Economic recovery with price-quantity dynamics in an agent-based input-output model

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Introduction

- Sector-wise input–output (io) tables are widely used to model the inter-industry dependencies of a national economy and how it responds to shocks, such as the economic recovery from disasters and pandemics.
- In conventional io analyses producers react to such (demand) shocks exclusively by adjusting the quantity of their produced goods while prices remain fixed.
- We propose: An agent-based extension to these dynamic io recovery models in which producers can adjust their economic behaviour to external shocks by price and quantity adjustments simultaneously.

Input/Output-Tables

An economy consists of N sectors. $(\mathrm{X_t})_{\mathrm{ij}}$ is the number of goods flowing from sector i to sector j:

$$\text{Sector i} \xrightarrow{(\mathrm{X_t})_{\mathrm{i,j}}} \text{Sector j}.$$

- $\mathrm{X_t} \in \mathrm{Mat}_{\mathrm{N\times N}}(\mathbb{R})$ Matrix of intersectoral flow
- $(\mathrm{D_t}) \in \mathbb{R}^{\mathrm{N}}$ are the goods used by the end consumer in each sector.
- $(\mathrm{v_t}) \in \mathbb{R}^{\mathrm{N}}$ is the value added by each sector when producing its goods. (e.g.: Labour costs and investement)
- $(\mathrm{Y_t}) \in \mathbb{R}^{\mathrm{N}}$ vector of total outputs

Flow consistency equations

Produced goods are either used by other sectors, or by the end consumer. Reversely, the output of a sector is given by the sum its inputs plus its value added:

$$\sum_{\mathrm{j=1}}^{\mathrm{N}} \underbrace{(\mathrm{X_t})_{\mathrm{ij}}}_{\text{Goods flowing to other sectors}} + \underbrace{(\mathrm{D_t})_{\mathrm{i}}}_{\text{Demand}} = \underbrace{(\mathrm{Y_t})_{\mathrm{i}}}_{\text{Total output}}, \quad \sum_{\mathrm{i=1}}^{\mathrm{N}} (\mathrm{X_t})_{\mathrm{ij}} + \underbrace{(\mathrm{v_t})_{\mathrm{j}}}_{\text{value added}} = (\mathrm{Y_t})_{\mathrm{j}}.$$

These equations express, that the money is conserved. i.e.: The monetary inflow of each sector equals its monetary outflow.

Assumption of constant technical coefficients

From now on, we will assume that the technical coefficients, defined by

$$\mathrm{A_{ij}} := (\mathrm{X_t})_{\mathrm{ij}}/(\mathrm{Y_t})_{\mathrm{j}}$$

stay constant. One sector j requires $\mathrm{A_{ij}}$ units from sector i to produce one of itself. e.g.: To produce 1 car, one always needs 4 wheels and 1 engine; 1 bicycle: 2 wheels and 0 engines.

Quantity/Price relationship

All variables in the IO-tables are in monetary therms

$$[(\mathrm{X_t})_{\mathrm{ij}}] = [(\mathrm{D_t})_{\mathrm{i}}] = [(\mathrm{v_t})_{\mathrm{i}}] = [(\mathrm{Y_t})_{\mathrm{i}}] = \$.$$

Defining the quantity as $\mathrm{Q_t}$ and the price as $\mathrm{P_t}$:

$$(\mathrm{Y_t})_{\mathrm{i}} =: (\mathrm{Q_t})_{\mathrm{i}} \cdot (\mathrm{P_t})_{\mathrm{i}} = (\mathrm{Q_t} \circ \mathrm{P_t})_{\mathrm{i}}.$$

(where $\circ$ is the element-wise multiplication)
The dimensions of each variable in this equation are the following

$$[(\mathrm{Y_t})_{\mathrm{i}}] = \$, \quad [(\mathrm{Q_t})_{\mathrm{i}}] = \mathrm{q_i}, \quad [(\mathrm{P_t})_{\mathrm{i}}] = \$/\mathrm{q_i}.$$

And we will define $\mathrm{E_t}$ by

$$\mathrm{D_t} =: \mathrm{E_t} \circ \mathrm{P_t}.$$

i.e.: $\mathrm{E_t}$ is the quantity of goods delivered to the end consumer.

