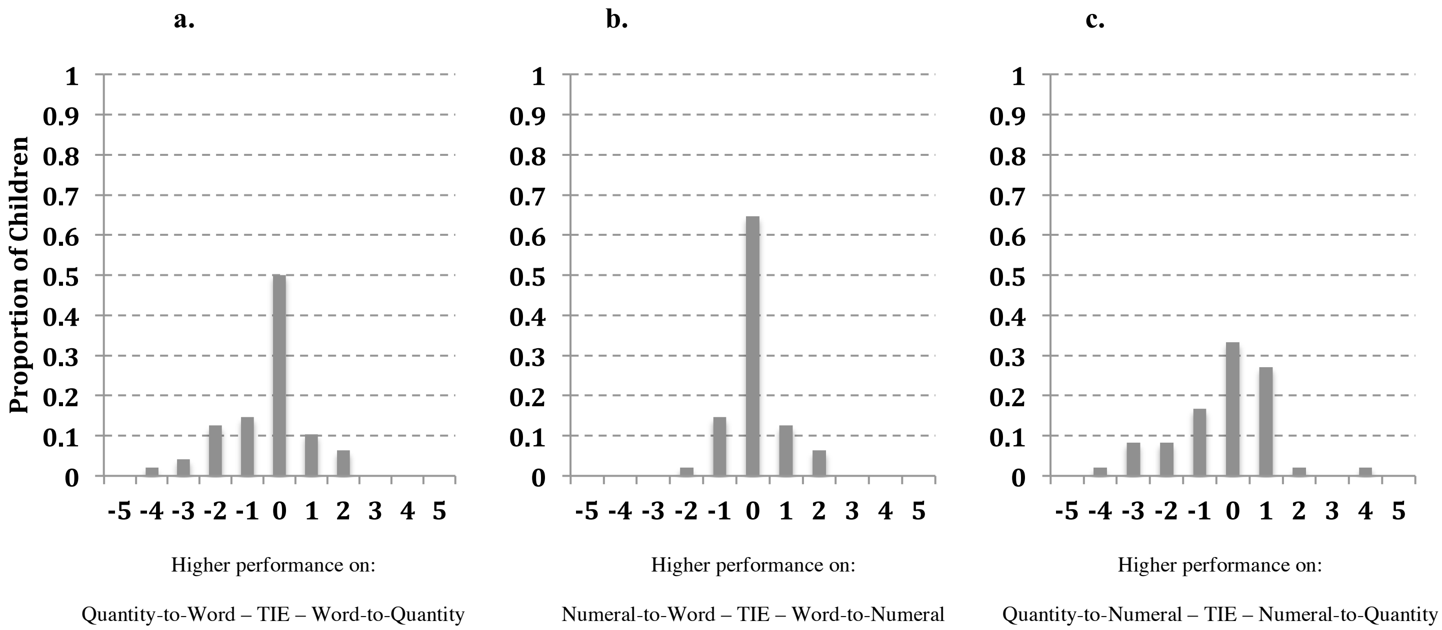

Figure 2. Proportion of children with each level difference score (range -5 to 5) for each of the three mapping dyads: (a) quantities-words, (b) words-numerals, and (c) quantities-numerals. "TIE" represents equal performance across the two directions.

