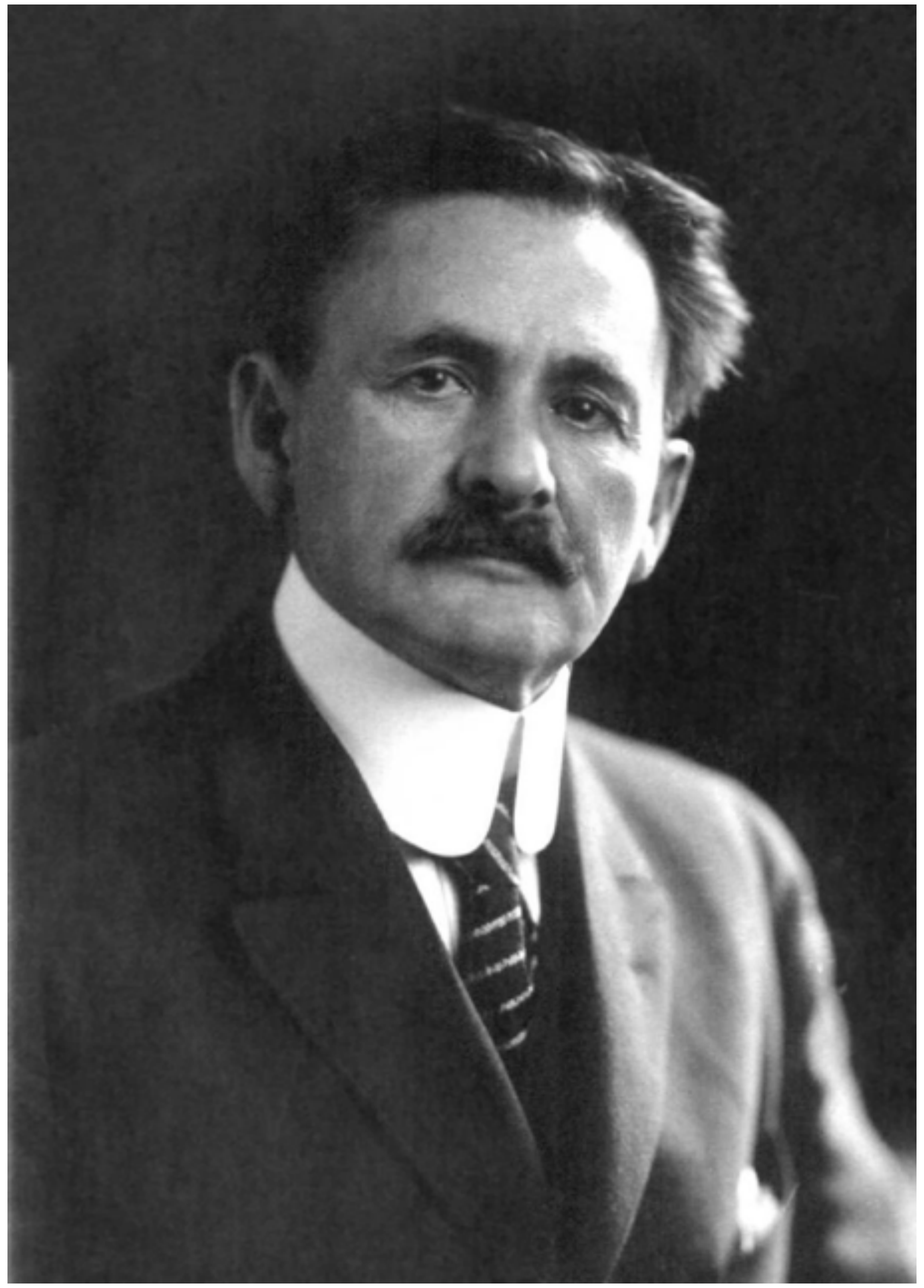
STATISTICAL THINKING IN PYTHON I

Probability density functions

Continuous variables

- Quantities that can take any value, not just discrete values

Michelson's speed of light experiment



measured speed of light (1000 km/s)

299.85	299.74	299.90	300.07	299.93
299.85	299.95	299.98	299.98	299.88
300.00	299.98	299.93	299.65	299.76
299.81	300.00	300.00	299.96	299.96
299.96	299.94	299.96	299.94	299.88
299.80	299.85	299.88	299.90	299.84
299.83	299.79	299.81	299.88	299.88
299.83	299.80	299.79	299.76	299.80
299.88	299.88	299.88	299.86	299.72
299.72	299.62	299.86	299.97	299.95
299.88	299.91	299.85	299.87	299.84
299.84	299.85	299.84	299.84	299.84
299.89	299.81	299.81	299.82	299.80
299.77	299.76	299.74	299.75	299.76
299.91	299.92	299.89	299.86	299.88
299.72	299.84	299.85	299.85	299.78
299.89	299.84	299.78	299.81	299.76
299.81	299.79	299.81	299.82	299.85
299.87	299.87	299.81	299.74	299.81
299.94	299.95	299.80	299.81	299.87