Motivation and Introduction of $\bar{\mathrm{X}}$

Motivation: Starting from an equilibrium state (all produced goods are sold at a set price), then producers experience a shock $\bar{\mathrm{X}}$. Dropping the often made assumption that producers "know" whether they are facing a quantity or a price shock, but rather have an inert behaviour (quantified by behavioural parameter $\mathrm{h_Q}$ and $\mathrm{h_v}$ to either adjus Q or P.

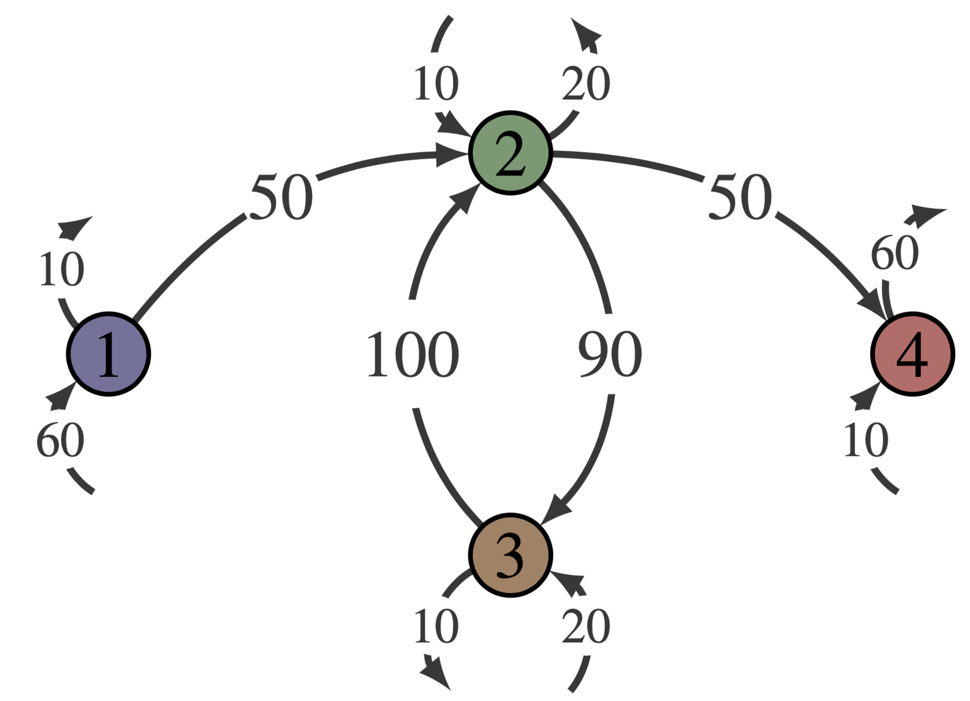
Goal: Update equations, which adjust the IO-table to a demand shock $\bar{\mathrm{X}}$ by adjusting both, price and quantity, whilst keeping the technical coefficients (for the quantity) constant, and the IO-table consistent (satisfying the equilibrium equations) and so that they converge such that $\mathrm{D_\infty} = \mathrm{D_0} + \bar{\mathrm{X}}_0$.

