



UNSUPERVISED LEARNING IN PYTHON

Visualizing the PCA transformation

Dimension reduction

- More efficient storage and computation
- Remove less-informative "noise" features
- ... which cause problems for prediction tasks, e.g. classification, regression



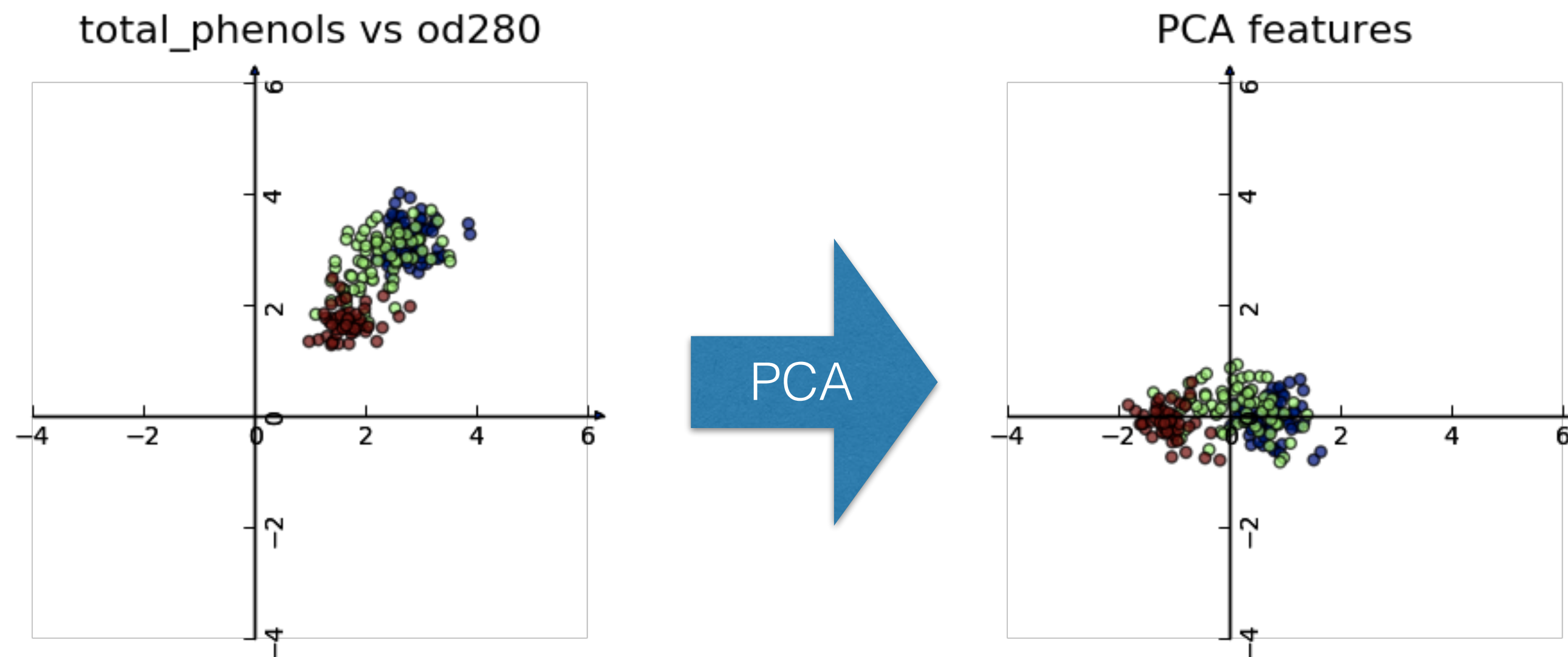
Principal Component Analysis

- PCA = "Principal Component Analysis"
- Fundamental dimension reduction technique
- First step "decorrelation" (considered here)
- Second step reduces dimension (considered later)



PCA aligns data with axes

- Rotates data samples to be aligned with axes
- Shifts data samples so they have mean 0
- No information is lost



PCA follows the fit/transform pattern

- PCA a scikit-learn component like KMeans or StandardScaler
- `fit()` learns the transformation from given data
- `transform()` applies the learned transformation
- `transform()` can also be applied to new data



Using scikit-learn PCA

- `samples` = array of two wine features (total_phenols & od280)

```
In [1]: print(samples)
```

```
[[ 2.8   3.92]
 [ 2.65   3.4 ]
 ...
 [ 2.05   1.6 ]]
```

```
In [2]: from sklearn.decomposition import PCA
```

```
In [3]: model = PCA()
```

```
In [4]: model.fit(samples)
```

```
Out[4]: PCA(copy=True, ...)
```

```
In [5]: transformed = model.transform(samples)
```



PCA features

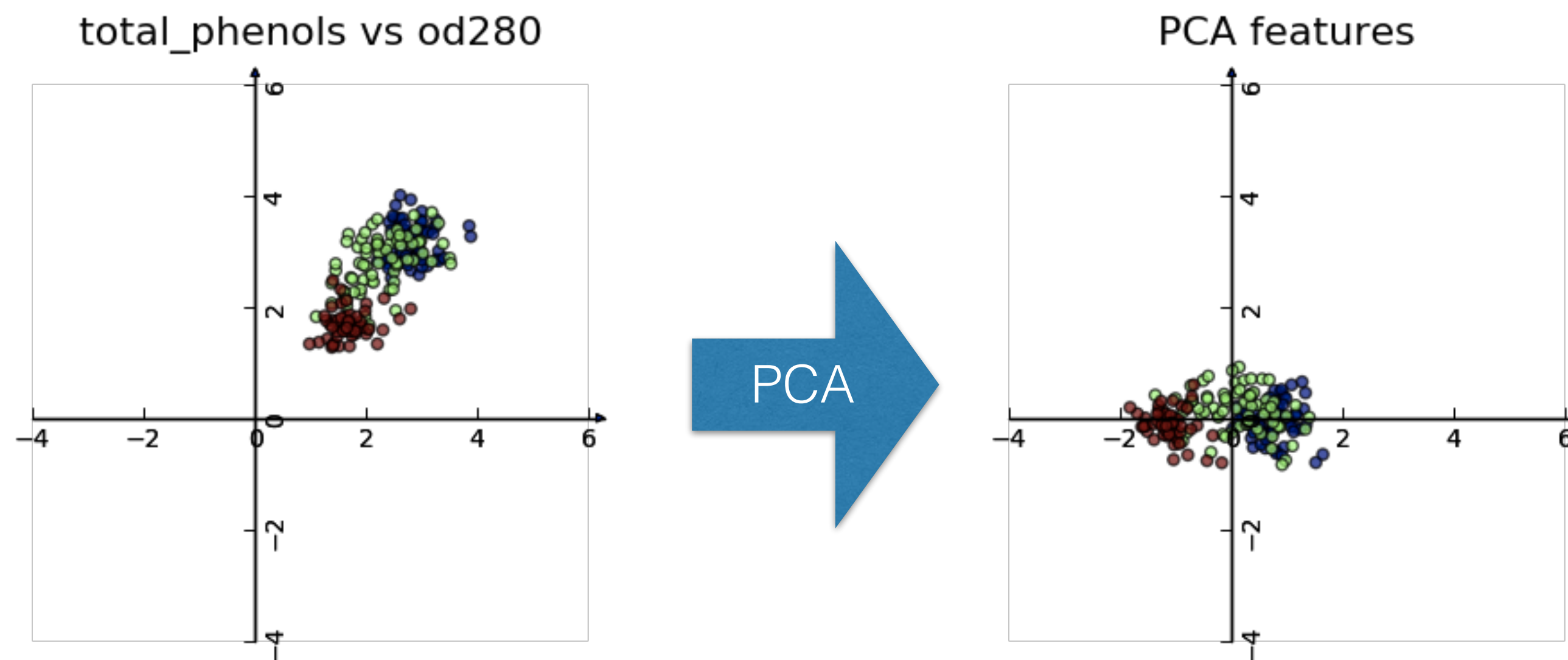
- Rows of transformed correspond to samples
- Columns of transformed are the "PCA features"
- Row gives PCA feature values of corresponding sample

```
In [6]: print(transformed)
[[ 1.32771994e+00  4.51396070e-01]
 [ 8.32496068e-01  2.33099664e-01]
 ...
 [-9.33526935e-01 -4.60559297e-01]]
```



