Chapter 9 Morphological Image Processing

- *Morphology*（*形态学*）denotes a branch of biology.
- *Mathematical morphology*： extracting image components that are useful in the representation and description of region shape.
- the language of mathematical morphology is set theory集合论.

Chapter 9 Morphological Image Processing

Main Content

1.Basic concepts from set theory

2. Dilation 膨胀 and erosion 腐蚀

3. Opening and closing

4. The Hit-or-Miss transformation 击中或不中变换

5. Some basic Morphological Algorithms

9.1 Preliminaries

Some basic concepts from set theory

-Subset 子集

-Union 并集

-Intersection 交集

-Complement 补集

-Difference 差

-Reflection 反射

-Translation 平移

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

$$(A)_z = \{c \mid c = a + z, a \in A\}$$

9.1 Preliminaries

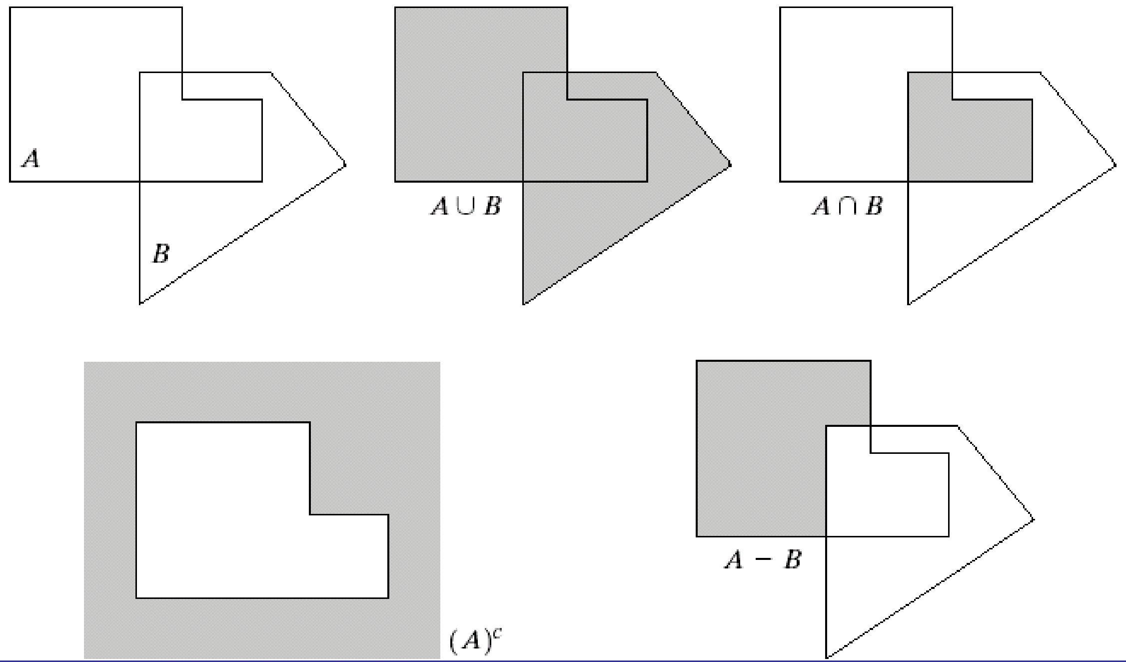


FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

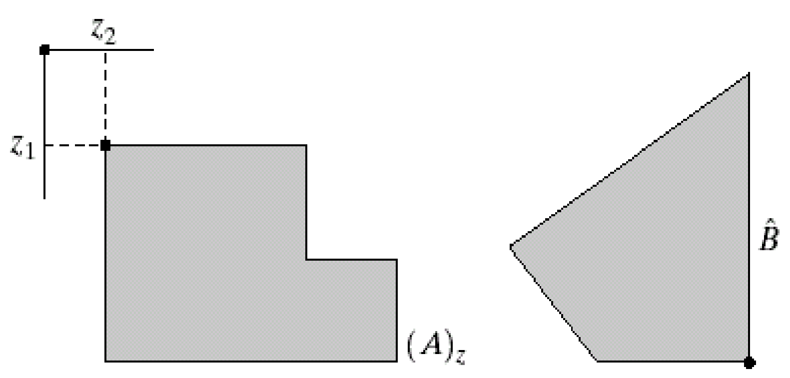


FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

9.1 Preliminaries

•Logic Operations Involving Binary Images

The principal logic operations: AND, OR, NOT

These operations are functions are *functionally complete*

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

9.1 Preliminaries

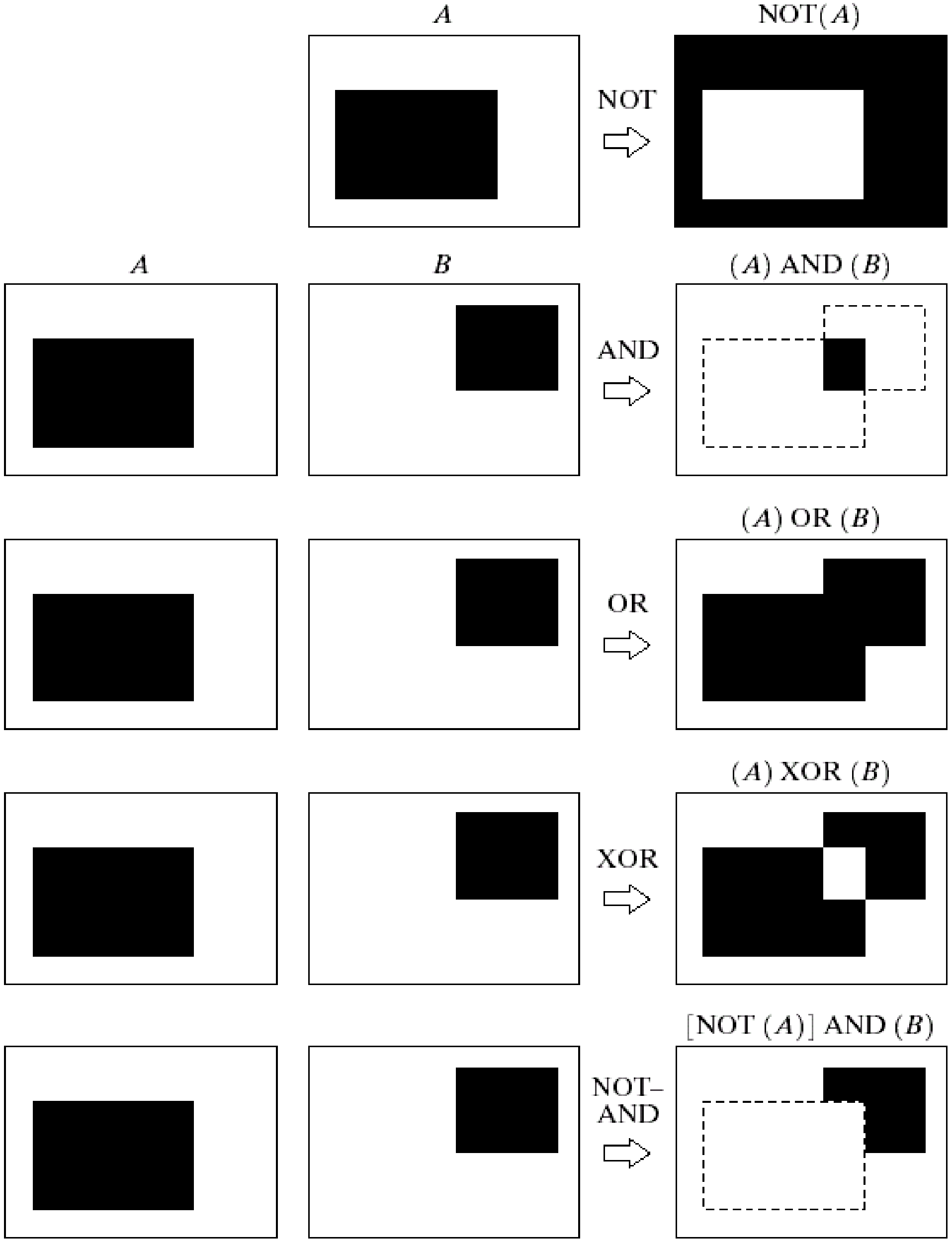


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

9.2 Dilation and Erosion

Dilation and Erosion are fundamental to morphological processing.