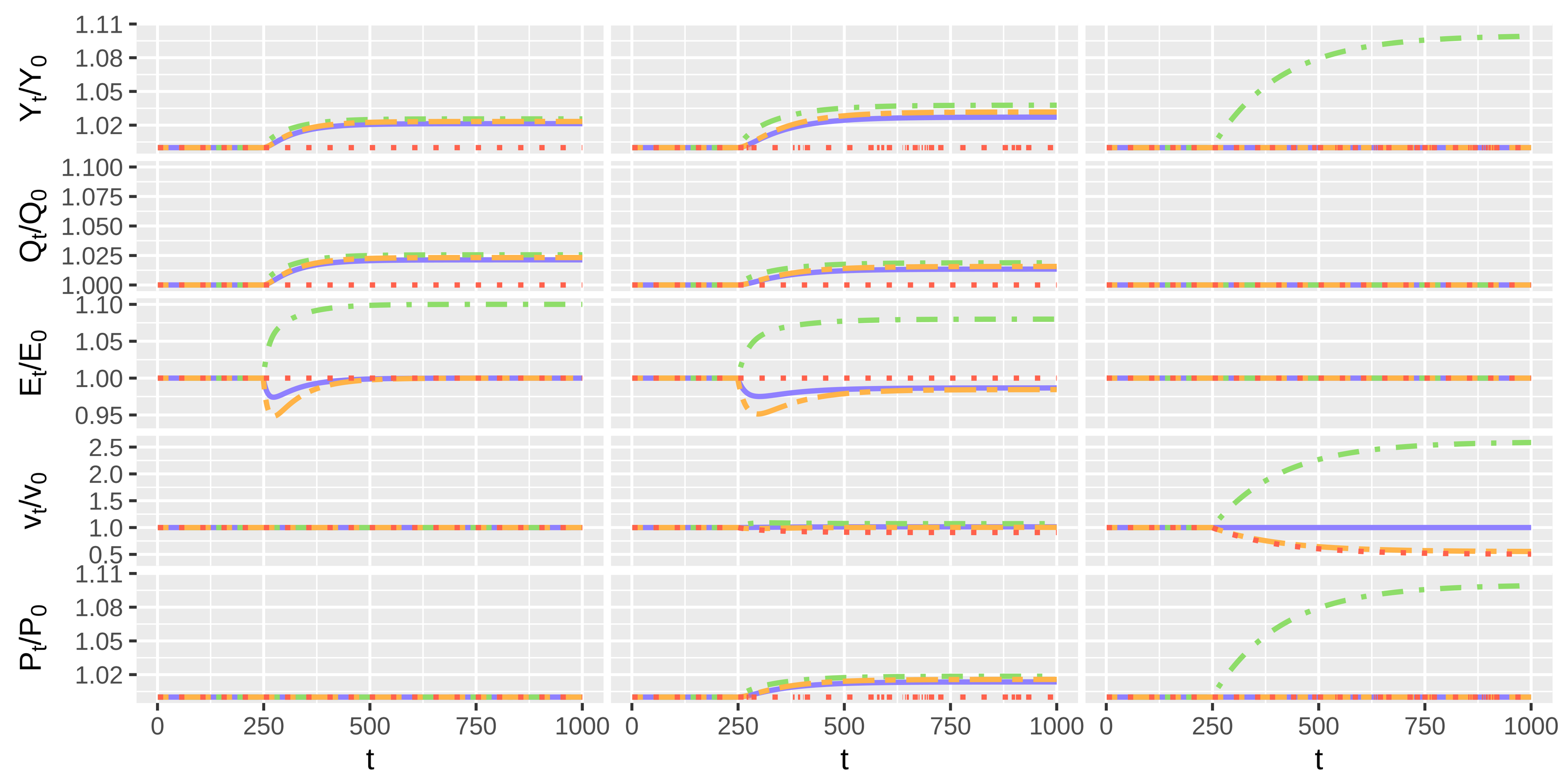
Update Equations

Defining $\mathrm{L} := \mathrm{diag}\left((\mathrm{v_t})_1/(\mathrm{Y_t})_1, \ldots, (\mathrm{v_t})_{\mathrm{N}}/(\mathrm{Y_t})_{\mathrm{N}}\right)$