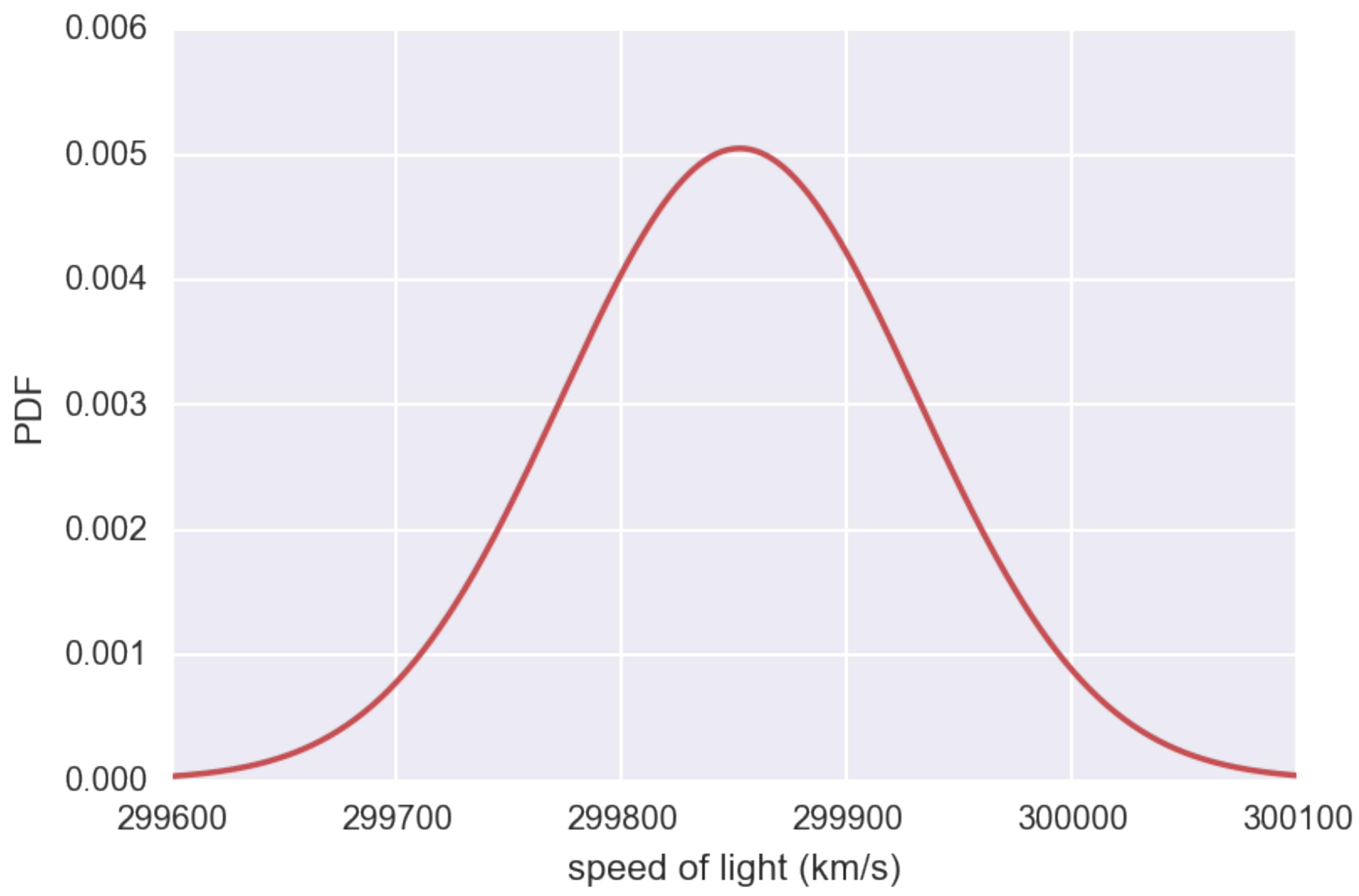
Image: public domain, Smithsonian

Data: Michelson, 1880

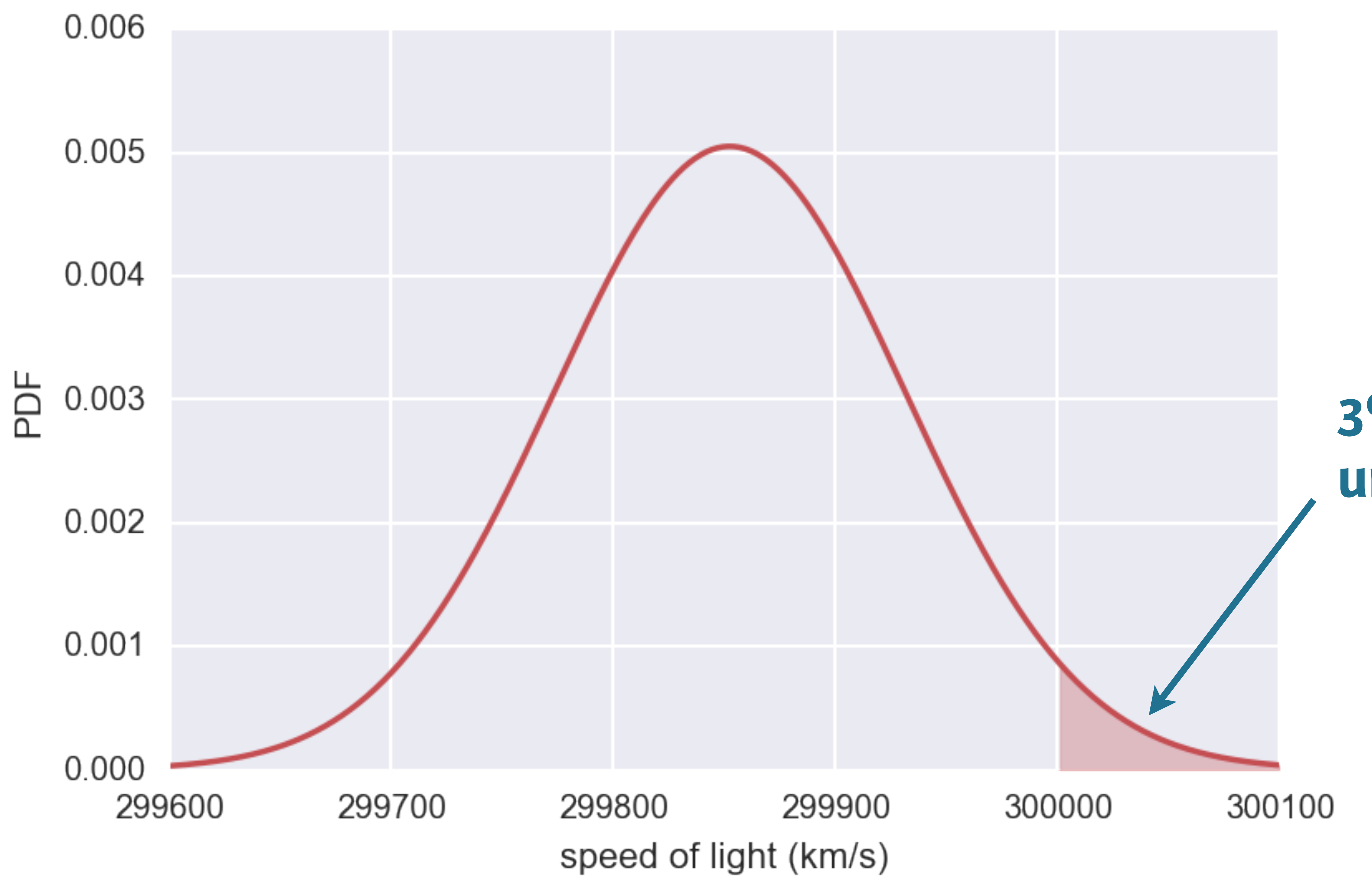
Probability density function (PDF)

- Continuous analog to the PMF
- Mathematical description of the relative likelihood of observing a value of a continuous variable

Normal PDF

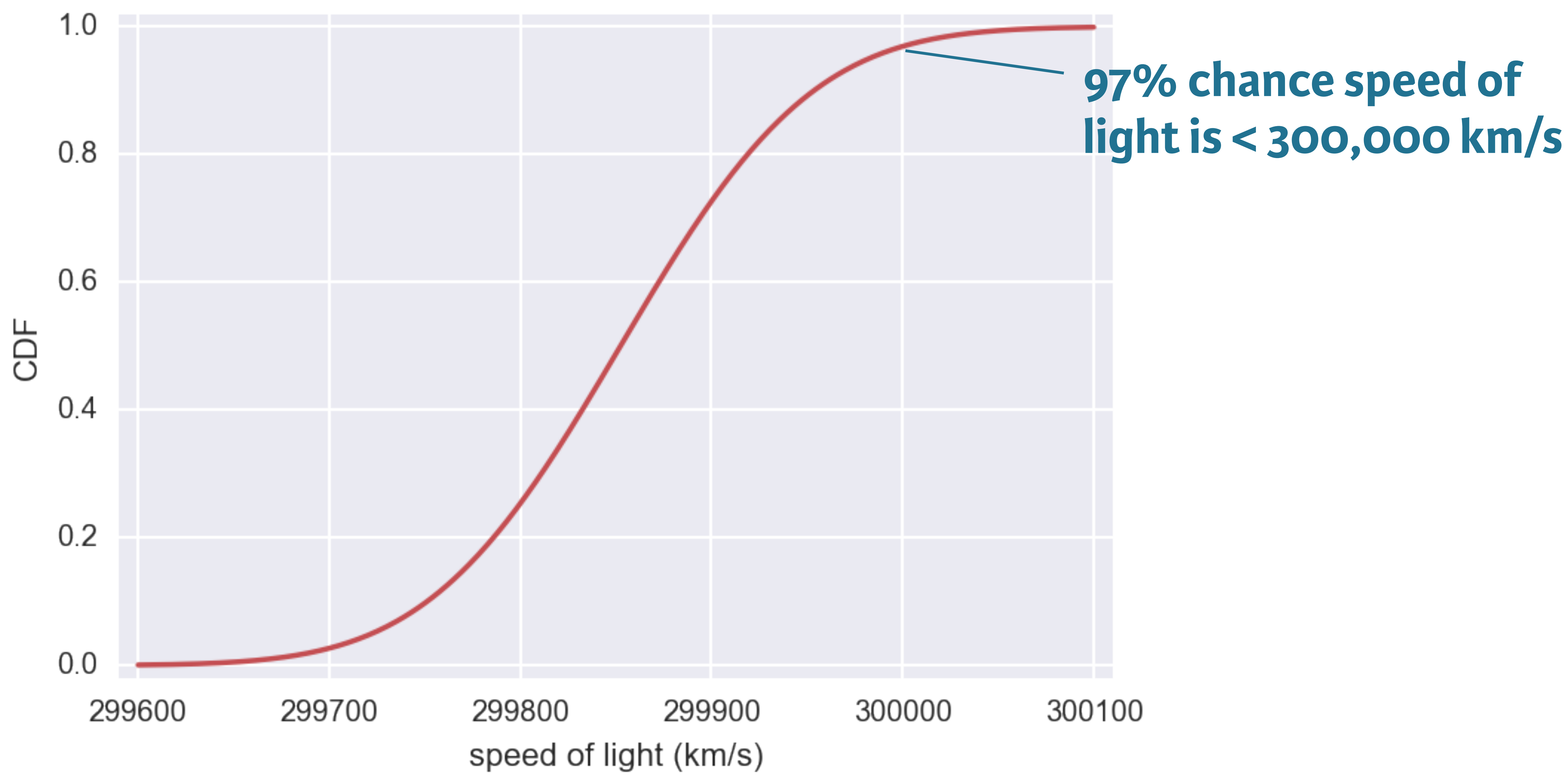

0.006
0.005
0.004
0.003
0.002
0.001
0.000
PDF
299600
299700
299800
299900
300000
300100
speed of light (km/s)