PCA features are not correlated

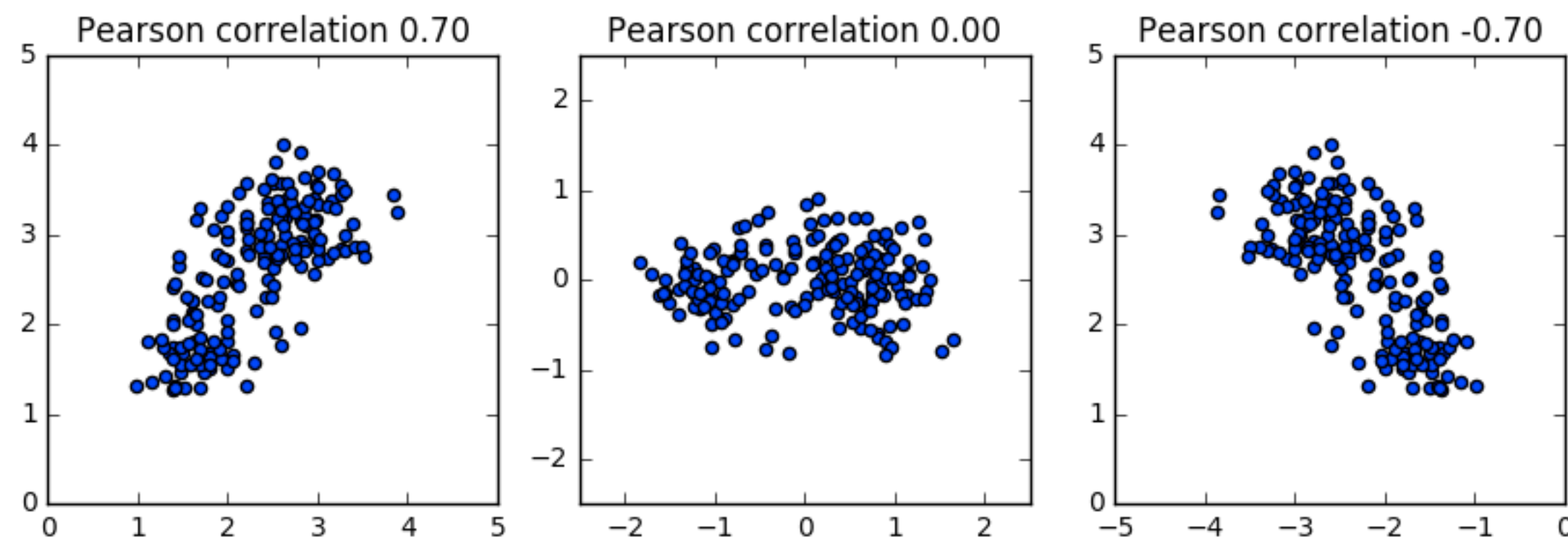
- Features of dataset are often correlated, e.g. total_phenols and od280
- PCA aligns the data with axes
- Resulting PCA features are not linearly correlated ("decorrelation")





Pearson correlation

- Measures linear correlation of features
- Value between -1 and 1
- Value of 0 means no linear correlation

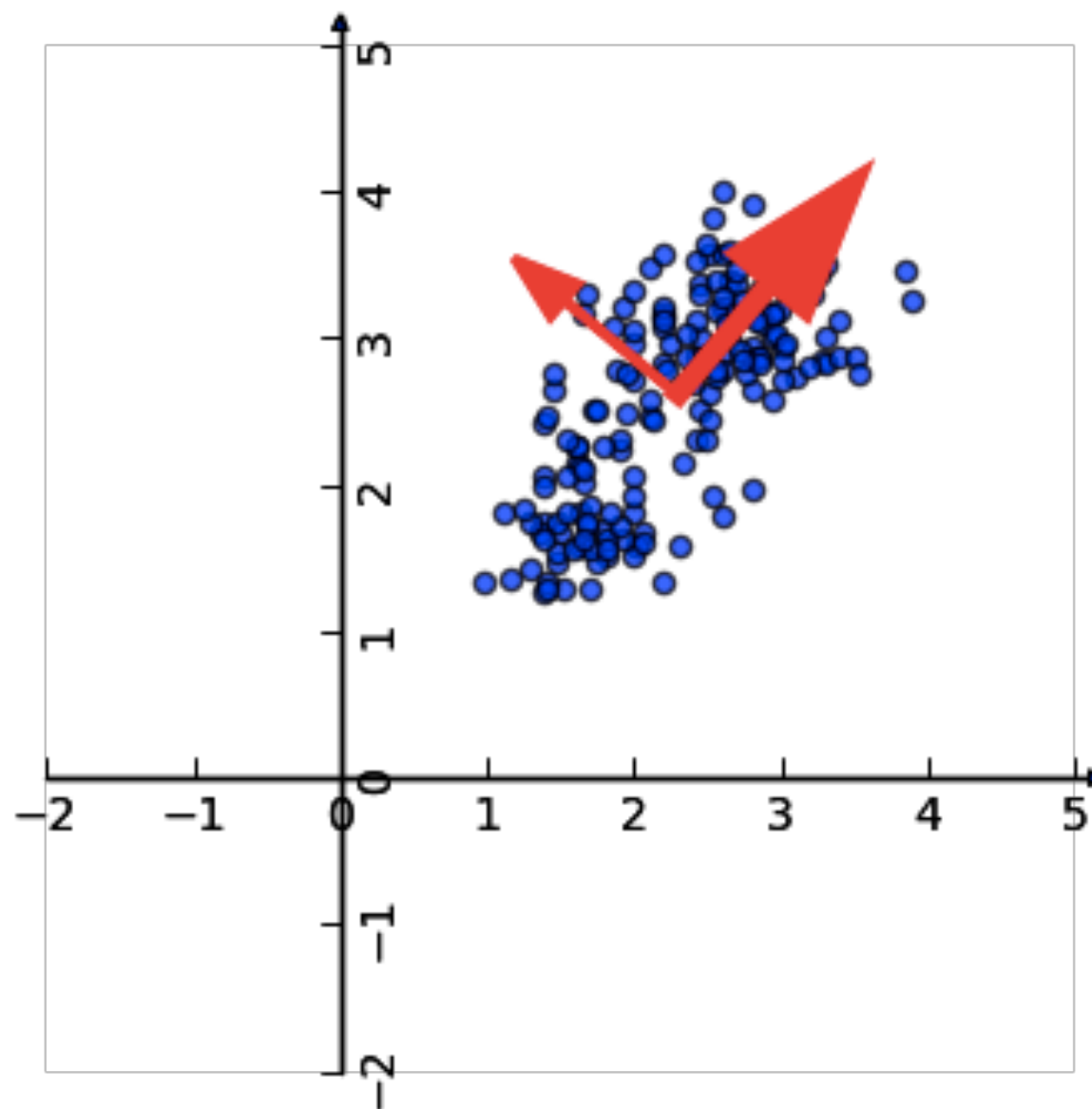




Principal components

- "Principal components" = directions of variance
- PCA aligns principal components with the axes
- Available as `components_` attribute of PCA object
- Each row defines displacement from mean

The Principal Components



```
In [7]: print(model.components_)  
[[ 0.64116665  0.76740167]  
 [-0.76740167  0.64116665]]
```



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Let's practice!



