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Problem 1

a)

converse: : if I will stay at home, then it snows tonight
Contrapositive: if I wont stay at home, then it doesn't snow tonight.
Inverse : if it doesn't snow tonight, then I won't stay at home.

b)

Converse : It is a sunny summer day whenever I go to the beach.
Contrapositive: It is not sunny summer day whenever I don't go to the beach.
Inverse : I don't go to the beach whenever it is not a sunny summer day.

c)

Converse : If I sleep until noon, then I stay up late.
Contrapositive: If I don't sleep until noon, then I stay up late.
Inverse : If I don't stay up late, then I don't sleep until noon.

Problem 2

a)

p	q	¬p	¬q	p ⊕ ¬q
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

b)

p	**q**	**r**	**¬p**	**¬q**	**¬r**	**p ⟺ q)**	**¬ p ⟺ ¬ r**	**(p ⟺ q) ⊕ (¬ p ⟺ ¬ r)**
0	0	0	1	1	1	1	1	0
0	0	1	1	1	0	1	0	1
0	1	0	1	0	1	0	1	1
0	1	1	1	0	0	0	0	0
1	0	0	0	1	1	0	0	0
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	1	0	1
1	1	1	0	0	0	1	1	0

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c)

p	**q**	**$\neg$ p**	**$\neg$ q**	**p $\oplus$ q**	**p $\oplus$ $\neg$ q**	**(p $\oplus$ q) $\Rightarrow$ (p $\oplus$ $\neg$ q)**
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	0	1	1

Problem 3

a) $\exists x\ (P(x) \land Q(x))$

b) $\exists x\ (P(x) \land \neg Q(x))$

c) $\forall x\ (P(x) \lor Q(x))$

d) $\neg \exists x\ (P(x) \lor Q(x))$

e) $\exists x\ (P(x) \land Q(x)) \to \exists x\ H(x)$

f) $\exists x\ H(x)$

g) Not everyone can speak English and knows python.

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Problem 4

=> We Solve this problem with Induction method.

Basis step => Let's apply $n=1$ on the equation

$$3 + 3.5 = \frac{3(5^2-1)}{4}$$

$$18 = 18 \checkmark$$

We prove that the equation true for $n=1$

Inductive Step → Apply $n=k$ on the equation and accept that the equation is true for $n=k$

$$\underbrace{3 + 3.5 + \dots + 3.5^k}_{a} = \underbrace{\frac{3(5^{k+1}-1)}{4}}_{a}$$

=> Apply $n=k+1$ on the equation and prove that equation is true based on $n=k$

$$\underbrace{3 + 3.5 + \dots + 3.5^k}_{a} + 3.5^{k+1} = \frac{3(5^{k+2}-1)}{4}$$

$$\frac{\cancel{3}(5^{k+1}-1)}{4} + \frac{4.\cancel{3}.5^{k+1}}{4} = \cancel{3}\left(\frac{5^{k+2}-1}{4}\right)$$

$$5.5^{k+1} - 1 = 5^{k+2} - 1$$

So, the equation is Proved

$$\boxed{5^{k+2} - 1 = 5^{k+2} - 1} \checkmark$$

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Problem 5

⇒ we know n is odd so Let's say $\boxed{n = 2x+1}$

Basis step ⇒ Apply $x=1$ on the equation

(1) $n^2-1 = (2x+1)^2-1$

$$\boxed{n^2-1 = \boxed{4x^2+4x}\cancel{+1-1}} \checkmark$$

(2) $\underline{4\cdot 1^2 + 4.1} = \frac{8}{8} = 1 \checkmark$

(3) Inductive step ⇒ Apply $X=k$ on the equation and accept that the equation is true for $x=k$

We accept

$\underbrace{4k^2+4k}_{a}$ → This is divisible by 8

(4) ⇒ Apply $x=k+1$ on the equation and prove that equation is true based on $x=k$

$$4(k+1)^2 + 4(k+1)$$

$$= 4(k^2+2k+1) + 4k+4$$

$$= 4k^2 + 8k + 4 + 4k + 4$$

a

$$= \boxed{4k^2+4k + 8k+8} \checkmark$$

This is a and as we accepted it is divided by 8

This one is already divided by 8

→ $\boxed{\text{So, the equation is proved.} \checkmark}$

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Problem 6

a) Roots of that equation 2 and 4 so the set is {2, 4}
b) There are infinite numbers between 2 and 3. The set is infinitive including 2 and 3
c) {2,4,5}
d) {2,4}
e) {2,4}

So, a = d = e .

Problem Bonus

a)

1- The upper side of the circuit we have a statement (p ∧ q)
2- The bottom side we have a statement (p v q’)
3- On the multiplexer 1 is selected so bottom side will be result.
4- So, the sentence will be “It is sunny or the flowers are not blooming.”

b)

I wrote the program in c++

```cpp
    #include<iostream>

    using namespace std;

int main()
{
    bool p,q;
    bool result, multiplexer;

    cout << "Enter the value of p, q and multiplexer:";
    cin >> p >> q >> multiplexer;

    if(multiplexer == 0)
        result = p & q;
    else
        result = p | (!q) ;

    cout << "Result is "<< result <<endl;

    return 0;
}
```

```
Terminal
Enter the value of p, q and multiplexer:0 1 0
Result is 0

Press enter to continue...
```