Dilation:

With A and B as sets in Z^2, the *dilation* of A by B, denoted $A \oplus B$, is defined as

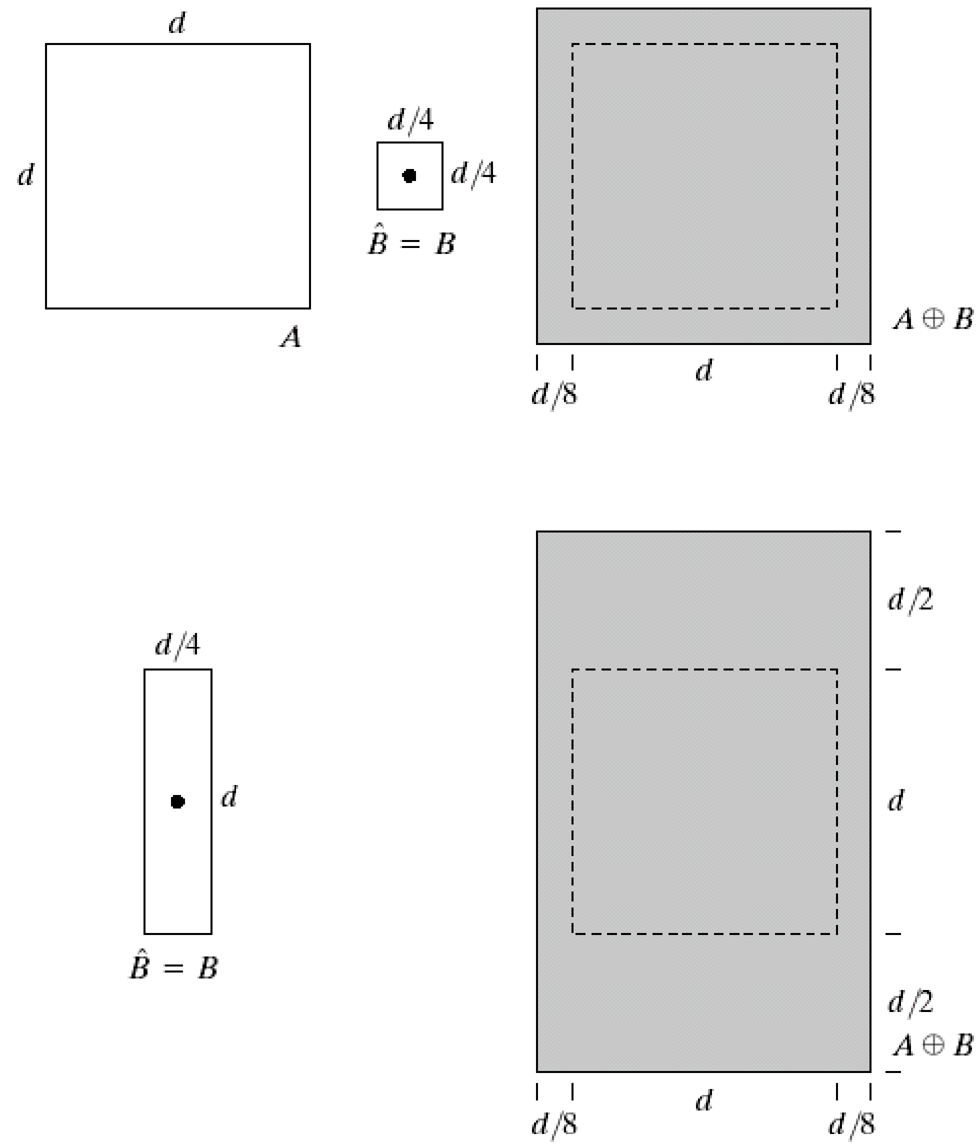
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}. \tag{9.2-1}$$

This equation is based on obtaining the reflection of B about its origin and shifting this reflection by z. The dilation of A by B then is the set of all *displacements*, z, such that $\hat{B}$ and A overlap by at least one element. Based on this interpretation, Eq. (9.2-1) may be rewritten as

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}. \tag{9.2-2}$$

Set B is commonly referred to as the *structuring element* in dilation, as well as in other morphological operations.

9.2 Dilation and Erosion

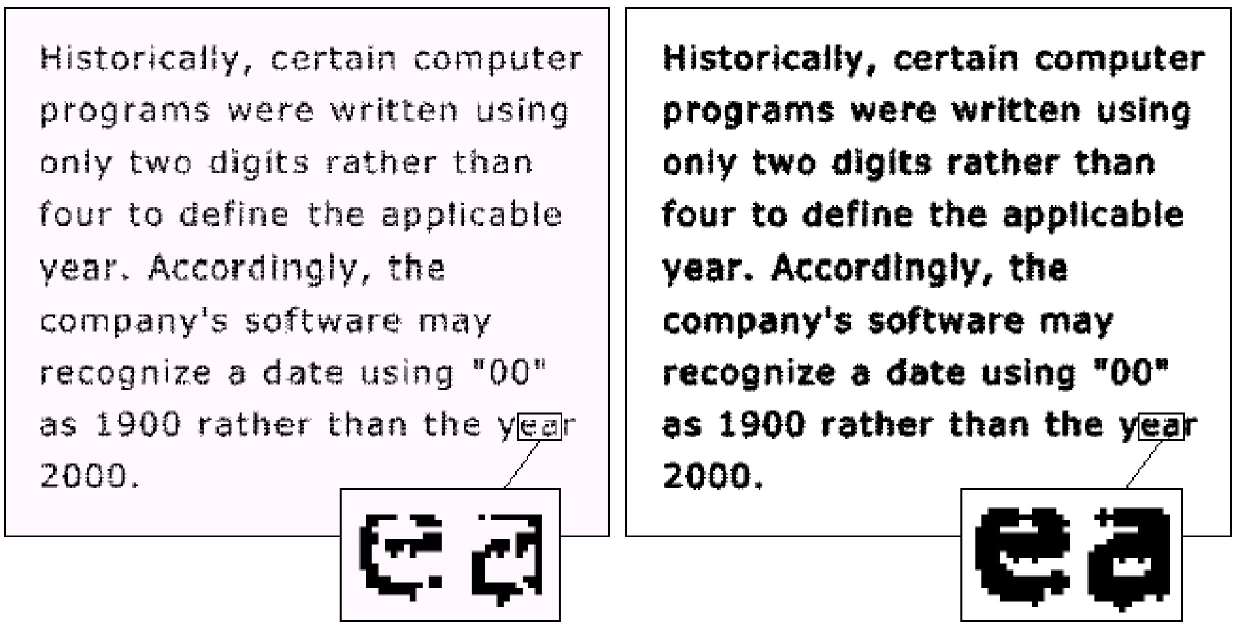


a b c
d e

FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

9.2 Dilation and Erosion

Example 9.1 Use of morphological dilation for bridging gaps.



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Advantage: directly produce a binary image

Low pass filters: produce a grey-level image

9.2 Dilation and Erosion

Erosion

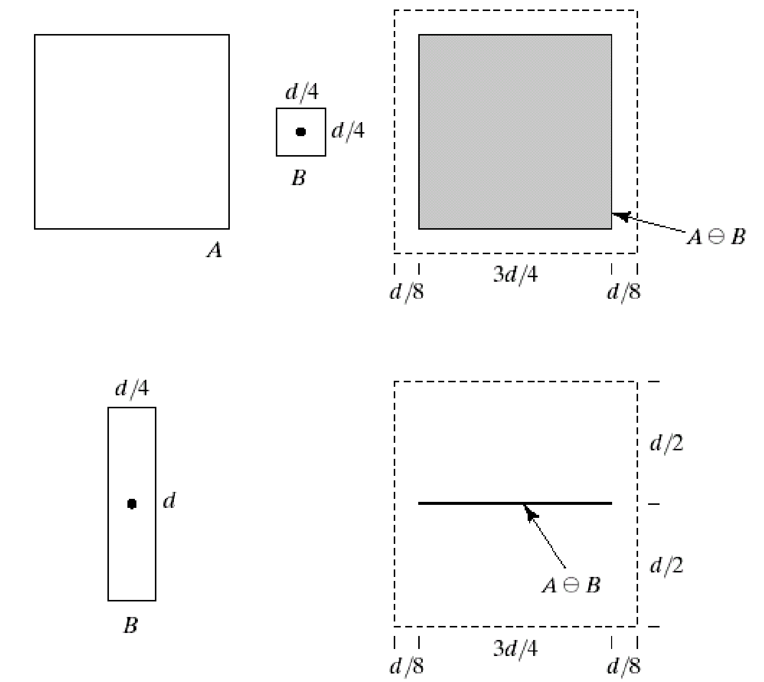
For sets A and B in Z^2 the erosion of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{z \mid (B)_z \subseteq A\}. \tag{9.2-3}$$

In words, this equation indicates that the erosion of A by B is the set of all points z such that B, translated by z, is contained in A. As in the case of dilation,

9.2 Dilation and Erosion

Example: Erosion



a b c
d e

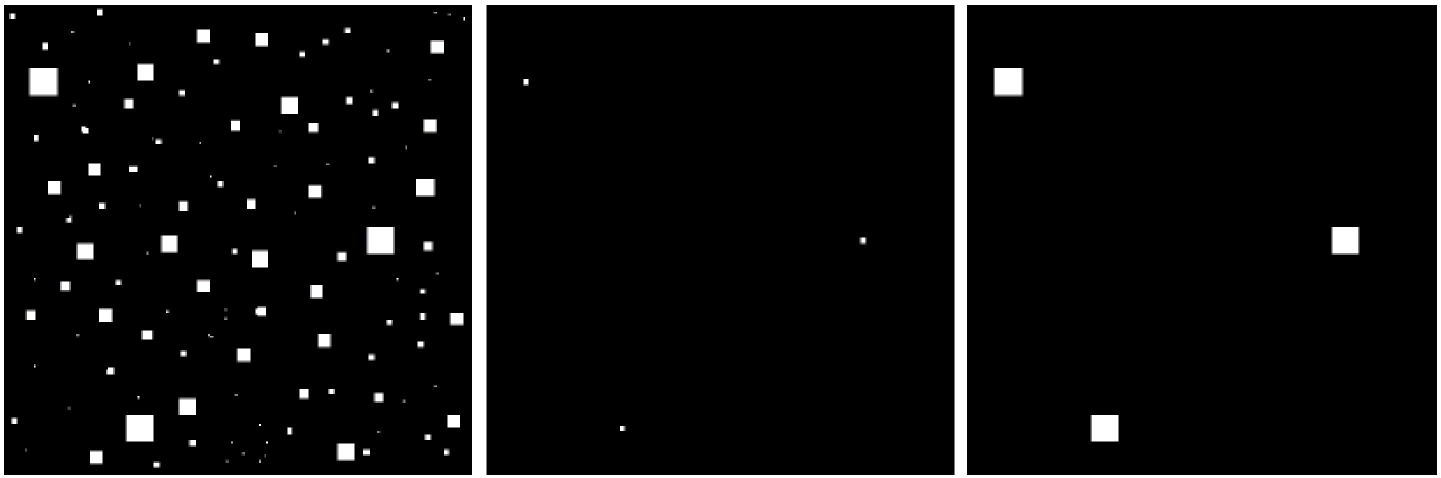
FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Dilation and Erosion are duals of each other with respect to set complementation and reflection.

$$(A \quad {}^{c} \oplus \hat{B}$$

9.2 Dilation and Erosion

Example 9.2 Use of morphological erosion for removing image components – retain the largest squares



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Dilation expands an image and erosion shrinks it.