$$\mathrm{Q_{t+1}} = \mathrm{Q_t} + \mathrm{h_Q}\left(\bar{\mathrm{X}}_{\mathrm{t}} \circ \mathrm{P_t^{-1}}\right)$$
$$\mathrm{E_{t+1}} = (\mathbb{I} - \mathrm{A})\mathrm{Q_{t+1}}$$
$$\bar{\mathrm{X}}_{\mathrm{t+1}} = \bar{\mathrm{X}}_{\mathrm{t}} - (\mathrm{E_{t+1}} - \mathrm{E_t}) \circ \mathrm{P_t}$$
$$\mathrm{v_{t+1}} = \mathrm{v_t} + \mathrm{h_v}\left(\mathrm{diag}(\mathrm{v_t})\mathrm{L}(\mathbb{I} - \mathrm{A^T})\left(\bar{\mathrm{X}}_{\mathrm{t}} \circ \mathrm{E_t^{-1}}\right)\right)$$
$$\mathrm{P_{t+1}} = (\mathbb{I} - \mathrm{A^T})^{-1}\mathrm{L}(\mathrm{v_0^{-1}} \circ \mathrm{v_{t+1}})$$
$$\bar{\mathrm{X}}_{\mathrm{t+1}} = \bar{\mathrm{X}}_{\mathrm{t}} - \mathrm{E_t} \circ (\mathrm{P_{t+1}} - \mathrm{P_t}).$$

4-sector sample-economy





These plots show the trajectory of the different variables for the 4-sector sample economy to a shock of $\bar{\mathrm{X}} = (0, 2, 0, 0)$ with $(\mathrm{h_Q}, \mathrm{h_v}) = \{(0.05, 0), (0.025, 0.025), (0, 0.05)\}$

Fitting process

Using the IO-tables of 2 consecutive years

$$\begin{array}{c|c} \mathrm{X_0} & \mathrm{D_0} \; \mathrm{Y_0} \\ \hline (\mathrm{v_0})^{\mathrm{T}} & \\ (\mathrm{Y_0})^{\mathrm{T}} & \end{array} \longrightarrow \begin{array}{c|c} \mathrm{X_1} & \mathrm{D_1} \; \mathrm{Y_1} \\ \hline (\mathrm{v_1})^{\mathrm{T}} & \\ (\mathrm{Y_1})^{\mathrm{T}} & \end{array}$$