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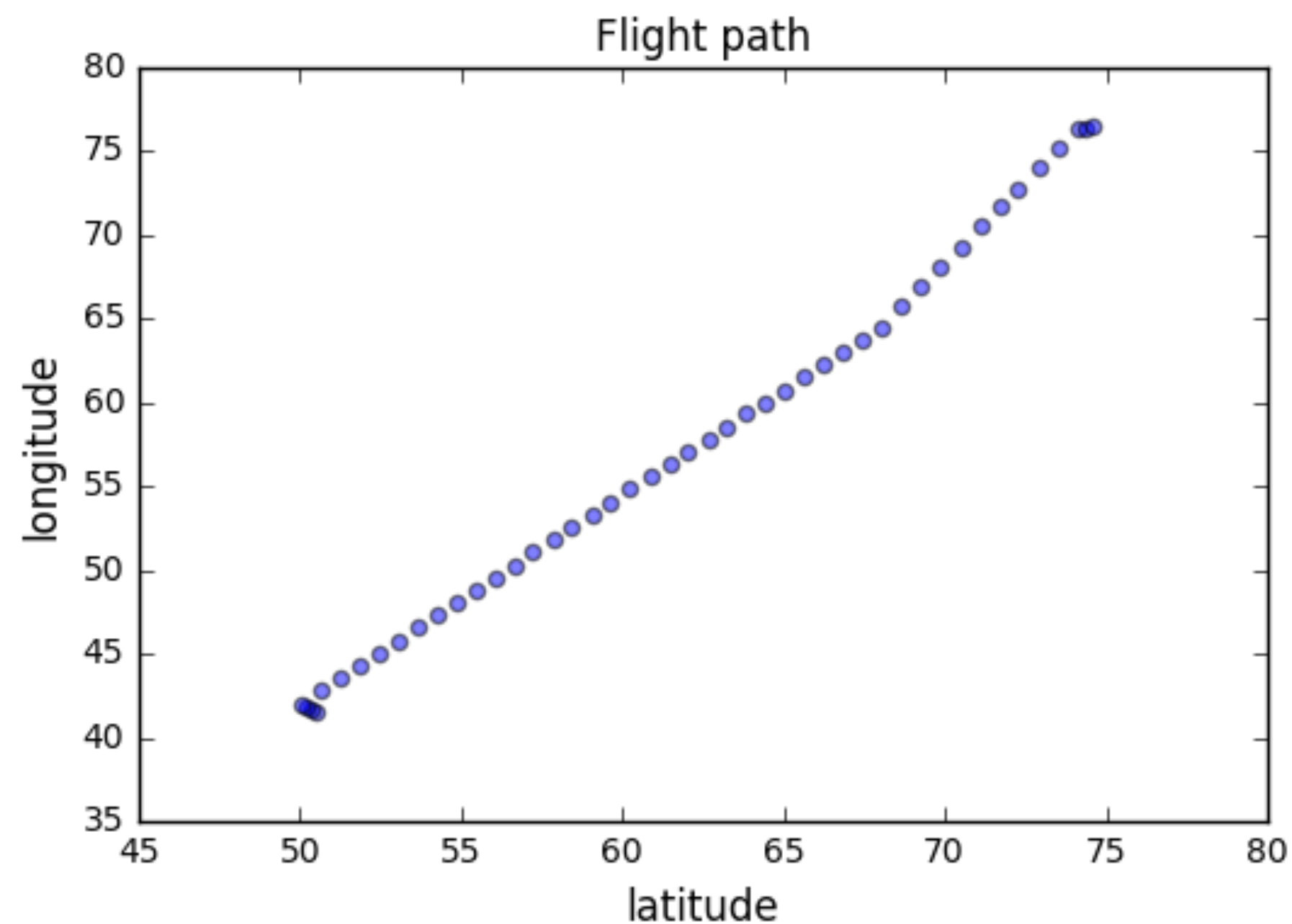
Intrinsic dimension



Intrinsic dimension of a flight path

- 2 features: longitude and latitude at points along a flight path
- Dataset *appears* to be 2-dimensional
- But can approximate using one feature: displacement along flight path
- Is intrinsically 1-dimensional

```
latitude longitude
50.529      41.513
50.360      41.672
50.196      41.835
50.035      42.015
50.678      42.796
51.281      43.526
...
```





Intrinsic dimension

- Intrinsic dimension = number of features needed to approximate the dataset
- Essential idea behind dimension reduction
- What is the most compact representation of the samples?
- Can be detected with PCA