9.3 Opening and Closing

- Opening smoothes the contour of an object, breaks narrow isthmuses(地峡), and eliminates thin protrusions（突出）

- Closing also tends to smooth sections of contours, but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gap in the contour.

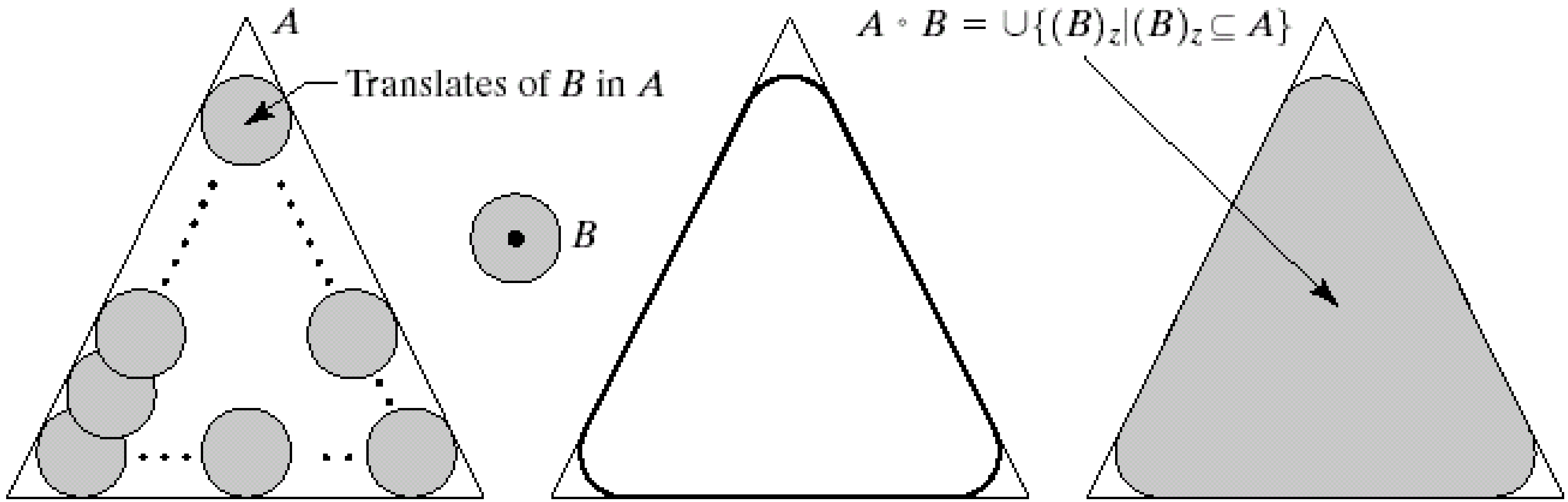
The opening of set A by structuring element B

A

The closing of set A by structuring element B

A

9.3 Opening and Closing



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

9.3 Opening and Closing

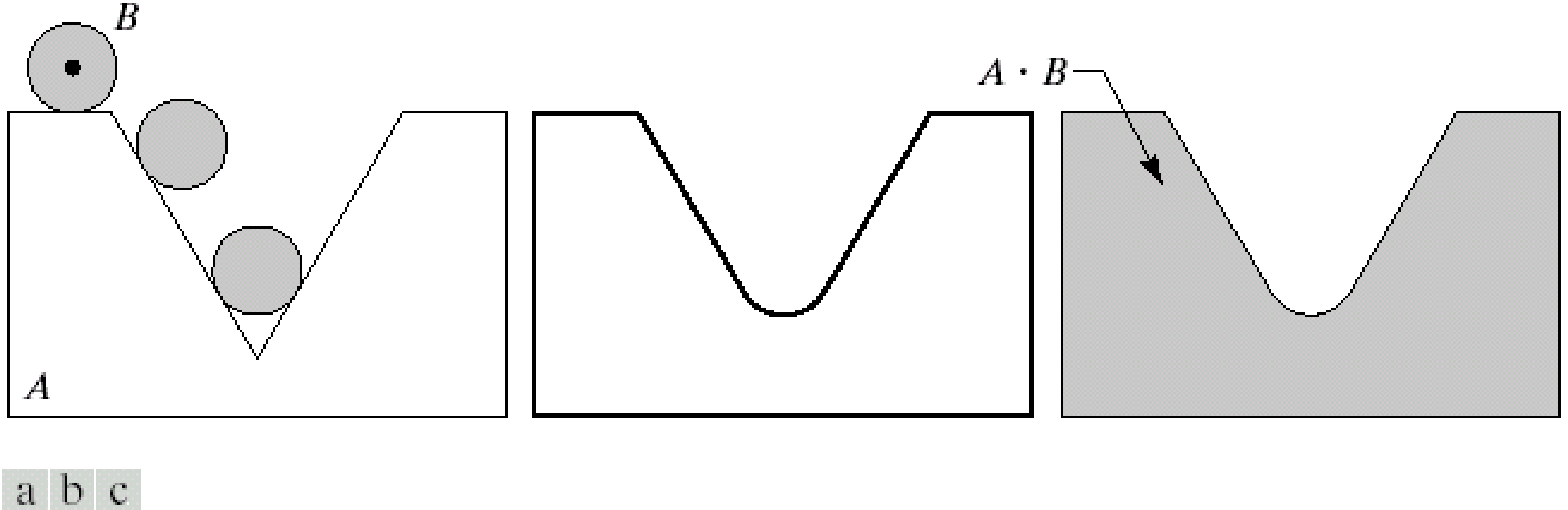


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

9.3 Opening and Closing

Example 9.3 A simple illustration of morphological opening and closing.

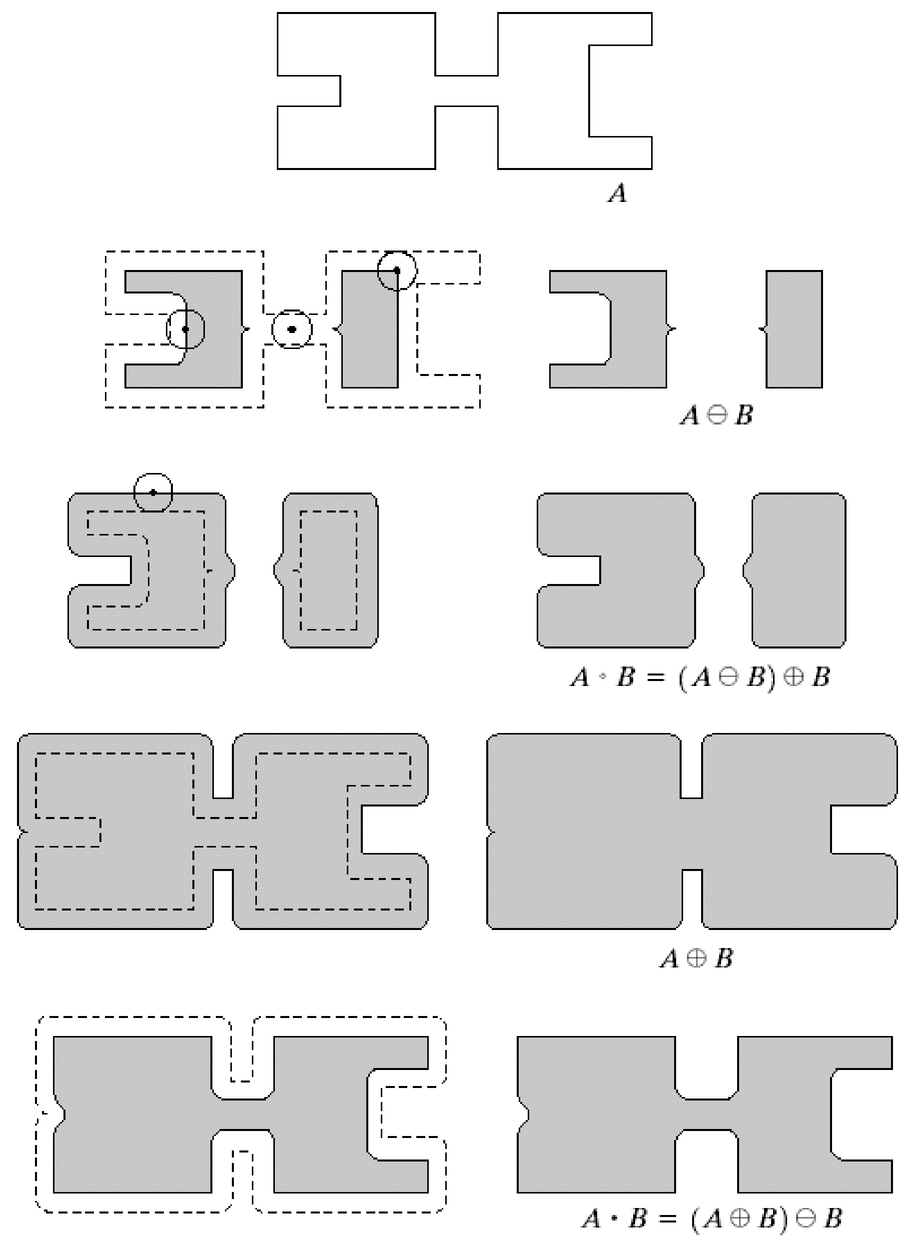


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Opening smoothes the contour of an object, breaks narrow isthmuses(地峡), and eliminates thin protrusions（突出）

Closing also tends to smooth sections of contours, but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gap in the contour.