Normal PDF



3% of total area under PDF

Normal CDF


1.0
0.8
0.6
0.4
0.2
0.0
CDF
299600
299700
299800
299900
300000
300100
speed of light (km/s)
97% chance speed of light is < 300,000 km/s

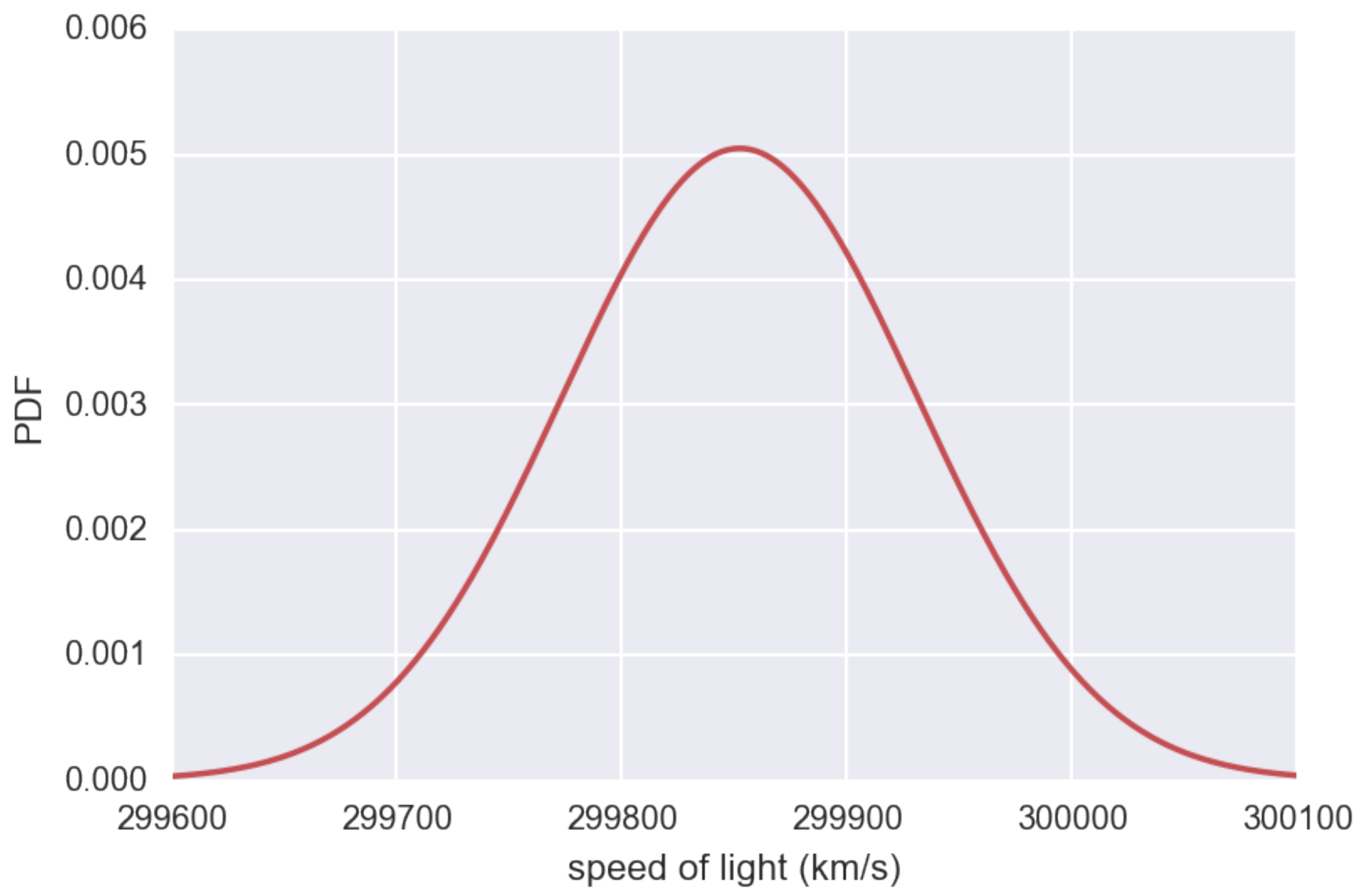
Let's practice!

Introduction to the Normal distribution

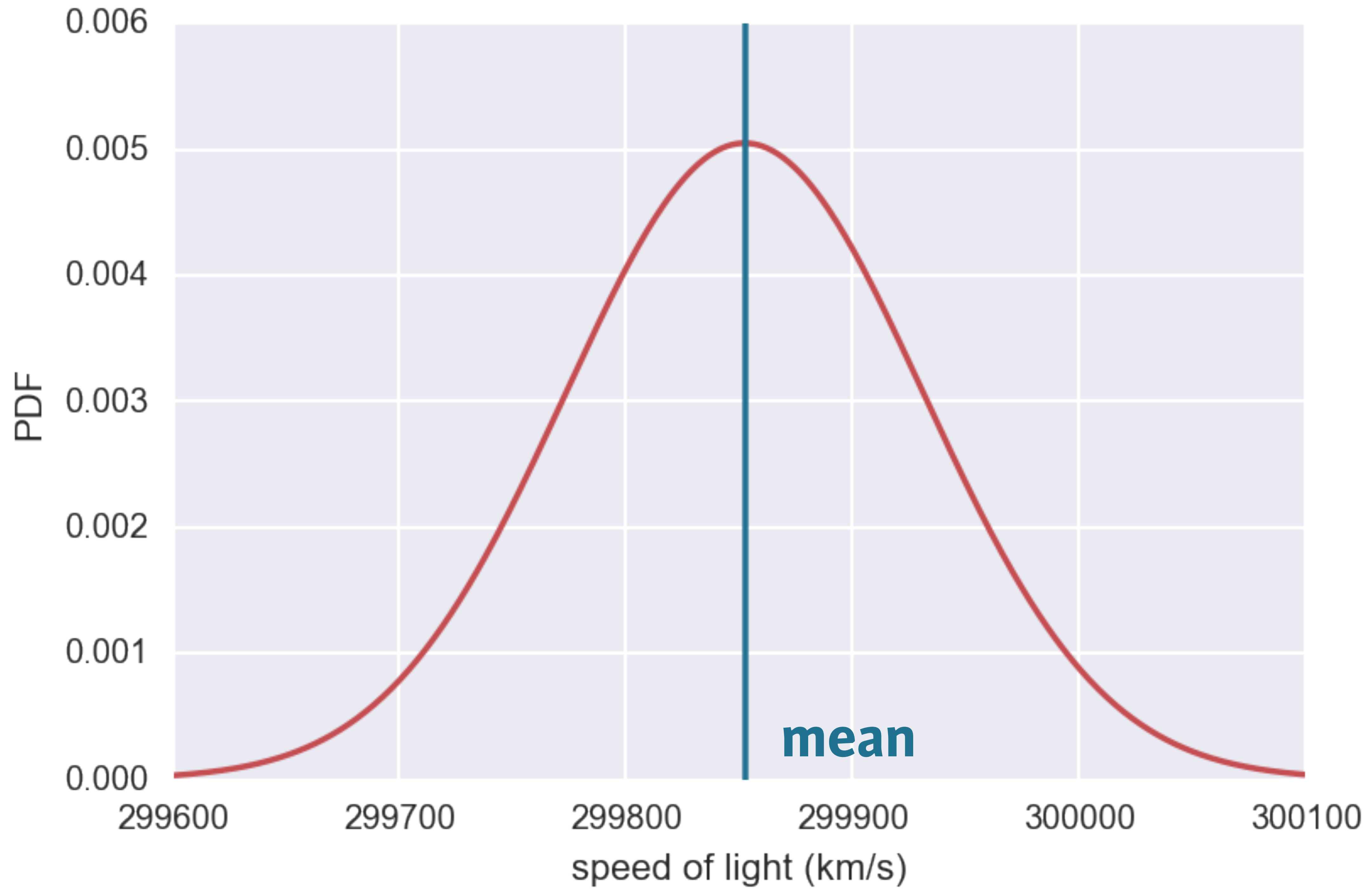
Normal distribution

- Describes a continuous variable whose PDF has a single symmetric peak.