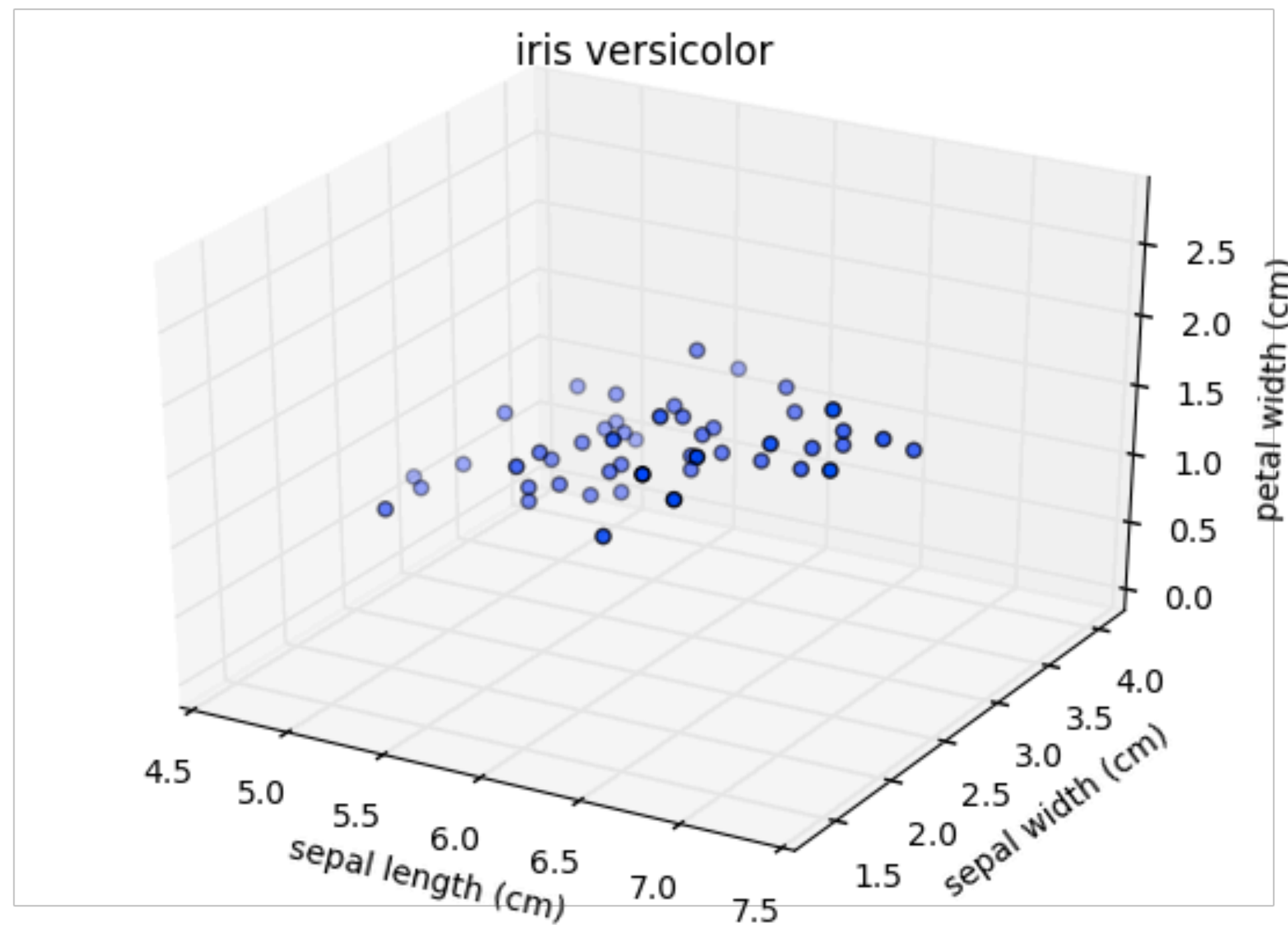
Versicolor dataset

- "versicolor", one of the iris species
- Only 3 features: sepal length, sepal width, and petal width
- Samples are points in 3D space



Versicolor dataset has intrinsic dimension 2

- Samples lie close to a flat 2-dimensional sheet
- So can be approximated using 2 features

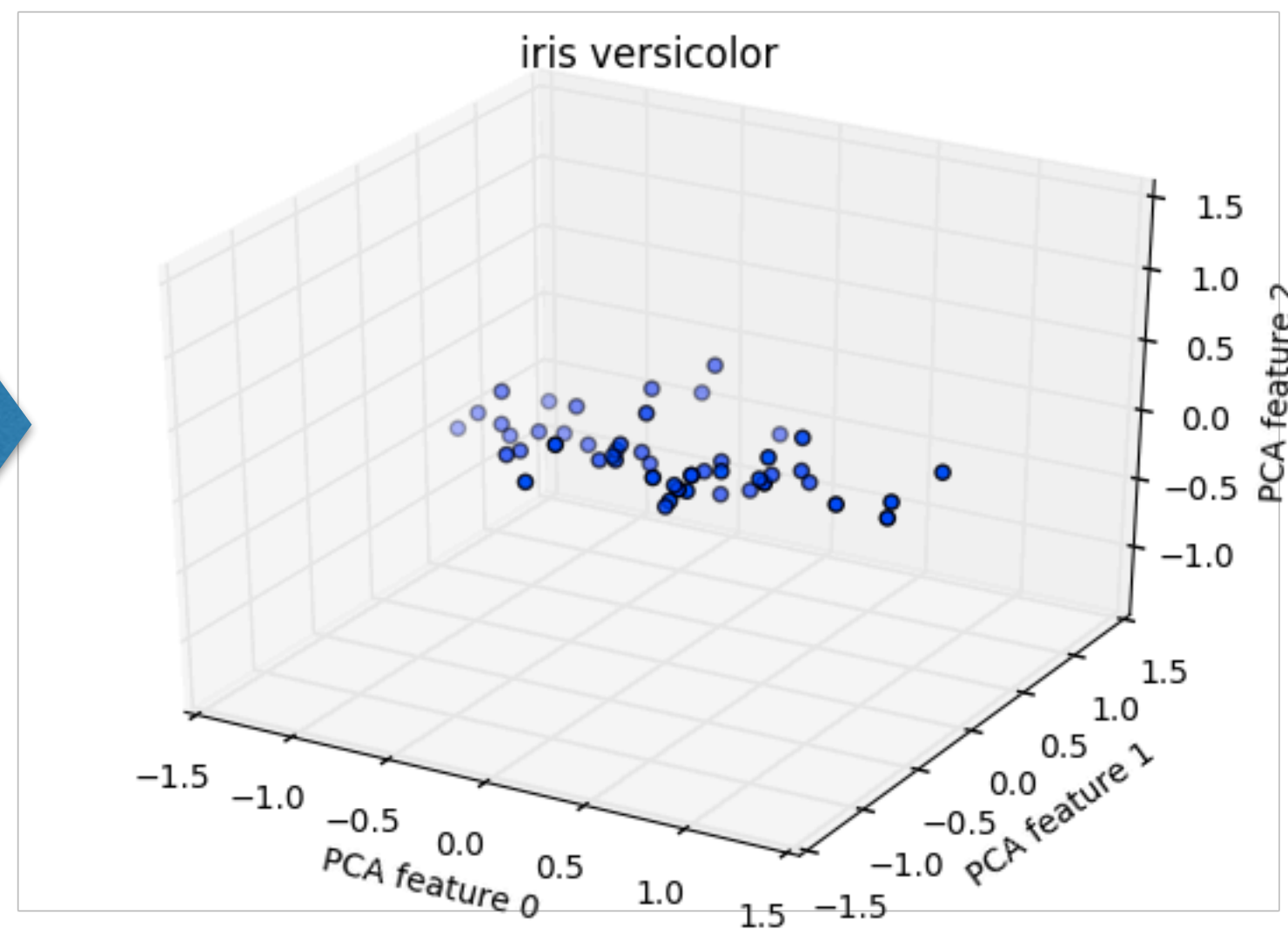
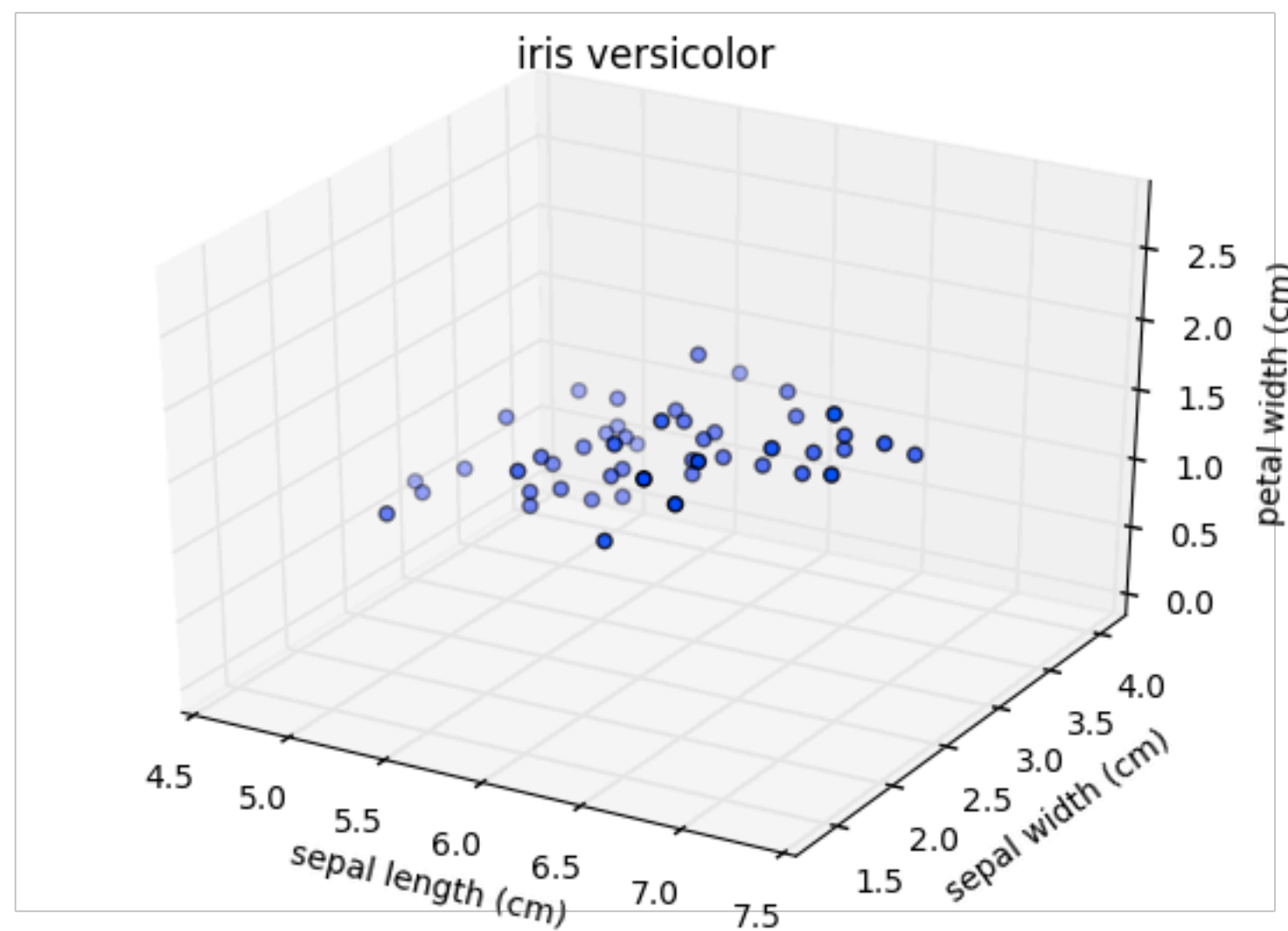