9.3 Opening and Closing

Opening and closing are duals of each other with respect to set complementation and reflection.

$$(A$$

We leave the proof of this result as an exercise (Problem 9.14).

The opening operation satisfies the following properties:

(i) $A \circ B$ is a subset (subimage) of A.
(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
(iii) $(A \circ B) \circ B = A \circ B$.

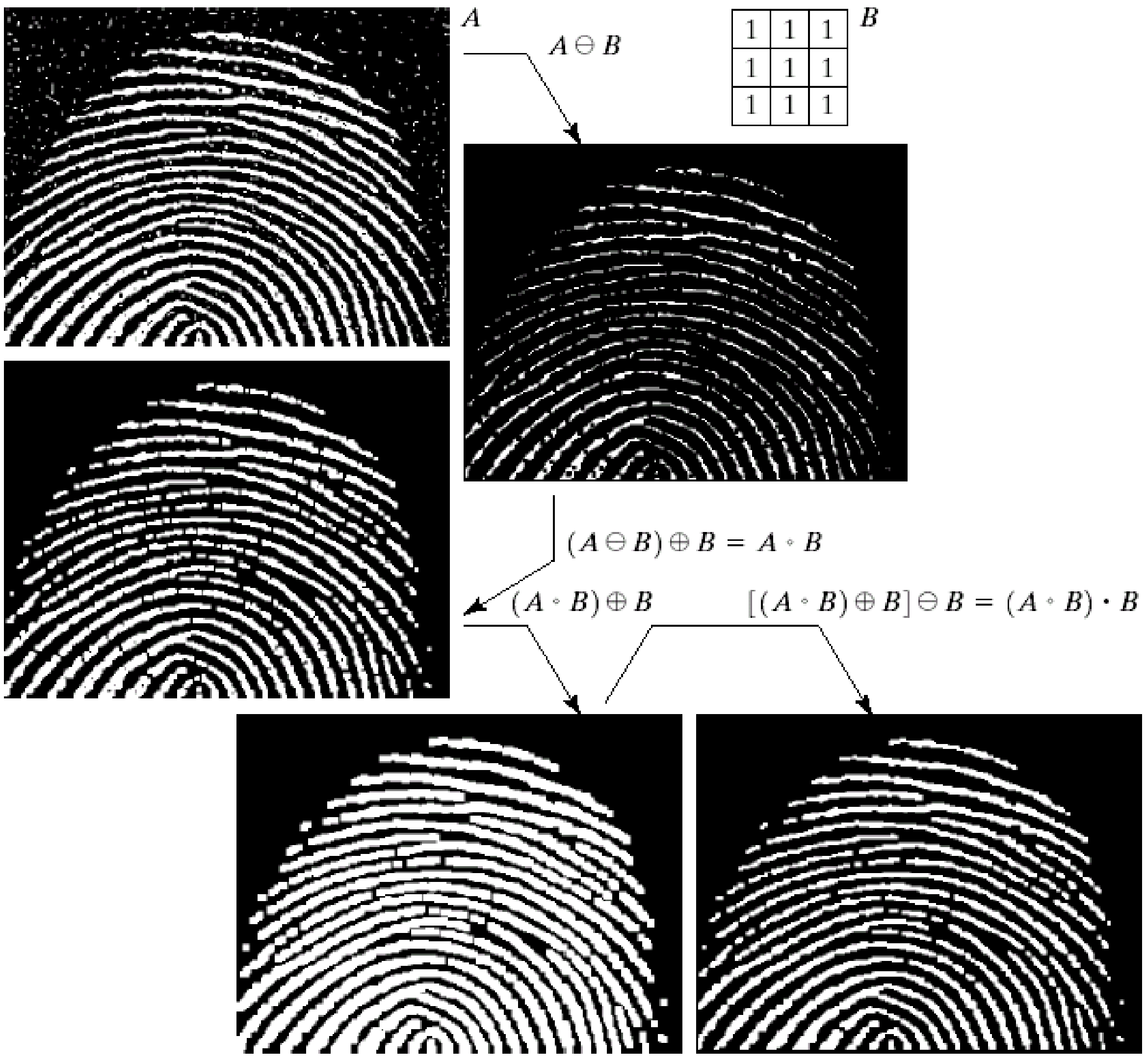
Similarly, the closing operation satisfies the following properties:

(i) A is a subset (subimage) of $A \bullet B$.
(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
(iii) $(A \bullet B) \bullet B = A \bullet B$.

Note from condition (*iii*) in both cases that multiple openings or closings of a set have no effect after the operator has been applied once.

9.3 Dilation and Erosion

Example 9.4 Use of opening and closing for morphological filtering.



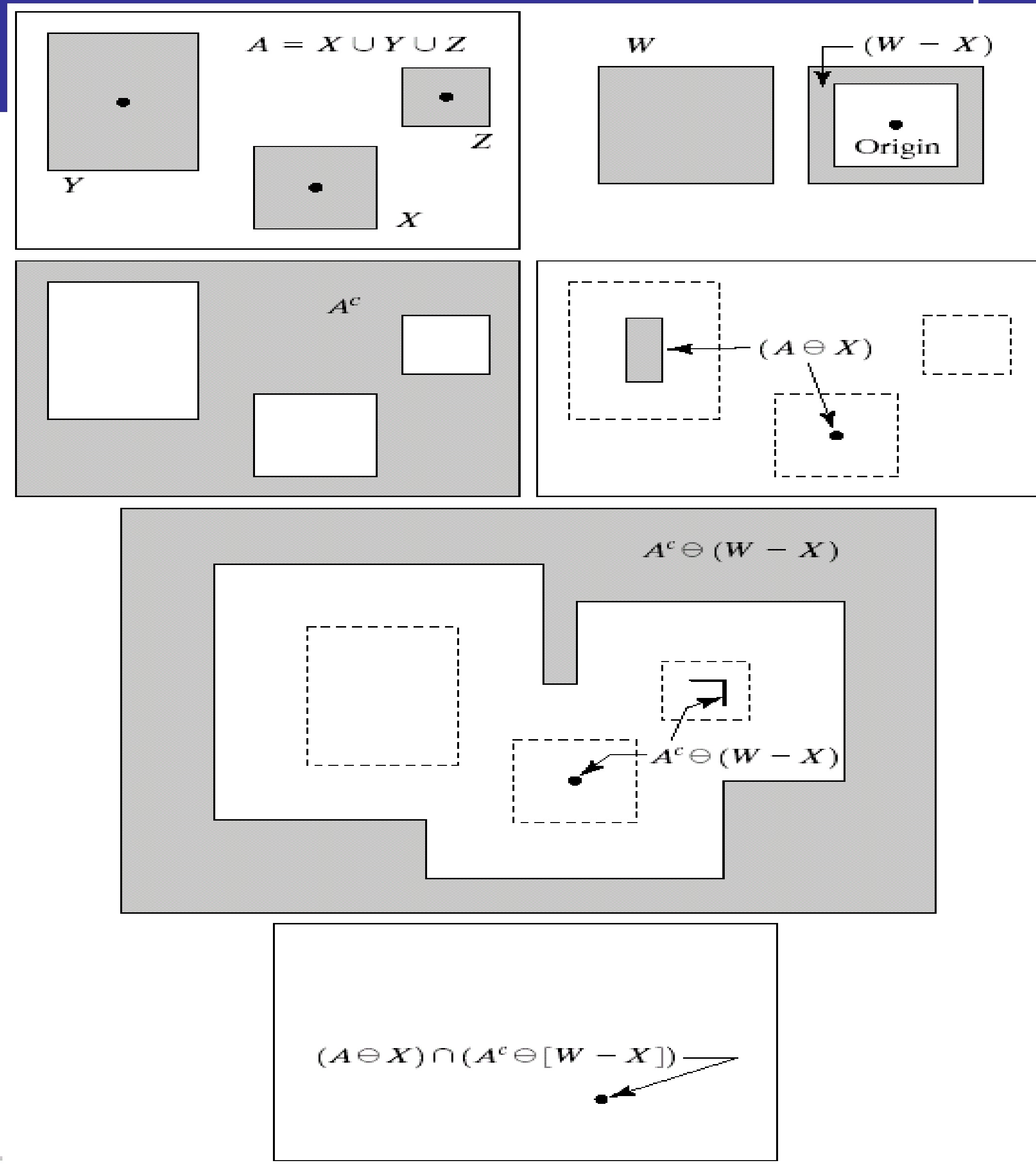
a b c d e f

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Noise reduction

9.4 The Hit-or-Miss Transformation

The morphological hit-or-miss transform is a basic tool for shape detection.



a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

9.5 Some Basic Morphological Algorithms

- For binary images, the morphology is used to extract image components that are useful in the representation and description of shape.
- extracting boundaries, connected components, the convex hull 凸壳， and the skeleton of a region.
- the images are binary, with 1's shown shaded and 0's shown in white (sometimes the other way round).