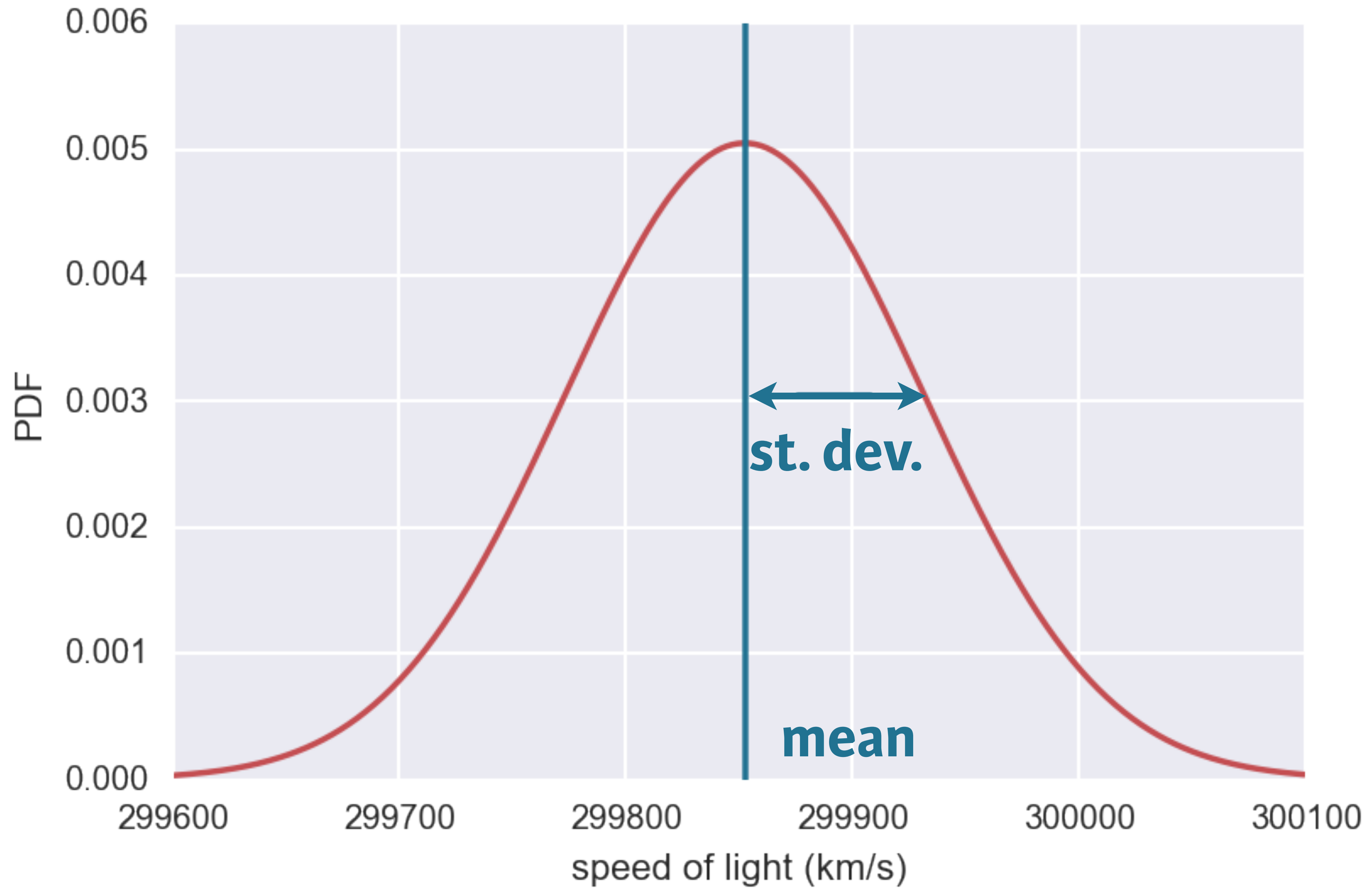
Normal distribution



Normal distribution

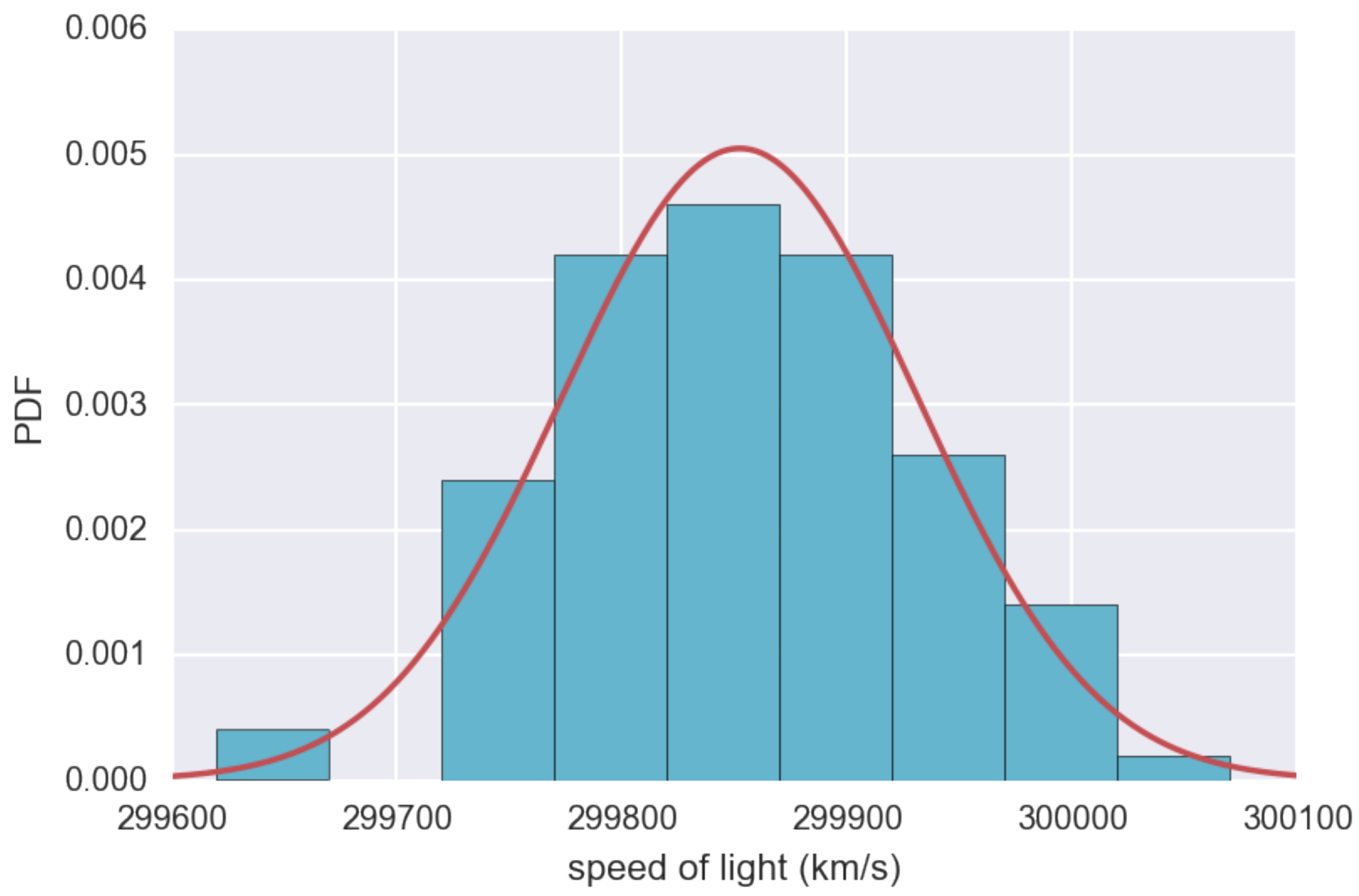


Normal distribution



Parameter		Calculated from data
mean of a Normal distribution	$\neq$	mean computed from data
st. dev. of a Normal distribution	$\neq$	standard deviation computed from data