PCA identifies intrinsic dimension

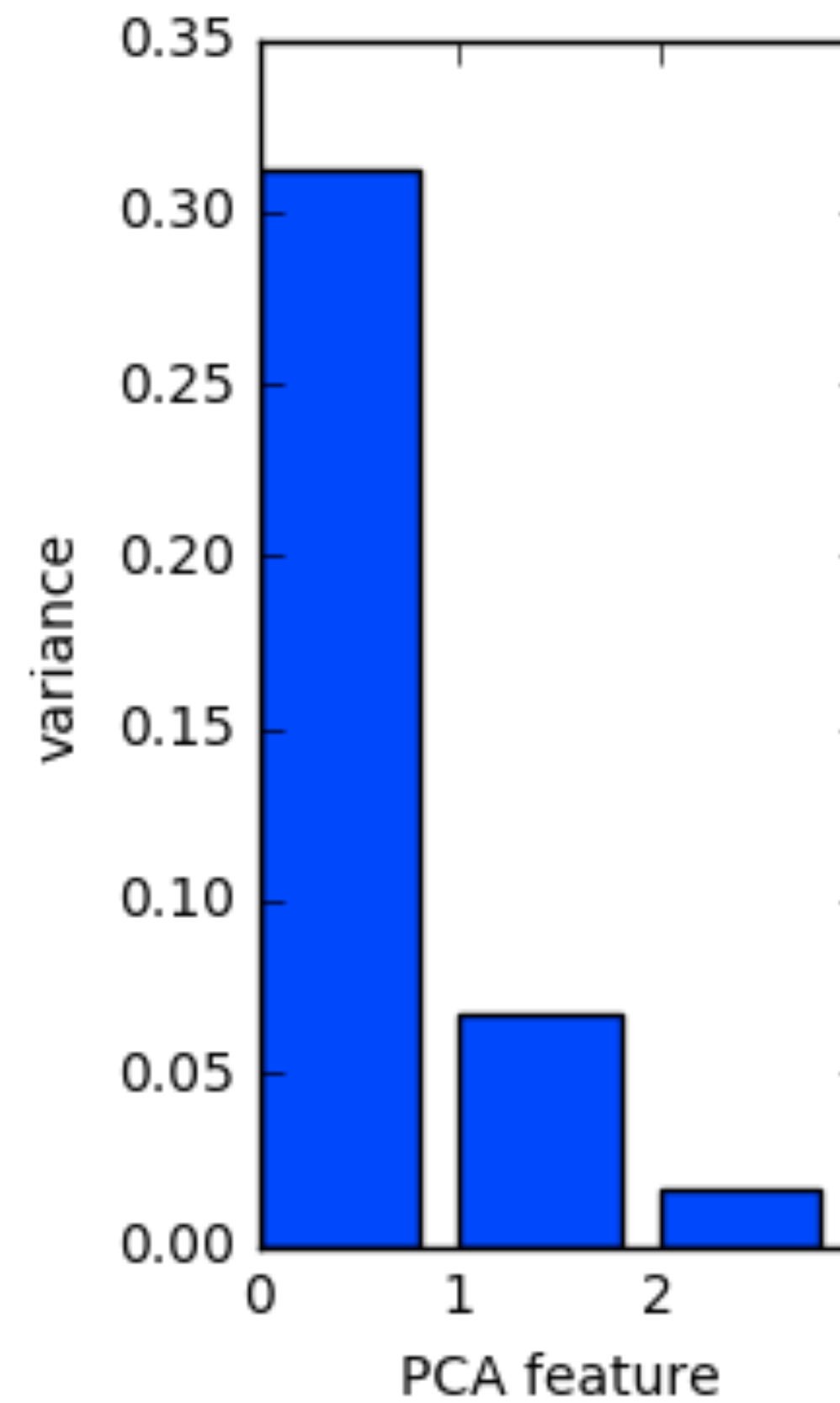
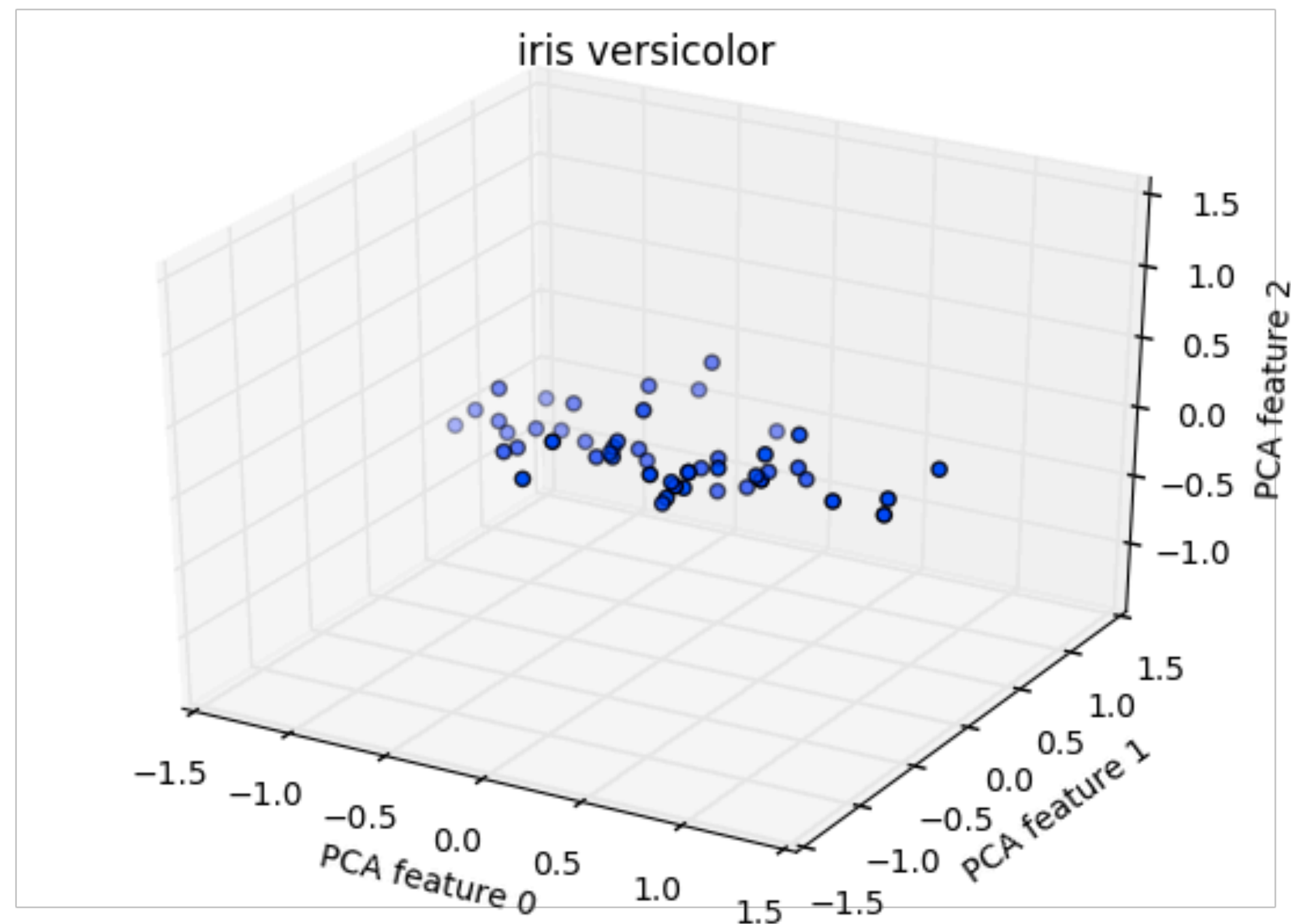
- Scatter plots work only if samples have 2 or 3 features
- PCA identifies intrinsic dimension when samples have *any number* of features
- Intrinsic dimension = number of PCA features with significant variance