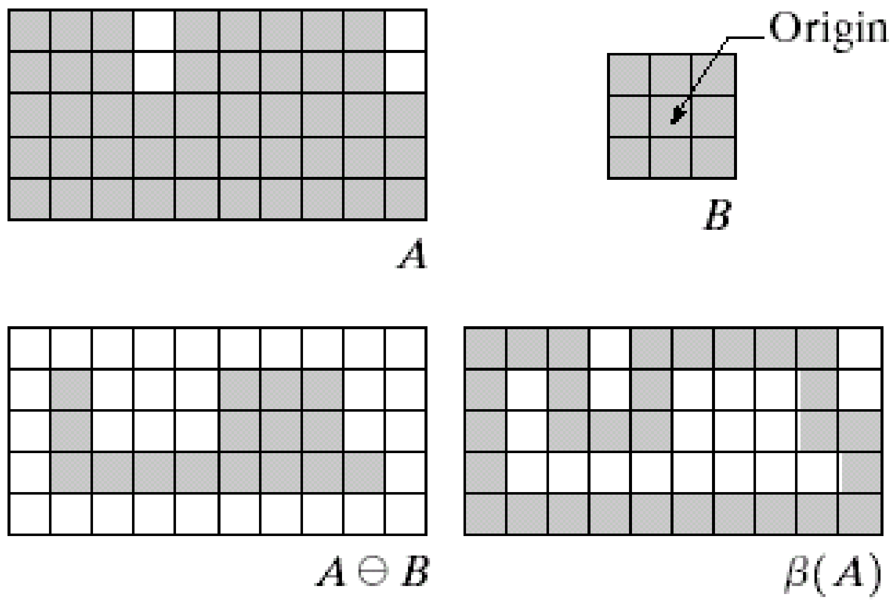
9.5.1 Boundary Extraction

• The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \qquad (9.5\text{-}1)$$

a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.

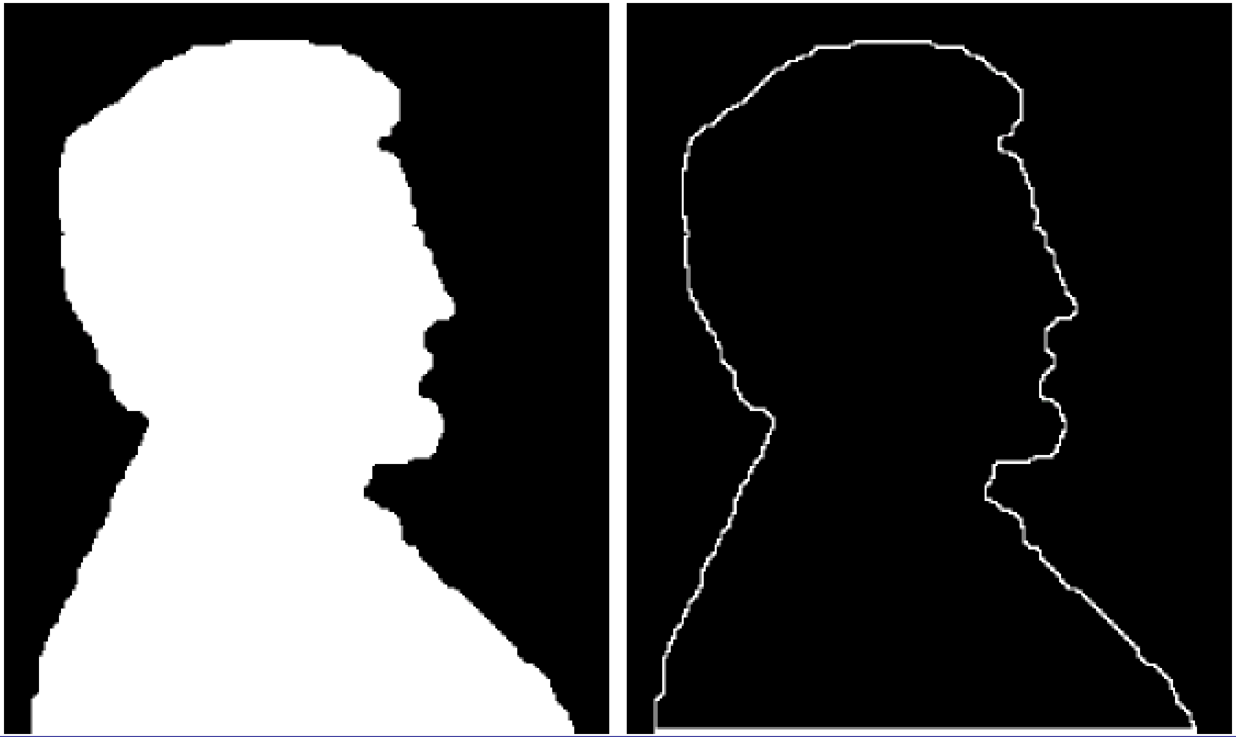


Although the structuring element shown in (b) is among the most frequently used, it is by no means unique.

A 5*5 structuring element of 1's would result in a boundary between 2 and 3 pixels thick.

9.5.1 Boundary Extraction

Example 9.5 Boundary extraction by morphological processing



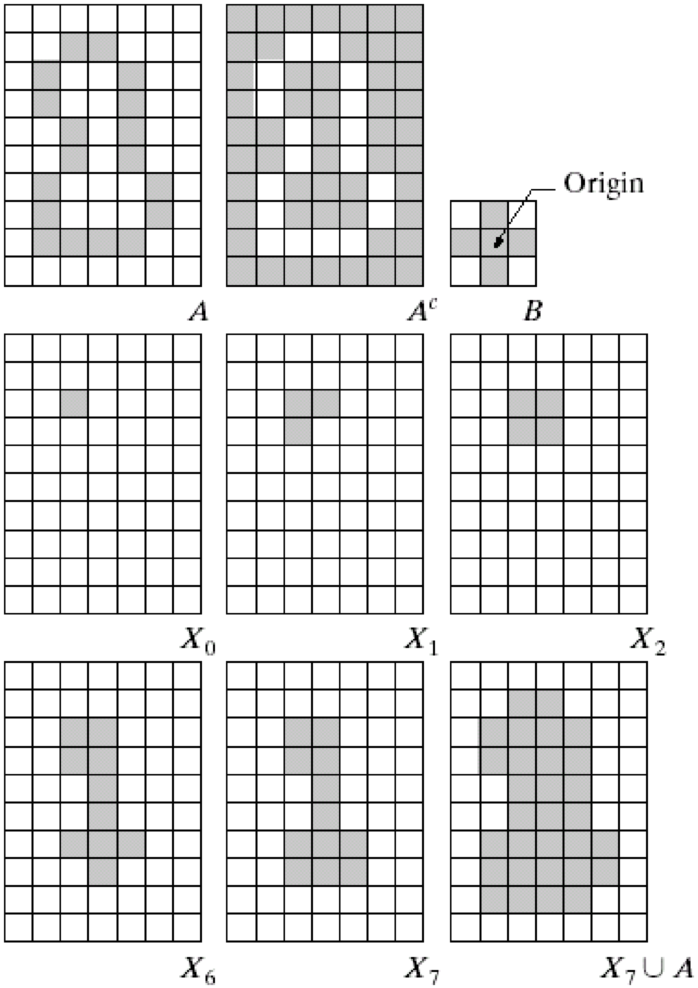
a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.5.2 Region Filling

The algorithm for region filling based on set dilations 膨胀, complementation补集, and intersections交集.

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$



a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

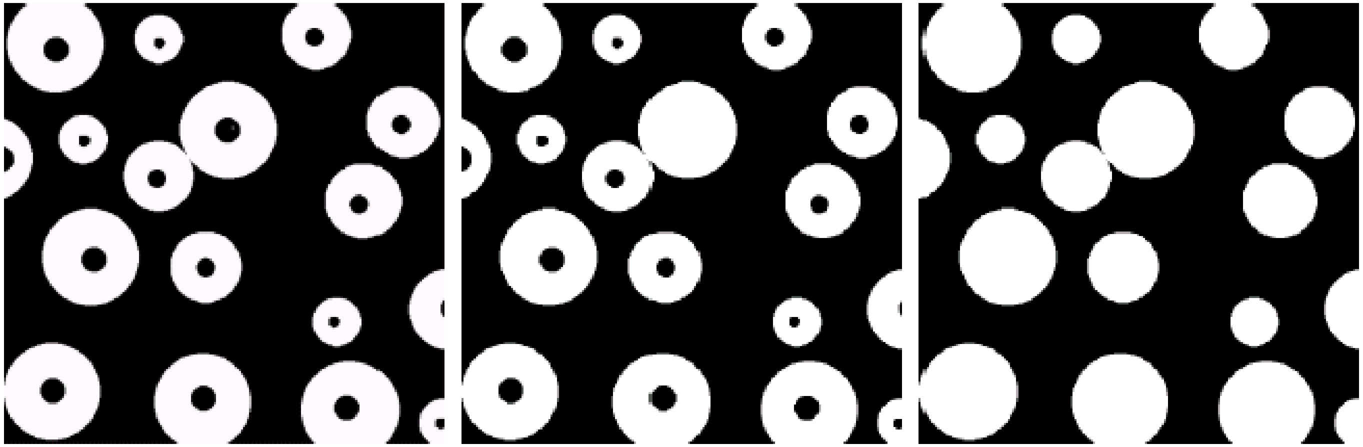
Nonboundary points are labeled 0

Assign a value of 1 to p (inside) to begin. X_0=p

The algorithm terminates at iteration step k if x_k=x_{k-1}

9.5.2 Region Filling

Example 9.6 Morphological region filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

9.5.3 Extraction of connected components

Let Y represent a connected component contained in a set A and assume that a point *p* of Y is known.