Comparing data to a Normal PDF



Data: Michelson, 1880

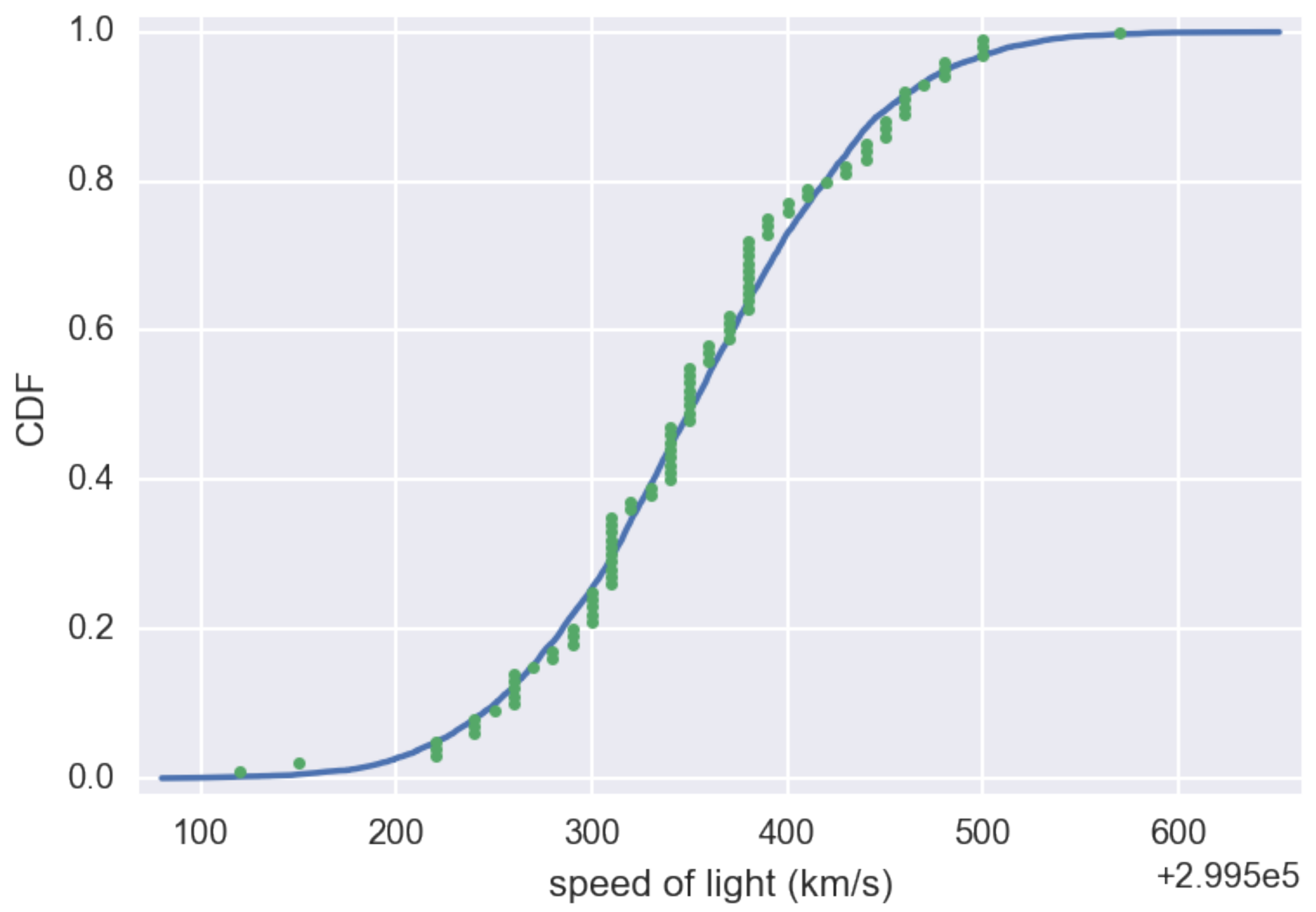
Checking Normality of Michelson data

```
In [1]: import numpy as np

In [2]: mean = np.mean(michelson_speed_of_light)

In [3]: std = np.std(michelson_speed_of_light)

In [4]: samples = np.random.normal(mean, std, size=10000)

In [5]: x, y = ecdf(michelson_speed_of_light)

In [6]: x_theor, y_theor = ecdf(samples)
```

Checking Normality of Michelson data

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: sns.set()

In [4]: _ = plt.plot(x_theor, y_theor)

In [5]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [6]: _ = plt.xlabel('speed of light (km/s)')

In [7]: _ = plt.ylabel('CDF')