PCA of the versicolor samples



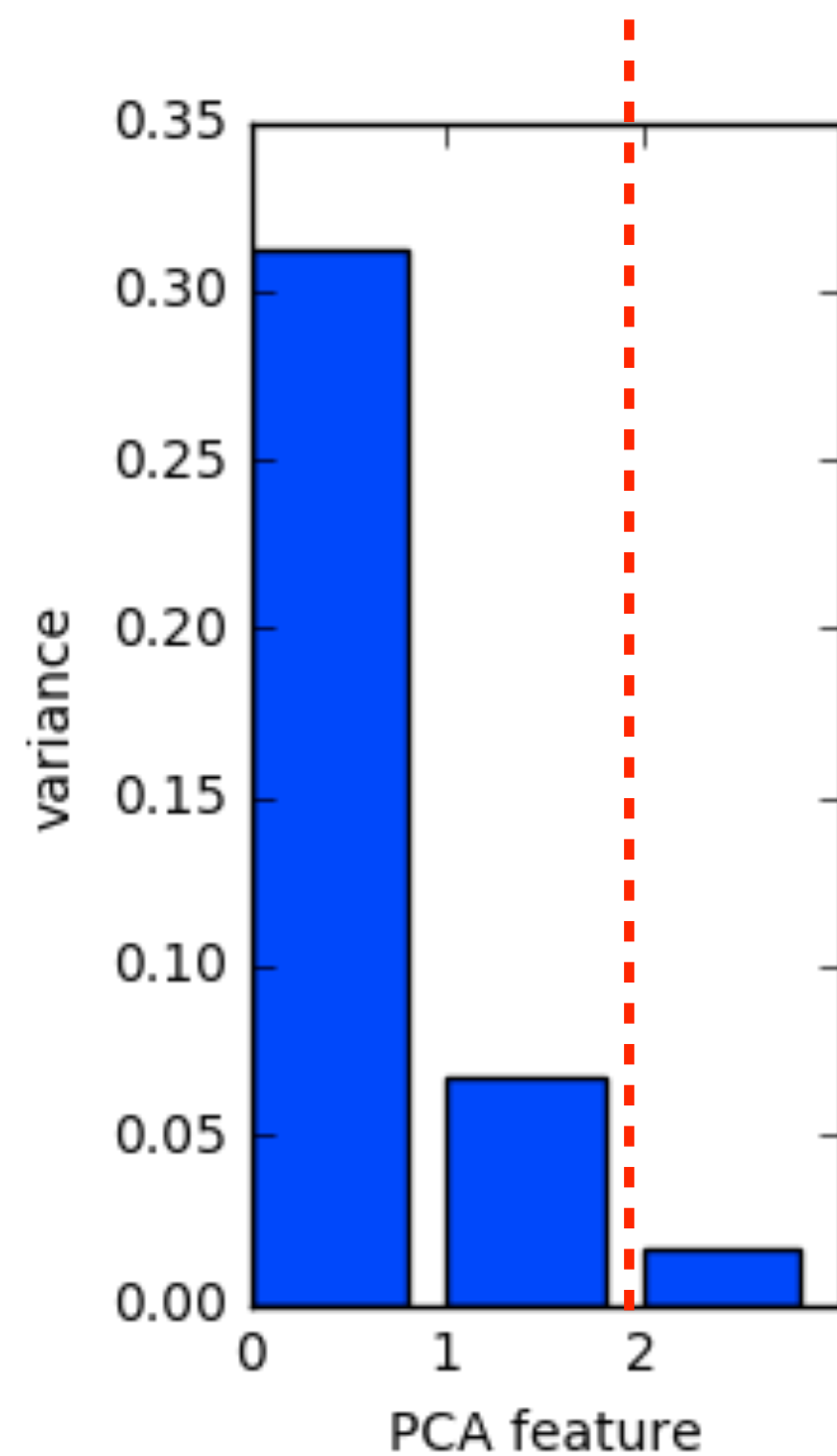
PCA features are ordered by variance descending





Variance and intrinsic dimension

- Intrinsic dimension is number of PCA features with significant variance
- In our example: the first two PCA features
- So intrinsic dimension is 2





Plotting the variances of PCA features

- `samples` = array of versicolor samples

```
In [1]: import matplotlib.pyplot as plt
```

```
In [2]: from sklearn.decomposition import PCA
```

```
In [3]: pca = PCA()
```

```
In [4]: pca.fit(samples)
```

```
Out[4]: PCA(copy=True, ... )
```

```
In [5]: features = range(pca.n_components_)
```



Plotting the variances of PCA features

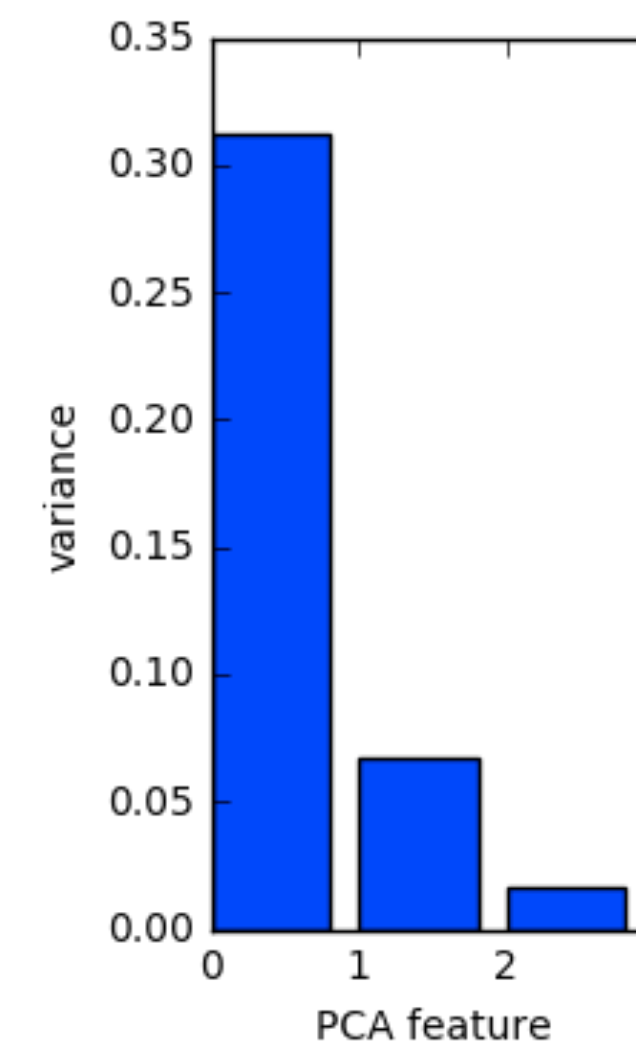
```
In [6]: plt.bar(features, pca.explained_variance_)
```

```
In [7]: plt.xticks(features)
```

```
In [8]: plt.ylabel('variance')
```

```
In [9]: plt.xlabel('PCA feature')
```

