Discrete Mathematic HW-4
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Problem 1

a) We put the value on the linear recurrence relation to show that the solution is true. If the both side is equal then we say the solution is true.

① $a_n = -2^{(n+1)} \longrightarrow a_{n-1} = -2^{(n-1+1)}$

$$= \underline{\underline{-2^n}}$$

Let's put these on the linear equation.

② $a_n = 3a_{n-1} + 2^n$

$$-2^{(n+1)} = 3(-2^n) + 2^n$$

$$-2^{(n+1)} = -3(2^n) + 2^n$$

$$-2^{(n+1)} = -2(2^n)$$

$$\boxed{-2^{(n+1)} = -2^{(n+1)}}$$

So the given solution is true.

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b)

1

$$a_n = \underbrace{3a_{n-1}}_{a_n^{(h)}} + \underbrace{2^n}_{a_n^{(p)}}$$

2

$$\underbrace{a_n^{(h)}}_{r} = \underbrace{3a_{n-1}}_{3} \longrightarrow \text{Root of characteristic equation}$$

$$\boxed{a_n^{(h)} = c_1(3)^n}$$

3

$$a_n^{(p)} = A.2^n$$
$$a_{n-1}^{(p)} = A.2^{(n-1)}$$

$$A.2^n = 3.A.2^{(n-1)} + 2^n$$

$$(A-1).\cancel{2^n} = 3.A.\cancel{2^{(n-1)}}$$

$$2A-2 = 3A$$

$$\boxed{a_n^{(p)} = -2^{(n+1)}} \xleftarrow{\text{so}} \boxed{A = -2}$$

4

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = c_1(3)^n - 2^{(n+1)}$$

$$a_0 = c_1 \cdot 3^0 - 2^1 =$$

$$a_0 = c_1 - 2 = 1$$

$$\boxed{c_1 = 3}$$

5

$$a_n = 3^{n+1} - 2^{n+1}$$

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Problem 2

$$F(n) = \underbrace{4f(n-1) - 4f(n-2)}_{F_n^{(h)}} + \underbrace{n^2}_{F_n^{(p)}}$$

$$r^2 - 4r + 4 = 0$$

$$\boxed{r = 2 \quad r = 2}$$

$$\boxed{F(n)^{(h)} = c_1(2)^n + c_2(2)^n . n}$$

$$F(n)^{(p)} = A.n^2 + Bn + C$$

$$F_{(n-1)}^{(p)} = A(n-1)^2 + B(n-1) + C$$

$$F(n-2)^{(p)} = A(n-2) + B(n-2) + C$$

$$F(n)^{(p)} = 4F(n)^{(p)} - 4F(n)^{(p)} + n^2$$

$$A.n^2 + 8n + C = A(-12 + 8n) + n^2 + 4B$$

$$\boxed{A = 1 \quad B = 8 \quad C = 20}$$

$$\boxed{F_n^{(p)} = n^2 + 8n + 20}$$

$$F(n) = c_1(2)^n + c_2(2)^n . n + 8n + 20$$

$$F(0) = c_1 + 20 = 2$$

$$F(1) = 2c_1 + 2c_2 + 29 = 5$$

$$\boxed{c_1 = -18 \quad c_2 = 6}$$

$$F(n) = -18 . 2^n + 6(2)^n . n + n^2 + 8n + 20$$

$$\boxed{F(n) = n^2 + 8n + 3 . 2^{(n+1)} . (n-3) + 20}$$

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Problem 3

1) finding characteristic roots of the equation

$$a_n = 2a_{(n-1)} - 2a_{(n-2)}$$

$$r^2 = 2r - 2$$

$$r^2 - 2r + 2 = 0$$

The roots are not reel, so we solve this by using discriminant equation.

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2-4.1.(-2)}}{2.1}$$

$$r = 1 \pm \sqrt{3}$$

$$\boxed{r_1 = 1+\sqrt{3}} \quad \boxed{r_2 = 1-\sqrt{3}}$$

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$$a_n = c_1(1+\sqrt{3})^n + c_2(1-\sqrt{3})^n$$

$$a_0 = c_1 + c_2 = 1$$

$$a_1 = c_1(1+\sqrt{3}) + c_2(1-\sqrt{3}) = 2$$

$$c_1 = \frac{\sqrt{3}+1}{2\sqrt{3}} \qquad c_2 = \frac{3-\sqrt{3}}{6}$$

$$\boxed{a_n = \frac{\sqrt{3}+1}{2\sqrt{3}}(1+\sqrt{3})^n + \left(\frac{3-\sqrt{3}}{6}\right)(1-\sqrt{3})^n}$$