In [8]: plt.show()
```

Checking Normality of Michelson data



Data: Michelson, 1880

Let’s practice!

The Normal distribution: Properties and warnings

Image: Deutsche Bundesbank

The Gaussian distribution

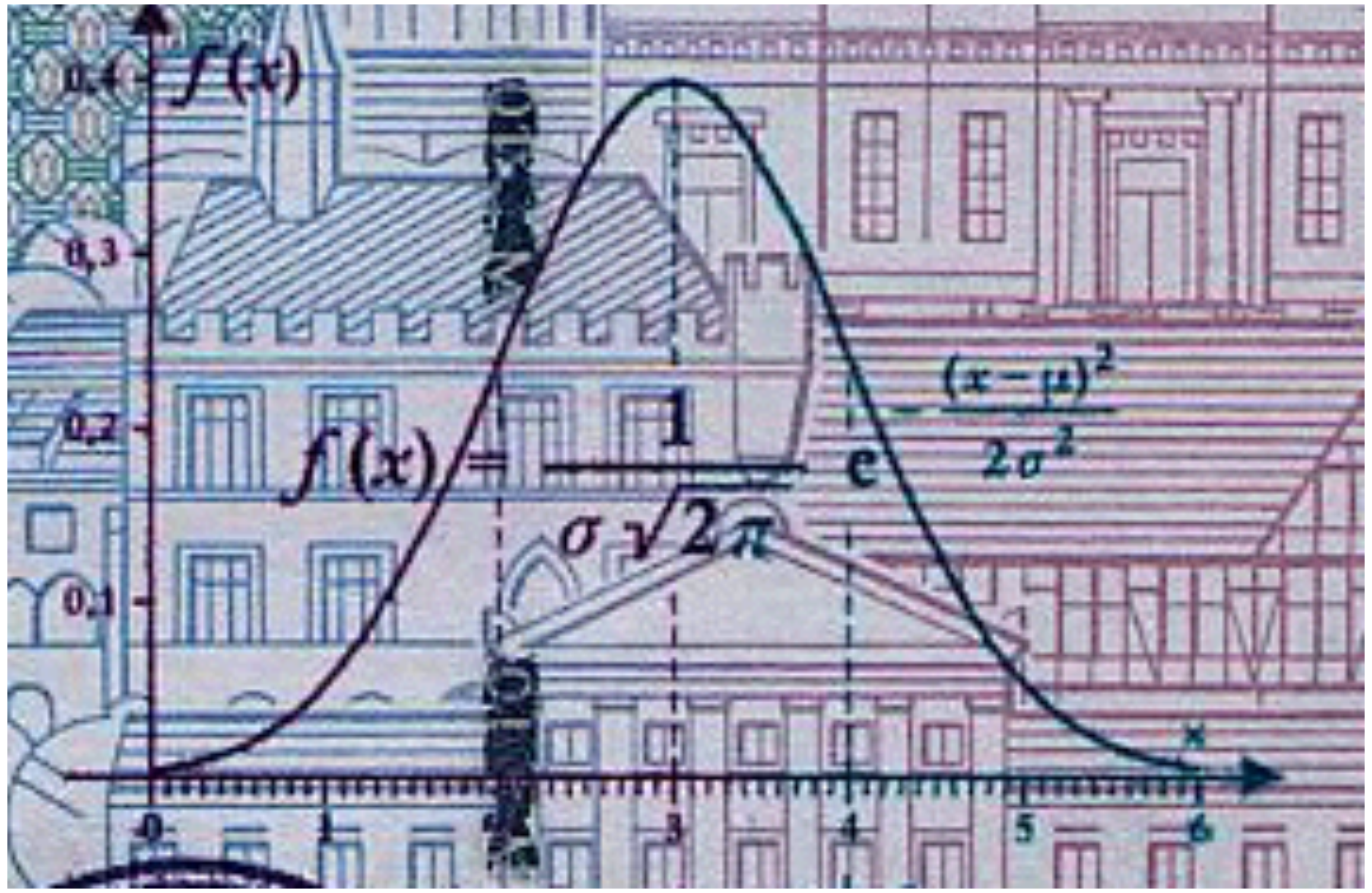
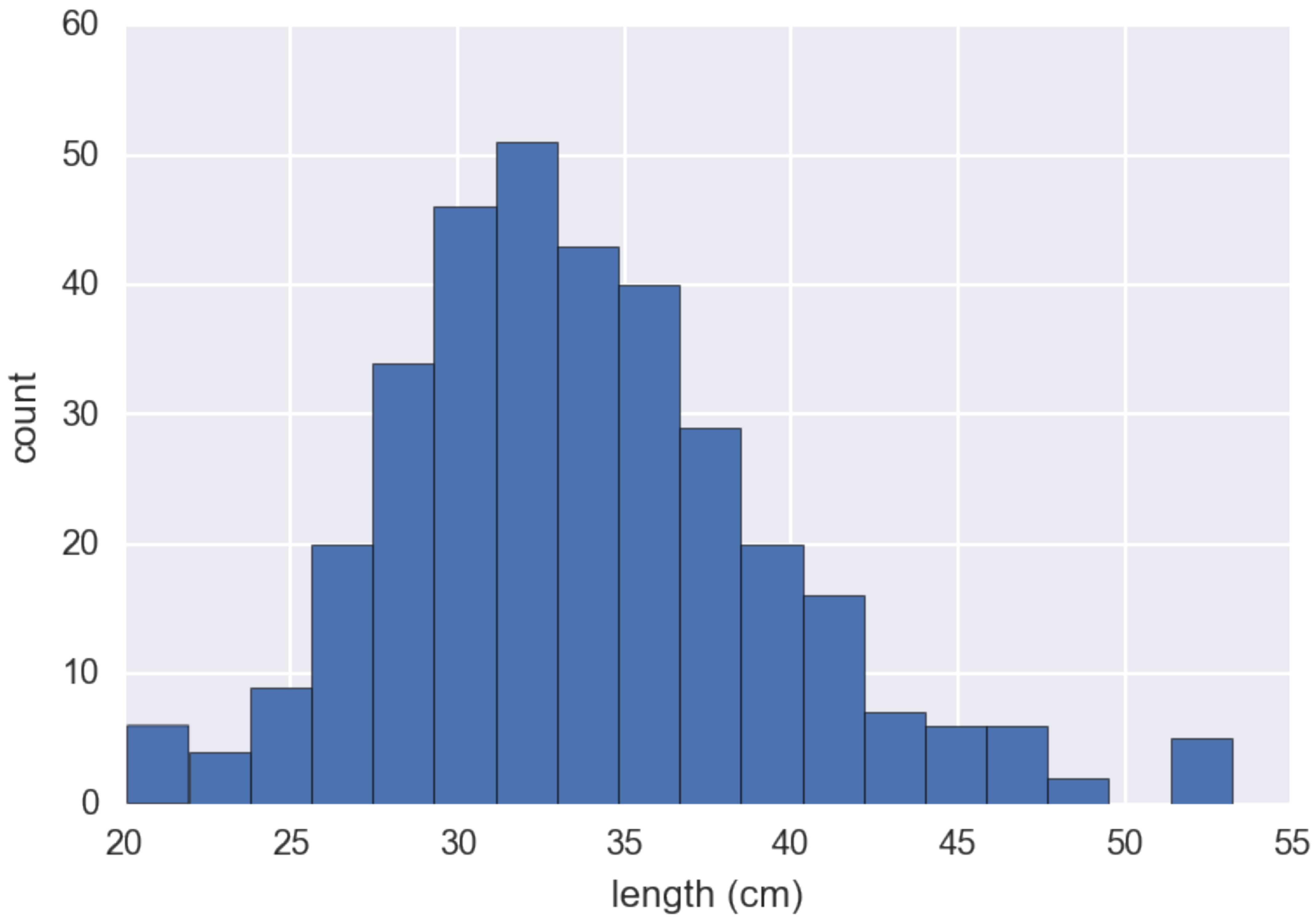


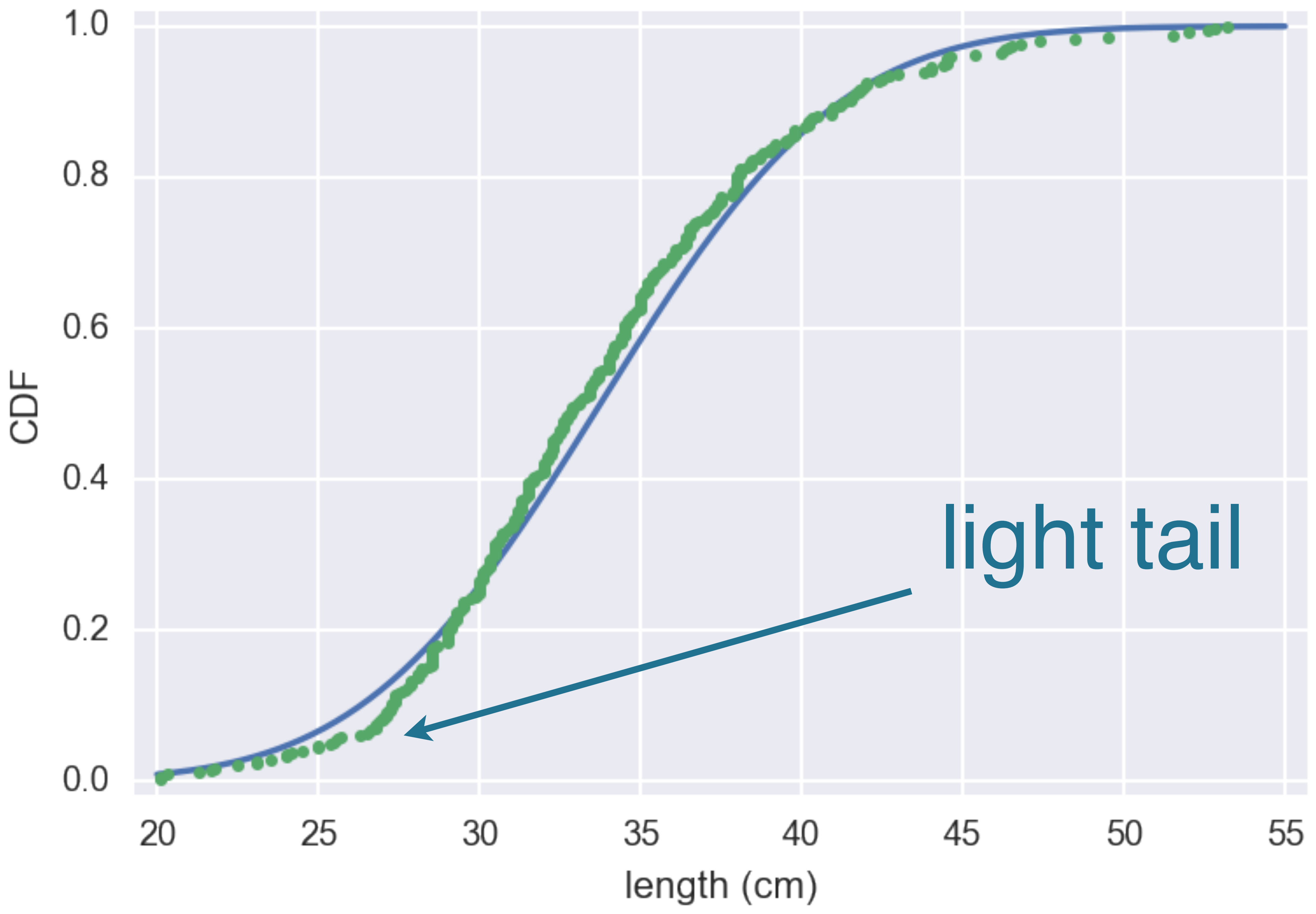
Image: Deutsche Bundesbank

Length of MA large mouth bass



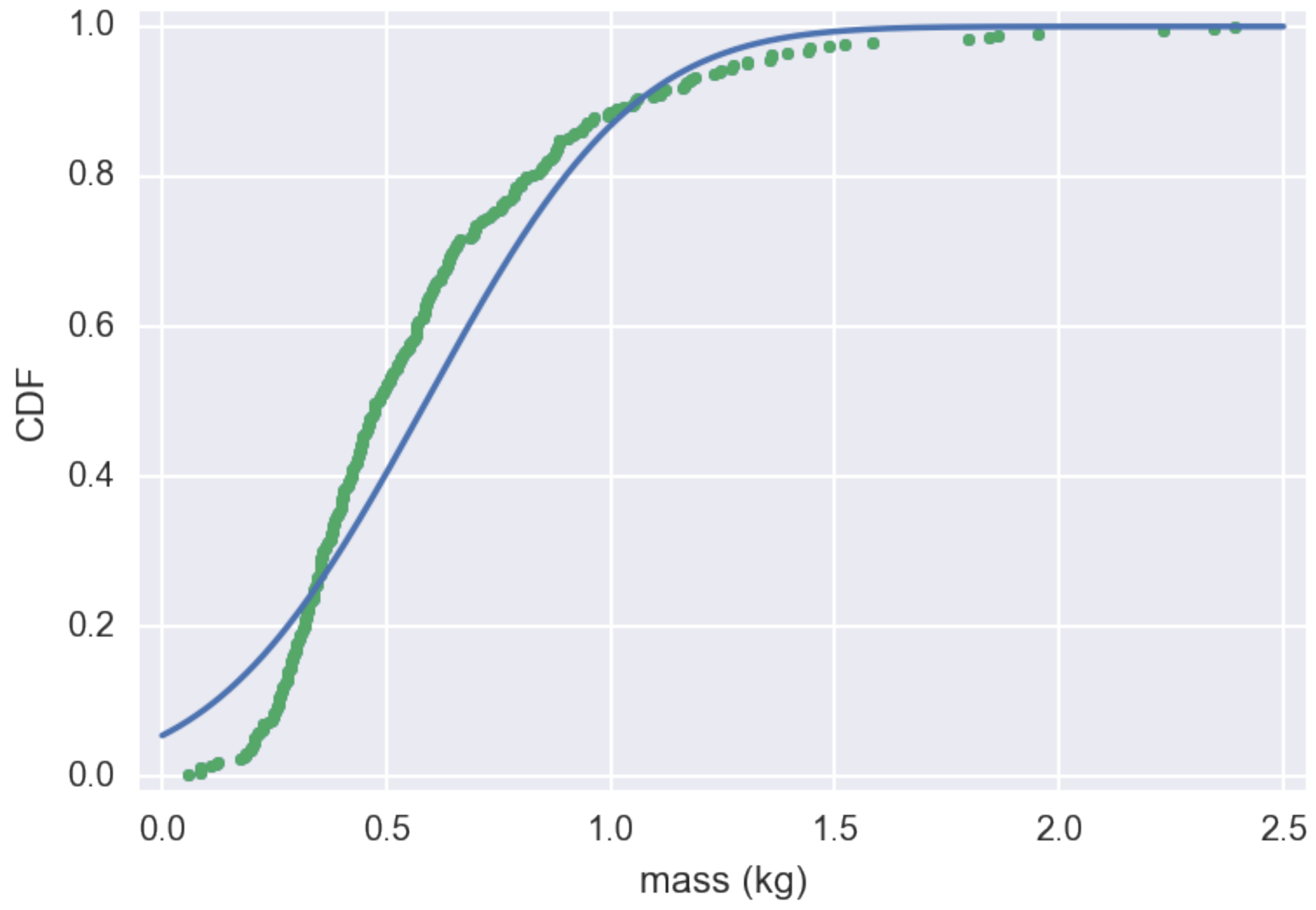
Source: Mass. Dept. of Environmental Protection

Length of MA large mouth bass



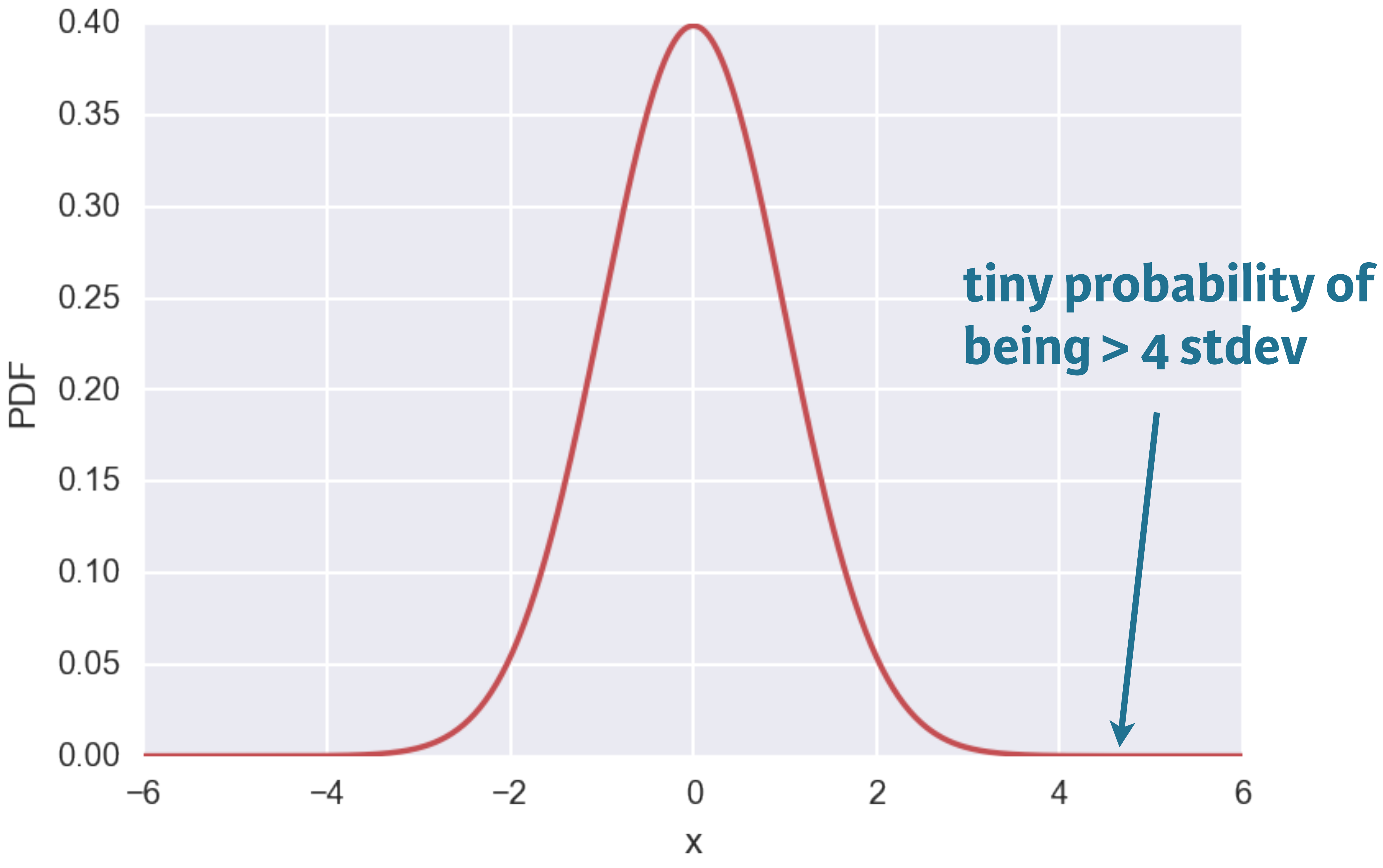
Source: Mass. Dept. of Environmental Protection

Mass of MA large mouth bass