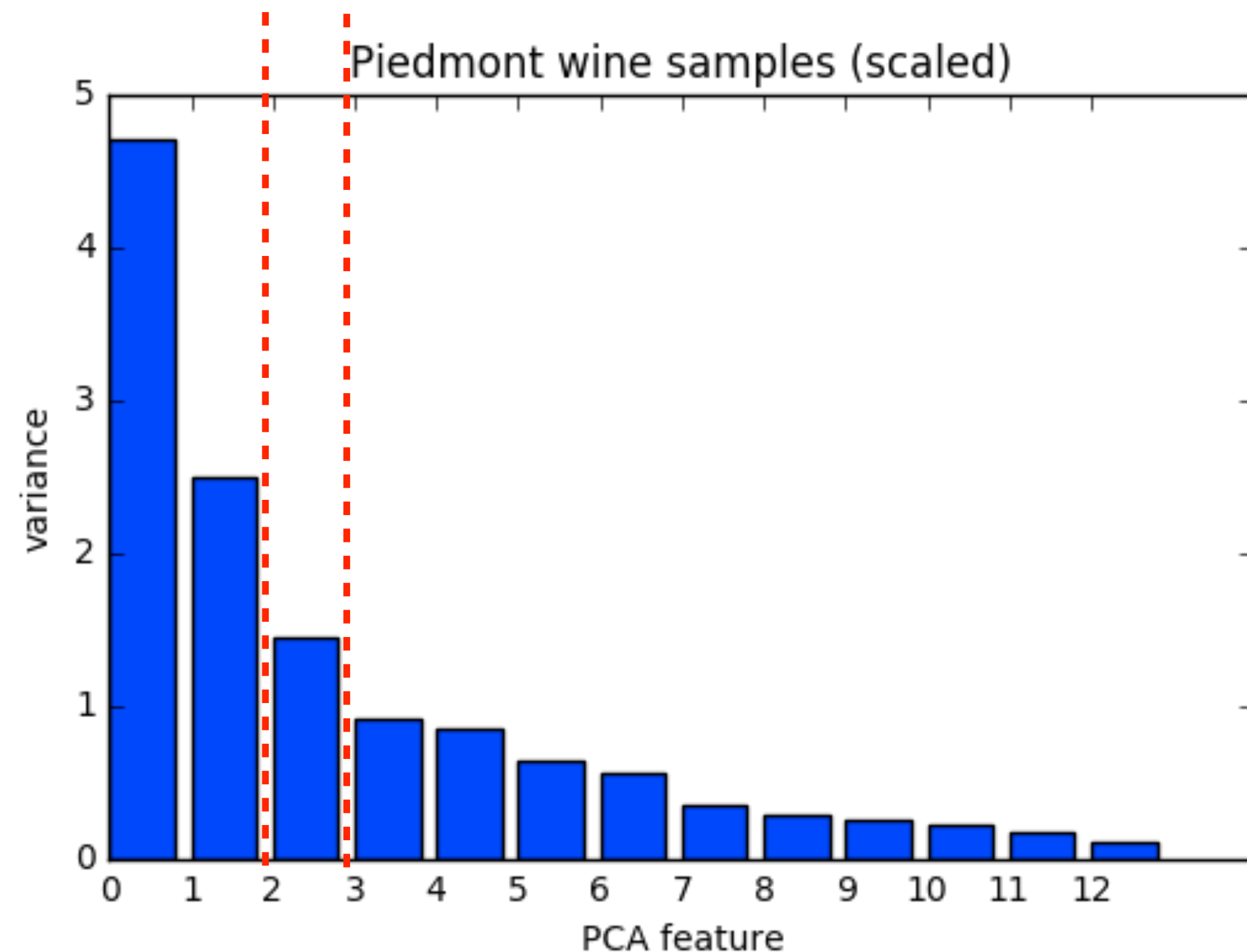
```
In [10]: plt.show()
```





Intrinsic dimension can be ambiguous

- Intrinsic dimension is an idealization
- ... there is not always one correct answer!
- Piedmont wines: could argue for 2, or for 3, or more





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Dimension reduction with PCA