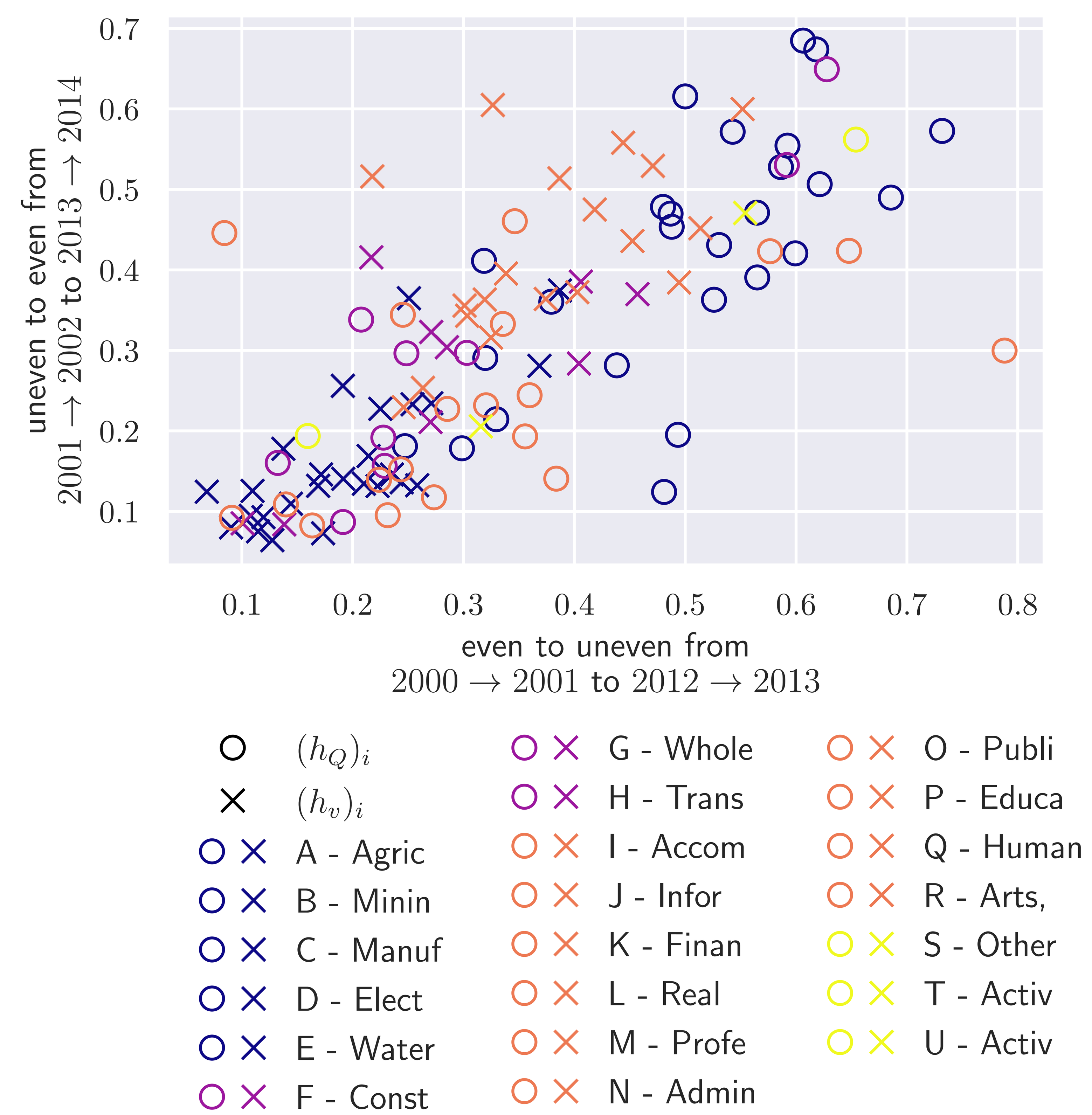
Assume a shock $\bar{\mathrm{X}} = \mathrm{D_1} - \mathrm{D_0}$ acts at $\mathrm{t} = 0$ on the system. Depending on $(\mathrm{h_Q}, \mathrm{h_v})$ the ABM will produce different io tables.

$$\begin{array}{c|c} \mathrm{X_0} & \mathrm{D_0} \; \mathrm{Y_0} \\ \hline (\mathrm{v_0})^{\mathrm{T}} & \\ (\mathrm{Y_0})^{\mathrm{T}} & \end{array} \xrightarrow[\mathrm{h_Q, h_v}]{\text{Run update steps}} \begin{array}{c|c} \tilde{\mathrm{X}}_1 & \tilde{\mathrm{D}}_1 = \mathrm{D_0} + \bar{\mathrm{X}} = \mathrm{D_1} \; \tilde{\mathrm{Y}}_1 \\ \hline (\tilde{\mathrm{v}}_1)^{\mathrm{T}} & \\ (\tilde{\mathrm{Y}}_1)^{\mathrm{T}} & \end{array}$$

Goal: Minimize the Residual

$$\min_{\mathrm{h_Q, h_v}} \|(\mathrm{Y_1} - \tilde{\mathrm{Y}}_1)\|_2 + \|(\mathrm{v_1} - \tilde{\mathrm{v}}_1)\|_2$$

Results



The ressource intensive sectors, such as manufacturing, agriculture and mining are more prone to adjust their quantity than their prices ($\mathrm{h_Q} > \mathrm{h_v}$), whilst service sectors, such as administration and finance, are more prone to adjust their prices and therefore their value added ($\mathrm{h_v} > \mathrm{h_Q}$).