Source: Mass. Dept. of Environmental Protection

Light tails of the Normal distribution



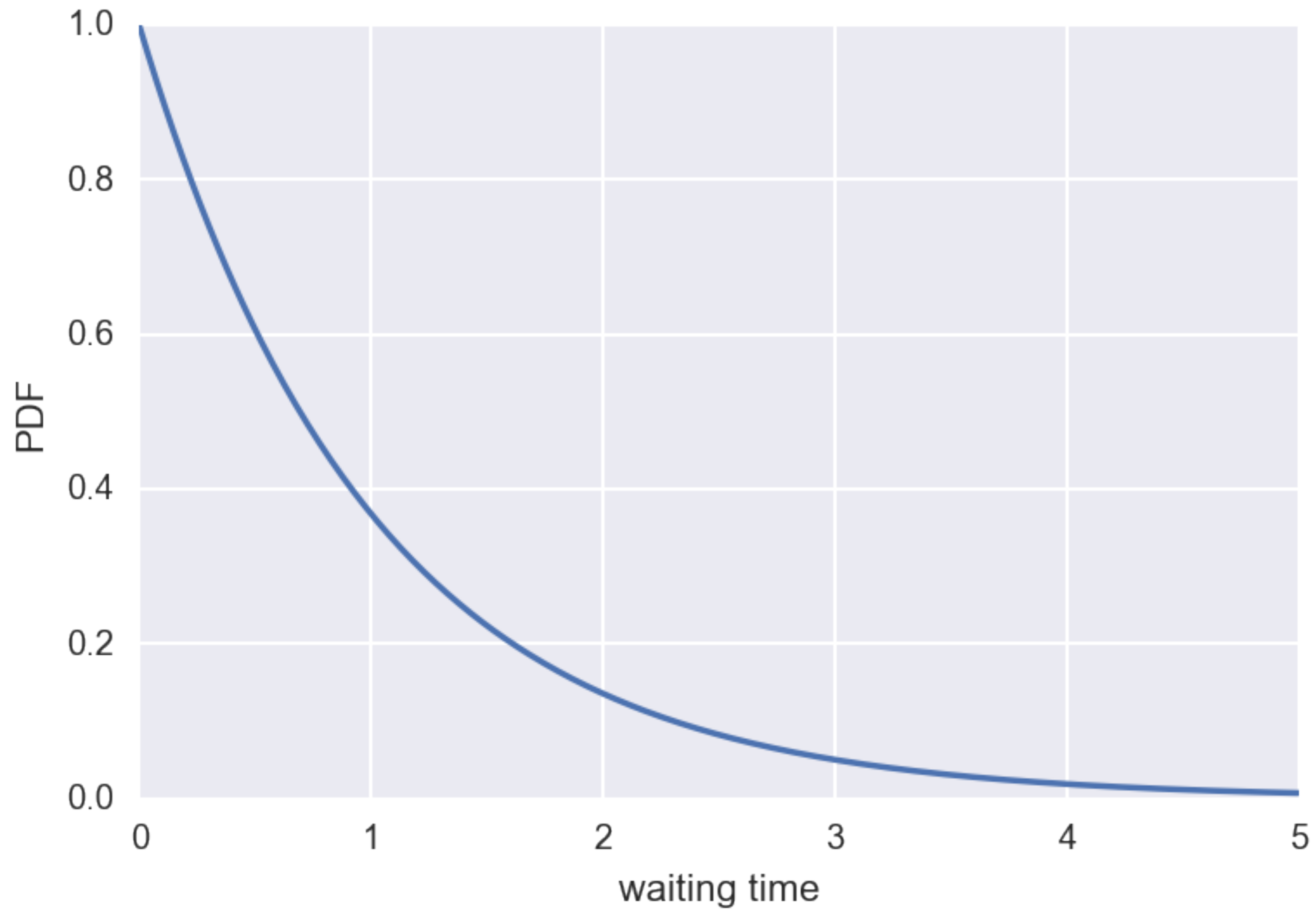
Let's practice!

The Exponential distribution

The Exponential distribution

- The waiting time between arrivals of a Poisson process is Exponentially distributed

The Exponential PDF



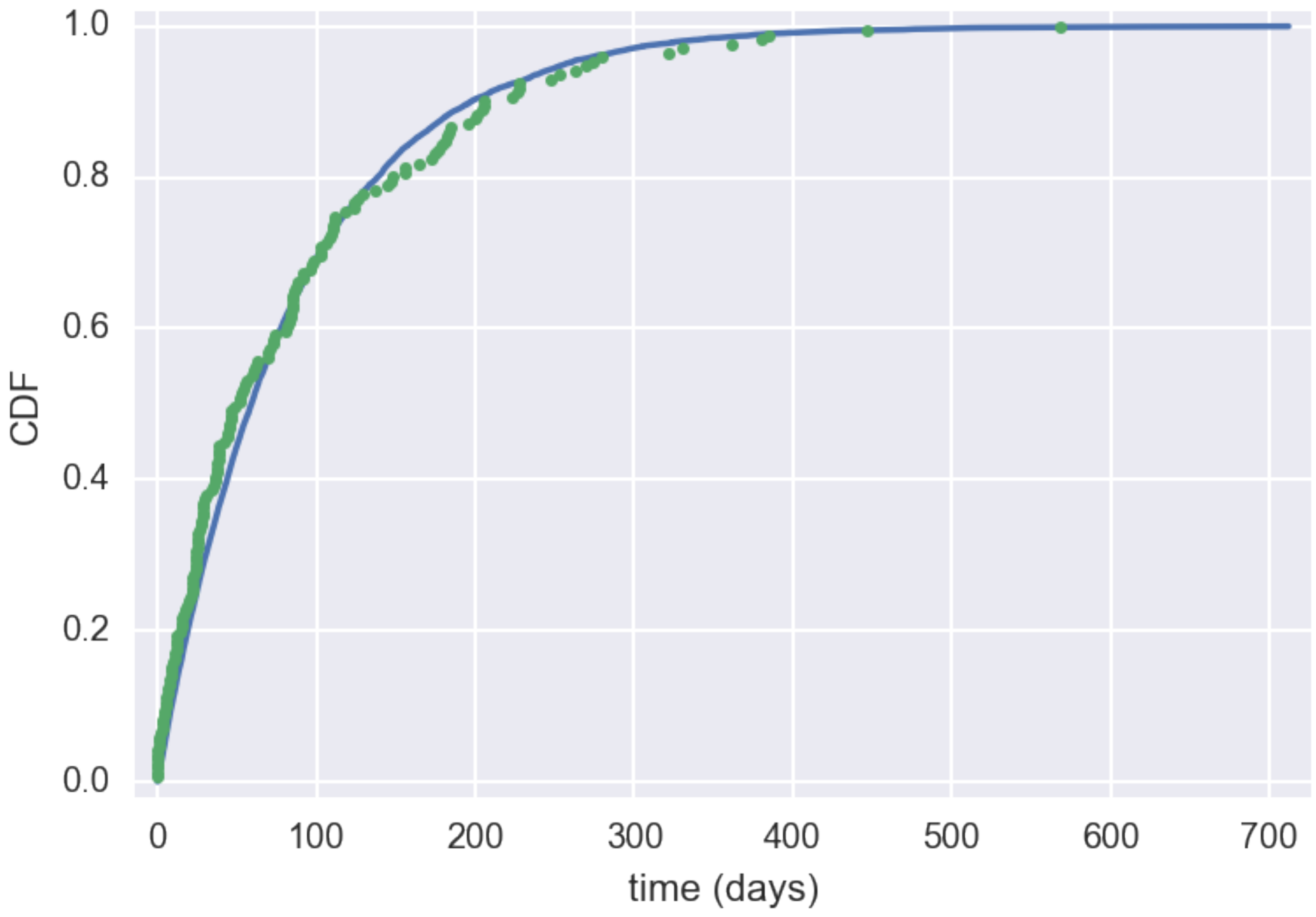
Possible Poisson process

- Nuclear incidents:
  - Timing of one is independent of all others

Exponential inter-incident times

```
In [1]: mean = np.mean(inter_times)

In [2]: samples = np.random.exponential(mean, size=10000)

In [3]: x, y = ecdf(inter_times)

In [4]: x_theor, y_theor = ecdf(samples)

In [5]: _ = plt.plot(x_theor, y_theor)

In [6]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [7]: _ = plt.xlabel('time (days)')

In [8]: _ = plt.ylabel('CDF')

In [9]: plt.show()
```

Exponential inter-incident times



Data Source: Wheatley, Sovacool, Sornette, Nuclear Events Database

Let’s practice!

Final thoughts

You now can...

- Construct (beautiful) instructive plots
- Compute informative summary statistics
- Use hacker statistics
- Think probabilistically

In the sequel, you will...

- Estimate parameter values
- Perform linear regressions
- Compute confidence intervals
- Perform hypothesis tests

See you in the sequel!