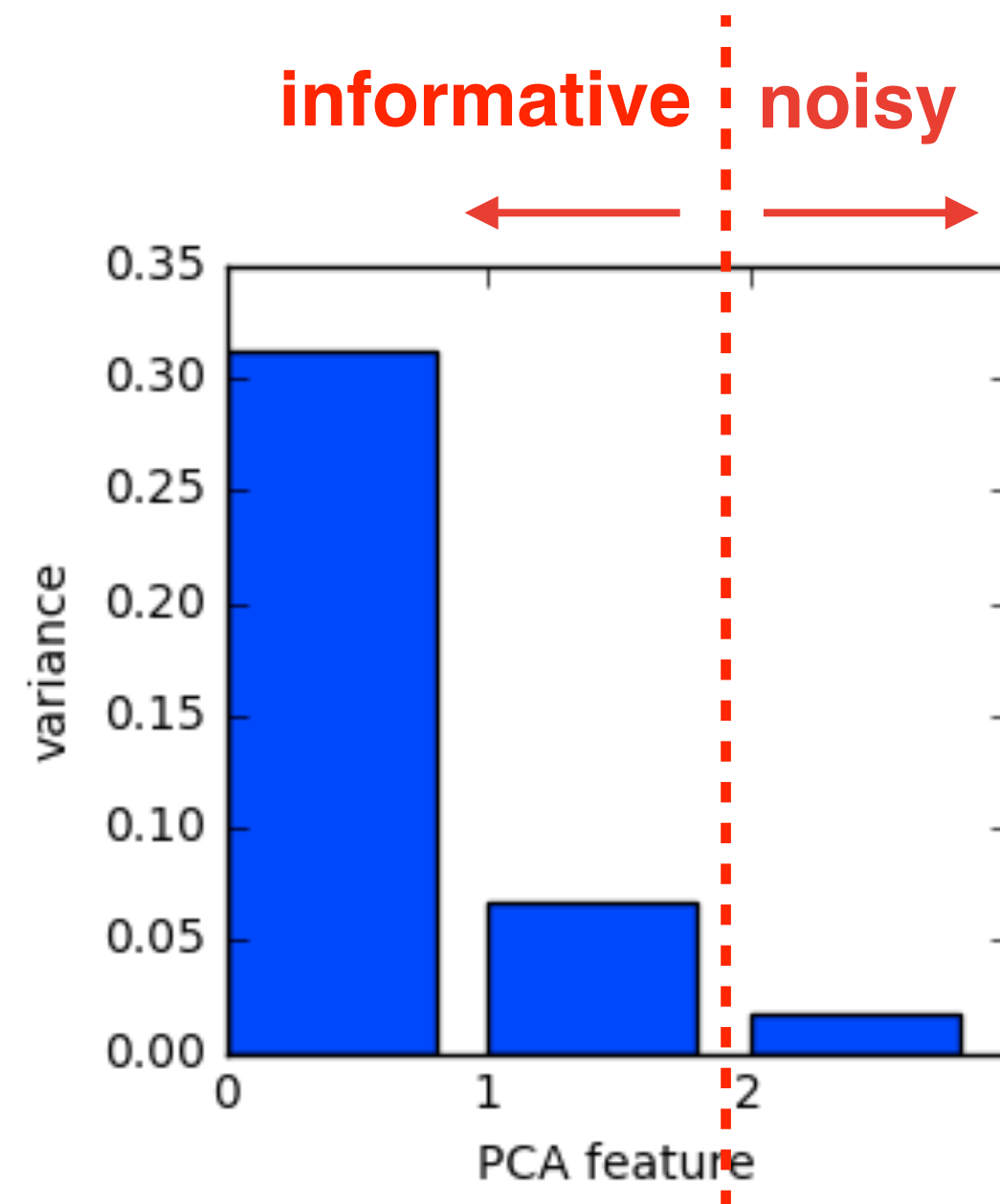
Dimension reduction

- Represents same data, using less features
- Important part of machine-learning pipelines
- Can be performed using PCA



Dimension reduction with PCA

- PCA features are in decreasing order of variance
- Assumes the low variance features are "noise"
- ... and high variance features are informative





Dimension reduction with PCA

- Specify how many features to keep
- E.g. `PCA(n_components=2)`
- Keeps the first 2 PCA features
- Intrinsic dimension is a good choice



Dimension reduction of iris dataset

- `samples` = array of iris measurements (4 features)
- `species` = list of iris species numbers

```
In [1]: from sklearn.decomposition import PCA
```

```
In [2]: pca = PCA(n_components=2)
```

```
In [3]: pca.fit(samples)
```

```
Out[3]: PCA(copy=True, ... )
```

```
In [4]: transformed = pca.transform(samples)
```

```
In [5]: print(transformed.shape)  
(150, 2)
```



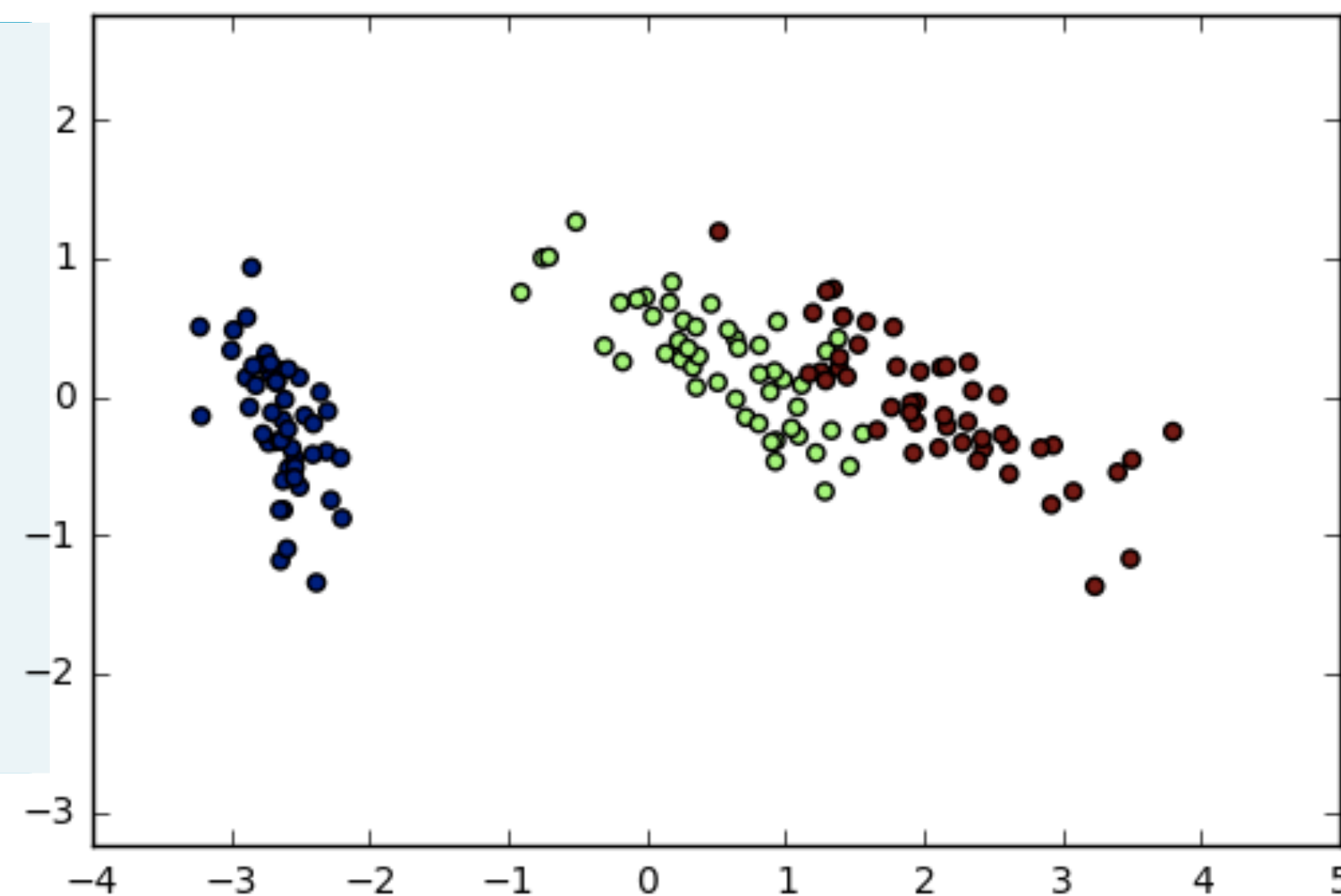
Iris dataset in 2 dimensions

- PCA has reduced the dimension to 2
- Retained the 2 PCA features with highest variance
- Important information preserved: species remain distinct

```
In [6]: import matplotlib.pyplot as plt
```

```
In [7]: xs = transformed[:,0]  
....: ys = transformed[:,1]
```

```
In [8]: plt.scatter(xs, ys, c=species)  
....: plt.show()
```



Dimension reduction with PCA

- Discards low variance PCA features
- Assumes the high variance features are informative
- Assumption typically holds in practice (e.g. for iris)



Word frequency arrays

- Rows represent documents, columns represent words
- Entries measure presence of each word in each document
- ... measure using "tf-idf" (more later)

	aardvark	apple	.	.	.	zebra
document0	0,	0.1,	...			0.
document1						
.						
.						
.						
.						

word frequencies ("tf-idf")



Sparse arrays and `csr_matrix`

- Array is "sparse": most entries are zero
- Can use `scipy.sparse.csr_matrix` instead of NumPy array
- `csr_matrix` remembers only the non-zero entries (saves space!)

	aardvark	apple	. . .	zebra
document0	0,	0.1,	...	0.
document1				
.				
.				
.				

word frequencies ("tf-idf")



TruncatedSVD and csr_matrix

- scikit-learn PCA doesn't support `csr_matrix`
- Use scikit-learn **TruncatedSVD** instead
- Performs same transformation

```
In [1]: from sklearn.decomposition import TruncatedSVD

In [2]: model = TruncatedSVD(n_components=3)

In [3]: model.fit(documents)  # documents is csr_matrix
Out[3]: TruncatedSVD(algorithm='randomized', ...)

In [4]: transformed = model.transform(documents)
```



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