CSE 211 Discrete Mathematics
Homework 2
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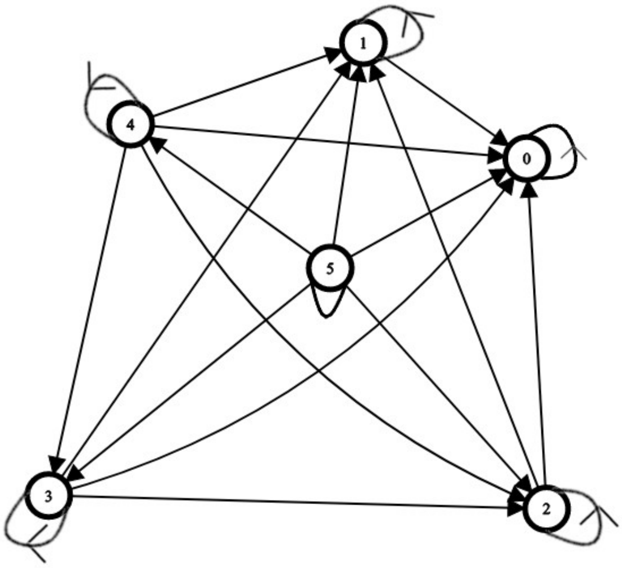
Problem 1

Our poset is like that ({0,1,2,3,4,5 } , >=)

1) This says that a relation (a,b) where a bigger equal b. Let the set is A. Then,

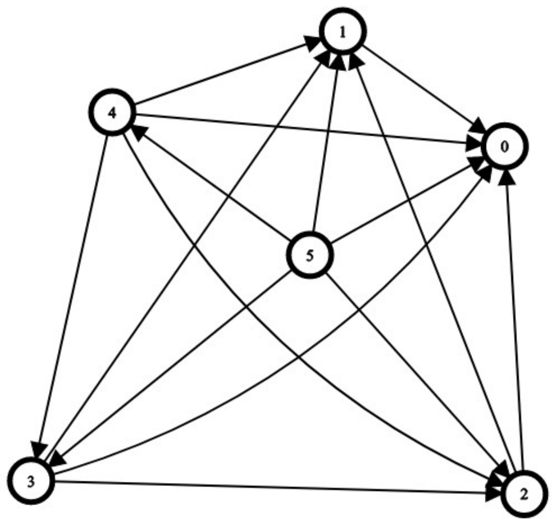
A = { ($0 \prec 0$), ($0 \prec 1$), ($0 \prec 2$), ($0 \prec 3$), ($0 \prec 4$), ($0 \prec 5$), ($1 \prec 1$), ($1 \prec 2$), ($1 \prec 3$), ($1 \prec 4$) , ($1 \prec 5$) , ($2 \prec 2$), ($2 \prec 3$), ($2 \prec 4$), ($2 \prec 5$) ($3 \prec 3$), ($3 \prec 4$), ($3 \prec 5$), ($4 \prec 4$), ($4 \prec 5$), ($5 \prec 5$) }

Let's draw the directed graph the relation:



2) Since we know that a poset must provide reflexivity, we also do not need the reflexive relations in A. So, A = { ($0 \prec 1$), ($0 \prec 2$), ($0 \prec 3$), ($0 \prec 4$), ($0 \prec 5$), ($1 \prec 2$), ($1 \prec 3$), ($1 \prec 4$) , ($1 \prec 5$) ,($2 \prec 3$), ($2 \prec 4$), ($2 \prec 5$), ($3 \prec 4$), ($3 \prec 5$), ($4 \prec 5$) }

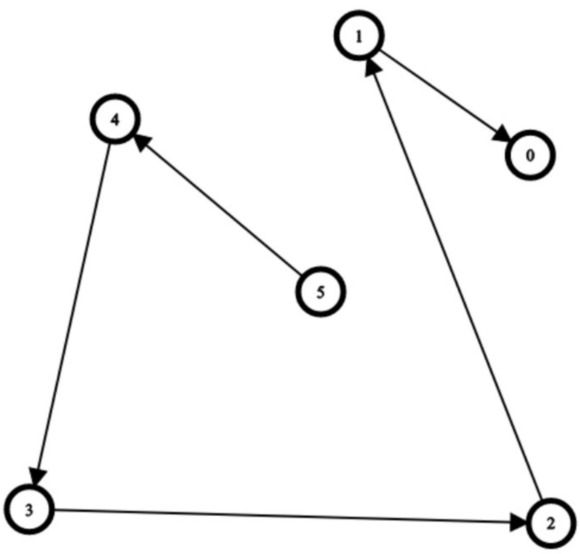
We remove the self loops in the diagram:



3) As a final step we remove the transivite edges :

A = {(0 $\prec$ 1), (1 $\prec$ 2), (2 $\prec$ 3), (3 $\prec$ 4), (4 $\prec$ 5) }

And the hasse diagram is like this:



Problem 2

a) Maximal elements: {1,2},{2,3,4},{1,3,4} (Because there is no living edge from them)

b) Minimal elements: {1}, {2}, {4} (Because there is no coming edge to them)

c) There is no greatest element since there is no element that has coming edge from all other elements.

d) The all upper bounds of {{2}, {4}} : {2,4}, {2,3,4} (Because both {2,4} and {2,3,4} have downward path to {2} and {4})

e) The least upper bound of {{2}, {4}} : {2,4} (Because it is less than the other upper bound)

f) The all lower bounds of {{1, 3, 4}, {2, 3, 4}} : {4}, {3,4} (Because both {4} and {3,4} have upward path to {1, 3, 4} and {2, 3, 4}) .

g) The greatest lower bound of {{1, 3, 4}, {2, 3, 4}} : {3,4} (Because {4} is under the {3,4})