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

Where X_0=p, and B is a suitable structuring element.

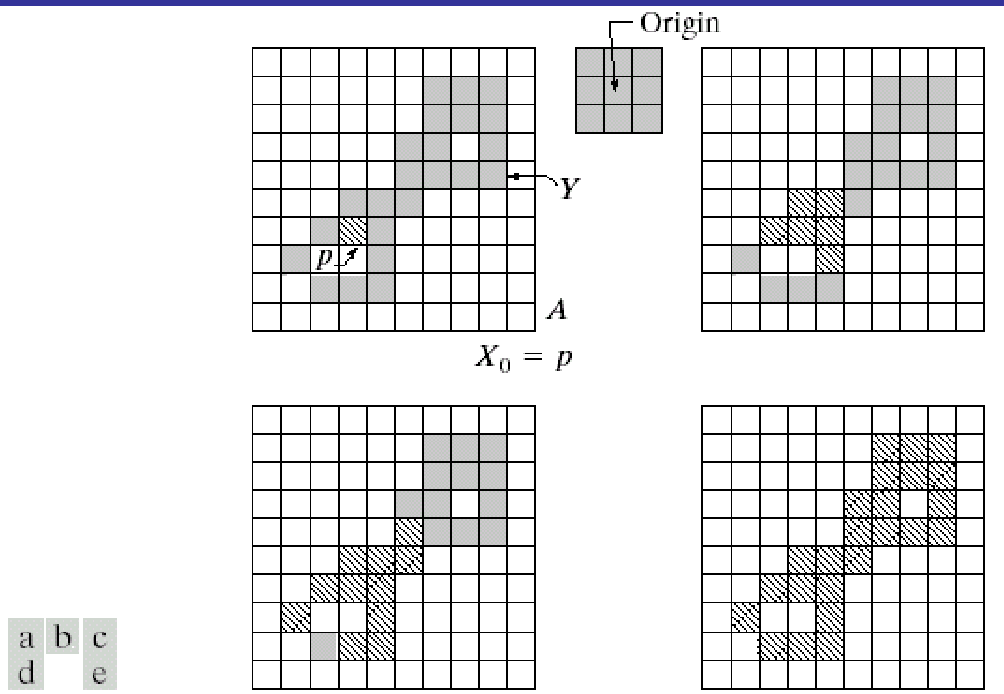


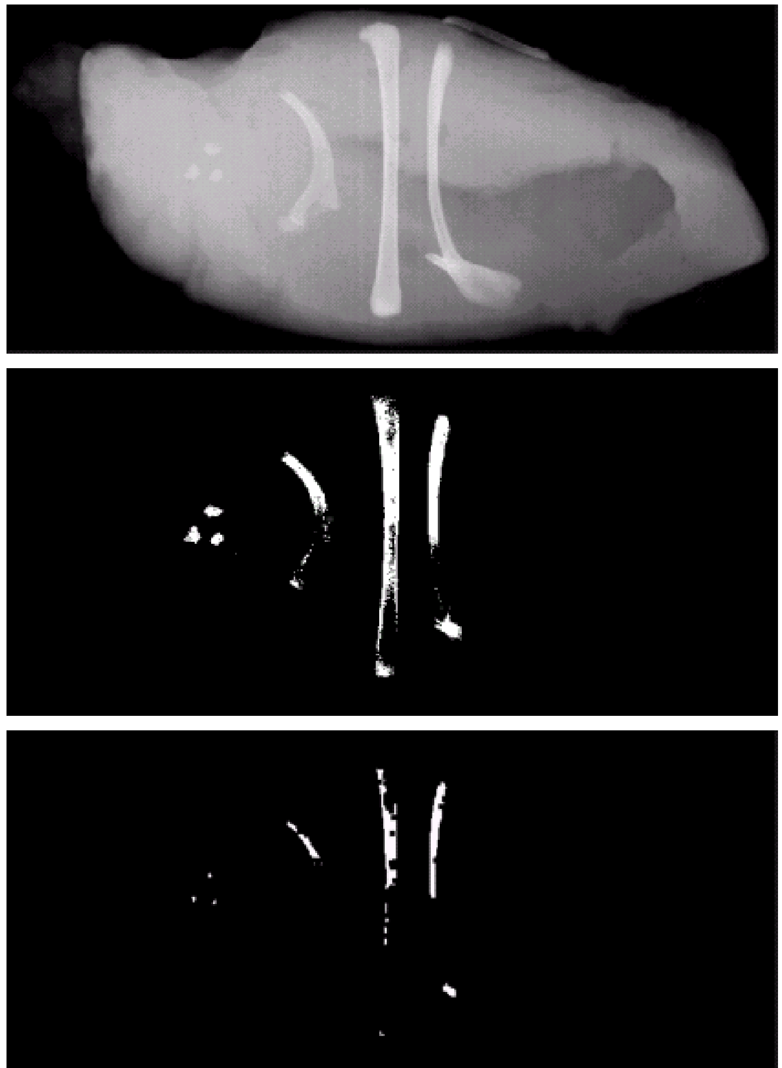
FIGURE 9.17 (a) Set *A* showing initial point *p* (all shaded points are valued 1, but are shown different from *p* to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

9.5.3 Extraction of connected components

Example 9.7 Using connected components to detect foreign objects in package food.

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

9.5.4 Convex Hull 凸壳

- The *convex Hull H* of an arbitrary set S is the smallest convex set containing S.
- Algorithm: Let Bi, i=1,2,3,4, represent the four structuring elements in 9.19(a)

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots \tag{9.5-4}$$

with $X_0^i = A$. Now let $D^i = X_{\text{conv}}^i$, where the subscript "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i. \tag{9.5-5}$$

9.5.4 Convex Hull 凸壳

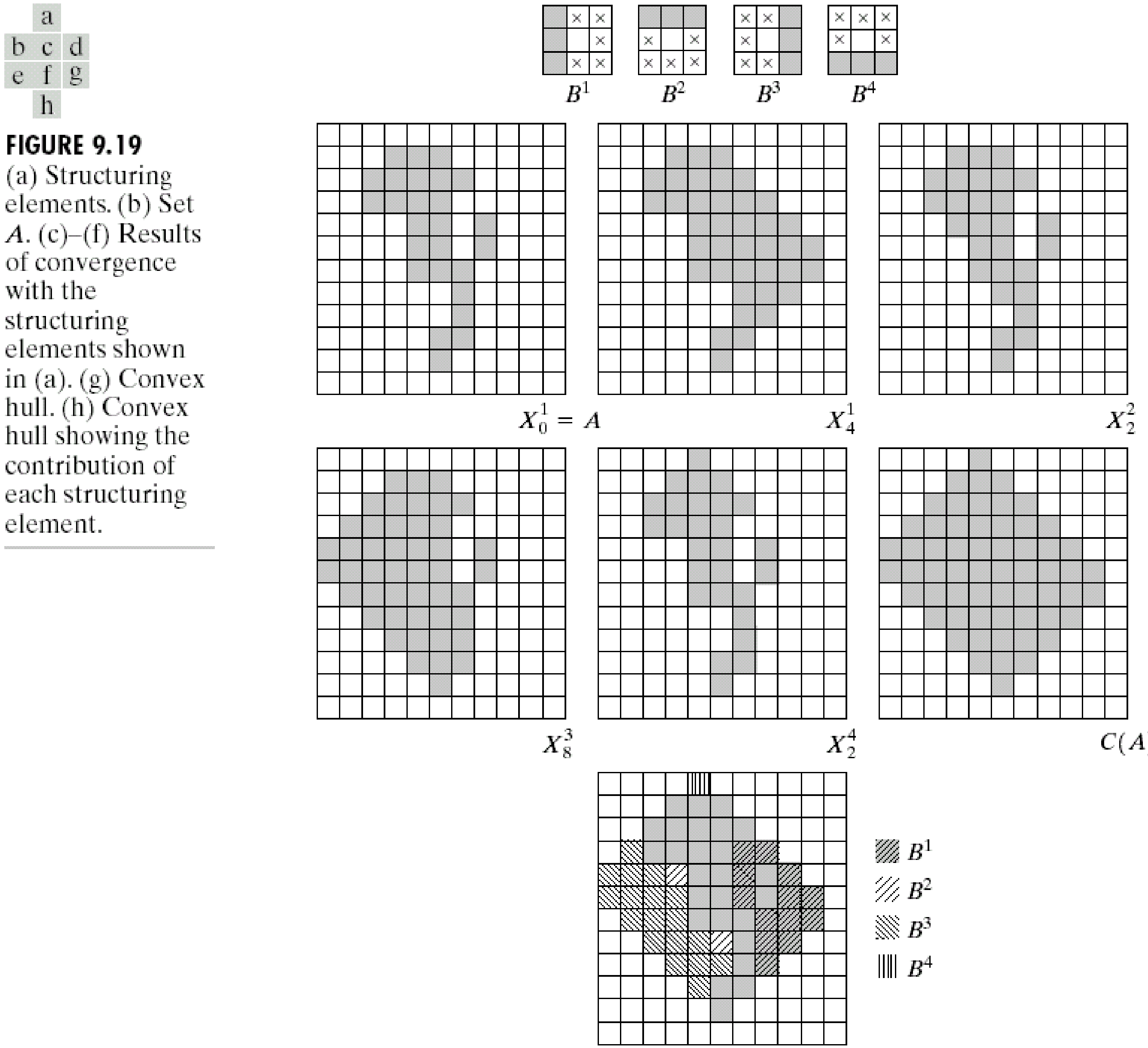


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

9.5.5 Thinning

The thinning of a set A by a structuring element B, can be defined in terms of the hit-or-miss transform

$$
\begin{aligned}
A \otimes B &= A - (A \circledast B) \\
&= A \cap (A \circledast B)^c. \qquad (9.5\text{-}6)
\end{aligned}
$$