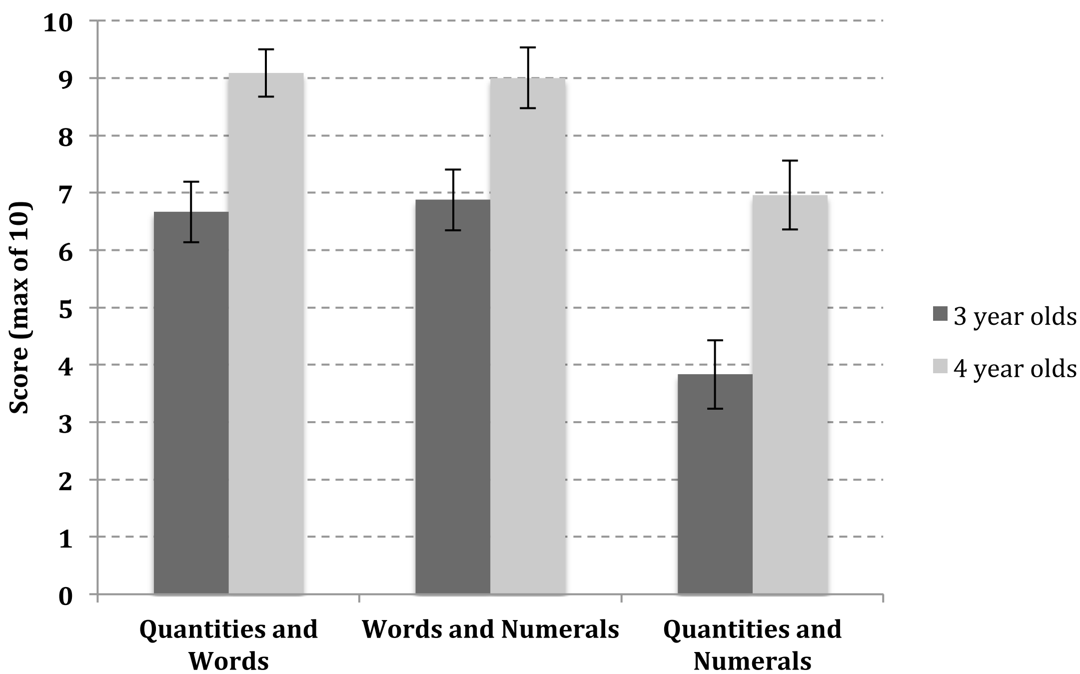

Figure 3. Accuracy scores on the three mapping tasks (combining both directions) separated by age. Error bars represent standard error of the mean.

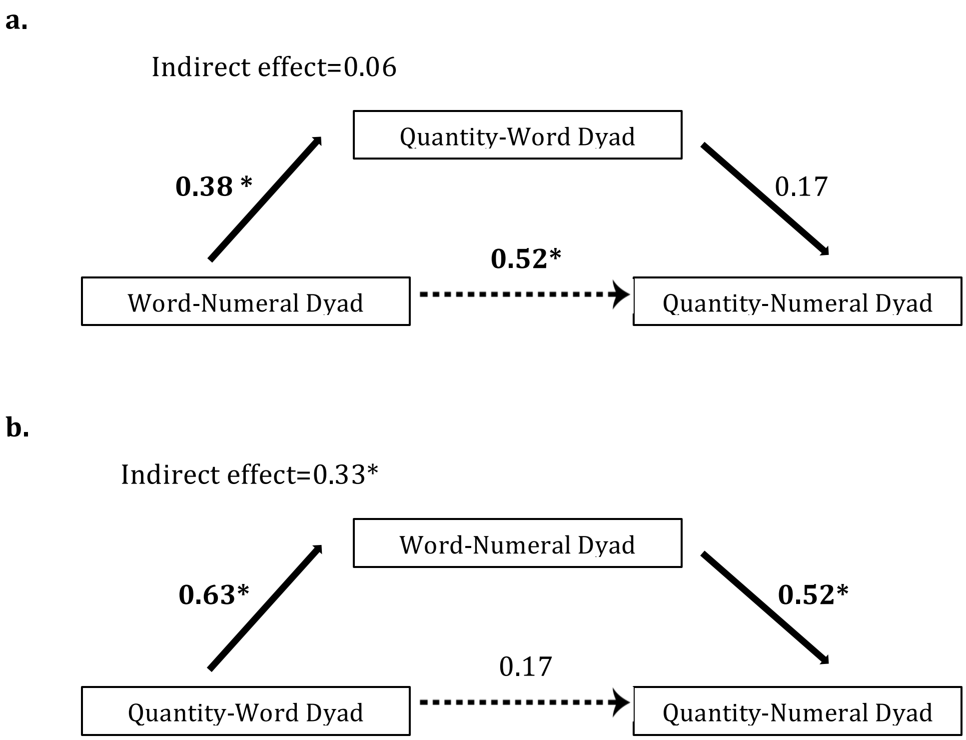

Figure 4. Mediation models testing: (a) the Quantity account and (b) the Symbol account for predicting children's mapping between numerals and quantities. Unstandardized coefficients are presented for each path, controlling for the other relevant predictor variables as well as age. * $p<0.05$.