The other expression:

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n).$$

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

9.5.5 Thinning

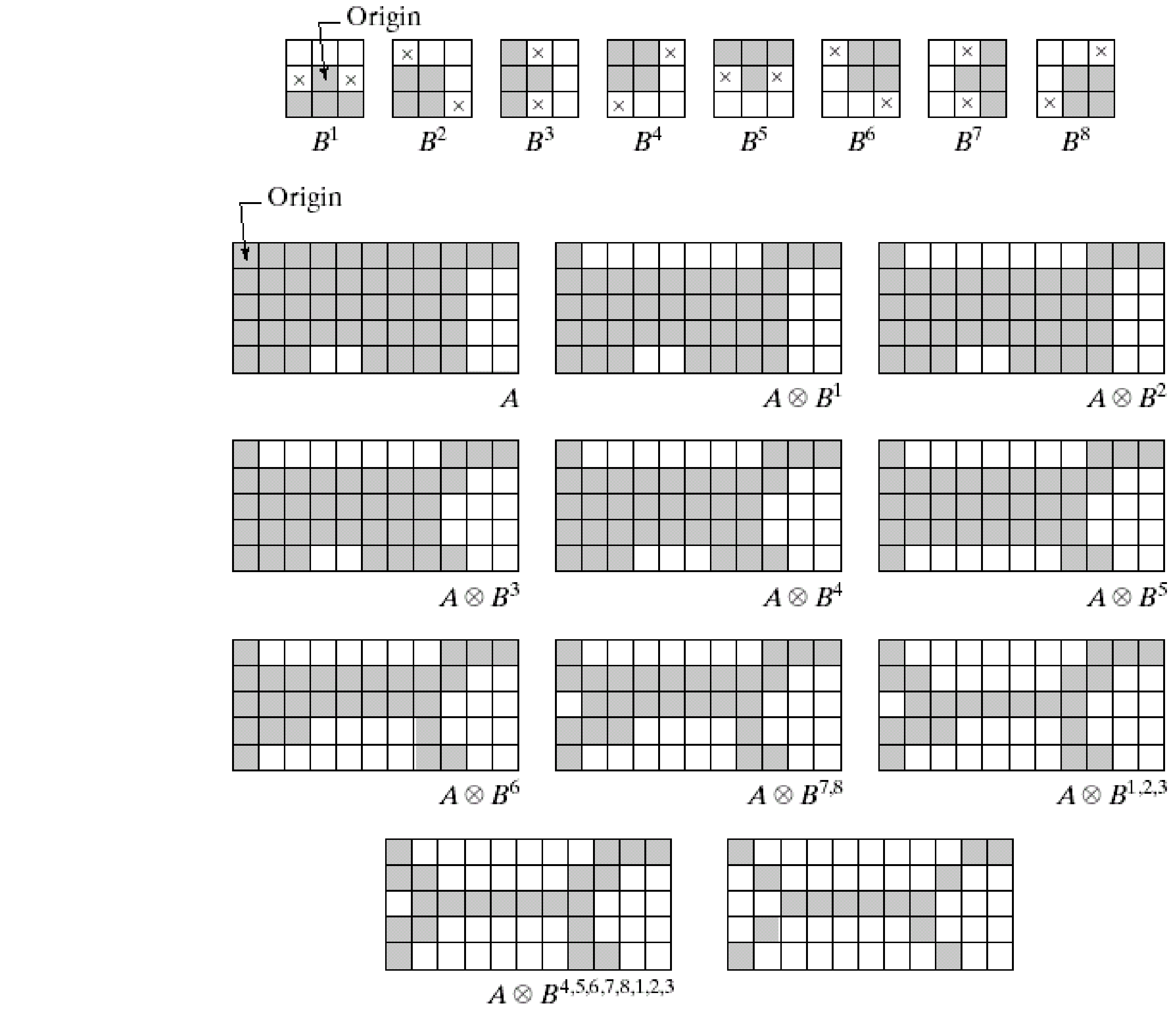


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

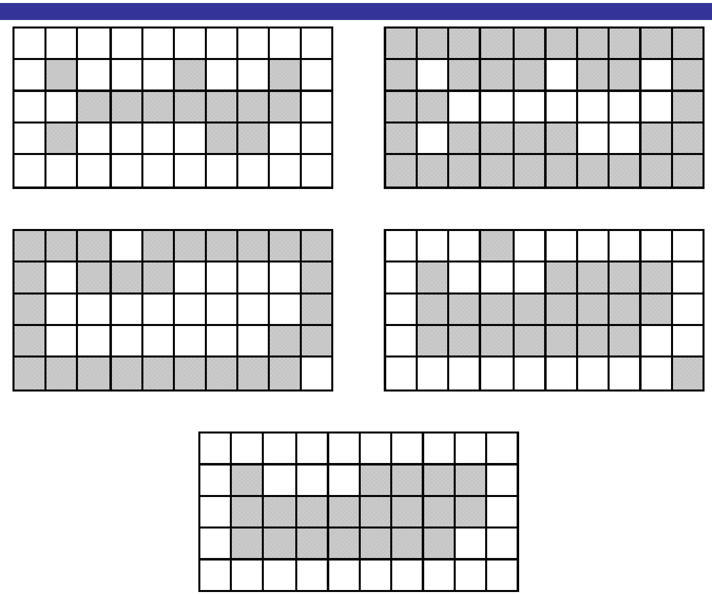
9.5.6 Thichening 粗化

Thickening is the morphological dual 对偶 of thinning

$$A \odot B = A \cup (A \circledast B) \tag{9.5-9}$$

and

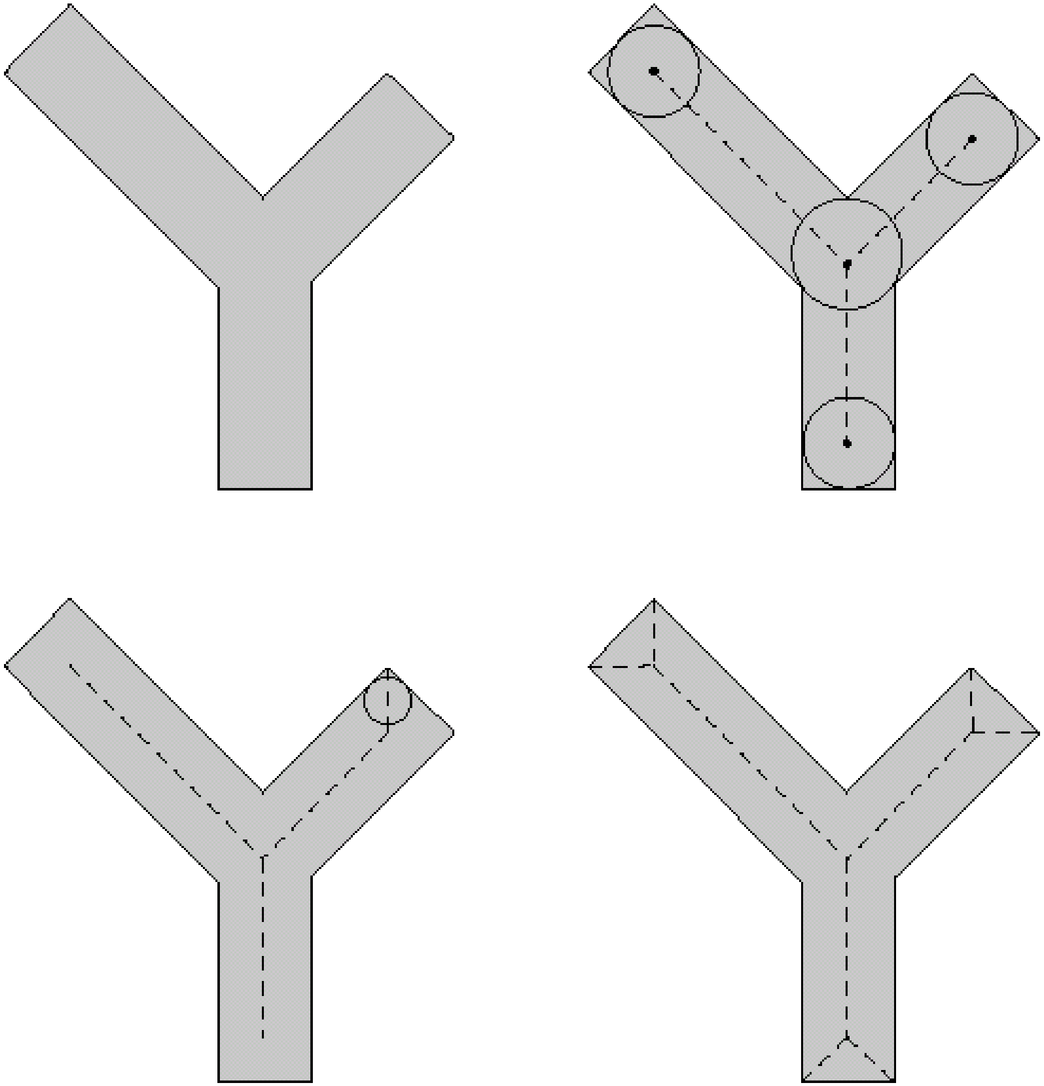
A



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

9.5.7 Skeletons 骨架



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

9.5.7 Skeletons 骨架

(a)If z is a point of S(A) and $(D)_z$ is the largest disk centered at z and contained in A, one can't find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a *maximum disk*.

(b) The disk $(D)_z$ touches the boundary of A at two or more different places.

9.5.7 Skeletons 骨架

Example 9.8 Computing the skeleton of a simple figure

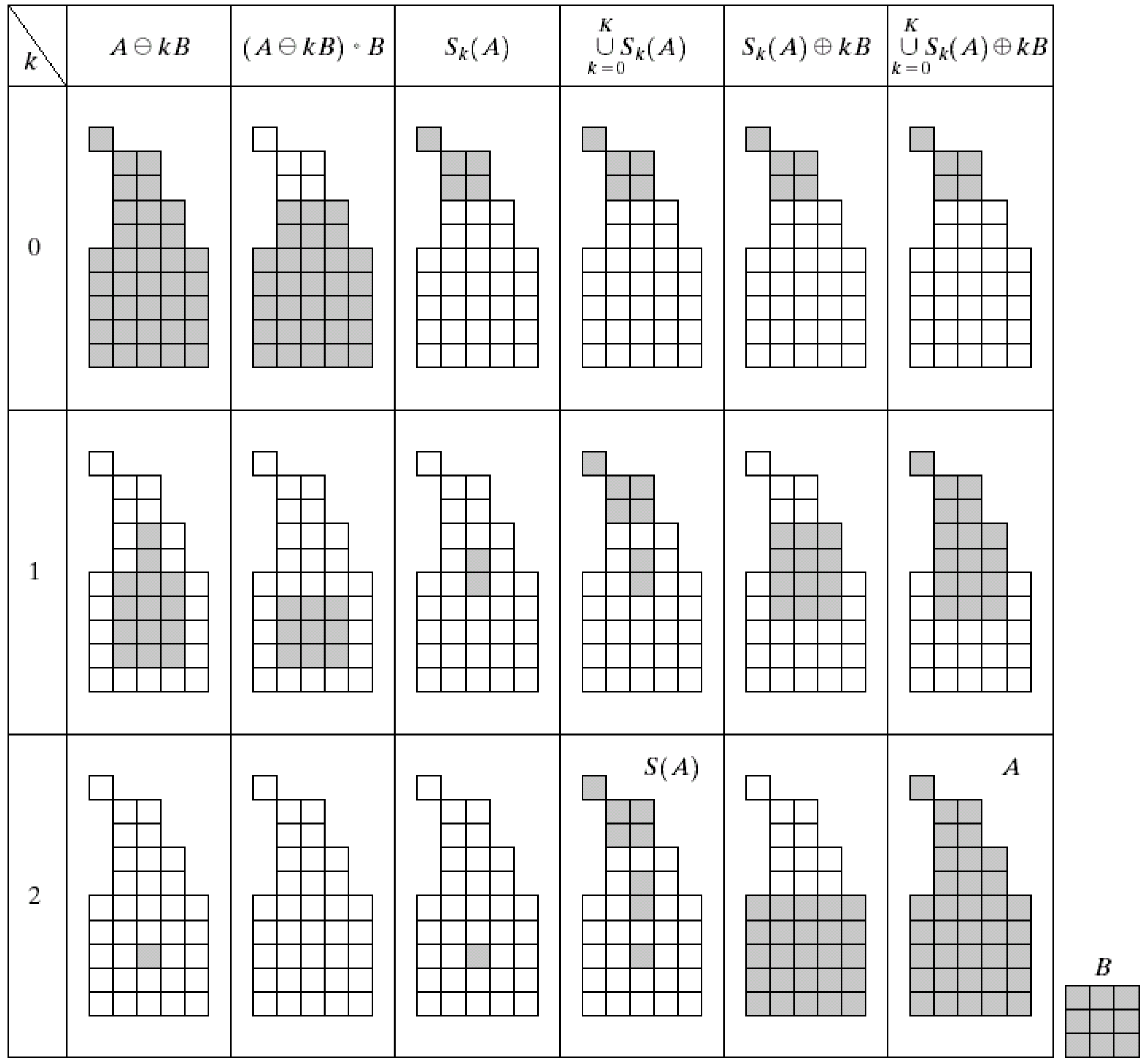


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

9.5.8 Pruning 裁剪

- clean up the “spurs”

a b
c
d e
f g

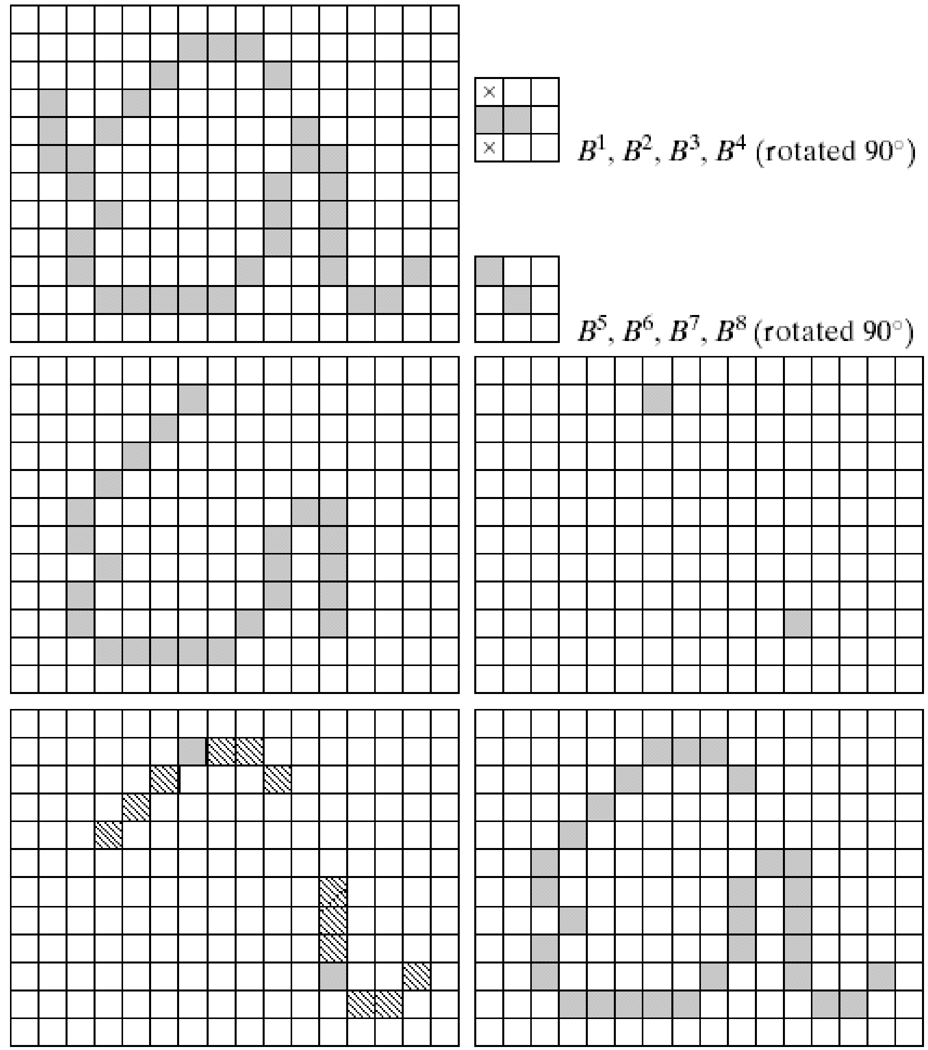


FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

9.5.9 Summary of Morphological Operations on Binary Image

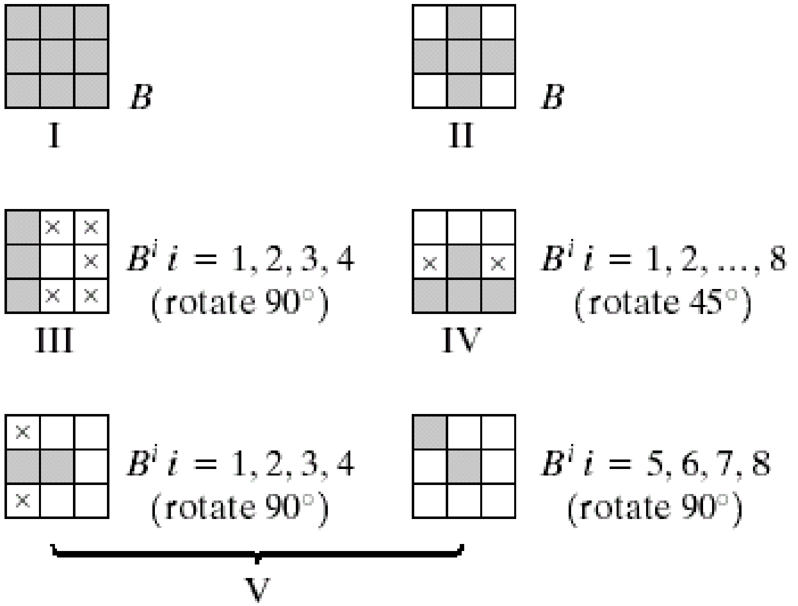


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.5.9 Summary of Morphological Operations on Binary Image

TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

9.5.9 Summary of Morphological Operations on Binary Image

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

9.5.9 Summary of Morphological Operations on Binary Image

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

Operation	**Equation**	**Comments** (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

9.5.9 Summary of Morphological Operations on Binary Image

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.